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Performance of a Novel Hybrid Model through Simulation and Historical Financial Data

(Prestasi Model Hibrid Novel melalui Simulasi dan Data Kewangan Sejarah)

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ABSTRACT

It is thoroughly acknowledged that the historical financial time series is not linear, exhibits structural changes, and is volatile. It has been noticed in the current literature that because of the existence of structural breaks in the historical time series, the GARCH family models provide misleading results and poor forecasts. Thus, it is unavoidable to incorporate models with nonlinearity in the conditional mean and conditional variance to capture volatility dynamics more precisely than the existing models. Therefore, inspiring in this matter, this study proposes a novel hybrid model of exponential autoregressive (ExpAR) with a Markov-switching GARCH (MSGARCH) model. This study also examines volatility dynamics and performances through simulation and real-world financial data. Moreover, this study investigates downside risk management performances using 5% VaR (Value-at-Risk) back-testing. The empirical findings showed that the proposed model outperforms the benchmark model for both simulation and real-world time series data. The VaR results also showed that the proposed model captures downside risk more meticulously than the benchmark model.

Keywords: ExpAR model; ExpAR-MSGARCH model; MSGARCH model; structural breaks; value-at-risk

ABSTRAK

Diakui secara benar bahawa siri masa kewangan masa lampau adalah tidak linear, menunjukkan perubahan struktur dan meruap. Dapat dilihat dalam kepustakaan semasa oleh kerana adanya putusan berstruktur dalam siri masa lampau, model keluarga GARCH memberikan hasil yang tidak benar dan ramalan yang lemah. Oleh itu, tidak dapat dielak untuk menggabungkan model yang tidak linear pada min dan varians bersyarat untuk menguasai dinamik kemeruapan dengan lebih tepat daripada model sedia ada. Maka, berinspirasi daripada hal ini, kajian ini mencadangkan model hibrid baharu eksponen autoregresif (ExpAR) dengan model pertukaran Markov GARCH (MSGARCH). Kajian ini juga mengkaji prestasi dan dinamik kemeruapan melalui simulasi dan data kewangan dunia yang betul. Lebih-lebih lagi, penyelidikan ini mengkaji prestasi pengurusan risiko penurunan menggunakan ujian semula 5% VaR (risiko pada nilai). Penemuan empirik menunjukkan bahawa model yang dicadangkan mengungguli model penanda aras untuk kedua-dua simulasi dan data siri masa yang betul. Hasil VaR juga menunjukkan bahawa model yang dicadangkan menangkap risiko penurunan lebih teliti daripada model penanda aras.

Kata kunci: Model ExpAR; model ExpAR-MSGARCH; model MSGARCH; putusan berstruktur; risiko pada nilai

INTRODUCTION

The study of time series in finance and economics is one of the topmost interests by academicians and researchers of various subjects. However, most of the research was concentrated primarily on linear modeling. The preference of linear models during investigating time-varying data is because of many advantages over the nonlinear ones such as unsophisticated, easy to estimate, capable of explaining many existing real-world (historical) time series data and forecasting capability is good enough than the alternative methods. The linear models containing the above characteristics and widely used are the autoregressive moving average (ARMA) and the integrated autoregressive moving average (ARIMA) were introduced by Box and Jenkins (1970). Though the real-world time series data exhibit not only linearities but also nonlinearities, a question has arisen whether the existing models are capable of enlightening and forecasting the volatility dynamics of such time series better than existing linear models.

The Threshold Autoregressive (TAR) model of Tong (1978), Self-Exciting Threshold Autoregressive (SETAR) model of Tong and Lim (1980), Smooth Transition Autoregression (STAR) model of Chan and Tong (1986), the Exponential Autoregressive (ExpAR) model of Ozaki (1980) and the Bilinear model of Granger and Andersen (1978) are typical nonlinear models in the conditional mean. In contrast, the most popular and representative nonlinear models in the conditional variance are the Autoregressive Conditional Heteroscedasticity (ARCH) model of Engle (1982) and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model of Bollerslev (1986). Nevertheless, combining linearities or nonlinearities in conditional mean and conditional variance has become another popular hybrid model class in the preceding two decades.

Baragona et al. (2002) used the Genetic Algorithm (GA) to find the gamma parameter of the ExpAR model and allowed more than one gamma value in the ExpAR model specification, which improved forecasting performances. They also argued that the order of the model is not necessary to be large, as the low-order ExpAR model is enough to get accurate multi-step forecasting. It is found that the SARIMA model does not adequately capture many seasonal data, however, it could be accurately captured through the periodic ExpAR model (Merzougui et al. 2016).

Some researchers also studied the simulation performance, geometrical ergodicity, stationarity, and forecast accuracy for ExpAR models (Allal & El Melhaoui 2006; Meitz & Saikkonen 20081). Moreover, this nonlinear model applied successfully in many areas of time series such as hydrology (Ghosh et al. 2014; Merzougui 2017), macroeconomic (Amiri 2012; Katsiampa 2014), ecology (Haggan & Ozaki 1981), and signal (Ishizuka et al. 2005). Ghosh et al. (2014) found evidence of limit cycle behavior and concluded that forecasting ability is better than the ARIMA model. Merzougui (2017) applied a periodic restricted ExpAR(1) model in the monthly water flow of Fraser River and concluded that it is better than the linear model. In recent work, Xu et al. (2019) used recursive search methods to estimate the ExpAR model parameters to improve accuracy and convergence rate.

Apart from that, structural breaks in the volatility are found in many financial and economic assets, and thus overlooking this characteristic can significantly affect the accuracy of the volatility forecasts. Many researchers and experts care about only a single-regime conditional volatility model, whereas Danielsson (2011) refers to these models as one reason for the financial crisis. Recent studies have shown that GARCH family models might fail to predict actual variants in the volatility during volatility dynamics changes regime over time (Ardia et al. (2018; Bauwens et al. 2014). A way out this issue is to allow the GARCH parameters to vary over time, corresponding to a hidden discrete Markov process, which is termed as Markov-switching GARCH (MSGARCH) model. This approach precedes volatility predictions that can promptly adapt to unconditional volatility changes (Marcucci 2005).

Volatility or price fluctuation in the financial market is a widespread phenomenon nowadays, especially price and stocks index. Volatility in digital currency, commodity, energy, exchange rates, and the stock markets are the natural consequence of variations in the movement of a market. These movements, such as market expectation, negative and positive news, new information, and trading volume, will cause changes in the financial market variance in daily returns. In the financial markets, clustering in volatility is known as stylized characteristics, which show small and significant shifts in the returns will be followed by other small and significant shifts (Hossain & Ismail 2021). The ARCH and GARCH models become very popular among academics, researchers, and practitioners to describe the variance in financial time series data. Combining the SETAR and Bilinear model with the ARCH model to produce a second-generation model SETAR-ARCH and Bilinear-ARCH was proposed by Tong (1990). Since then, combining the conditional mean model with the conditional variance model has become popular among researchers and practitioners. This hybrid model is being applied more extensively in time series data.

In recent decades, second-generation models have become more popular than first-generation models because they can capture volatility dynamics and forecasts more accurately. In its continuity, Abdollahi and Ebrahimi (2020) combined the Adaptive Neuro-Fuzzy Inference System (ANFIS), ARFIMA, and Markov-Switching (MS) models, which was able to capture many features of crude oil price. Lin et al. (2020) observed in their study that Markov regime-switching (MRS)

GARCH family models perform well compared to the regular GARCH family models. Their results showed that the Hidden Markov (HM)-EGARCH model performs better than the usual GARCH-type models. There is evidence that financial and economic time series exhibit structural breaks in the volatility, which is observed in many studies. As a result, there are significant outcomes on modeling volatility dynamics. More significantly, by applying various structural transmissions, many findings showed that forecasting results improve rather than those not considering this issue (Mohammadi & Su 2010). By applying the MS long memory model, Sanzo (2018) found that out-of-sample results show a more consistent prediction counter to those obtained from the chosen GARCH models in particular time horizons. Herrera et al. (2018) also confirmed that the MSGARCH model exhibits a more accurate forecasting counter to GARCH-type models when forecasting crude oil volatility. Arellano and Rodríguez (2020) found that a high-volatility regime was significantly persistent and the leverage effect was not observed in Forex markets. Gao et al. (2020) observed in their empirical studies that the MRS-LMGARCH model did better than the LMGARCH and MRS-GARCH models.

Financial crisis throughout the world is not a rare event; almost all stock markets faced a crash for various reasons. The volatility of the world stock market has increased radically from 2006, according to IMF's World Uncertainty Index (WUI) (Ali et al. 2019). For instance, previous financial crises, the stock market crash in 1987, the global financial crisis (GFC) in 2007-2008, the Eurozone debt crisis in 2010, witness massive capital loss and bankruptcy of big financial institutions. The common reasons behind these crises were poor measurement methods, meager risk management, and lack of knowledge of governing risks, specifically miscalculating risk measures. Hull (2018) argued that most of the previous enormous financial losses could be avoided if reliable VaR modeling was appropriately implemented.

Uncertainty in financial markets is a regular trend; however, the recent financial world has become more volatile because of various reasons such as trade war, natural disaster, questionable security in banking sectors, and virus spread (like COVID-19). Moreover, in many empirical studies, evidence of structural changes in the regime and neglecting it produces misleading results. Therefore, effective modeling is needed with the combination of nonlinear conditional mean and nonlinear conditional variance model. In the previous study, only Katsiampa (2014) combined the ExpAR model with ARCH and GARCH but did not consider structural changes. Therefore, to fill the research gap in the literature, this study combines the nonlinear ExpAR model with MSGARCH.

This study conduct simulation and real-world data application, compare with benchmark model ARMA(1,1)-MS(2)GARCH(1,1) and also study the performance of capturing downside risk. The ExpAR(1) model of Ozaki (1980) was considered in the conditional mean equation. This nonlinear model has some notable features such as limit cycle, amplitude-dependent frequency, and jump phenomena. The aim of this study is threefold. First, to simulate approximately similar data for the proposed and benchmark models, then estimate results from the simulation data and compare the two models. Second, considering structural breaks to investigate the proposed model's performance to the real-world data. Thus, the questions arise, (i) can the real-world data fit using the proposed model? (ii) can it capture and explain volatility perfectly? Third, using VaR back-testing to capture downside risk and verify the volatility forecast accuracy for both in-sample and out-of-sample.

The following parts of this paper are organized as methods section, the data collection and transformation process, discussion on the proposed model and benchmark model, analysis of simulation results and real-world data application, and a concise summary of all sections.

Methods

This section contains insight into the methods, which provides an overview of mathematical formulation. Various econometric models and their mathematical formulations are covered in this section. Only the models utilized throughout this study are discussed here.

ARMA Model

An ARMA model is a combination of the autoregressive (AR) model and the moving average (MA) model. Let y_t observations at time t, y_{t-i}, lagged terms at time t-i and ε_{t-j} error terms at time t-j, then the ARMA(p, q) model can be written as

$$y_t = \omega + \sum_{i=1}^{p} \varphi_i y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t$$

where φ_i and θ_j are AR and MA terms, respectively. ω is constant and the error terms or white noise $\varepsilon_t \sim N(0, \sigma^2)$ and independently identically distributed (iid). The ARMA(1,1) model can be written as

$$y_t = \omega + \varphi_1 y_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \quad (1)$$

ExpAR MODEL

Ozaki and Oda (1977) were the first to describe the Exponential Autoregressive (ExpAR) model and then more explicitly explained by Ozaki (1980) and Haggan and Ozaki (1981) to illustrate nonlinear stochastic phenomena. An ExpAR(p) model can be written as

$$y_t = c + \{\phi_1 + \pi_1 exp(-\gamma y_{t-1}^2)\}y_{t-1} + \cdots\cdots +$$
$$\{\phi_p + \pi_p exp(-\gamma y_{t-1}^2)\}y_{t-p} + \varepsilon_t$$

where $\varepsilon_t \sim n.\,i.\,d.\,(0,1)$, c is constant, ϕ_i's, π_i's, $i = 1,2,3,\cdots$, p and γ are parameters that need to be determined. Here γ is known as scaling factor. The value of γ can be chosen such that $e^{-\gamma y_{t-1}^2}$ differs from both zero (0) and one (1) for most of the values of y_{t-1}.
The first order ExpAR(1) model is as

$$y_t = c + \phi_1 + \pi_1 exp(-\gamma y_{t-1}^2)y_{t-1} + \varepsilon_t \qquad (2)$$

MSGARCH MODEL

A comprehensive with detailed description and working procedure of a flexible Markov switching (MS) model presented by Hamilton (1994). Let y_t log-return of sample data (in percentage) at time t with $\mathbb{E}[y_t] = 0$ and $\{y_t\}$ is serially uncorrelated. According to Ardia et al. (2018), the MSGARCH model can be stated as:

$$y_t|(s_t = k, I_{t-1}) \sim F(0, h_{k,t}, \theta_k)$$

Here $F(\cdot)$ is a continuous distribution with zero (0) mean and $h_{k,t}$ is conditional variance, which is time-dependent in k regimes. While θ_k are vectors that conserve shape parameters and s_t is state variables, which are changed according to 1st order homogeneous Markov chain. The state variables s_t changes are pertaining to a first-order homogeneous Markov chain. The transition probability matrix $P = \{p_{ij}\}$, $i, j = 1, ..., K$ with $p_{ij} = P[s_t = j|\, s_{t-1} = i]$ and at time t-1, I_{t-1} denote information set. Hence, the transition probability matrix is written as:

$$P = \begin{bmatrix} p_{11} & \cdots & p_{1K} \\ \vdots & \ddots & \vdots \\ p_{K1} & \cdots & p_{KK} \end{bmatrix}$$

where i^{th} row and j^{th} column govern switching probability from state i to j. For instance, p_{12} implies that the probability switches from state 1 to 2 at time t and t + 1. The probability of keeping in the same state is governed as p_{11} for state 1, similarly to p_{22} for state 2. This is known as the MS model central-point structure, where switching in the same state is also a stochastic process. Generally, transition probabilities are invariable but allow time-varying is also possible. However, these conditional probabilities are obeying the constraints $0 < p_{ij} < 1$, i, $j = 1, ..., K$ and $\sum_{j=1}^{K} p_{ij} = 1$ ∀i∈{1, ..., K}. Referring to Haas et al. (2004), conditional variance $h_{k,t} = h(y_{t-1}, h_{k,t-1}, \theta_k)$ of the GARCH process of regimes $s_t = k$, stated as the function of past lagged returns and the vectors θ_k is regime dependent. Therefore, GARCH (1,1) process can be composed as:

$$h_{k,t} = \alpha_{0,k} + \alpha_{1,k} y_{t-1}^2 + \beta_k h_{k,t-1} \qquad (3)$$

where $\alpha_{1,k}$ and β_k are parameters at finite regime k and $\alpha_{1,k} + \beta_k < 1$. To make sure positivity, we required that $\alpha_{0,k} > 0$, $\alpha_{1,k} > 0$, and $\beta_k \geq 0$.

HYBRID MODEL

For the Markov chain illustration of our nonlinear autoregressive model with MSGARCH errors, the proposed assumptions are sufficient to prove geometric ergodicity and the presence of specific moments. Majority of the assumptions needed apply to the conditional mean and conditional variance separately, making verifying whether the assumptions hold. The following assumptions are imposed:
Assumption 1 The zt is iid(0,1) with a (Lebesgue) density that is positive and continuous on ℝ. Moreover, for some $k > 0, E[|z_t|^{2k}] < \infty$.
Assumption 2 The matrix norm $\|\cdot\|^*$ that is given by a vector norm, which is also represented by $\|\cdot\|^*$, such that $\|A\|^* \leq \rho$ for all $A \in \mathcal{A}_*$, where $\mathcal{A}_* = \{A(x): x \in \mathbb{R}^p\}$ and $0 < \rho < 1$ (Meitz & Saikkonen 2008).
Assumption 3 $\alpha_i > 0$, $\beta_i > 0$, the Markov chain is homogeneous, and $p_{ij} \in (0,1)$ for all $i, j = \{1,2, \ldots, k\}$.
Assumption 4 Let $\rho(\cdot)$ be the spectral radius of a matrix, i.e. its largest eigenvalue in modulus. Then $\rho\ (\Omega) < 1$ (Haas et al. (2004).
The ARMA-MSGARCH model is already existed in the literature. Only the mathematical formulation is discussed here. An ARMA(1)-MSGARCH(1,1) model is defined as

$$y_t = \omega + \varphi_1 y_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \qquad (4)$$

where $\varepsilon_t = z_t\sqrt{h_{k,t}}$, $z_t \sim N(0,1)$ iid, and $\varepsilon_t \sim N(0, \sqrt{h_{k,t}})$. Then MSGARCH(1,1) model becomes

$$h_{k,t} = \alpha_{0,k} + \alpha_{1,k}\varepsilon_{t-1}^2 + \beta_k h_{k,t-1} \qquad (5)$$

where $\alpha_{1,k}$ and β_k are parameters at finite regime k and $\alpha_{1,k} + \beta_k < 1$.

An ExpAR(1)-MSGARCH(1,1) model is defined as

$$y_t = c + \{\phi_1 + \pi_1 exp(-\gamma y_{t-1}^2)\} y_{t-1} + \varepsilon_t \quad (6)$$

where $\varepsilon_t = z_t\sqrt{h_{k,t}}$, $z_t \sim N(0,1)$ iid, and $\varepsilon_t \sim N(0, \sqrt{h_{k,t}})$. Then MSGARCH(1,1) model becomes

$$h_{k,t} = \alpha_{0,k} + \alpha_{1,k}\varepsilon_{t-1}^2 + \beta_k h_{k,t-1} \quad (7)$$

where $\alpha_{1,k}$ and β_k are parameters at finite regime k and $\alpha_{1,k} + \beta_k < 1$.

RESULTS AND DISCUSSION

SIMULATION

Since an alternative model was proposed, it is crucial to simulate the model for various parameter's values to see significant differences with the benchmark model, which this model could illustrate. Regarding comparability purposes among the two models, the standard parameter value was taken as the same value across the model (Haggan & Ozaki 1981; Katsiampa 2014; Ozaki 1980). The parameter's value was chosen arbitrarily so that it can be close to actual values and consider the distribution is iid with mean 0 and variance 1. Different possible combinations were considered during the selection of arbitrary parameter values. Only the first-order model was considered for the simulation process to avoid the complexity, and it can explain real-world data's nonlinear behaviour (Hansen & Lunde 2005).

The simulation process was executed using R programming software, where different packages (such as strucchange, tseries, psych, FinTS, arima, lmtest, MSGARCH, GAS) were used. Here 1000, 2000, and 2800 observations were generated for ExpAR(1)-MSGARCH(1,1) and ARMA(1,1)-MSGARCH(1,1) model with burn-in phase is 500 and replicated 500 times for the case of ExpAR model. During the simulation process and estimation of results, the scale parameter gamma (γ) of the ExpAR(1) model was the same for both cases. The primary aim was to generate similar time series data. Then, the two models were compared from estimated results using simulated time series data by considering how close the initial values are to the estimated values, the deviation of standard errors, and the model selection indicators based on log-likelihood (LL) and information criteria.

The two models' simulated time series data are displayed in Figures 1, 2, and 3 and their autocorrelation plots in Supplementary Figures 1, 3, and 5. Table 1 reported the initial value of parameters taken during simulations. In order to compare two model performances; it is vital to produce similar data series. From the simulated series plot, it was observed that approximately similar data is generated in the three simulations, which is one of our objectives. It has been observed from descriptive statistics in Table 2; almost all indicators are the same with only a slight difference in standard deviation value which is expected. The mean of the simulated series are slightly deviates from zero when the number of observations increased. The Jarque-Bera statistics of p-value suggest that the two simulated series are normally distributed. The skewness value is close to zero, and the excess kurtosis value approached zero when the number of observations increased. The skewness and excess kurtosis value are closed to zero means that the distribution is approximately symmetric and light-tailed.

TABLE 1. Initial value of parameters for each simulation

Simulation	ExpAR			ARMA		MSGARCH	Observations
	ϕ_1	π_1	γ	φ_1	θ_1		
1	0.05	0.63	9.1	-0.23	0.29	(0.02, 0.007, 0.97, 0.02, 0.007, 0.97, 0.92, 0.12)	1000
2	0.04	0.25	14.5	0.44	-0.4	(0.02, 0.0001, 0.98, 0.02, 0.0001, 0.98, 0.92, 0.13)	2000
3	0.024	-0.14	14.49	0.5	-0.5	(0.28, 0.0004, 0.7, 0.28, 0.0004, 0.7, 0.92, 0.13)	2800

TABLE 2. Descriptive statistics of simulated series

Model	Mean	SD	Minimum	Maximum	Skewness	Excess Kurtosis	JB	Observations
ExpAR-MSGARCH	0	0.97	-2.76	2.92	0.03	-0.11	0.75	1000
ARMA-MSGARCH	0	0.99	-2.84	3.01	0.03	-0.11	0.75	
ExpAR-MSGARCH	-0.01	0.99	-2.83	2.99	0.04	-0.08	0.58	2000
ARMA-MSGARCH	-0.01	1	-2.86	3.03	0.04	-0.08	0.58	
ExpAR-MSGARCH	-0.01	0.95	-2.78	3.21	0.01	-0.04	0.88	2800
ARMA-MSGARCH	-0.01	0.98	-2.87	3.31	0.01	-0.04	0.87	

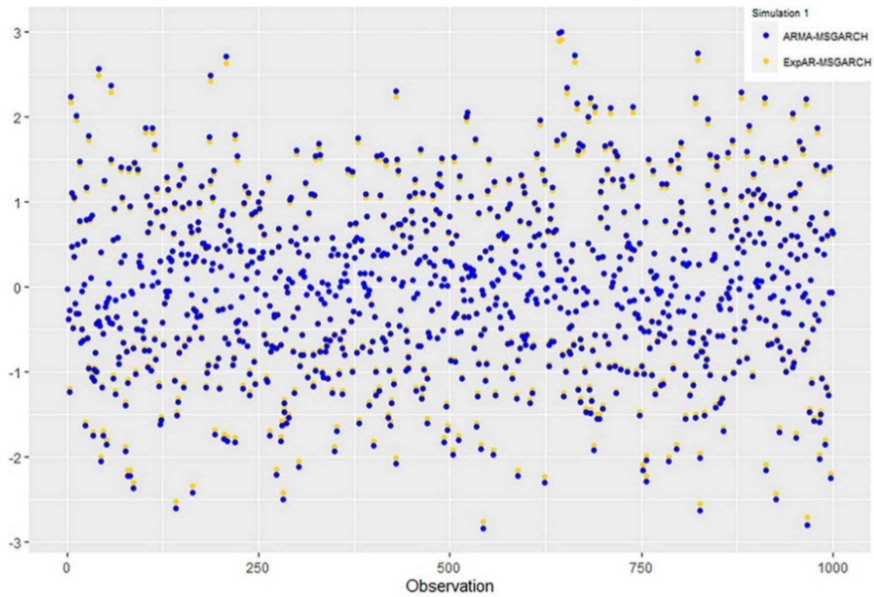

FIGURE 1. Simulated series plot of ExpAR-MSGARCH vs ARMA-MSGARCH model

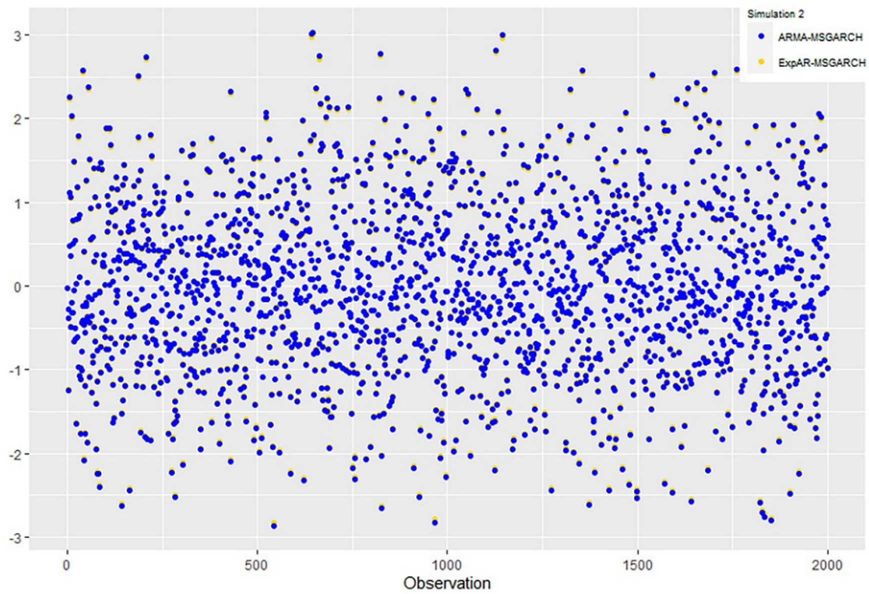

FIGURE 2. Simulated series plot of ExpAR-MSGARCH vs ARMA-MSGARCH model

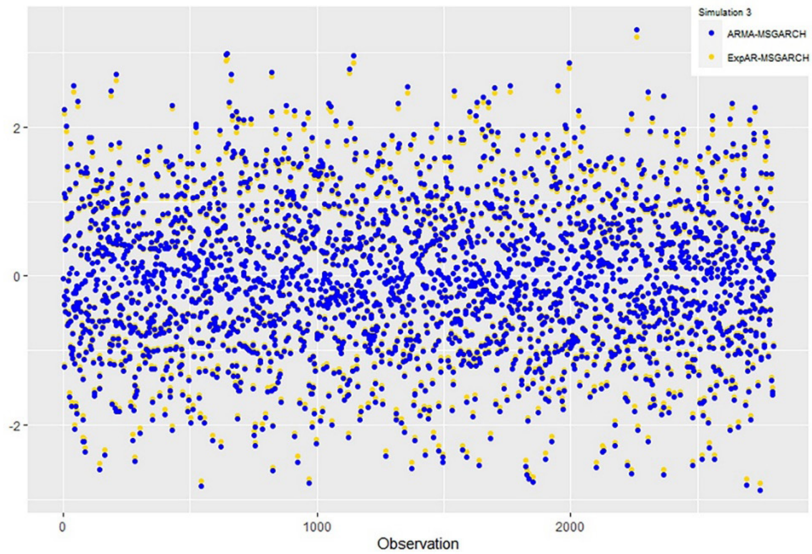

FIGURE 3. Simulated series plot of ExpAR-MSGARCH vs ARMA-MSGARCH model

From Table 3, it has been seen that the initial value of parameters is very close to the estimated values, which is one of our objectives. The arch and garch parameter values hold positivity constraints, and the sum of these parameters values is less than one for both models. The standard errors of the ExpAR-MSGARCH model are lower than of the ARMA-MSGACRH model. Only a slight change in the estimated results of the second simulation.

TABLE 3. Estimated results from simulated series of ExpAR(1)-MS(2)GARCH(1,1) and ARMA(1,1)-MS(2)GARCH(1,1)

Parameters	1000 observations		2000 observations		2800 observations	
	ExpAR-MSGARCH	ARMA-MSGARCH	ExpAR-MSGARCH	ARMA-MSGARCH	ExpAR-MSGARCH	ARMA-MSGARCH
ϕ_1/φ_1	0.0469 (0.0322)	-0.2336 (0.3734)	0.0411 (0.0226)	0.4359 (0.5445)	0.0243 (0.0191)	0.5174 (0.4345)
π_1/θ_1	0.6299 (0.4718)	0.2896 (0.367)	0.2483 (0.4746)	-0.3963 (0.556)	-0.1398 (0.3773)	-0.4942 (0.4412)
$\alpha_{0,1}$	0.0214 (0.2194)	0.0262 (1.121)	0.0216 (0.358)	0.0221 (0.3448)	0.2908 (4.1567)	0.2496 (5.2641)
$\alpha_{1,1}$	0.0067 (0.0963)	0.007 (0.4388)	0.0002 (0.0082)	0.0001 (0.0057)	0.0005 (0.0232)	0.0002 (0.0082)
β_1	0.9704 (0.2383)	0.9665 (1.0739)	0.9778 (0.3649)	0.9778 (0.3446)	0.6787 (4.604)	0.7389 (5.502)
$\alpha_{0,2}$	0.0214 (0.2527)	0.0262 (2.3942)	0.0216 (0.4124)	0.0221 (0.429)	0.2908 (3.3969)	0.2496 (4.1494)
$\alpha_{1,2}$	0.0067 (0.1099)	0.007 (0.9372)	0.0002 (0.0108)	0.0001 (0.0078)	0.0005 (0.0156)	0.0002 (0.0058)
β_2	0.9704 (0.2751)	0.9665 (2.2921)	0.9778 (0.4196)	0.9778 (0.4289)	0.6787 (3.7698)	0.7389 (4.3356)
$p_{1,1}$	0.9259	0.9912	0.9224	0.9227	0.9222	0.9226
$p_{2,2}$	0.8795	0.981	0.8665	0.8657	0.865	0.8645
LL	**-1379.0478**	-1409.7439	**-2814.1774**	-2836.9539	**-3832.6253**	-3909.7799
AIC	**2774.0956**	2835.4877	**5644.3547**	5689.9077	**7681.2506**	7835.5598
BIC	**2813.3497**	2874.7498	**5689.158**	5734.715	**7728.7468**	7883.0588
Stable probabilities						
State 1	0.6191	0.6829	0.6323	0.6346	0.6346	0.6365
State 2	0.3809	0.3171	0.3677	0.3654	0.3654	0.3635

The standard error is shown in the parentheses

The two model's stable probabilities of the two states are relatively similar. The proposed model's log-likelihood value is greater than the benchmark model, and the information criteria AIC (Akaike's Information Criteria) and BIC (Bayesian Information Criteria) of the proposed model are lower than the benchmark model in each simulation. The volatility plots (in Supplementary Figures 2, 4, and 6) of the two models are annualized considering 250 trading days and show the same pattern. The volatility of the ARMA-MSGACRH model is higher than the volatility of the ExpAR-MSGACRH model. From the above analysis, it can be concluded that our proposed model performs well than the benchmark model.

HISTORICAL DATA'S SUMMARY STATISTICS

The daily adjusted closing price was collected from http://finance.yahoo.com and DSEX from http://dsebd.org. Missing data and dates were omitted during the filtering process to avert any confusing results (Moritz & Bartz-Beielstein 2017). Therefore, the number of observations becomes different for a different sample. The sample name, period, and observation size is reported in Supplementary Table 1. The ending date is the same for all samples, but the starting date is different because of the availability of the data. For example, DSEX began on January 27, 2013, because Bangladesh Stock Exchange started counting the DSEX index from this date, while Bitcoin (BTC) price was over $100 on April 01, 2013; therefore, this date is considered as the starting date. All prices are taken as in US dollars.

The sample data was transferred into the percentage of the first difference of natural logarithmic return on daily adjusted closing prices. Let be denoted as daily log-return and be daily adjusted closing price at time . Then, continuously compounded return series is defined as

$$r_t = 100*(ln(P_t/P(_{t-1})$$

To demonstrate the proposed model's practical capability by considering financial time series data to inspect whether this model can explain real-world data. Six samples were chosen to represent all financial sectors in order to compare the two models. The descriptive statistics are presented in Table 4. The mean of DSEX is zero, and crude oil WTI is negative, while Bitcoin possesses the highest mean. USDEUR possesses the lowest volatility than other indices, whereas Bitcoin possesses the highest volatility which is about sixty-four times more volatile. DSEX and USDEUR are positively skewed, signify right tail disturbance and others are negatively skewed, signify left tail disturbance. All sample's kurtosis value is positive and greater than zero implies that heavy-tailed distribution. To deal with heavy-tailed distribution, one should consider student's-t distribution (Rahim et al. 2018; Zahid & Iqbal 2020). However, only normal distribution was considered this time around, student's-t and other distributions will be considered for future work. The p-value of the ADF test confirms the returns are stationary and the p-value of the Jarque-Bera (JB) test suggests the returns are not normally distributed.

TABLE 4. Descriptive statistics of six financial data

Index	Mean	SD	Variance	Skewness	Kurtosis	ADF	JB
NASDAQ	0.06	1.24	1.54	-0.78	10.81	<0.01	<2.2e-16
DSEX	0.00	0.9	0.81	0.45	13.26	<0.01	<2.2e-16
Bitcoin	0.18	4.46	19.89	-0.56	12.34	<0.01	<2.2e-16
Gold	0.02	1.03	1.06	-0.66	6.72	<0.01	<2.2e-16
C. Oil WTI	-0.02	2.82	7.95	-1.91	64.88	<0.01	<2.2e-16
USD-EUR	0.01	0.56	0.31	0.09	2.2	<0.01	<2.2e-16

It is vital to test whether ARCH effects exist on the residuals before executing the MSGARCH model (Abdulsalam & Bouresli 2019; Theiri & Ati 2020). If there are no ARCH effects, then any linear model is enough to explain time series characteristics. Chi-squared statistics, degrees of freedom (df), and p-value are presented in Table 5 for six samples. Since the null hypothesis of no ARCH effects is rejected, it shows the presence of ARCH effects. Therefore, there is no restriction on executing the MSGARCH model.

TABLE 5. ARCH test results on residuals for six samples

	NASDAQ		DSEX		Bitcoin	
	ExpAR	ARMA	ExpAR	ARMA	ExpAR	ARMA
Chi-squared	685.31	773.95	467.95	443.74	240.59	264.6
df	12	12	12	12	12	12
p-value	<2.2e-16	<2.2e-16	<2.2e-16	<2.2e-16	<2.2e-16	<2.2e-16
	USD-EUR		Gold		C. Oil WTI	
	ExpAR	ARMA	ExpAR	ARMA	ExpAR	ARMA
Chi-squared	137.5	137.85	114.55	114.34	527.69	483.6
df	12	12	12	12	12	12
p-value	<2.2e-16	<2.2e-16	<2.2e-16	<2.2e-16	<2.2e-16	<2.2e-16

APPLICATION IN REAL-WORLD TIME SERIES DATA

The estimated results of six samples from two models ExpAR(1)-MS(2)GARCH(1,1) and ARMA(1,1)-MS(2)GARCH(1,1) are presented in Table 6, where coefficients, the standard error in parentheses and t value in the bracket. Two models estimated parameters values (both states) are very close to each other, even the constant coefficients. A similar result was also found in Katsiampa (2014), who studied ExpAR-ARCH, AR-ARCH, ExpAR-GARCH and AR-GARCH models, where common coefficients are close to each other. In our studies, according to the highest log-likelihood and the lowest information criteria, ExpAR(1)-MS(2)GARCH(1,1) model outperform ARMA(1,1)-MS(2)GARCH(1,1) model except for Bitcoin, where ARMA(1,1)-MS(2)GARCH(1,1) model is better than the proposed model. In USDEUR, the proposed model's LL, AIC, and BIC values are -2148.7453, 4313.4906, 4361.055, and the benchmark model -2149.6417, 4315.2834, 4362.8507, respectively. These values are very close; therefore, these two models can explain volatility dynamics very well. Most of the standard error value of the proposed model is lower than the benchmark model. Both regimes of DSEX and USDEUR are highly persistent, meanwhile for Bitcoin, Gold, and crude oil WTI, only the first regime is highly persistent. For NASDAQ, both regimes are moderately persistent. This regimes-persistence is shown in Figure 4, where the smoothed probabilities, $\mathbb{P}[s_t = k \mid I_T]$ for low and high volatility regimes (k = 1 and k = 2, respectively) are displayed. The high volatility regime covered by the red line and the low volatility regime shown uncovered area. The annualized volatility of each sample was extracted from two models (display in Supplementary Figure 7). The volatility plot shows that the two models capture volatility very well, and volatility movement is similar. It has been seen from the volatility plot that the volatility moving between 10 and 30 for NASDAQ, 10 to 20 for DSEX, 50 to 180 for Bitcoin, 4 to 15 for USDEUR, 10 to 27 for Gold, and 20 to 78 for WTI. This result is consistent with the sample's returns volatility. The stable probabilities (unconditional distribution) are higher for state 1 than state 2. In precise, the 1st regime is illustrated as low volatility and low persistence in the volatility dynamics. Alternatively, the 2nd regime is illustrated as high volatility and high persistence in the volatility dynamics. Visibly, the investors will identify regime one as a tranquil market with low volatility and low persistence, whereas regime two is a chaotic market with high volatility and high persistence.

Downside risk VaR (Value-at-Risk) at 5% risk level for both in-sample and out-of-sample are also reported in Table 6, where three tests results, namely unconditional coverage (UC) of Kupiec (1995), conditional coverage (CC) of (Christoffersen 1998), and dynamic quantile (DQ) of Engle and Manganelli (2004) are presented. The in-sample VaR was estimated over the full sample period, and the out-of-sample VaR was estimated over the 200 draws of point forecast value. The highlighted bold means correct coverage under the null hypothesis at 5% VaR level. The p-value of 5% VaR level for both in-sample and out-of-sample of the proposed model is comparatively higher than the benchmark model. Also, it can be added that the VaR back-testing results suggesting the two models fit well for six samples and explain and capture volatility very well. In-sample VaR plot at 5% level for six samples are displayed in Figure 5, where a red dashed line shows the ExpAR-MSGARCH model, and the solid blue line shows the ARMA-MSGARCH model. From Figure 5, it is evidence that the downside risk capture by the two models perfectly. If the two model's performance are compared based on 5% VaR, then, it is clear from the figure that the proposed model can capture the downside risk more accurately than the benchmark model. From the above discussion and results analyses, the proposed model outperforms the benchmark model for the six samples.

Now, based on simulation performance and practical application, the following summary may be formed. Approximately similar data was successfully generated from the two models ExpAR(1)-MS(2)GARCH(1,1) and ARMA(1,1)-MS(2)GARCH(1,1), where the value of the common parameters were the same. After that, the simulated data were fitted using the two models. Then, it was observed from the estimated results that the common parameters value was very close to each other, the standard error of the proposed model was smaller than the benchmark model, the LL of the proposed model was greater than the benchmark model,

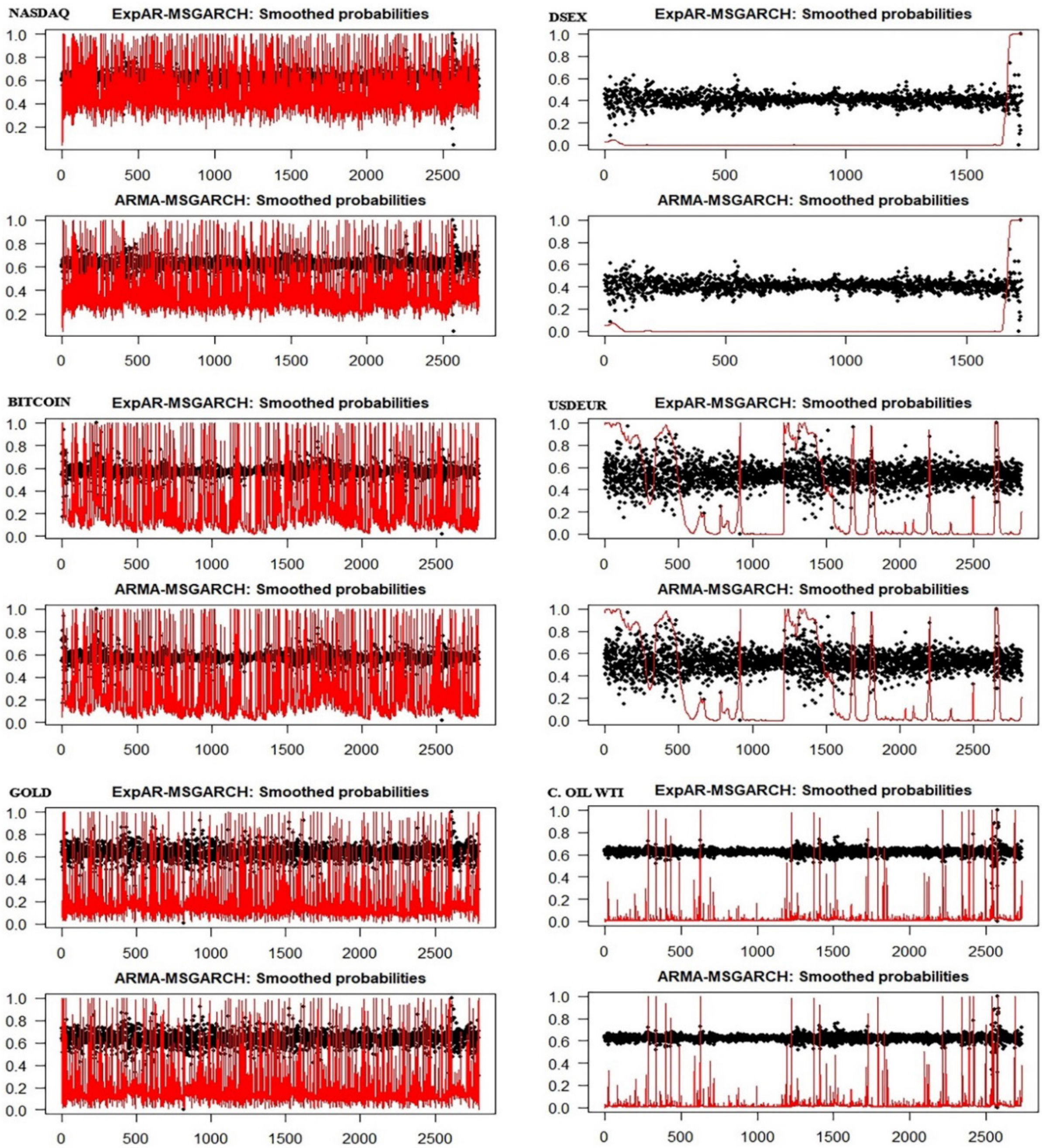

FIGURE 4. Smoothed probabilities of six samples plotted from ExpAR-MSGARCH and ARMA-MSGARCH models. High volatility regime covered by the red line. The name of the sample is shown in the top left corner of each plot

AIC, and BIC of the proposed model were lower than the benchmark model. Based on the simulation results, the proposed model was better than the benchmark model.

To find the answer to the second aim questions, real-world data were fitted using the proposed model and benchmark model. For this purpose, six different sample's daily adjusted closing prices were considered. There was evidence of structural breaks in all sample series (reported in Supplementary Table 2). After fitting the real-world data into our models, it was observed that the proposed model captures volatility dynamics and explains it very well. The proposed model outperforms all financial sectors over the benchmark model except Bitcoin. From the analysis, it is clear that one model cannot perfectly describe the volatility dynamics of all financial sectors.

To verify the volatility forecasting accuracy of the two models, VaR back-testing at 5% level was used. The estimated results showed that the two models correctly capture the downside risk. In this case, the proposed model capture downside risk very well compared to the benchmark model.

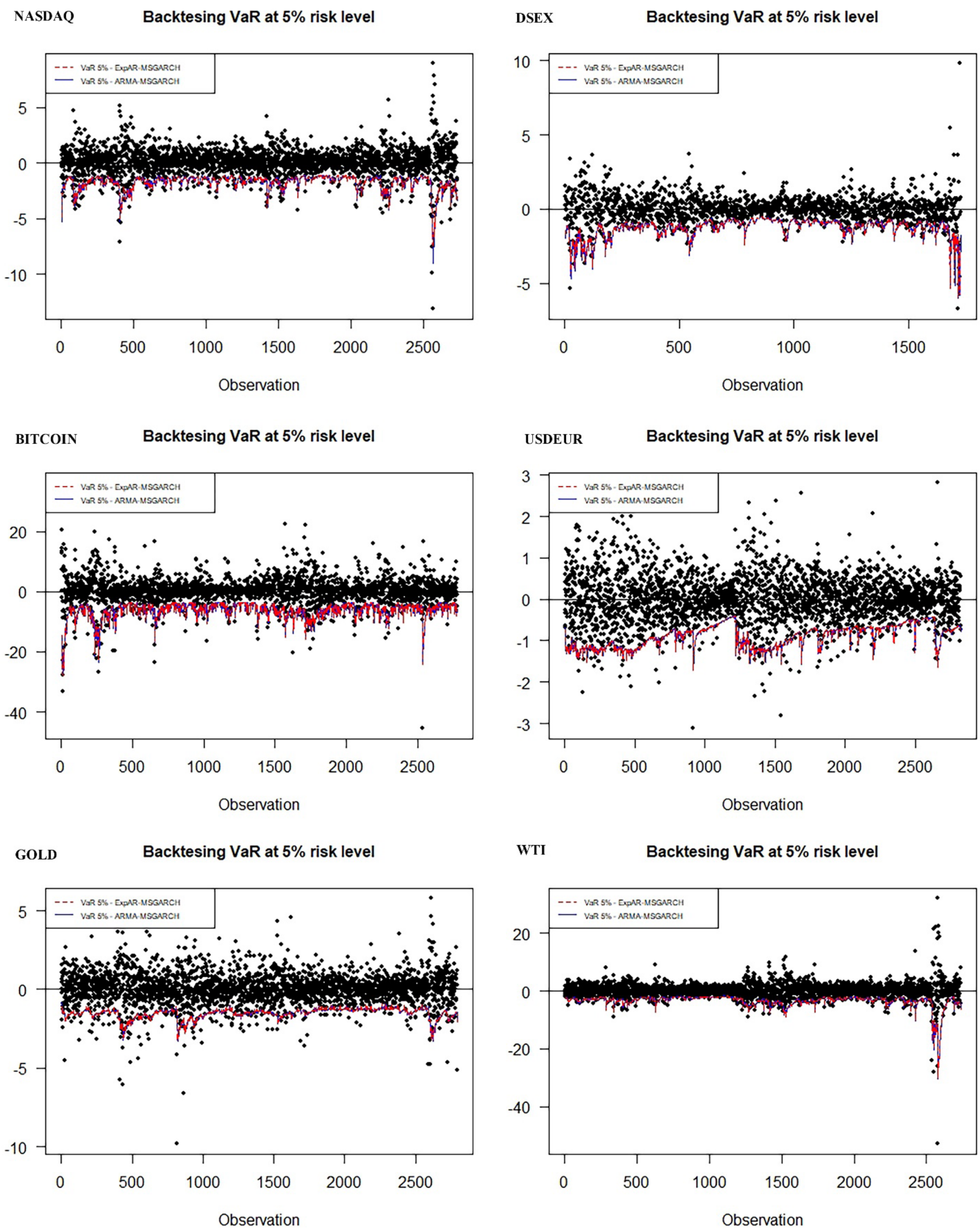

FIGURE 5. In-sample 5% VaR back-testing plot of six samples. The name of the sample is shown in the top left corner of each plot

TABLE 6. Estimated results of six samples from ExpAR(1)-MS(2)GARCH(1,1) and ARMA(1,1)-MS(2)GARCH(1,1) model

Variable	NASDAQ		DSEX		Bitcoin		USDEUR		Gold		C. Oil WTI	
	ExpAR-MSGARCH	ARMA-MSGARCH	ExpAR-MSGARCH	ARMA-MSGARCH	ExpAR-MSGARCH	ARMA-MSGARCH	ExpAR-MSGARCH	ARMA-MSGARCH	ExpAR-MSGARCH	ARMA-MSGARCH	ExpAR-MSGARCH	ARMA-MSGARCH
γ	0.015	-	0.01	-	0.001	-	0.04	-	8.0	-	0.02	-
ϕ_1/φ_1	-0.8089	-0.4663	-0.5687	-0.1413	-0.2547	-0.8425	-0.1562	-0.4605	-0.0272	0.08	-0.0329	0.4772
	(0.0654)	(0.0797)	(0.1325)	(0.1779)	(0.0684)	(0.2072)	(0.2694)	(0.288)	(0.0193)	(0.3964)	(0.0274)	(0.1134)
	[-12.364]	[-5.8491]	[-4.292]	[-0.7942]	[-3.724]	[-4.0655]	[-0.58]	[-1.5986]	[-1.407]	[0.2018]	[-1.203]	[4.2071]
π_1/θ_1	0.8449	0.3405	0.7746	0.273	0.3339	0.8362	0.1312	0.4281	-0.2114	-0.1109	0.0165	-0.5292
	(0.0777)	(0.0837)	(0.145)	(0.1729)	(0.0827)	(0.2052)	(0.2863)	(0.2936)	(0.2408)	(0.395)	(0.0505)	(0.1087)
	[10.87]	[4.0672]	[5.341]	[1.5785]	[4.039]	[4.0754]	[0.458]	[1.4582]	[-0.878]	[-0.2808]	[0.327]	[-4.8671]
$\alpha_{0,1}$	0.0032	0.0062	0.013	0.0129	0.0253	0.0243	0.0001	0.0001	0.0035	0.0036	0.0715	0.0708
	(0.0061)	(0.0035)	(0.0041)	(0.004)	(0.0121)	(0.012)	(0.0002)	(0.0002)	(0.0018)	(0.0018)	(0.017)	(0.0167)
	[0.5259]	[1.7525]	[3.1847]	[3.1902]	[2.0971]	[2.0145]	[0.287]	[0.3356]	[1.9822]	[2.0247]	[4.2077]	[4.2486]
$\alpha_{1,1}$	0.0682	0.0791	0.1591	0.1665	0.0464	0.0433	0.0107	0.0108	0.0265	0.0265	0.0748	0.074
	(0.0308)	(0.026)	(0.0833)	(0.0961)	(0.012)	(0.0118)	(0.0067)	(0.0066)	(0.0093)	(0.0093)	(0.0176)	(0.0172)
	[2.2159]	[3.0407]	[1.9099]	[1.7234]	[3.8639]	[3.6808]	[1.6097]	[1.642]	[2.8625]	[2.8557]	[4.2609]	[4.2973]
β_1	0.8446	0.8488	0.8249	0.8208	0.8801	0.8832	0.9861	0.986	0.9469	0.9465	0.8932	0.8939
	(0.0315)	(0.0192)	(0.0097)	(0.0093)	(0.0097)	(0.01)	(0.0012)	(0.0012)	(0.0055)	(0.0056)	(0.0062)	(0.0061)
	[26.7764]	[44.1513]	[85.4411]	[88.1608]	[90.4132]	[88.2292]	[789.8821]	[812.5294]	[171.5031]	[168.9141]	[143.9148]	[145.6266]
$\alpha_{0,2}$	0.1359	0.2137	0.9519	0.9798	4.7341	4.7239	0.3477	0.3131	2.5413	2.5314	28.8879	25.2268
	(0.0509)	(0.0811)	(0.2969)	(0.3006)	(1.4709)	(1.3949)	(0.3337)	(0.3474)	(0.3202)	(0.3067)	(18.4851)	(13.9973)
	[2.6718]	[2.6362]	[3.2061]	[3.2598]	[3.2185]	[3.3865]	[1.0418]	[0.9012]	[7.9364]	[8.2531]	[1.5628]	[1.8023]
$\alpha_{1,2}$	0.1967	0.2327	0.9999	0.9998	0.1642	0.1685	0.0565	0.055	0.552	0.547	0.9546	0.9396
	(0.1192)	(0.1753)	(0.0055)	(0.0065)	(0.1275)	(0.1306)	(0.0792)	(0.09)	(0.1937)	(0.1904)	(1.1073)	(1.2474)
	[1.6498]	[1.3276]	[181.9528]	[153.009]	[1.287]	[1.2906]	[0.7135]	[0.6119]	[2.8491]	[2.8738]	[0.8621]	[0.7533]
β_2	0.7939	0.758	0.0000	0.0000	0.8305	0.8274	0.3571	0.4152	0.0007	0.0003	0.0452	0.0601
	(0.0065)	(0.0084)	(0.0002)	(0.0024)	(0.0043)	(0.0034)	(0.5692)	(0.5971)	(0.0212)	(0.0139)	(0.0041)	(0.0046)
	[121.3536]	[89.9594]	[0.0035]	[0.0206]	[195.3375]	[241.9694]	[0.6274]	[0.6954]	[0.0311]	[0.0205]	[10.9866]	[13.1552]
$p_{1,1}$	0.4115	0.5887	0.9994	0.9994	0.8142	0.8097	0.994	0.994	0.7863	0.7831	0.9759	0.9741
$p_{2,2}$	0.4048	0.3023	0.9967	0.9967	0.4403	0.4458	0.9856	0.9857	0.058	0.0565	0.1869	0.1939
LL	**-3851.3557**	-3858.0254	**-1853.5262**	-1860.2662	-7188.989	**-7181.4312**	**-2148.7453**	-2149.6417	**-3700.2463**	-3704.8763	**-5711.8689**	-5715.0868
AIC	**7718.7114**	7732.0508	**3723.0523**	3736.5324	14393.978	**14378.8624**	**4313.4906**	4315.2834	**7416.4926**	7425.7525	**11439.7379**	11446.1736
BIC	**7766.0137**	7779.356	**3766.6901**	3780.1748	14441.4168	**14426.3041**	**4361.055**	4362.8507	**7463.9716**	7473.2344	**11487.049**	11493.4876
Stable probabilities												
State 1	0.5028	0.6291	0.8534	0.8494	0.7508	0.7444	0.7047	0.7038	0.8151	0.8131	0.9712	0.9689
State 2	0.4972	0.3709	0.1466	0.1506	0.2492	0.2556	0.2953	0.2962	0.1849	0.1869	0.0288	0.0311
VaR back-testing (In sample)												
LRuc	**0.055**	0.0047	**0.7057**	**0.5443**	**0.934**	**1.0000**	**0.1094**	**0.0899**	**0.19**	**0.2552**	0.0162	**0.0692**
LRcc	**0.1581**	0.0167	**0.9312**	**0.7549**	**0.4905**	**0.5168**	**0.1424**	**0.198**	**0.3546**	**0.5183**	0.0342	**0.1202**
DQ	**0.1249**	**0.055**	**0.2316**	0.0036	**0.2947**	**0.2075**	**0.3324**	**0.4659**	**0.7719**	**0.8035**	**0.0829**	**0.1471**
VaR back-testing (Out-of-sample)												
LRuc	**0.1622**	0.0275	**0.0737**	**0.0737**	**1.0000**	**1.0000**	0.0002	0.0002	**0.7416**	**0.7416**	**0.3047**	**0.3047**
LRcc	**0.1366**	**0.0812**	**0.1776**	**0.1776**	**0.8042**	**0.8042**	0.0009	0.0009	**0.6682**	**0.6682**	**0.4574**	**0.4574**
DQ	**0.2423**	**0.3015**	**0.7292**	**0.7295**	**0.1657**	**0.1608**	**0.2811**	**0.2809**	**0.7693**	**0.7603**	**0.8736**	**0.8739**

CONCLUSION

Overall, our proposed model is well enough to explain both simulation and real-world data's volatility dynamics considering structural breaks in the real-world data. Furthermore, it can capture the downside risk splendidly. Therefore, the proposed model is useful in fitting, illustrating, and capturing the downside risk of nonlinear behavior in economic and financial time series data.

At the same time, three objectives of this study were successfully achieved. It is of prime importance to select a reliable model for capturing volatility and forecasting the risk before an investment. Since the proposed model explains real-world data relatively more thoroughly than the benchmark model and captures the downside risk accurately, it would be beneficial for the investors to make a correct decision. This study opens a wide range of potential research opportunities, like the Markov chain Monte Carlo method instead of the Maximum Likelihood estimation method and student's-t distribution to deal with leptokurtic distribution.

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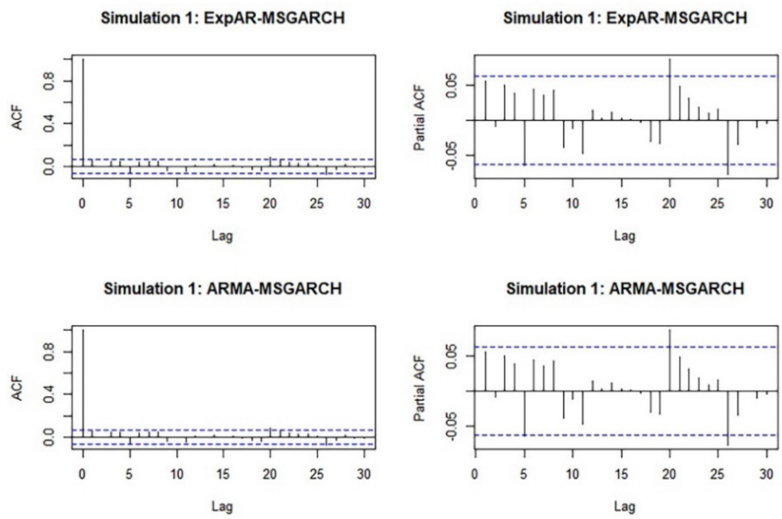
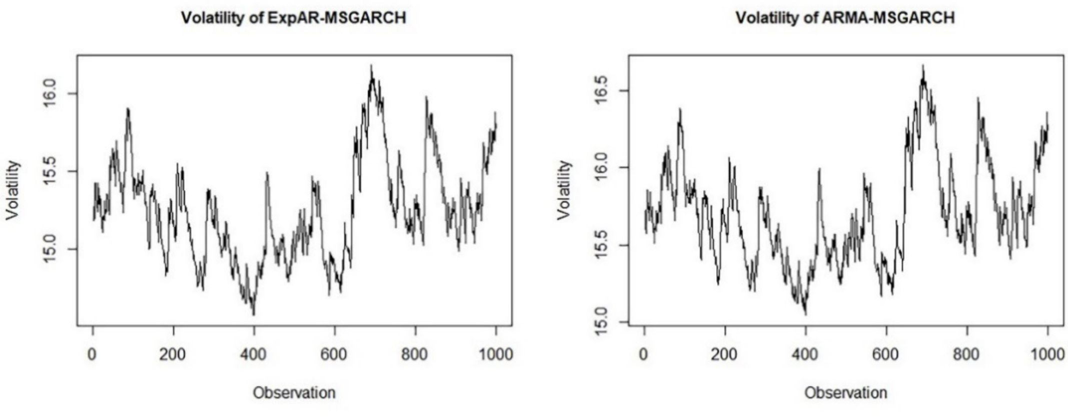
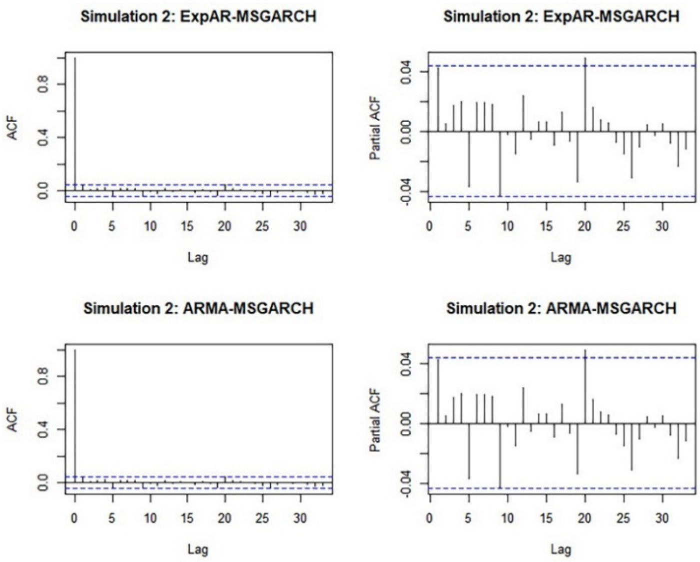


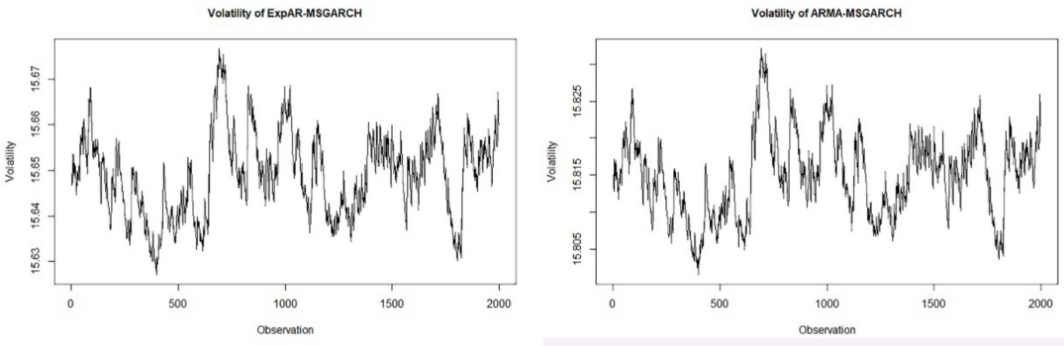
SUPPLEMENTARY FIGURE 1. Simulation 1: Autocorrelation and partial autocorrelation of simulated series



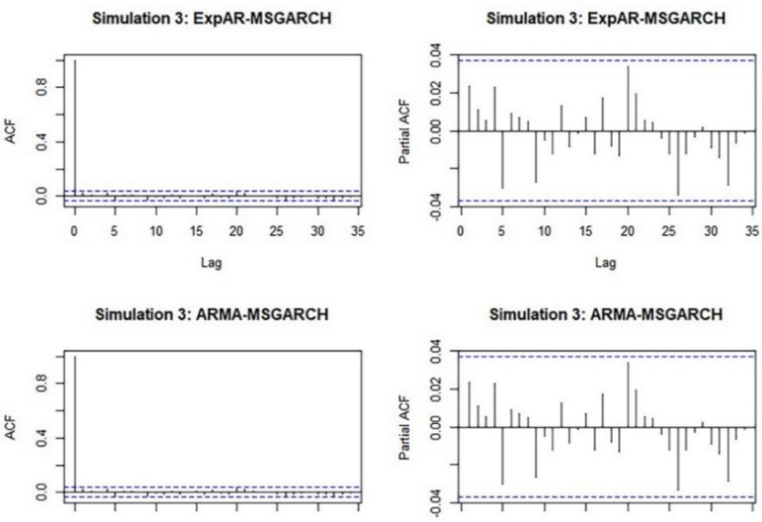
SUPPLEMENTARY FIGURE 2. Simulation 1: Annualized volatility plot of ExpAR-MSGARCH and ARMA-MSGARCH model



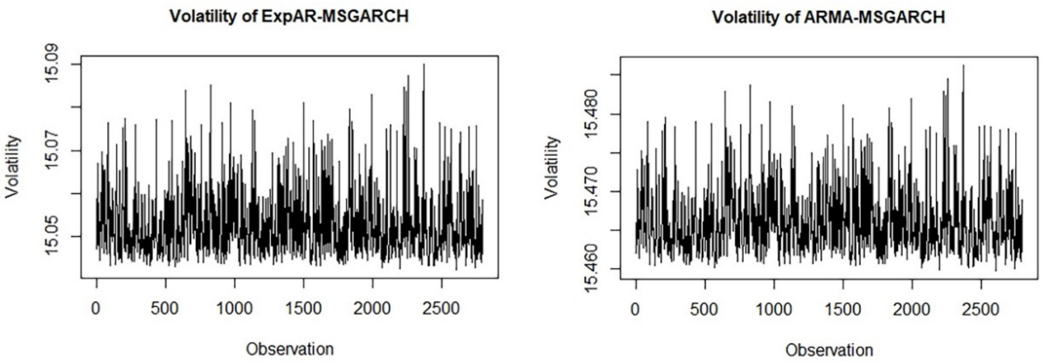
SUPPLEMENTARY FIGURE 3. Simulation 2: Autocorrelation and partial autocorrelation of simulated series



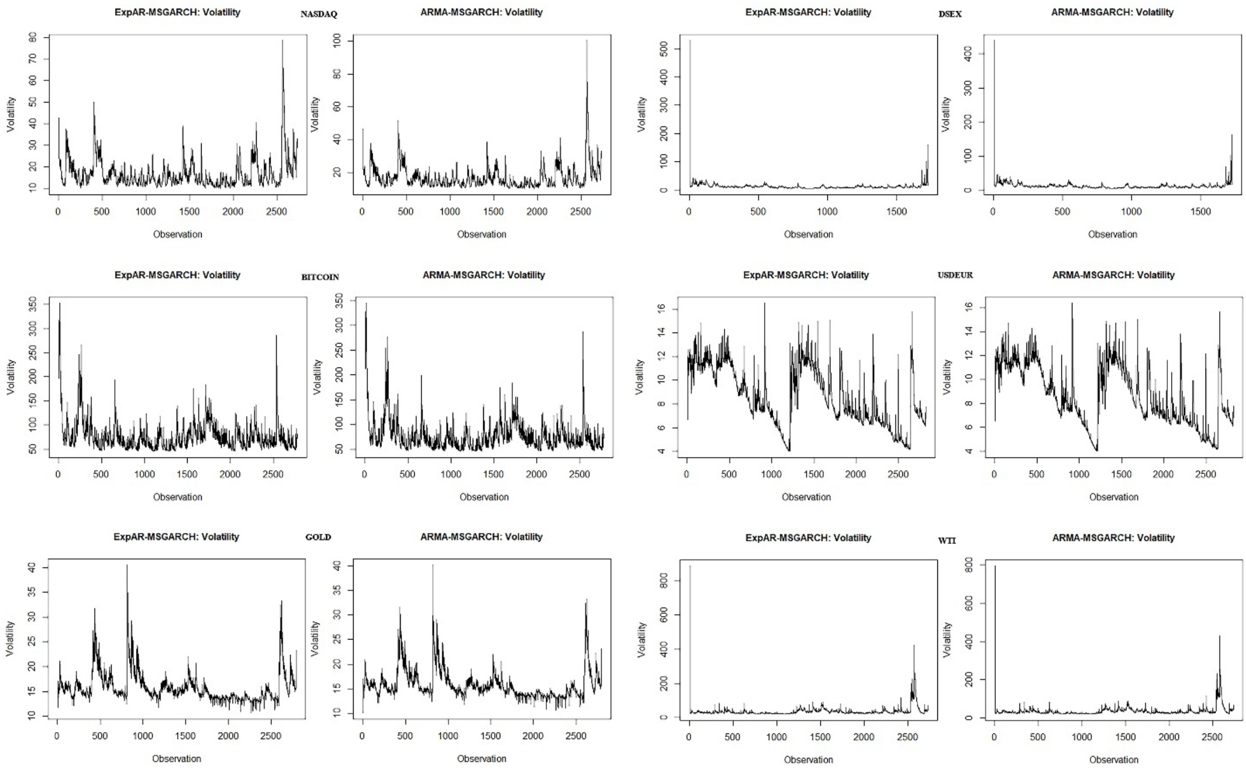
SUPPLEMENTARY FIGURE 4. Simulation 2: Annualized volatility plot of ExpAR-MSGARCH and ARMA-MSGARCH model



SUPPLEMENTARY FIGURE 5. Simulation 3: Autocorrelation and partial autocorrelation of simulated series



SUPPLEMENTARY FIGURE 6. Simulation 3: Annualized volatility plot of ExpAR-MSGARCH and ARMA-MSGARCH model



SUPPLEMENTARY FIGURE 7. Annualized volatility plot of six financial data from ExpAR-MSGARCH and ARMA-MSGARCH model. The name of the sample is placed middle of the plot