

## STUDY OF TRANSMISSION OF TUBERCULOSIS BY SIR MODEL USING RUNGE-KUTTA METHOD

(Kajian Transmisi Tuberkulosis oleh Model SIR Menggunakan Kaedah Runge-Kutta)

FARAH LIYANA AZIZAN, SARATHA SATHASIVAM\*, MAJID KHAN MAJAHAR ALI, THING CHEE HONG & CLEMENT HII KIING YION

### ABSTRACT

This project is conducted to see the prediction of the transmission of the tuberculosis disease's trend with demography and without demography. It is carried out by the SIR model with the Runge-Kutta fourth-order technique using mathematical modelling to analyse Tuberculosis transmission. Furthermore, this project examines the Tuberculosis disease prediction performance of the two SIR models by comparing the data and also to predict the future trend of Tuberculosis transmission in Malaysia in the year 2021 by calculating its incidence rate for each 100 thousand people. We discovered that combining the SIR Model with demography improves the prediction of Tuberculosis disease spread. We also discovered that the higher the transmission rate, the lower the incidence rate per 100 thousand people, and the higher the incidence rate per 100 thousand people, the lower the recovery rate. As a result, it is acceptable to argue that these variables play a significant impact in determining epidemic growth rates.

*Keywords:* Runge-Kutta; SIR model; Tuberculosis disease

### ABSTRAK

Projek ini dijalankan untuk melihat ramalan penularan trend penyakit tuberkulosis dengan demografi dan tanpa demografi. Ia dijalankan oleh model SIR dengan teknik Runge-Kutta tahap empat menggunakan pemodelan matematik untuk menganalisis penularan Tuberkulosis. Selain itu, projek ini mengkaji prestasi ramalan penyakit tuberkulosis bagi dua model SIR dengan membandingkan kedua-dua data dan juga untuk meramalkan trend masa depan penularan Tuberkulosis di Malaysia pada tahun 2021 dengan mengira kadar kejadian bagi setiap 100 ribu orang. Kami mendapati bahawa menggabungkan Model SIR dengan demografi menambahbaik ramalan penularan penyakit Tuberkulosis. Kami juga mendapati bahawa semakin tinggi kadar penularan, semakin rendah kadar penyakit setiap 100 ribu orang, dan semakin tinggi kadar penyakit setiap 100 ribu orang, semakin rendah kadar pemulihan. Kesimpulannya, pembolehubah ini memainkan kesan yang signifikan dalam menentukan kadar pertumbuhan wabak

*Kata kunci:* Runge-Kutta; model SIR; penyakit tuberkulosis

### 1. Introduction

Tuberculosis (TB) disease is an infectious disease that is currently a global pandemic disease occurring worldwide for the last few decades. It spreads quickly from person to person in the air due to a bacterium called Mycobacterium Tuberculosis (Abdul Halim 2013). These bacteria will become dormant and remain in the body without causing any symptoms when infected by these bacteria. It is referred to as latent Tuberculosis disease at this point. However, if the immune system is weakened or harmed, i.e., infected by HIV, the

Tuberculosis disease can start becoming active and damaging their lungs. Fatigue, weight loss, fever, and chest pain are some of the symptoms that can be arisen (Churchyard *et al.* 2017).

Tuberculosis can sometimes be fatal, especially for patients with weakened immune systems, such as young children, the elderly, people with diabetes, or certain cancers. The World Health Organization (WHO) estimates 1,500,000 people will pass on Tuberculosis in 2020, including 214,000 people infected with HIV. In addition, Tuberculosis is the second most common cause of death globally after COVID-19 (World Health Organization 2021).

Malaysia has been fighting against Tuberculosis by implementing the directly observed short-term treatment strategy (DOTS) since 2002. Figure 1 shows the total cases and notification rate of Tuberculosis disease in Malaysia from 1990 to 2015 (Ministry of Health Malaysia 2016). We discovered that the number of cases of tuberculosis disease grew by 17.2 per cent in 2015 from the statistics provided in the National Strategic Plan for Tuberculosis Control (2016-2020). Besides, there were 24220 total cases reported. The notification rate was 79 cases for each 100 thousand population in 2015 (Ministry of Health Malaysia 2016), making Malaysia a country of concern for Tuberculosis disease.

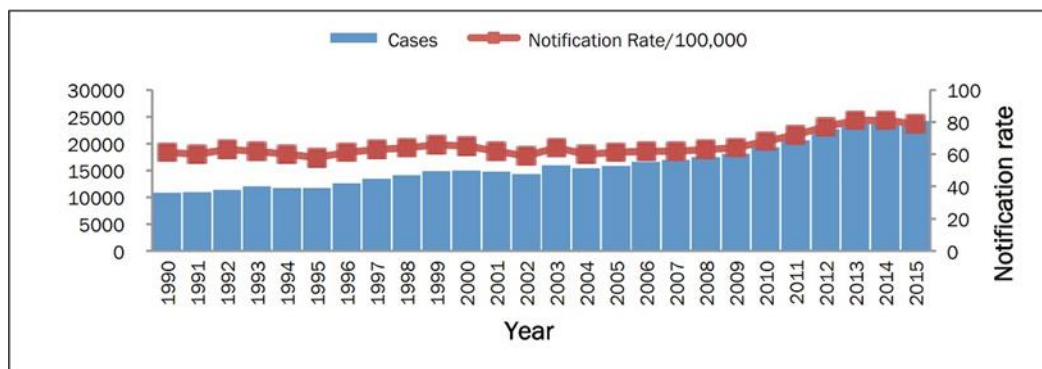


Figure 1: Total cases and notification rate of Tuberculosis disease in Malaysia from 1990 to 2015.

Due to the increasing Tuberculosis disease notification rate since 2010, the Malaysian Ministry of Health (MOH) has issued the National Tuberculosis Strategic Plan (2016-2020). The plan's objective is to provide guidelines for more effective Tuberculosis disease prevention and control strategies in Malaysia. Tan Sri Dato' Seri Dr Noor Hisham bin Abdullah, the general director of Health Malaysia, hopes to eliminate TB in Malaysia by 2035 through the National TB Strategic Plan (Ministry of Health Malaysia 2016). We carry out a study on the transmission of Tuberculosis disease and determine the possibility of predicting its trend in 2021.

Many researchers or scientists have carried out SIR models to control and predict the spread of Tuberculosis disease (Adamu *et al.* 2019; Bahari *et al.* 2022; Iskandar *et al.* 2017). We are interested in applying the SIR model with mathematical modelling to examine the tuberculosis transmission pattern in Malaysia and predict outcomes about its spreading in the future. Mathematical simulations such as SIR models are valuable and powerful tools for adequately predicting the dynamical behaviour of disease and providing insight into how to build effective control techniques (Khan *et al.* 2019). Using mathematical simulations to investigate the patterns of contagious diseases has gotten a lot of interest in recent years because they help us better understand how disease spreads and give a simple overview of epidemiological data. As a result, it can assist predict development outbreaks as the disease spreads and thinking critically about the underlying disease's control. Because they analyse the development of infectious diseases through time, models used to research the transmission

of contagious diseases are called dynamic epidemiological models. Side *et al.* (2018) showed that the SIR model with the Runge-Kutta fourth-order approach could interpret the transmission of Tuberculosis. Hence, the SIR model's transmission model of Tuberculosis disease is being studied and discussed in our project. The fourth-order Runge-Kutta approach is used to provide numerical results and SIR structural models in this study. Then, employing real data from the Ministry of Health Malaysia (MOH), we build the variables and initial values. After that, the model is used to predict and visualise the amount of tuberculosis cases in Malaysia.

For predicting the transmission or spreading of Tuberculosis in Malaysia over time, a Susceptible-Infectious-Recovered (SIR) mathematical model is used to obtain a good result in predicting the transmission trend of the disease in the future (Adamu *et al.* 2019). The SIR model of epidemic illness propagation was employed in this investigation. As a result, the overall population will be separated into three categories in the modelling. The first category, known as the suspicion group, is those who have not yet been infected with the tuberculosis virus. Those who are infected with the tuberculosis virus make up the second group. The third group includes persons who recovered or were killed after contracting Tuberculosis. In the mathematical formulation, clearly that the independent variable is time, which can be measured in days, months, or years, whereas the dependent variables are as follows:

- $S = S(t)$  which indicates a susceptible group at a given time
- $I = I(t)$  which indicates the infected group at a given time
- $R = R(t)$  which indicates a recovered group at a given time

It can be mathematically formulated with and without demography, vaccination, etc (Kousar *et al.* 2016). In our project, we formulate SIR Model without demography and with demography. Susceptible groups  $S$ , Infected groups  $I$ , and Recovered groups  $R$  are sum up to be the total population  $N$  (Ndiaye *et al.* 2020). We then use the fourth-order Runge-Kutta method as a numerical solution to the SIR model. Since the numerical method only discovered results close to real solutions, numerical solutions are indeed known as an approximation. One-step technique, such as the Runge-Kutta approach, can be used to solve numerical differential equations. The objectives of our project are listed below.

- (1) Study of Tuberculosis disease transmission using the SIR model with the Runge-Kutta fourth-order (RK4) method.
- (2) To discuss the TB prediction performance of the 2 SIR models by comparing with the current data, such as the incidence of Malaysia.
- (3) To predict the future trend of Tuberculosis transmission in Malaysia in the year 2021 by calculating its incidence rate for each 100 thousand population.

The findings suggest that the spread of Tuberculosis disease can be managed by slowing epidemic transmission and speeding up recovery. One strategy to limit the infection rate is to prevent tuberculosis-infected people away from vulnerable groups, whereas maximum treatment is required to enhance the healing rate. Epidemic processes are stochastic in nature, particularly at the individual level. Mathematical models and simulations aid in understanding that variations in disease propagation can be explained by chance fluctuations alone, rather than changes in virulence (Becker 1979).

The formulation of the SIR model using the fourth-order Runge-Kutta method is presented in Section 2.1 and Section 2.2. We faced a minor problem in obtaining the initial number of Recovered individuals due to the lack of information provided by the Ministry of

Health Malaysia (MOH). However, we have encountered the problem by calculating the success rate of a disease. Our results and discussions predict the future trend of Tuberculosis disease in Malaysia by considering the incidence rate for each 100 thousand population. The ways in obtaining the number of recovered individuals from success rate and calculating the incidence rate of a disease have been discussed in Section 2.3. Also, we have provided results such as plotting the graphs of Suspected, Infected and Recovered at different times using python for the SIR Model with and without demography in Section 3. Lastly, in Section 4, we provide the conclusion.

## 2. Methodology

### 2.1. SIR model without demography for Tuberculosis disease

We assume that the population remains unchanged for SIR Model without demography during the required investigating period. This means that we do not consider the migration, the mortality rate, and the birth rate in this period. Figure 2 is an illustration of such a model.

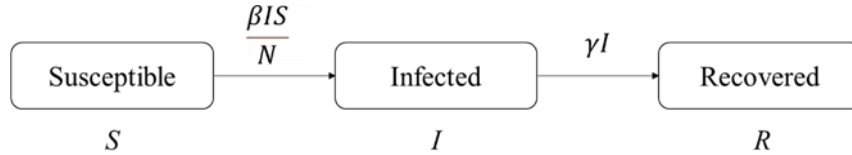


Figure 2: Illustration of SIR model without demography (Ndiaye *et al.* 2020)

Figure 2 shows that the number of susceptible people is decreased by the number of infected individuals who have close contact with the patient. The number of infected people is then increased by the disease's transmission rate and lowered by the number of people who recover. Lastly, the number of the recovered group is increased by the speed of recovery of individuals, and any factors do not reduce it.

#### 2.1.1. Formulation of SIR model without demography

The formulation of the SIR model without demographics is shown below (Kousar *et al.* 2016). Kermack Mckendrick's classic SIR model is:

$$\begin{aligned}
 \frac{dS}{dt} &= -\frac{\beta SI}{N} \\
 \frac{dI}{dt} &= \frac{\beta SI}{N} - \gamma I \\
 \frac{dR}{dt} &= \gamma I
 \end{aligned} \tag{1}$$

with an initial guess  $S(0) = S_o, I(0) = I_o$  and  $R(0) = R_o$ . Table 1 below shows the notations and parameters used in the illustration and formulation.

Table 1: The notations and the parameters used in the SIR model without demography

<b>S</b>	Susceptible group
<b>I</b>	Infected group
<b>R</b>	Recovered group
<b><math>\beta</math></b>	The transmission rate of the disease
<b><math>\gamma</math></b>	The recovery rate from the disease

### 2.1.2. Numerical method of SIR model using RK4 method

Note the modelling differential equation for SIR Model without demography as below. We let

$$\begin{aligned}
 k_1 &= f_s(t_i, I_i, S_i) = -\frac{\beta S_i I_i}{N} \\
 f_I(t, I, S) &= \frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I \\
 f_R(t, I) &= \frac{dR}{dt} = \gamma I
 \end{aligned} \tag{2}$$

We need to find the coefficient in every iteration  $i$ . The coefficient of every iteration  $i$  is given below (Side *et al.* 2018).

$$\begin{aligned}
 k_1 &= f_s(t_i, I_i, S_i) = -\frac{\beta S_i I_i}{N} \\
 l_1 &= f_I(t_i, I_i, S_i) = \frac{\beta S_i I_i}{N} - \gamma I_i \\
 m_1 &= f_R(t_i, I_i) = \gamma I_i
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 k_2 &= f_s\left(t_i + \frac{h}{2}, I_i + \frac{hl_1}{2}, S_i + \frac{hk_1}{2}\right) = -\frac{\beta\left(S_i + \frac{hk_1}{2}\right)\left(I_i + \frac{hl_1}{2}\right)}{N} \\
 l_2 &= f_I\left(t_i + \frac{h}{2}, I_i + \frac{hl_1}{2}, S_i + \frac{hk_1}{2}\right) = \frac{\beta\left(S_i + \frac{hk_1}{2}\right)\left(I_i + \frac{hl_1}{2}\right)}{N} - \gamma\left(I_i + \frac{hl_1}{2}\right) \\
 m_2 &= f_R\left(t_i + \frac{h}{2}, I_i + \frac{hl_1}{2}\right) = \gamma\left(I_i + \frac{hl_1}{2}\right)
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 k_3 &= f_s\left(t_i + \frac{h}{2}, I_i + \frac{hl_2}{2}, S_i + \frac{hk_2}{2}\right) = -\frac{\beta\left(S_i + \frac{hk_2}{2}\right)\left(I_i + \frac{hl_2}{2}\right)}{N} \\
 l_3 &= f_I\left(t_i + \frac{h}{2}, I_i + \frac{hl_2}{2}, S_i + \frac{hk_2}{2}\right) = \frac{\beta\left(S_i + \frac{hk_2}{2}\right)\left(I_i + \frac{hl_2}{2}\right)}{N} - \gamma\left(I_i + \frac{hl_2}{2}\right) \\
 m_3 &= f_R\left(t_i + \frac{h}{2}, I_i + \frac{hl_2}{2}\right) = \gamma\left(I_i + \frac{hl_2}{2}\right)
 \end{aligned} \tag{5}$$

$$\begin{aligned}
 k_4 &= f_s(t_i + h, I_i + hl_3, S_i + hk_3) = -\frac{\beta(S_i + hk_3)(I_i + hl_3)}{N} \\
 l_4 &= f_I(t_i + h, I_i + hl_3, S_i + hk_3) = \frac{\beta(S_i + hk_3)(I_i + hl_3)}{N} - \gamma(I_i + hl_3) \\
 m_4 &= f_R(t_i + h, I_i + hl_3) = \gamma(I_i + hl_3)
 \end{aligned} \tag{6}$$

Thus, the numerical solution for the SIR Model without demography is

$$\begin{aligned}
 S_{i+1} &= S_i + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4) \\
 I_{i+1} &= I_i + \frac{1}{6}h(l_1 + 2l_2 + 2l_3 + l_4) \\
 R_{i+1} &= R_i + \frac{1}{6}h(m_1 + 2m_2 + 2m_3 + m_4)
 \end{aligned} \tag{7}$$

where  $h$  is the step size.

## 2.2. SIR model with demography for Tuberculosis disease

We consider the death rate and the birth rate in the experimental period for the SIR model with demography while ignoring minor migration. The birth rate and mortality rate must be identical for the population size to be assumed constant throughout the whole time (Widyarningsih *et al.* 2018). The model's illustration is then given in Figure 3.

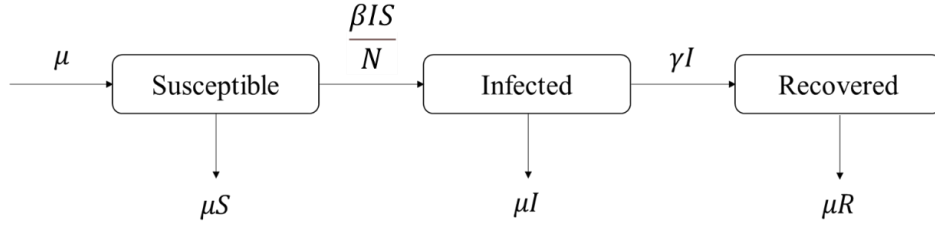


Figure 3: Illustration of SIR model with demography (Bahari *et al.* 2022)

From Figure 3, the number of the susceptible group,  $S$ , is increased by the birth rate while it is decreased by the death rate and the number of infected individuals who have closely contacted the patients. Next, the disease's transmission rate can increase the number of infected people, while the disease's death rate and the recovery rate can decrease. Finally, the recovery rate increases the number of the recovered group, whereas the death rate decreases.

### 2.2.1. Formulation of SIR model with demography

The formulation of the SIR model with demography is given below (Side *et al.* 2018).

$$\begin{aligned}
 \frac{dS}{dt} &= \mu - \frac{\beta SI}{N} - \mu S \\
 \frac{dI}{dt} &= \frac{\beta SI}{N} - \gamma I - \mu I \\
 \frac{dR}{dt} &= \gamma I - \mu R
 \end{aligned} \tag{8}$$

with initial conditions  $S(0) = S_0, I(0) = I_0$  and  $R(0) = R_0$ . The notations and the parameters used in the illustration and formulation are listed in Table 2.

Table 2: The notations and the parameters used in the SIR model with demography

<b>S</b>	Susceptible group
<b>I</b>	Infected group
<b>R</b>	Recovered group
<b><math>\beta</math></b>	The transmission rate of the disease
<b><math>\gamma</math></b>	The recovery rate from the disease
<b><math>\mu</math></b>	The birth rate and the death rate

### 2.2.2. Numerical method of SIR model using RK4 method

Note the modelling differential equation for the SIR model with demography as below. We let

$$\begin{aligned}
 f_s(t, I, S) &= \frac{dS}{dt} = \mu - \frac{\beta SI}{N} - \mu S \\
 f_I(t, I, S) &= \frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I - \mu I \\
 f_R(t, I, R) &= \frac{dR}{dt} = \gamma I - \mu R
 \end{aligned} \tag{9}$$

We need to find the coefficient of  $k_j, l_j$  and  $m_j$  for  $j \in \{1, 2, 3, 4\}$  in every iteration  $i$ . The coefficient of  $k_j, l_j$  and  $m_j$  for  $j \in \{1, 2, 3, 4\}$  in every iteration  $i$  is given below (Side *et al.* 2018).

$$\begin{aligned}
 k_1 &= f_s(t_i, I_i, S_i) = \mu - \frac{\beta S_i I_i}{N} - \mu S_i \\
 l_1 &= f_I(t_i, I_i, S_i) = \frac{\beta S_i I_i}{N} - \gamma I_i - \mu I_i \\
 m_1 &= f_R(t_i, I_i, R_i) = \gamma I_i - \mu R_i
 \end{aligned} \tag{10}$$

$$\begin{aligned}
 k_2 &= f_s\left(t_i + \frac{h}{2}, I_i + \frac{hl_1}{2}, S_i + \frac{hk_1}{2}\right) = \mu - \frac{\beta\left(S_i + \frac{hk_1}{2}\right)\left(I_i + \frac{hl_1}{2}\right)}{N} - \mu\left(S_i + \frac{hk_1}{2}\right) \\
 l_2 &= f_I\left(t_i + \frac{h}{2}, I_i + \frac{hl_1}{2}, S_i + \frac{hk_1}{2}\right) \\
 &= \frac{\beta\left(S_i + \frac{hk_1}{2}\right)\left(I_i + \frac{hl_1}{2}\right)}{N} - \gamma\left(I_i + \frac{hl_1}{2}\right) - \mu\left(I_i + \frac{hl_1}{2}\right) \\
 m_2 &= f_R\left(t_i + \frac{h}{2}, I_i + \frac{hl_1}{2}, R_i + \frac{hm_1}{2}\right) = \gamma\left(I_i + \frac{hl_1}{2}\right) - \mu\left(R_i + \frac{hm_1}{2}\right)
 \end{aligned} \tag{11}$$

$$\begin{aligned}
 k_3 &= f_s \left( t_i + \frac{h}{2}, I_i + \frac{hl_2}{2}, S_i + \frac{hk_2}{2} \right) = \mu - \frac{\beta \left( S_i + \frac{hk_2}{2} \right) \left( I_i + \frac{hl_2}{2} \right)}{N} - \mu \left( S_i + \frac{hk_2}{2} \right) \\
 l_3 &= f_l \left( t_i + \frac{h}{2}, I_i + \frac{hl_2}{2}, S_i + \frac{hk_2}{2} \right) \\
 &= \frac{\beta \left( S_i + \frac{hk_2}{2} \right) \left( I_i + \frac{hl_2}{2} \right)}{N} - \gamma \left( I_i + \frac{hl_2}{2} \right) - \mu \left( I_i + \frac{hl_2}{2} \right)
 \end{aligned} \tag{12}$$

$$\begin{aligned}
 m_3 &= f_R \left( t_i + \frac{h}{2}, I_i + \frac{hl_2}{2} + R_i + \frac{hm_2}{2} \right) = \gamma \left( I_i + \frac{hl_2}{2} \right) - \mu \left( R_i + \frac{hm_2}{2} \right) \\
 k_4 &= f_s(t_i + h, I_i + hl_3, S_i + hk_3) = \mu - \frac{\beta(S_i+hk_3)(I_i+hl_3)}{N} - \mu(S_i + hk_3) \\
 l_4 &= f_l(t_i + h, I_i + hl_3, S_i + hk_3) \\
 &= \frac{\beta(S_i + hk_3)(I_i + hl_3)}{N} - \gamma(I_i + hl_3) - \mu(I_i + hl_3) \\
 m_4 &= f_R(t_i + h, I_i + hl_3, R_i + hm_3) = \gamma(I_i + hl_3) - \mu(R_i + hm_3)
 \end{aligned} \tag{13}$$

Thus, the numerical solution for the SIR model with demography is

$$\begin{aligned}
 S_{i+1} &= S_i + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4) \\
 I_{i+1} &= I_i + \frac{1}{6}h(l_1 + 2l_2 + 2l_3 + l_4) \\
 R_{i+1} &= R_i + \frac{1}{6}h(m_1 + 2m_2 + 2m_3 + m_4)
 \end{aligned} \tag{14}$$

where  $h$  is the step size.

### 2.3. Incidence rate and success rate

#### 2.3.1. Incidence rate

The incidence rate of Tuberculosis (TB) is defined as the number of newly enrolled Tuberculosis (TB) cases for each 100 thousand population. Thus, we can estimate its incidence rate,  $rate_i$  by using the formula.

$$\text{Incidence rate, } rate_i = \frac{\text{No. of new TB cases reported in the period}}{\text{Total population at risk in period}} \times 100,000 \tag{15}$$

#### 2.3.2. Success rate

We obtain information on the success rate of Tuberculosis (TB) disease in Malaysia from World Bank World Development Indicators (World Bank 2020). The formula in calculating the success rate,  $rate_s$  from the disease is given below.



$$\text{Success rate, rate}_s = \frac{\text{No. of recovered individuals in the period}}{\text{Total number of infected individuals in the period}} \times 100 \quad (16)$$

### 3. Results and Discussion

In Section 3.1, we analyse the efficiency and availability of the SIR model with and without demography. In Section 3.2, we project the trajectory of Tuberculosis disease transmission in Malaysia for 2021.

The estimated success treatment rate for Tuberculosis disease of Malaysia in the year 2015 was about 78% (World Bank 2020). We can obtain the estimated number of recovered individuals using the formula discussed in Section 2.3.2. Thus, we estimate that 18892 individuals recovered from the Tuberculosis disease in 2015. Thus, we have collected all the information needed to generate the SIR Model. Table 3 shows the number of susceptible, infected, and recovered individuals in 2015.

Table 3: Number of susceptible, infected and recovered people in the year 2015

Year	Number of Susceptible people	Number of Infected people	Number of Recovered people
2015	189,337	24,220	18892

We need the numeric transmission rate  $\beta$ , recovery rate  $\gamma$ , and birth/mortality rate  $\mu$ . We predict that a patient can bring infection to around 20.6 people, and the patient takes an average of 140 days to recover from the disease with the help of medication. The number of tuberculosis deaths was 5.5 for each 100 thousand people in 2015 (World Bank 2020). As a result, we have the physical parameter  $\beta = 0.04857, \gamma = 0.007143$  and  $\mu = 0.000055$ .

#### 3.1. Efficiency and availability of SIR model

Let the data set in Table 3 be our initial guess with  $\beta = 0.04857, \gamma = 0.007143$  and  $\mu = 0.000055$ . Now, we predict and observe the transmission of Tuberculosis disease in 2016 by using SIR Model without and with demography.

##### 3.1.1. SIR model without demography

The value of initial conditions and each parameter's value are given in Table 4. The output is shown below.

Table 4: Initial conditions and value for each parameter used in the model without demography

Parameter	Value
$S(0)$	189,337
$I(0)$	24,220
$R(0)$	18892
$\beta$	0.04857
$\gamma$	0.007143

From Figure 4 below, we observe a peak of infection between the 90<sup>th</sup> to 120<sup>th</sup> days. Then, the number of infected people began to decline in 2016. Next, the susceptible individuals

attain their maximum initially and keep decreasing while the recovered individuals increase steadily and reach their maximum at the end of 2016. We assume the population involved to be constant throughout the whole period.

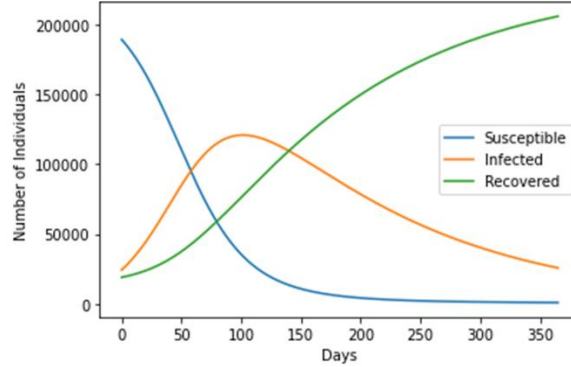


Figure 4: Graph generated by SIR model without demography for 2016

From Figure 5 below, we can obtain the number of infected individuals for 2016 at 365<sup>th</sup> row, about 25629 people. As a result, the estimated incidence rate of Tuberculosis disease in Malaysia for the year 2016 can be computed by the given formula with the total population in 2016 is roughly 30.68 million.

$$I_1 = \frac{25629}{30.68 \times 10^6} \times 100,000 = 83.54 \tag{17}$$

SIR Data of Tuberculosis in Malaysia (2016)			
Days	S	I	R
1	1.89E+05	24220	18892
2	1.88E+05	25018	19068
3	1.87E+05	25836	19250
4	1.86E+05	26676	19438
5	1.85E+05	27537	19633
.	.	.	.
.	.	.	.
.	.	.	.
100	36012	1.21E+05	75628
101	35112	1.21E+05	76494
102	34234	1.21E+05	77359
103	33378	1.21E+05	78225
104	32544	1.21E+05	79090
105	31731	1.21E+05	79956
.	.	.	.
.	.	.	.
.	.	.	.
360	816.09	26542	2.05E+05
361	811.58	26357	2.05E+05
362	807.13	26173	2.05E+05
363	802.73	25991	2.06E+05
364	798.39	25809	2.06E+05
365	794.1	25629	2.06E+05

Figure 5: SIR data of Tuberculosis in Malaysia (2016)

3.1.2. SIR model with demography

The value of initial conditions and value for each parameter is below. Table 5 below is the output.

Table 5: Initial conditions and value for each parameter used in the SIR model without demography

Parameter	Value
$S(0)$	189,337
$I(0)$	24,220
$R(0)$	18892
$\beta$	0.04857
$\gamma$	0.007143
$\mu$	0.000055

From Figure 6, we notice that it is similar to the graph in Figure 4. There is a peak infection of Tuberculosis disease between the 90<sup>th</sup> and 120<sup>th</sup> days. The number of susceptible individuals and recovered individuals decreases and increases throughout 2016, respectively. The graph seemed somehow weird as we also assume the population involved to be constant throughout the whole period.

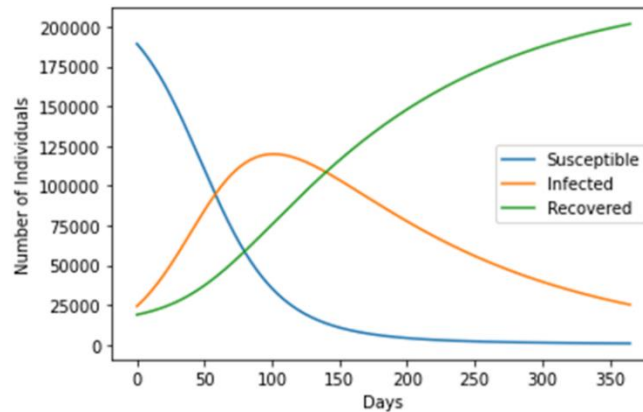


Figure 6: Graph generated by SIR model with demography for 2016

From Figure 7 below, at the end of 2016, the number of affected people was estimated to be around 25161 people. Given that the total population in 2016 is about 30.68 million, the predicted Tuberculosis disease incidence rate in Malaysia for 2016 is calculated as

$$I_2 = \frac{25161}{30.68 \times 10^6} \times 100,000 = 82.01 \tag{18}$$

Days	S	I	R
1	1.89E+05	24220	18892
2	1.88E+05	25016	19067
3	1.87E+05	25833	19248
4	1.86E+05	26672	19435
5	1.85E+05	27531	19628
.	.	.	.
.	.	.	.
.	.	.	.
90	46388	1.18E+05	66623
91	45249	1.19E+05	67467
92	44135	1.19E+05	68314
93	43046	1.19E+05	69162
94	41982	1.19E+05	70012
95	40943	1.19E+05	70863
96	39928	1.20E+05	71715
.	.	.	.
.	.	.	.
.	.	.	.
360	840.1	26063	2.01E+05
361	835.5	25880	2.01E+05
362	830.95	25699	2.01E+05
363	826.46	25518	2.02E+05
364	822.02	25339	2.02E+05
365	817.64	25161	2.02E+05

Figure 7: SIR data of Tuberculosis in Malaysia (2016)

Table 6 shows the number of infections and tuberculosis incidence for each 100 thousand people in 2016.

Table 6: Number of infected people and incidence rate of disease for each 100 thousand people

Year	Number of Infected people	The incidence rate of disease (for each 100 thousand people)
2016	25739	81.30

To ease in determining the efficiency of 2 SIR models, we summarise the actual and the estimated incidence rate obtained from Sections 3.1.1 and 3.1.2 in Table 7. Their relative percentage error is also given.

Table 7: The actual and the estimated incidence rate

Type of SIR Model	Predicted Incidence Rate of TB for each 100 thousand population	Actual Incidence Rate of TB for each 100 thousand population	Error (%)
Without Demography	83.54	81.30	2.76
With Demography	82.01	81.30	0.87

Notice that for both SIR Model, the error in predicting the incidence rate of Tuberculosis are 2.76% and 0.87%, respectively, which is considered a small error. Thus, it is clear that predicting the transmission trend of Tuberculosis disease using the SIR Model with the RK4 method is efficient. On the other hand, the SIR Model with demography is more efficient due

to its lower error. Hence, we would like to predict the transmission trend of Tuberculosis in 2021 using the SIR model with demography.

### 3.2. Prediction on the transmission of Tuberculosis disease in 2021

We estimate that there were around 190,000 individuals who carried out contact examinations for Tuberculosis disease in the year 2020. Besides, the treatment success rate of Tuberculosis in Malaysia from 2015 to 2019 is about 78% to 81%. Hence, we estimate the treatment success rate in 2020 is 80% as average. Then, by the formula (Section 2.3.2), we obtain the estimated number of recovered individuals in 2020, around 18915. As a result, Table 8 shows the expected total number of susceptible people, infected people, and healed people in 2020.

Table 8: Total number of susceptible, infected and recovered people in Malaysia for 2020

Year	Estimated number of Susceptible people	Number of Infected people	Estimated number of Recovered people
2020	190,000	23,644	18,915

We consider different  $\beta$  and  $\gamma$  in the prediction of transmission trend of Tuberculosis disease in 2021. The effects of altering  $\beta$  and  $\gamma$  are detailed in Section 3.2.1 and 3.2.2, respectively.

#### 3.2.1. Changing $\beta$ while other parameters remain unchanged

We consider 4 different values of transmission rate  $\beta = 0.1, 0.05, 0.04857, 0.03333$ .

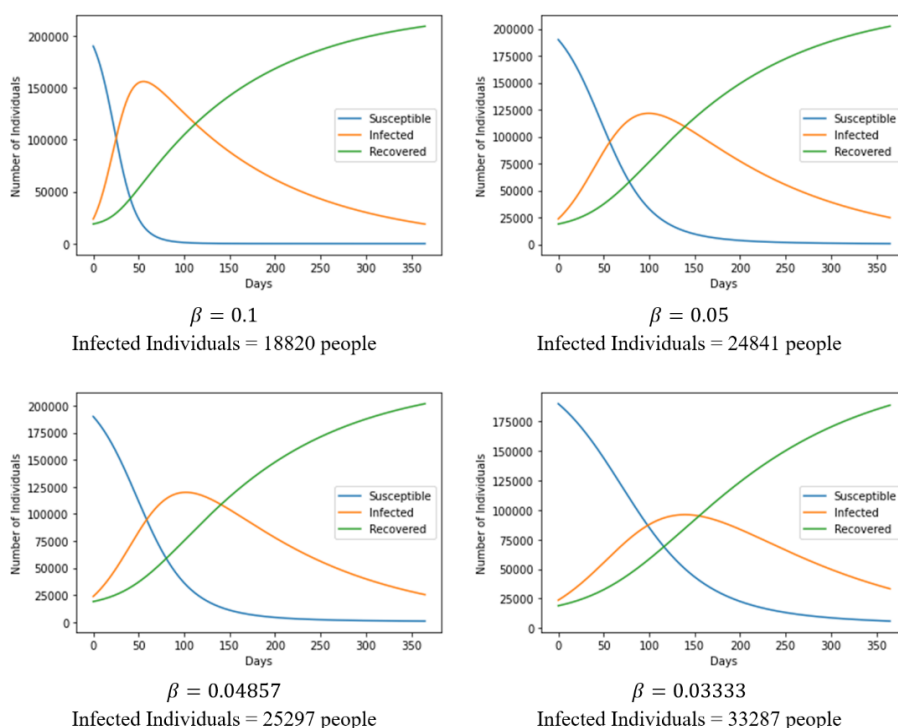


Figure 8: Graph generated by SIR model with different  $\beta$

We observed that the peak infection is lower for a smaller value of  $\beta$ , and the infected graph is more skew to the left. This means that the lower the transmission rate  $\beta$ , the lower the possibility for an individual to get infected.

Given that Malaysia's population will be around 33.45 million in 2021, then we use the methodology in Section 2.3.1. Table 9 shows the incidence rate of Tuberculosis for each 100 thousand people.

Table 9: Estimated incidence rate for different  $\beta$

$\beta$	Estimated Incidence Rate of TB for each 100 thousand population (2021)
0.1	56.26
0.05	74.26
0.04857	75.63
0.03333	99.51

Therefore, the greater the transmission rate  $\beta$ , the lower the incidence rate of Tuberculosis in Malaysia by 2021.

### 3.2.2. Changing $\gamma$ while other parameters remain unchanged

We consider 4 different values of transmission rate,  $\gamma = 0.01, 0.007143, 0.005556$ .

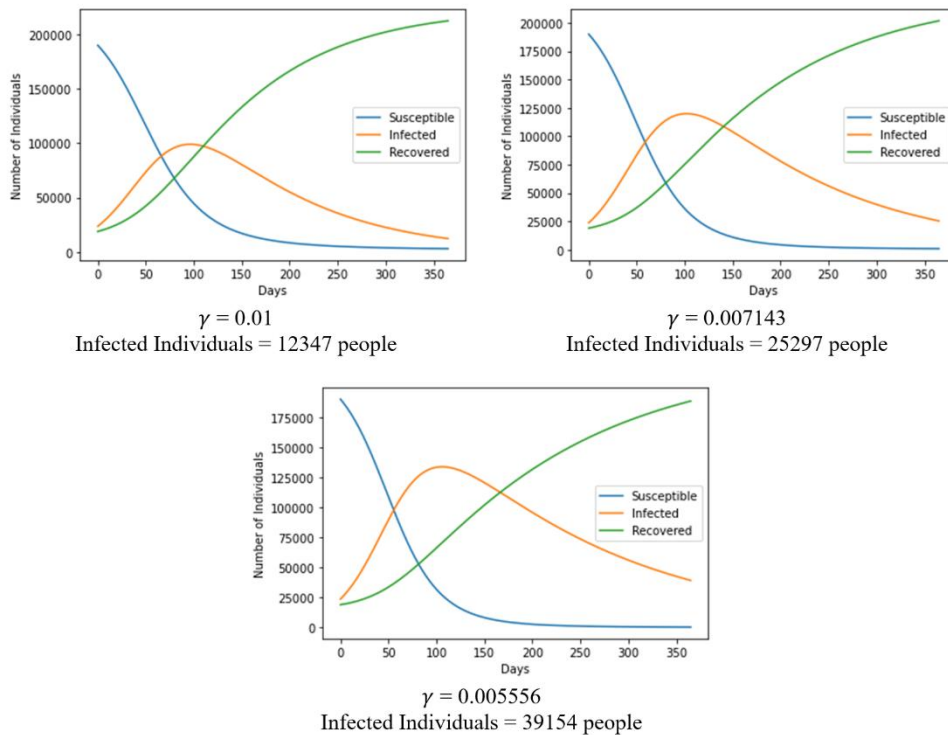


Figure 9: Graph generated by SIR model with different  $\gamma$

We can notice that the lower the value of recovery rate  $\gamma$ , the higher the infection peak of Tuberculosis disease. However, the period for the peak infection to occur is similar for three different  $\gamma$ .

Table 10 shows the incidence rate of Tuberculosis for each 100 thousand people. We apply the formula in Section 2.3.1, given that the total population in Malaysia in the year 2021 is around 33.45 million

Table 10: Estimated incidence rate for different  $\gamma$

$\gamma$	Estimated Incidence Rate of TB for each 100 thousand population (2021)
0.01	36.91
0.007143	75.63
0.005556	117.05

Therefore, the lower the recovery rate  $\gamma$  the greater the incidence rate of Tuberculosis in Malaysia by the end of 2021.

In short, this is our prediction on the transmission trend of Tuberculosis disease in Malaysia for the year 2021 by considering different values of parameters in the SIR Model. It enables us to estimate the number of infected individuals, although it is most likely to vary at any given time.

#### 4. Conclusion

In this project, we studied the transmission of Tuberculosis in Malaysia by using SIR Model with demography and without demography. The numerical method used is Runge-Kutta 4 method, and Python programming coding was used in the simulation. We compared the two models based on their efficiency in predicting the future transmission trend of Tuberculosis disease. We found that the SIR Model with demography gives a better prediction for the spreading of Tuberculosis disease. This could be due to various events such as birth, death, immigration, etc. Furthermore, we forecasted the Tuberculosis disease transmission pattern through 2021 by calculating the disease's incidence rate for each 100 thousand people. The effect of physical parameters  $\beta$  and  $\gamma$  on their transmission trend has also been discussed in detail. We found out that the greater the transmission rate  $\beta$ , the lower the incidence rate for each 100 thousand population, whereas the lower the recovery rate  $\gamma$ , the higher the incidence rate for each 100 thousand people. Therefore, it is reasonable to conclude that these parameters play an important role in determining the rate of epidemic growth. Tuberculosis is controllable by injecting a Mycobacterium Bovis (BCG) vaccine. Therefore, we can predict disease transmission using the SIR demographic model with vaccination and discuss the vaccine's effectiveness in controlling its transmission trend. The vaccination process would eventually improve the recovery rate and reduce the death rate of Tuberculosis disease. We also recommend quarantining the infected people as a preventive measure as the BCG vaccine has limited effect against the epidemic in the developing world.

#### Acknowledgement

This research is supported by Research University Grant (RUI) (1001/PMATHS/8011131) by Universiti Sains Malaysia.

## References

- Abdul Halim N. 2013. Tuberculosis model: A mathematical analysis. Masters Thesis. Universiti Malaya.
- Adamu H.A., Muhammad M., Jingi A. & Usman M. 2019. Mathematical modelling using improved SIR model with more realistic assumptions. *International Journal of Engineering and Applied Sciences (IJEAS)* **6**(1): 64–69.
- Bahari M.F., Utami R. & Rosyida A. 2022. SIR model for the spread of Tuberculosis in Kudus Regency. *World Scientific News* **163**: 128–138.
- Becker N. 1979. The uses of epidemic models. *Biometrics* **35**(1): 295.
- Churchyard G., Kim P., Shah N.S., Rustomjee R., Gandhi N., Mathema B., Dowdy D., Kasmar A. & Cardenas V. 2017. What we know about Tuberculosis transmission: An overview. *The Journal of Infectious Diseases* **216**(suppl\_6): S629–S635.
- Iskandar T., Chaniago N.A., Munzir S., Halfiani V. & Ramli M. 2017. Mathematical model of tuberculosis epidemic with recovery time delay. *AIP Conference Proceedings* **1913**(1), pp. 020021.
- Khan M.A., Ahmad M., Ullah S., Farooq M. & Gul T. 2019. Modeling the transmission dynamics of tuberculosis in Khyber Pakhtunkhwa Pakistan. *Advances in Mechanical Engineering* **11**(6): 1687814019854835.
- Kousar N., Mahmood R. & Ghalib M. 2016. A numerical study of SIR epidemic model. *International Journal of Sciences: Basic and Applied Research* **25**(2): 354–363.
- Ministry of Health Malaysia. 2016. *National Strategic Plan For Tuberculosis Control*. Putrajaya: Disease Control Division, Ministry of Health Malaysia.
- Ndiaye B.M., Tendeng L. & Seck D. 2020. Analysis of the COVID-19 pandemic by SIR model and machine learning technics for forecasting. *arXiv preprint arXiv: 2004.01574*.
- Side S., Utami A.M. & Pratama M. I. (2018). Numerical solution of SIR model for transmission of tuberculosis by Runge-Kutta method. In *Journal of Physics: Conference Series* **1040**(1), pp. 012021.
- Widyaningsih P., Nugroho A.A. & Saputro D.R.S. 2018. Susceptible infected recovered model with vaccination, immunity loss, and relapse to study tuberculosis transmission in Indonesia. *AIP Conference Proceedings* **2014**(1), pp. 020121.
- World Bank. 2020. Tuberculosis treatment success rate (% of new cases) - Malaysia. <https://data.worldbank.org/indicator/SH.TBS.CURE.ZS?locations=MY> (15 December 2021).
- World Health Organization. 2021. Tuberculosis. <https://www.who.int/news-room/fact-sheets/detail/tuberculosis> (2 January 2022)

*School of Mathematical Sciences*

*Universiti Sains Malaysia*

*11800, USM, Penang*

*MALAYSIA*

*E-mail: aflyana@unimas.my, saratha@usm.my\*, majidkhanmajaharali@usm.my, cheehong98@student.usm.my, clementhiikiingyion@student.usm.my*

*Centre for Pre-University Studies*

*Universiti Malaysia Sarawak*

*94300, Kota Samarahan, Sarawak*

*MALAYSIA*

*Email: aflyana@unimas.my*

Received: 27 January 2022

Accepted: 21 May 2022

---

\*Corresponding author