Acoustic Mach number, jet Mach number or jet velocity: Choosing the optimal control property for jet noise experiments at different test rigs

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Wind tunnel experiments are great in order to study jet noise. Yet, the heat demand even for ISA cold engine thrust conditions is rather high. The additional heating and/or cooling of air in order to meet atmospheric norm conditions increases both energy demand, test time and test cost. Some energy lean experimental pressureized air test rigs make compromises on the temperature control system. Hence, norm conditions cannot be met and a way must be found how to optimally deal with the shortcomings of the test rig.

This paper starts with an ideal test rig and the implications of testing especially subsonic jets under ISA norm conditions. Then, the shortcomings due to low-energy designs will be investigated by examining two factors: The jet temperature as well as the temperature of the acoustic chamber (below or above ISA norm temperature).

The Ffowcs-Williams analogy for jet noise will be rewritten without any temperature terms. This will show that jet noise scales approximately with $I \propto M_j^2 \cdot U_j^1 \cdot M_{ac}^5$.

The derivation will be used in order to evaluate measurement strategies on two different pressureized air test rigs: (case 1) the remotely located compressor which is characterized by a constant total jet temperature, but - if unheated - is typically too cold for ISA norm conditions, and (case 2) a closely located compressor, where the compression heat is preserved, yet causes runaway temperatures within the jet and possibly within the acoustic test room.

The aim of this paper is to show a solution to the discussion on which operational parameter is optimal for any test facility: the acoustic Mach number M_{ac}, jet Mach number M_j or jet velocity U_j. A suitable test parameter produces a small error (e.g. less than 12 % or 0.5 dB) in comparison to ISA norm conditions over a wide range of any subsonic operation.

Assuming that the test room is warmer than ISA, unheated (too cold) jet test rigs like AWB make small errors when using the jet velocity whereas slightly too hot jet test rigs like JExTRA are better of using jet Mach number or acoustic Mach number for the definition of their jet operations. The latter case is demonstrated using a data example from JExTRA.

I. Introduction

An initial study on the topic has been conducted by Jente[1] where the case 1 test conditions for cold unheated jet air supply test rigs were investigated. This paper complements the topic by also including case 2 test conditions.

It is also worth to mention that this paper discusses rather small errors of test conditions (pressure, temperature) compared to ISA conditions. Such a small error could be for example a deviation of even up to 20 % in total jet temperature.

However, this paper does not aim to propose an answer for more substantial deviations. Especially, the question will not be answered on how to deal with the violation of similarity laws for intensive physical properties, e.g. when measuring hot core flow of dual stream engines under cold experimental conditions. The error between hot and cold temperature could be for example, $800\,\mathrm{K}/273\,\mathrm{K} - 1 \approx 193\,\%$, which is much higher than the small error of 20 % within this paper.

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Nomenclature

Name	Unit	Meaning
Δf	$[Hz]$	bandwidth of narrowband frequencies
Δf_{sc}	$[Hz]$	bandwidth (general term)
γ	[-]	adiabatic index
ρ_0	[kg/m^3]	(static) density of medium in acoustic room
ρ_j	[kg/m^3]	static density of jet
ρ_m	[kg/m^3]	static mixed density of discharged jet
θ	[°]	polar angle of microphone, from engine exhaust (aft-front)
a_0	[m/s]	speed of sound in acoustic room
c_P	[J/kg − K]	specific heat capacity
f	$[Hz]$	frequency
f_m	$[Hz]$	third-octave band center frequency
I	[W/m^2]	Acoustic intensity
M_{ac}	[-]	acoustic Mach number
M_{ac}	[-]	convection Mach number
M_j	[-]	jet Mach number
p_0	[Pa]	(static) ambient pressure of acoustic room
P	[W]	power / heat flow rate
R	[J/kg − K]	specific gas constant (of air)
R_0	[m]	distance source-observer
SPL	[dB]	sound pressure level
Sr	[-]	Strouhal number
$T_{t,j}$	[K]	total jet temperature
T_0	[K]	(static) temperature of acoustic room
T_j	[K]	static jet temperature
T_m	[K]	static temperature of discharged jet
$TTR_{a/t}$	[-]	ratio of actual to target total temperature
U_j	[m/s]	jet velocity

II. Ideal test rig

The jet noise measurement test rig (see figure 1) consists of a pressureized air supply system which delivers jet flow at a certain total temperature $T_{t,j}$ via a compressor.

The jet propagates into an acoustic chamber or room which is characterized by its (static) pressure p_0 and (static) temperature T_0.

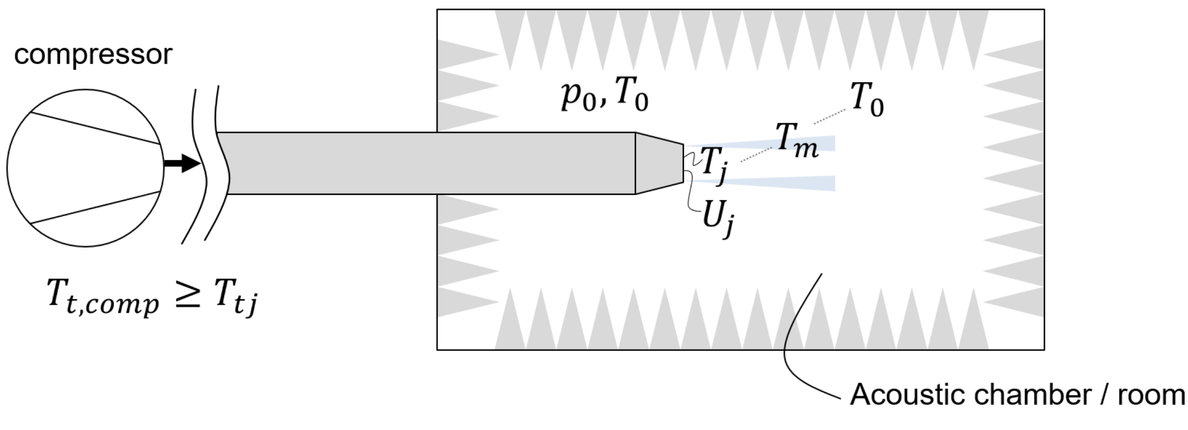

Figure 1: jet noise test rig, image from Jente[1]

III. Implications due to use of ISA-norm atmospheric condition

For reason of good and fair data comparability to other facilities, normed atmospheric conditions are defined. Here, the ISA-standard atmosphere is used:

$$p_0 := 101325\,\mathrm{Pa} \tag{1}$$

$$T_0 := 288.15\,\mathrm{K}(15\,^\circ\mathrm{C}) \tag{2}$$

$$T_j := T_0 \text{ (isothermal velocity profile)} \tag{3}$$

The measurement of a certain jet velocity or jet Mach number requires a moderately heated pressureized air supply (figure 2) - even for cold testing at ISA norm conditions of 15 °C:

$$T_{t,j}\left(U_j\right) := T_j + \frac{U_j^2}{2c_p} \tag{4}$$

$$\text{or: } T_{t,j}\left(M_j\right) := T_j \cdot \left(1 + \frac{\gamma - 1}{2} M_j^2\right) \tag{5}$$

$$T_{t,comp.out} = T_{t,j} \text{ (if adiabatic)} \tag{6}$$

Since the compression of air comes with compression heat, a closely located compressor can use the

T0=Tj =	15 °C		required ISA heat	
Mj = Mac [-]	Ttj [°C]	Uj [m/s]	P* [kW] Ø50mm	P* [kW] Ø100mm
0.5	29.4	170.1	12	47
0.6	35.7	204.2	20	82
0.7	43.2	238.2	33	130
0.8	51.9	272.2	49	194
0.9	61.7	306.3	69	276
1	72.6	340.3	95	379

*assume that compression heat is gone and flow must be heated from 15°C to target Ttj

Figure 2: required heating for cold jets to achieve ISA norm conditions

compression heat in order to (partially) deliver the wanted total temperature. Contrary to this, the compression heat of a remotely located compressor with non-insulated supply pipes will likely be "lost along the way", i.e. transferred from the jet to the pipe material and ambient.

For simplicity reasons, it is assumed that a certain jet Mach number M_j is to be tested. The (jet) Mach number definition helps to identify the corresponding ISA jet velocity U_j:

$$U_j := M_j \cdot a_j \tag{7}$$

$$U_j = M_j \cdot \sqrt{\gamma R T_j} \tag{8}$$

where γ is the adiabatic index (for cold flow of air $\gamma(T < 400\,\mathrm{K}) = 1.4$) and R is the specific gas constant (for air: $R = 287.058\,\mathrm{J/kgK}$).

The acoustic Mach number M_{ac} is defined as the ratio between jet velocity U_j and the speed of sound a_0 of the medium within the acoustic chamber. Hence, this property is defined wrt. the propagation of noise rather than the formation or origination of the noise source.

$$M_{ac} := \frac{U_j}{a_0} \tag{9}$$

$$M_{ac} = \frac{U_j}{\sqrt{\gamma R T_0}} \tag{10}$$

Some test facilities measure the acoustic room temperature T_0 within their wind tunnel ("flight nozzle") module and the jet velocity in an pressureized air ("jet nozzle") module, which is only used optionally when jet air is needed. Therefore, some test facilities do not show the acoustic Mach number in their control board as an in-situ control parameter. The property can be calculated post measurement, but is often not available in-situ.

Jet Mach number and acoustic Mach number are linked by the temperature ratio between static jet temperature T_j and acoustic room temperature T_0 (compare equations 8 and 10):

$$M_{ac} = M_j \sqrt{\frac{T_j}{T_0}} \tag{11}$$

For isothermal conditions (this includes the ISA norm definition) the acoustic and jet Mach number are the same by definition.

IV. Jet noise defined by Ffowcs-Williams

Ffowcs-Williams[2] modified Lighthill's classical theory for an estimation of the acoustic intensity I of a jet issuing into a quiescent medium. The analogy (equation 12) has been stated and used by the Georgia Tech researchers around Massey, Ahuja and Gaeta[3] for jet noise scaling, albeit with a slightly different purpose than within this paper. For reason of simplicity, an overhead microphone $\theta = 90\,^\circ$ is assumed. This allows to cross out the directivity term.

$$I \propto \frac{\rho_m^2 U_j^8}{\rho_0 a_0^5} \left(\frac{R_0}{D_j}\right)^{-2} \tag{12}$$

The density of the mixed jet ρ_m will here be determined by using the arithmetic mean (equation 14a) of static jet temperature T_j and acoustic room temperature T_0. This statement is also the boldest assumption of this paper and it is good to go two steps back and challenge it. The term ρ_m stems from the estimation of the turbulence stress tensor. The density ρ_m depends on the jet, but also on the quiescent medium. The temperature T_m adapts from internal flow properties (jet at nozzle exit/duct $T_m = T_j$) to the new external flow properties (of the acoustic room, $T_m = T_0$). The adjusting of the jet to ambient conditions is the reason why the mixed jet temperature may depend on directivity: For acoustic jet noise sources located at the nozzle exit, $T_m = T_j$ might be a better candidate than for broadband peak jet noise a few diameters downstream the jet axis, where a mix temperature (see equation 14) or even $T_m = T_0$ might be a better match. Questions about a hypothetic directivity of T_m will be addressed in the data example of section VII. If it is possible to test with isothermal conditions ($T_0 = T_j$), then such questions can be avoided.

With this side note, all the information is available for the correction of a spectrum to ISA conditions:

$$I \propto p_0 \frac{T_0 U_j^8}{R T_m^2 a_0^5} \left(\frac{R_0}{D_j}\right)^{-2} \tag{13}$$

Nevertheless, a transformation of the equations helps to gain some more theoretical understanding and helps answers the questions about any optimal operational parameter of the test facility. For mere simplicity in the following derivations, the geometric mean (equation 14b) of both temperatures is preferred over the use of the arithmetic one.

$$T_m = \begin{cases} 1/2 \cdot (T_0 + T_j) & \text{arithmetic mean (plots: dashed lines)} \\ \sqrt{T_0 \cdot T_j} & \text{geometric mean (plots: solid lines)} \end{cases} \tag{14}$$

With the definition of the ideal gas law $p = \rho R T_m$ and the speed of sound, the formula can be rewritten in terms of temperatures. Another assumption is a subsonic jet with the subsonic outlet condition $p_j = p_0$:

$$I \propto \frac{\gamma_j}{\gamma_j} \frac{U_j^2}{R T_j^{2/2}} \cdot U_j^1 \cdot \frac{U_j^5}{(\gamma_0 R T_0)^{5/2}} \cdot p_0 \left(\frac{R_0}{D_j}\right)^{-2} \tag{15}$$

$$I \propto \gamma_j M_j^2 \cdot U_j^1 \cdot M_{ac}^5 \cdot p_0 \left(\frac{R_0}{D_j}\right)^{-2} \tag{16}$$

Equation 16 is a crucial result. It states that jet noise scales with jet Mach number to the power of 2, jet velocity to the power of 1 and the acoustic Mach number to the power of 5. It is good to check this result for validity:

- The testing of same Mach number jets at different temperatures (hot and cold) results in a velocity scaling with exponent 6 instead of 8. The jet Mach number remains constant and cancels out whereas the jet velocity and acoustic Mach number vary depending on the temperature.

$$I(M_j = \text{const.}) \propto \gamma_j \cdot U_j^1 \cdot M_{ac}^5 \tag{17}$$

- The conditions for same speed jets at different Mach numbers can only be created for very high supersonic Mach numbers (e.g. $M_j \rightarrow 100$). This is likely only of theoretical interest. If the ambient temperature T_0 remains constant, equation 16 collapses to a scaling coefficient of 2. This does not fully align with theory which rather suggests a scaling coefficent of 3 for supersonic jet noise.

$$I(U_j = \text{const.}) \propto \gamma_j \cdot M_j^2 \tag{18}$$

When testing jet noise, a facility has to decide to go for one control parameter, either jet Mach number M_j, acoustic Mach number M_{ac} or jet velocity U_j. Therefore, the equations should be rewritten in terms of these physical properties. Microphone distance R_0 and jet diameter D_j and the adiabatic index for the acoustic room γ_0 (as $T_0 < 400\,\mathrm{K}$) will be treated as constants and therefore be neglected. The adiabatic index γ_j may not be constant when comparing very hot and very cold jets. The ambient pressure p_0 will also be assumed constant here, yet depends on daily weather conditions.

$$I \propto U_j^8 \cdot \frac{1}{T_j T_0^{5/2}} \tag{19}$$

$$I \propto M_j^8 \cdot \frac{T_j^3}{T_0^{5/2}} \cdot \gamma_j^4 \tag{20}$$

$$I \propto M_{ac}^8 \cdot \frac{T_0^{3/2}}{T_j} \tag{21}$$

The equations 19 to 21 show that jet noise scales with power 8 of any operational parameter, i.e. jet velocity, jet Mach number or acoustic Mach number, if the static temperatures in the jet and the acoustic room are equal (isothermal, $T_j = T_0$) and constant over the duration of the test campaign (e.g. ISA norm temperature of $T_{ISA} = 288.15\,\mathrm{K}$).

The SPL must be corrected according to the following formula:

$$\Delta SPL = 80 \lg\left(\frac{U_{j.meas}}{U_{j,target}}\right) - 10 \lg\left(\frac{T_j}{288.15\,\mathrm{K}}\right) - 25 \lg\left(\frac{T_0}{288.15\,\mathrm{K}}\right) \tag{22}$$

$$\Delta SPL = 80 \lg\left(\frac{M_{j.meas}}{M_{j,target}}\right) + 30 \lg\left(\frac{T_j}{288.15\,\mathrm{K}}\right) - 25 \lg\left(\frac{T_0}{288.15\,\mathrm{K}}\right) \tag{23}$$

$$\Delta SPL = 80 \lg\left(\frac{M_{ac.meas}}{M_{ac,target}}\right) - 10 \lg\left(\frac{T_j}{288.15\,\mathrm{K}}\right) + 15 \lg\left(\frac{T_0}{288.15\,\mathrm{K}}\right) \tag{24}$$

In the following two sections, two different test environments will be described which are named "Case 1" and "Case 2".

V. Case 1: constant total temperature of jet flow

The constant total temperature condition is relevant for test rigs where the total temperature of the test rig remains constant even though the (ISA) requirements for different target Mach number require an adjustment in total temperature. There are at least three relevant test cases:

1. Some jet noise test rigs are supplied with pressureized air by a remotely located compressor, but do not use any active tempering system. The compression heat is lost over the long supply pipe length and/or due to missing insulation. The total temperature of the jet then happens to depend on a temperature which is not related to the compressor temperature, e.g. the soil temperature (underground piping) or ambient outside or room temperature (supply pipes above ground) and hence, remains constant even if compressor conditions change.

2. A huge pressureized air reservoir or tank could be heated to a certain constant hot total temperature. A close distance to the rest room or good insulation would ensure nearly adiabatic conditions. Measurements would be conducted at the same total temperature even though operations are changing.

3. The slow response of some temperature control systems can be the reason why temperature requirements may be compromised for the sake of economic testing. Let us assume that an arbitrary temperature control system requires 30 minutes to fully adjust to a new temperature. For the benefit of quicker testing, test points with minimally different total jet temperatures may be tested at the same total temperature, even though this is not ideal.

The constant total temperature condition is very relevant for jet air supplies in test facilities like the Aeroacoustic Wind Tunnel Braunschweig (AWB), Niedergeschwindigkeitswindkanal Braunschweig (NWB), the DOAK test facility in Southampton or the cold operated SHJAR test rig at NASA Glenn.

The static jet temperature in equations 19 to 21 are replaced with the total jet temperature $T_{t,j}$ because this is the defining property.

$$I \propto U_j^8 \cdot \frac{1}{T_{t,j} - \frac{U_j^2}{2c_p}} \frac{1}{T_0^{5/2}} \tag{25}$$

$$I \propto M_j^8 \cdot \left(\frac{T_{t,j}}{1 + \frac{\gamma-1}{2} M_j^2} \right)^3 \frac{1}{T_0^{5/2}} \tag{26}$$

$$I \propto M_{ac}^8 \cdot \frac{1}{T_{t,j} - \frac{\gamma-1}{2} M_{ac}^2 T_0} T_0^{3/2} = M_{ac}^8 \cdot \frac{1}{\frac{T_{t,j}}{T_0} - \frac{\gamma-1}{2} M_{ac}^2} T_0^{5/2} \tag{27}$$

The equation set contains the same scaling as in equation 16. The total jet temperature can cause jet noise to scale with power 6 of jet velocity (see U_j terms in equation 25) or acoustic Mach number (see M_{ac} terms in equation 27) as well as power 2 of jet Mach number (see M_j terms in equation 26).

A neat illustration of the jet air supply system behavoir with constant total jet temperature is depicted in figure 3. The three different operational parameters are investigated within the AWB test environment ($T_{t,j} = 13\,°\text{C}$, $T_0 = 22\,°\text{C}$). Operating the test rig by jet velocity as test parameter produces the smallest error compared to ISA norm conditions. The testing of jet Mach numbers causes the largest deviations, close to 3 dB for $M_j = 1$.

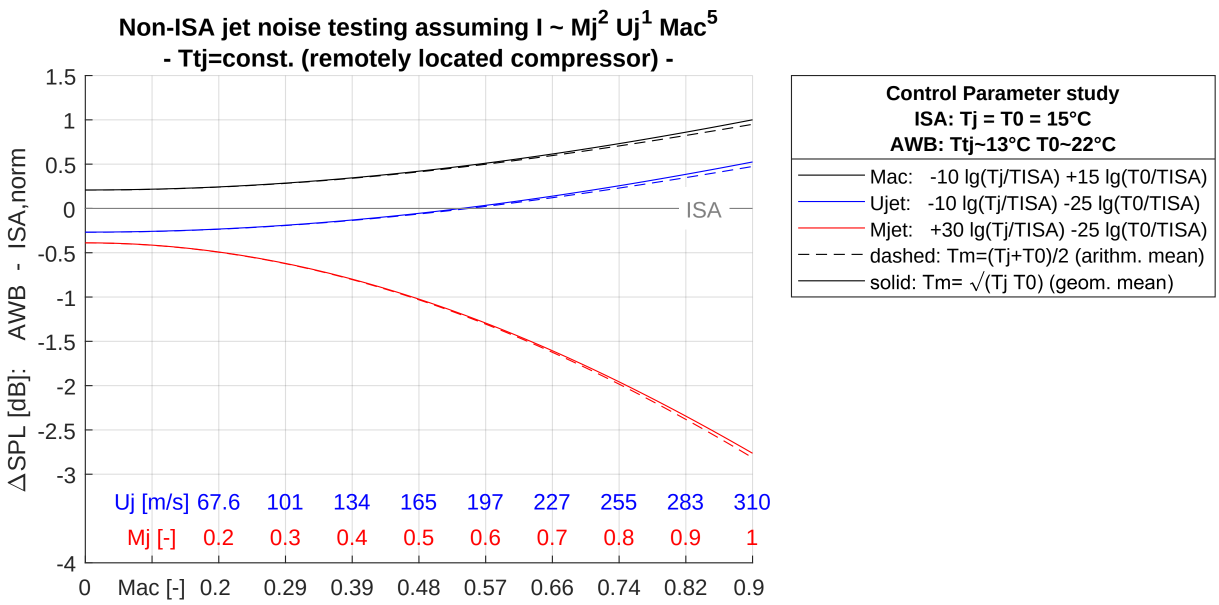

Figure 3: Operation parameter chart for typical AWB test rig conditions. solid lines = geometric mean, dashed lines = arithmetic mean

VI. Case 2: constant total temperature ratio

In some test rigs, the supplied jet air is passively heated, e.g. by preservation of the compression heat. The compression heat is a by-product of the compression process where ambient air is sucked in by a compressor and compressed to a target pressure. The good part about the compression is the favorable relation between compressed pressure and heat requirements for an operation: The higher the operation, the higher the compression heat. Short air supply pipes and good insulation help to preserve the heat.

Take for example the JExTRA test rig of DLR Berlin. A centrifugal compressor is closely located to the test section and heat losses can therefore be neglected. The compressor temperature increases with time (runaway temperatures) and quickly reaches a compressor outlet temperature above the required ISA total temperature.

It is nevertheless very useful and recommended to not allow the total temperature to settle for a high temperature which is way above the ISA target, but to test with runaway temperatures instead. The overall change in total temperature is only in the range of 1 K over a measurement time of 30 seconds. This is very small and can be dimensionally neglected (1 K/288 K = 0.3 %). The change of total temperature from low to high does support to measure low speed settings first and high speed settings thereafter.

The nature of a runaway temperature systems makes it appealing to use a different physical property than in case 1. Therefore, the ratio of the actual total temperature of the jet to the ISA target total temperature $TTR_{a/t}$ is defined. This ratio tells whether the jet air supply pipes is constantly heated enough or constantly heated too less in order to meet the ISA target.

$$TTR_{a/t} := \frac{T_{t,j}}{T_{t,ISA}} = \frac{T_{t,comp.in} + \frac{\dot{Q}_j}{\dot{m}_j c_{p,j}}}{T_{t,comp.in} + \frac{\dot{Q}_{ISA}}{\dot{m}_{ISA} c_{p,ISA}}} \tag{28}$$

$$\text{where } T_{t,ISA} = 288.15\,\text{K} \cdot \left(1 + \frac{\gamma - 1}{2} M_{ISA}^2\right) \tag{29}$$

Total temperatures were chosen because a typical (low speed) temperature measurement in the supply pipe with fixed-to-wall thermocouples measures the total temperature. Hence, the static jet temperature T_j can be expressed by the actual-to-target total temperature ratio (2) and other missing terms (1, 3, T_{ISA}) to balance the equation:

$$T_j = \underbrace{\frac{T_j}{T_{t,j}}}_{1} \underbrace{\frac{T_{t,j}}{T_{t,ISA}}}_{2} \underbrace{\frac{T_{t,ISA}}{T_{ISA}}}_{3} \underbrace{T_{ISA}}_{288.15\text{K}} \tag{30}$$

Jet Mach number operation. Assume that the control strategy is to measure a certain target Mach number $M_{ISA} = M_j$, but the temperature ratio of the compressor is only slightly higher or lower than target (assume $\gamma(T_j) \approx$ const.). Then, terms 1 and 3 cancel out against each other. Term 2 is the before defined total temperature ratio $TTR_{a/t}$ which determines whether there is too much or too less heat and $T_{ISA} = 288.15\,\text{K}$.

$$T_j(M_j, TTR_{a/t}) = \underbrace{\frac{1 + \frac{\gamma-1}{2} M_{ISA}^2}{1 + \frac{\gamma-1}{2} M_j^2}}_{=1} \cdot TTR_{a/t} \cdot 288.15\,\text{K} \tag{31}$$

$$T_j(M_j, TTR_{a/t}) = TTR_{a/t} \cdot 288.15\,\text{K} \tag{32}$$

Jet velocity operation. The convenient cancellation of terms 1 against term 3 in equation 31 does not work out as nicely in equation 33 when inserting the velocity definition into equation 30:

$$T_j(U_j, TTR_{a/t}) = \underbrace{\frac{1 + \frac{U_{ISA}^2}{2 c_p T_{ISA}}}{1 + \frac{U_j^2}{2 c_p T_j}}}_{=f(T_j)} \cdot TTR_{a/t} \cdot 288.15\,\text{K} \tag{33}$$

Nevertheless, the static jet temperature can be expressed with the velocity U_j and the total temperature ratio (equation 34).

Acoustic Mach number operation. Similar to this expression, it is also possible to express the jet temperature T_j with help of the acoustic Mach number M_{ac} and the room temperature T_0. To proof check the equations 32, 34 and 35, the total temperature ratio is set to unity (heat requirement is exactly met). This causes the different terms to collapse in order that the static jet temperature equals the ISA norm temperature.

$$T_j(U_j, TTR_{a/t}) = TTR_{a/t} \cdot T_{ISA} + \frac{U_j^2}{2 c_p} \cdot \left(TTR_{a/t} - 1\right) \tag{34}$$

$$T_j(M_{ac}, TTR_{a/t}, T_0) = TTR_{a/t} \cdot T_{ISA} + \frac{\gamma - 1}{2} T_0 M_{ac}^2 \cdot \left(TTR_{a/t} - 1\right) \tag{35}$$

$$= \begin{cases} TTR_{a/t} \cdot T_{ISA} & \text{no thrust} \\ TTR_{a/t} \cdot T_{ISA} \cdot \left(1 + \frac{\gamma-1}{2}\left(TTR_{a/t} - 1\right)\right) & \text{sonic jet} \end{cases} \tag{36}$$

The flow property analogies can be rewritten in terms of the total temperature ratio. T_{ISA} is a constant which can also be left out. The proportionality for jet velocity and acoustic Mach number include a small factor which depends on jet nozzle operation.

$$I \propto U_j^8 \cdot \frac{1}{TTR_{a/t} \cdot T_{ISA} T_0^{5/2}} \cdot \underbrace{\frac{1}{1+\frac{\gamma-1}{2} M_j^2 \left(TTR_{a/t}-1\right)}}_{\text{ops factor: } M_j=0\ldots1} \tag{37}$$

$$I \propto M_j^8 \cdot \frac{\left(TTR_{a/t} \cdot T_{ISA}\right)^3}{T_0^{5/2}} \tag{38}$$

$$I \propto M_{ac}^8 \cdot \frac{T_0^{3/2}}{TTR_{a/t} \cdot T_{ISA}} \cdot \underbrace{\frac{1}{1+\frac{\gamma-1}{2} M_j^2 \left(TTR_{a/t}-1\right)}}_{\text{ops factor: } M_j=0\ldots1} \tag{39}$$

The operations factor can be neglected for showing the sensitivies in a general/qualitative manner, but in order to be precise, the dependency on operations is included in figure 4. The sensitivity maps (figure 4)

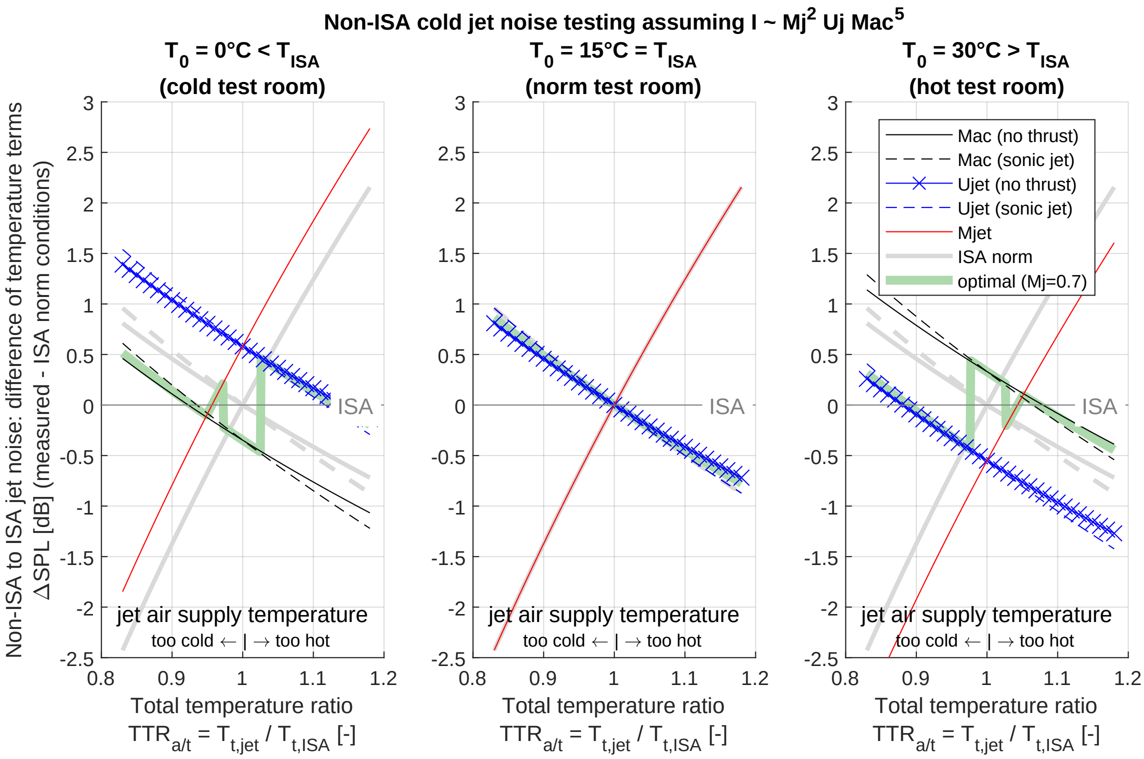

Figure 4: Sensitivity map of operational test parameters (according to equations 37 to 39) for measurement of ISA jet noise for various jet temperatures and three different acoustic test room temperatures.

provide SPL corrections of temperature terms assuming that the intended operational property was exactly measured. Let us focus on the norm test room (center diagram) first:

- One general result is that jet Mach number operations do positively correlate with the jet temperature. The hotter the jet, the higher the measured jet noise (see the slope of red curve).
- Contrary to this, jet velocity as well as acoustic Mach number operations do negatively correlate with the jet temperature. The hotter the jet, the lower the measured jet noise (see the slope of blue and black curve).
- The sensitivity of the two operations, jet velocity and acoustic Mach number, is the same in the norm test room.
- The slope of jet Mach number operations is steeper than the slope of the other two operational parameters. Hence, in an AWB-like wind tunnel with constant total jet temperature ($TTR = 0.8\ldots1$), the testing of jet velocities produces less correction (in terms of absolute ΔSPL) of jet noise than jet Mach number operations.

- The operational parameter (jet velocity, jet Mach number or acoustic Mach number) does not matter as long as the jet is heated to the correct temperature.

A test room which is cooler than norm temperature (figure 4, left) may not be a very common scenario. This could reflect the start conditions of a (winter) test day. However, especially a closed room is often heated by the warm-up of system components. The following sensitivities are observed:

- In comparison to the norm test room (gray) at same total temperature ratios, the jet noise corrections are larger for jet velocity and jet Mach number operations and smaller for acoustic Mach number operations (compare gray curves to blue, red and black).
- Jet air which is constantly too cold (including the majority of unheated case 1 rigs) should be operated with the acoustic Mach number in order to produce a small measurement error compared to ISA norm conditions.
- Jet air which is constantly too hot should be tested with jet velocity as an operational parameter. However, in practical terms it is likely that a hot static jet can heat a closed test room comparably fast. However, if flight jets are to be measured and the flight nozzle is constantly colder than ISA conditions, this case might have its real life application.
- Test rigs which prefer to operate on jet Mach number (e.g. because of runaway temperatures), should try to make fast measurements at cooler than ISA jet air supply total temperatures (here: $TTR_{a/t} \approx 95\,\%$).

A test room which is hotter than norm temperature (figure 4, right) is rather common. Closed test rooms typically heat up and if there is no temperature control or break for airing the room, those temperatures can quickly rise. The following sensitivities are observed:

- In comparison to the norm test room (gray) at same total temperature ratios, the jet noise corrections are smaller for jet velocity and jet Mach number operations and larger for acoustic Mach number operations.
- Jet air which is constantly too cold (including the majority of unheated case 1 rigs) should be operated with jet velocities in order to produce only a small measurement error compared to ISA norm. However, a static cold jet should also start to cool the warm closed room. If flight operations are to measured and the flight nozzle is warmer than ISA temperature, this case could also be applicable.
- Jet air which is too hot should be tested with the acoustic Mach number as an operational parameter.
- In the JExTRA test facility, the operational parameter is the jet Mach number. The test room temperatures of $T_0 = 30\,^\circ$ are representative for the measurement campaign in the summer of 2021. The error compared to ISA jet noise is comparably low as long as the jet is just slightly overheated ($TTR_{a/t} \approx 105\,\%$).

Some facilities have the ability to control jet temperatures to some extent. Their operation aim is to produce an isothermal (static) temperature profile, i.e. $T_0 = T_j$, even if this cannot be done at 288.15 K. Isothermal conditions are achieved when jet Mach number (red) and acoustic Mach number (black) match. The intersection points of both curves for the cold and hot test room in figure 4 indicate remarkable low correction terms of $\Delta SPL(T_0 = (288.15 \pm 15)\,\mathrm{K}) = \pm 0.1\,\mathrm{dB}$.

VII. Data example for the three operational parameters

In this data example, static ISA jet noise for $M_{ISA} = 0.605 (U_{ISA} = 205.9\,\mathrm{m/s})$ was measured at the JExTRA facility of DLR Berlin. The test room is too warm $T_0 \approx 29\,^\circ\mathrm{C}$ and so is the jet ($T_j \approx 27\,^\circ\mathrm{C}$ or $TTR_{a/t} \approx 1.04$). The microphones are located in the acoustic mid-field at $R/D = 11.46 Dj$.

The test point 445 operates at the same jet Mach number ($M_j \approx 0.605$), test point 479 operates at the same jet speed ($U_j = 205.9\,\mathrm{m/s}$) and test point 481 at the same acoustic Mach number ($M_{ac} \approx 0.605$) as the ISA test target.

Figure 5 lists the different test rig operational parameters. The ISA target operations can be compared to the actual test points. The bottom half of the table evaluates how well the measurement is expected to fit the spectrum according to equation 12.

Test rig operations			ISA (target)	Mj=const 445	Uj=const 479	Mac=const 481	Comment
Total temp. Ratio: actual/ISA	TTR_a/t	[-]	**1**	1.043	1.034	1.051	
Total jet temperature	Ttj	[K]	**309.2**	322.5	319.7	324.9	*jet too warm*
Static jet temperature	Tj	[K]	**288.15**	300.5	298.6	302.8	
Static jet temp. (arithm mean)	Tm	[K]	**288.15**	301.0	300.1	302.5	
Test room temperature	T0	[K]	**288.15**	301.5	301.6	302.2	*room too warm*
Test room pressure	P0	[hPa]	**1013**	1015	1015	1015	
Jet Speed	Uj	[m/s]	**205.9**	210.1	205.8	211.1	
Jet Mach number	Mj	[-]	**0.605**	0.605	0.594	0.605	
Acoustic Mach number	Mac	[-]	**0.605**	0.604	0.591	0.606	
Differences to ISA (according to derivations)							
SPL equivalent Mach number	M_eq	[-]	**0.605**	0.606	0.594	0.608	
SPL difference to ISA (log)	ΔSPL_a/t	[dB]	**0**	+0.04	-0.66	+0.15	
SPL difference to ISA (percent)		[-]	**0%**	+1%	-14%	+3%	
Sr difference to ISA (log)	10lg(Sr_a/t)	[dB]	**0**	-0.09	+0.00	-0.11	
Sr difference to ISA (percent)	Sr_a/t - 1	[-]	**0%**	-2%	+0%	-2%	

Figure 5: Test rig operations for ISA target and JExTRA test points

The shear amount of test parameters may be overwhelming. However, it is possible to calculate the theoretical/or equivalent Mach number for which the measured SPL value does not need any correction relative to the expected ISA-SPL value ($\Delta SPL = 0$). The formula chosen is similar to equation 23:

$$M_{eq} = \left[M_{j,meas}^8 \cdot \left(\frac{T_{j,meas}}{288.15\,\mathrm{K}} \right)^3 \left(\frac{288.15\,\mathrm{K}}{T_{0,meas}} \right)^{5/2} \left(\frac{p_{0,meas}}{1013\,\mathrm{hPa}} \right) \right]^{1/8} \tag{40}$$

The equivalent Mach number is a fine property, since it shows immediately that testpoint 445 is closest to target and thus defines the SPL of the datapoint. It is possible to also calculate the equivalent jet velocity. However, it is crucial to be very careful when stating this property in the data. Jet noise data files often contain a data table with frequency and SPL information as well as a header with operational data. While the uncorrected spectrum corresponds by definition to its equivalent jet velocity (or Mach number), the frequency information still depends on the measured jet velocity. Mix-ups between the two velocities should be avoided, e.g. by either not stating the equivalent jet velocity or clearly explaining the property.

A graphical solution to find the best operating condition is depicted by a small modification of the sensitivity map (figure 4, right) to the test conditions (T_0 and M_{ISA}). At $TTR_{a/t} \approx 1.04$, the expectation is that jet noise is highest for acoustic Mach number and jet Mach number and lowest for jet velocity operation. The correction term is low for jet Mach number and rather high for jet velocity operation.

Overhead position The question is how well the theory compares to the measured data. Spectra at the overhead position are displayed in figure 7. The diagram shows narrowband and third-octave band data. Since especially the frequencies are normed[4] according to equation 41, they collapse nicely on top of each other.

$$\Delta f_{sc} = \begin{cases} \Delta f = \text{const.} & \text{narrowband} \\ \frac{2^{1/3}-1}{\sqrt{2^{1/3}}} f_m = 0.2316 f_m & \text{third-octave band} \end{cases} \tag{41}$$

The Strouhal number range was chosen to feature the jet noise peak. On the right, the correction term for slightly different operations, temperatures and pressures is applied. The curves for all operations collapse very well. Since the "spectral noise", i.e. the deviation of SPL around its mean spectrum, is in the range of $\pm 0.25\,\mathrm{dB}$, it is not very easy to distinguish scaling deviations from spectral noise below the $0.25\,\mathrm{dB}$.

The third-octave band data indicates that jet velocity and jet Mach number collapse better than acoustic Mach number. It seems that the correction for the acoustic Mach number operation is a bit too high.

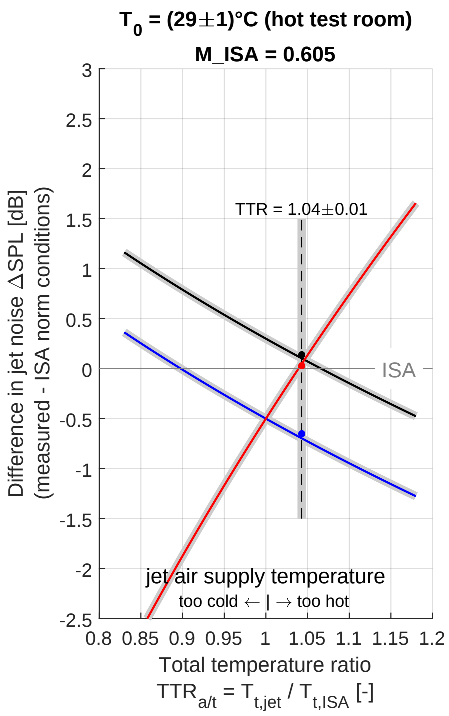

Figure 6: Comparison of the three operations (red: jet Mach number, blue: jet velocity, black: acoustic Mach number) in the TTR-Diagram.

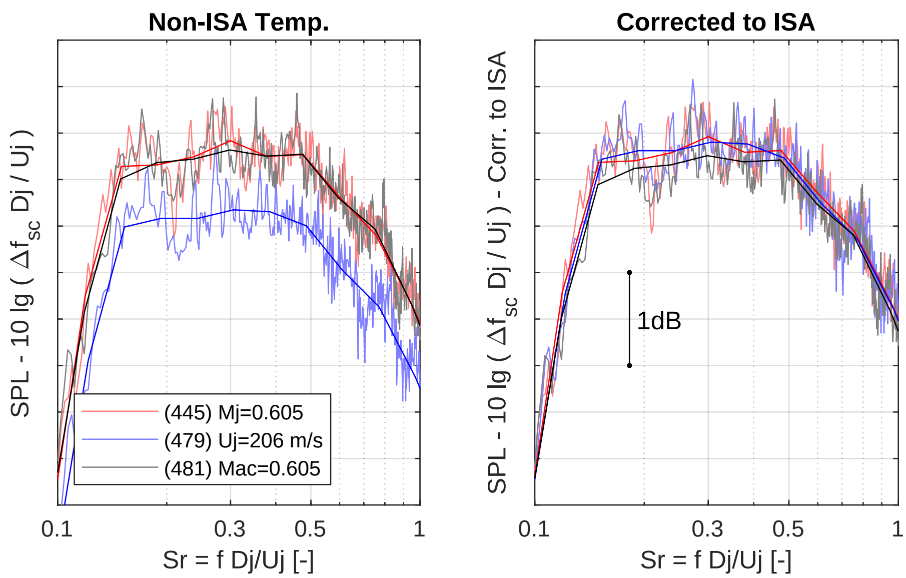

Figure 7: Normed spectrum of $M_{ISA} = 0.605$ ($205.9\,\mathrm{m/s}$) at $\theta = 90\,°$ (overhead position). Narrowband (faded color) and third-octave band data (bright color).

Rearward arc position The correction has also been tested on a rearward arc microphone (figure 8), i.e. downstream the engine outlet. The rearward arc microphones pick up the jet noise peak which is generated in the aerodynamic far-field of the jet. The figure shows that the Mach number test points collapse very well whereas there is an offset of $\approx -0.3\,\mathrm{dB}$ for the jet velocity test point.

A plausible reason behind this offset is that Strouhal number and SPL need a correction term for the directivity. Unfortunately, this term has been initially excluded within the paper (see equation 12).

The difficulty is that there are different terms which account for directivity effects: the simple or modified Doppler shift as well as directivity factor (see Viswanathan[5]). Let us shortly demonstrate that the directivity terms should help to close the gap between the normed spectra w/o directivity term. For reason of simplicity, the simple Doppler shift (term A in equation 42) is evaluated. The convective Mach number M_c is here interpreted as 65 % of the jet velocity relative to the ambient speed of sound. In other words, the convective Mach number is 65 % of the acoustic Mach number M_{ac}.

$$I \propto \ldots \cdot \underbrace{\left[\frac{1}{|1 - M_c \cos\theta|}\right]^5}_{A} \tag{42}$$

The simple Doppler shift correction requires no correction (compared to ISA target) for the acoustic Mach number test point (0 dB) and the jet Mach number test point (0.01 dB) wrt. the simple Doppler shift. Contrary to this, the jet speed test point is corrected by 0.23 dB due to the simple Doppler shift (dashed line in figure 8).

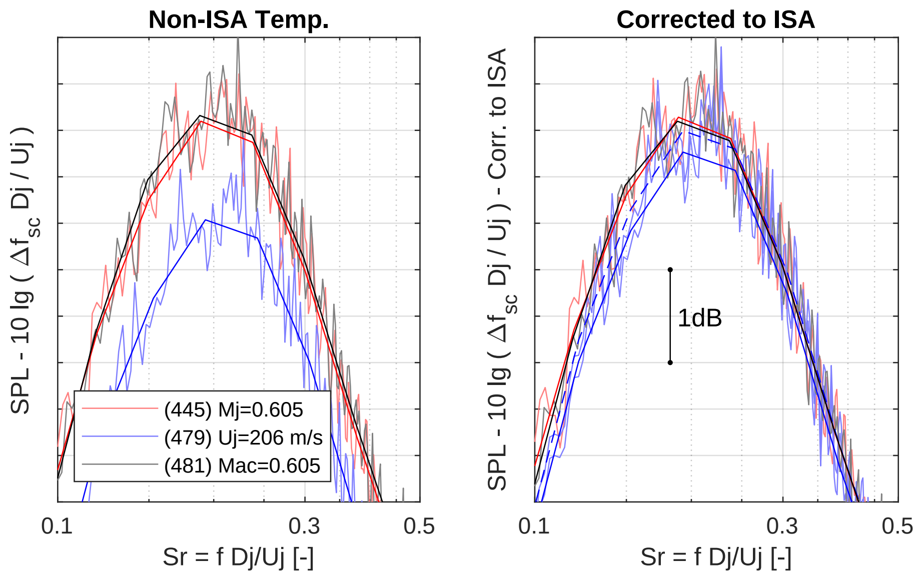

Figure 8: Normed spectrum of $M_{ISA} = 0.605$ ($205.9\,\mathrm{m/s}$) at $\theta = 21°$ (rearward arc). Narrowband (faded color) and third-octave band data (bright color). The correction to ISA in SPL and Strouhal number either lacks the directivity term (solid lines) or includes only the simple Doppler shift (dashed line).

VIII. Results

The Ffowcs-Williams analogy for jet noise without temperature terms corresponds to $I \propto M_j^2 \cdot U_j^1 \cdot M_{ac}^5$. This defines the three candidates for test rig operations: jet Mach number M_j, jet velocity U_j, as well as acoustic Mach number M_{ac}.

Two temperature behaviors of jet supply air have been described: The constant total jet temperature system (Case 1) for unheated flow and remotely located compressors, as well as a constant total jet temperature ratio system (Case 2) for closely located compressors. Furthermore, the state of the acoustic test room can be described as colder or warmer than ISA temperature.

The temperature sensitivity of the Ffowcs-Williams analogy has been evaluted for an overhead position. Assuming that the test room is warmer than ISA, unheated (too cold) jet test rigs like AWB make small errors when using the jet velocity whereas slightly too hot jet test rigs like JExTRA are better of using jet Mach number or acoustic Mach number for the definition of their jet operations.

The latter case was demonstrated using a data example from JExTRA where the testrig was operated with either jet Mach number, jet velocity or acoustic Mach number. The corrected spectra fit very well to the prediction, i.e. the corrected spectra collapse within a band of 0.1 . . . 0.2 dB. The data example shows that the predicted best-performing operational parameter (here: the jet Mach number) needed almost no correction. Contrary to this, the temperature behavior of the JExTRA test rig requires greater corrections when operating on jet velocities. Since both, jet and test room, are warmer than ISA and jet velocity the same as ISA, the Mach number is lower than ISA.

An equation has been proposed to calculate the SPL-equivalent Mach number, i.e. the Mach number for which the correction to ISA norm conditions is zero. The question is whether the equivalent Mach number could be an alternative candidate for being the decisive test rig operations parameter. Since this property fits only the special case of isolated jet noise according to the Ffowcs-Williams analogy at the overhead position, this may not be the best idea, e.g. imagine a change of the velocity scaling law to power 5 or 6 when testing a dedicated installed engine build.

The derivations in this paper are not sufficient to fully capture directivity effects. State of the art terms for the simple or modified Doppler shift and directivity factor are rather complex wrt. deriving the sensitivity of operational parameters. Nevertheless, the data example for jet noise in the rear-ward arc has here shown how much more effort it takes to correct the use of an operational parameter which is less suited for the temperature behavior of the test rig.

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References

[1] Jente, C., "Acoustic Mach number, jet Mach number or jet speed - what is the optimal control property for jet noise experiments at AWB," *1st DJINN Conference: Industrially oriented jet noise reduction technologies*, EU H2020.

[2] Ffowcs Williams, J. E., "The noise from turbulence convected at high speed," *Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences*, Vol. 255, 1963, pp. 469–503.

[3] Massey, K., Ahuja, K., and Gaeta, R., "Noise Scaling for Unheated Low Aspect Ratio Rectangular Jets," *10th AIAA/CEAS Aeroacoustics Conference*, American Institute of Aeronautics and Astronautics, Reston, Virigina, 05102004.

[4] Gaeta, R. and Ahuja, K., "Subtle Differences in Jet Noise Scaling with Narrow Band Spectra Compared to 1/3-Octave Band," *9th AIAA/CEAS Aeroacoustics Conference and Exhibit*, Aeroacoustics Conferences, American Institute of Aeronautics and Astronautics, 2003.

[5] Viswanathan, K., "Mechanisms of Jet Noise Generation: Classical Theories and Recent Developments," *International Journal of Aeroacoustics*, Vol. 8, No. 4, 2009, pp. 355–407.