Enhancing Mathematical Fluency in the Elementary Classroom

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Introduction

The purpose of this research was to investigate mathematical fluency in a third grade classroom by using a game involving multiplication. We evaluated how students solved these problems and what strategies they used to better understand their level of knowledge and if they have reached mathematical fluency.

In order to determine the purpose of my research, I first asked myself what kinds of questions I would like answered about fluency. The first question I wanted to know was “are the strategies students choose very similar or are they inventive?” I felt this question would be answered through data analysis. The next two questions I had were “why do students choose the strategies they do?” and “are students more apt to work out the problem using arithmetic or use a strategy and which do they prefer?” These questions could be answered through personal interviews with the students after the lesson. The last question I had was “how can teachers expand students’ use of strategies?” This question would be answered through my review of literature.

Review of Literature

Math Fluency: An Introduction

Fluency, in the context of mathematics is defined as "the efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently" (Gojak, 2012). "Fluency rests on a well-built mathematical foundation with three parts" (Kansas College and Career Ready Standards, 2013). The three parts are accuracy, efficiency, and flexibility. Accuracy is the ability to solve the problem correctly and get the correct answer. Efficiency means that they are choosing to use an effective strategy for solving that problem. This may look different as students progress in their learning and learn to solve problems in new ways. How a kindergartener solves 5+2 and how a third grader solves that same problem may look completely different. A third grader will probably know more strategies and will know how to choose the one that is most efficient. The third part, flexibility, describes the ability for the student to solve problems in a variety of ways and be flexible in their strategies. In case they forget how to solve it a certain way, they can solve it using another strategy. They may also be able to switch strategies if they realize their initial one will not work for a particular problem. These parts include understanding what operations mean and how they relate to each other, an understanding of a large number of number relationships, and a thorough understanding of the base ten number system. Fluency is about much more than being fast and accurate. It demands much more of the students; it requires them to be critical thinkers of mathematics. Simply knowing the answer is not good enough. They must be able to understand deeper relationships to be able to explain why an answer is the way it is.

Teaching Fluency in Mathematics

One reason students do not choose the most efficient method (ex: counting on fingers) is because they do not have conceptual understanding of how to choose a method for solving a problem. They simply choose the only method they know how to use. Common Core State Standards fro Mathematics (CCSS-M) (NG…, 2010) challenges teachers to help students develop procedural fluency, which is the ability to choose a procedure and be able to switch methods if the first one is not working. “Students must know when, as opposed to just how, to use a procedure” (Kansas College and Career Ready Standards, 2013). Teachers can build this procedural fluency by demonstrating multiple solution strategies for students and allowing them to choose their own strategy based on the context of the problem. They should also be sure to build connections between previous learning and the new concepts to be taught. Accessing prior knowledge is a key component of student retention. One key thing to remember about procedural fluency is that it “is always AFTER conceptual understanding,” meaning that the student needs to learn the concept before they are expected to gain fluency (Kansas College and Career Ready Standards, 2013). A student cannot be procedurally fluent in a concept they do not have a good basis in. Teachers should make sure to provide students with many opportunities to work with, think about, and build understanding of numbers and their relations. “Teachers must create learning experience that produce opportunities for students to make sense of and organize number relations. [These learning experiences] should provide space for students’ attention to being accurate, to being flexible in their thinking, and to developing efficiency in their reasoning and processes" (Matney, 2014). Educators should strive to build the three parts of mathematical fluency (as mentioned in the introduction) in their students. Additionally, teachers should not stop when the student "gets" the concept. Fluency is more than just understanding the problem, so the teacher could do things such as provide "experiences for students to solve word problems of slightly higher difficulty [to promote] their development of clear models and explanations for ideas that they will become fluent with during the next year" (Matney, 2014). Fluency is an ongoing process. Most of the time, a student will not learn a concept and then become fluent in that concept the same year. The skills taught in one school year will me made solid the next school year. Students need time to think about and become comfortable with each topic. Teachers should avoid rushing through one topic just to move onto the next.

Teacher Strategies

Students learn in many different ways, so having a multitude of strategies to use in a classroom is essential to reach all learners. Some students may love participating in a class discussion and working in groups, while others may prefer to explore new topics on their own. Teaching things in multiple ways also allows students to develop deep connections and understanding within and across subjects. Developing computational fluency does not happen overnight. It takes continuous work on the teacher's part to get the student thinking about numbers in many ways.

One tool a teacher can use to provide students with opportunities to develop computational fluency is to implement a routine of number talks. These are "short, ongoing daily routine that provides students with meaningful ongoing practice with computation" (Math Perspectives). A teacher displays a math problem on the board and allows the students to think deeply about how to solve it, how it relates to other math problems, or the relationship between the numbers. The focus is not on the correct answer, but the thinking process. Students are free to talk and share their thinking with others. It is a collaborative process. The teacher honors all thinking, regardless of if its right or wrong. Making mistakes is part of the learning process. The learning is also primarily student led. Instead of the teacher telling the students how to solve a problem, the teacher is simply the recorder of information. The students talk through their thinking, and the teacher writes things on the board. The students learn from each other. (Parrish, 2014) Number talks are an easy way for students to gain a great deal of knowledge on virtually any math concept, and requires little preparation from the teacher.

A Conducive Classroom Environment

In order to successfully implement a learning tool such as number talks, teachers need to make sure that their classroom is conducive to learning and sharing ideas. Students need to be okay with sharing their answers and being wrong sometimes. Since fluency is not striving for just a right answer, students need to be able to share their thought process with their peers. This requires vulnerability on the students' part. Students may have opposing opinions, which is great for discussion, but students must understand this. "The classroom should have established norms and a classroom ecology that promotes safety in constructing and sharing viable arguments and critiquing other's thinking" (Matney, 2014). Using number talks in a math curriculum allows students to build on all three parts of math fluency. Classroom discussion is a vital part of conceptual understanding because getting to verbalize opinions, strategies, and solutions increases awareness and understanding.

References

Boaler, J. (2014). Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts. Youcubed at Stanford.

Gojak, L. (2012, November 1). Fluency: Simply Fast and Accurate? I Think Not!

Kansas State Department of Education. (2013). Fluency is More Than Mere Speed.

Matney, G. (2014). Early Mathematics Fluency with CCSSM. Teaching Children Mathematics, 21(1), 27-33.

Parrish, S. (2014). Number talks: Helping children build mental math and computation strategies, grades K-5. Sausalito, CA: Math Solutions.

Method

This project is both qualitative and quantitative in nature, so the way I conducted the research had to be able to collect both types of data. After going over the literature and discussing the topic of mathematical fluency with my mentor, Dr. Chepina Rumsey, we created a lesson plan. We designed a game that asked third graders to solve multiplication problems in unique ways. After an introduction where students were asked to think about how to solve multiplication problems with various strategies, the students would play a modified game of "war" using notecards that had multiplication problems on them. Students would take turns flipping over a card to reveal a multiplication problem. Whoever correctly answered the problem first got to keep the cards. When the students got to a card with a red star on it, they had to record some information on a worksheet. The worksheet asked for the students to show how they solved the problem and give the correct answer. We encouraged students to not use the algorithm, but instead show unique ways of solving these problems. After everyone had completed the game, we came back together as a class and had students share what types of strategies they used and discuss them as a whole class. While the students were playing the game, Chepina and I made note of students that we wanted to share and/or be interviewed. This allowed us to get a variety of responses.

The worksheets that the students completed contained the quantitative data. This was then analyzed to give me answers to some of my questions I had asked in the beginning. In order to collect qualitative data, we decided to conduct a few interviews after the lesson was over with individual students. These interviews consisted on asking the students further questions about what they wrote on their worksheet. Students expanded on their answers and gave us further insight into why they did what they did.

Hypothesis

When creating the multiplication problems that would be used on the game cards, Chepina and I made sure to have a variety of problems that we predicted would elicit a variety of strategies. For example, for the problem 17 x 4, the strategy we predicted the students would use (the anticipated strategy) was to decompose. This means that we thought the students would break apart the 17 into a 10 and a 7, then multiply 10 and 4 to get 40, multiply 7 and 4 to get 28, and then add those two numbers together to get a product of 68. We assigned an anticipated strategy to each of the multiplication problems encountered on the worksheets and made sure that each of the three problems on the worksheet had a different anticipated strategy. The following table shows the multiplication problems that appeared in the game (with a red star on the card) and on the worksheets along with their anticipated strategy.

Problem	Anticipated Strategy
6 x 15	double then multiply
5 x 9	step count
17 x 4	decompose
19 x 3	round up
4 x 16	double than multiply
6 x 7	close to double
17 x 3	decompose
6 x 10	step count

4 x 19	round up
50 x 4	step count
13 x 3	decompose
2 x 18	round up
10 x 4	step count
9 x 3	round up
5 x 6	double than multiply

Results

Problem	Anticipated Strategy (AS)	Used AS?	Actual Strategy Used
6 x 15	double than multiply	no	step count
		no	algorithm
		no	step count (with dots)
		no	algorithm
5 x 9	step count	yes?	groups
		no	algorithm
		yes?	step count (with dots)
		no	algorithm
17 x 4	decompose	no	groups
		no	algorithm
		no	step count (with dots)
		no	algorithm
19 x 3	round up	no	algorithm
		no	step count (with dots)
		no	what you know to what you don't
		no	step count
4 x 16	double than multiply	no	round up
		no	round up
		no	decompose (turned 16 into 5, 5, 1, then counted parts)
		no	algorithm
6 x 7	close to double	no	round up
		no	round up
		yes?	took 6 x 6 = 36 and then added 6 more
		no	break into groups of two sixes (12) and then add pairs
17 x 3	decompose	yes	
		no	step count & step count with dots
		yes	
		no	step count (with dots)

6 x 10	step count	yes	
		no	mental math
		yes	also did 30 + 30 = 60
		yes	
4 x 19	round up	yes	
		yes	
		no	step count? confused by strategies
		yes	
50 x 4	step count	no	algorithm
		no	algorithm
		yes	
		yes	
13 x 3	decompose	no	used a strategy incorrectly
		no	
		no	step count (with dots)
		no	step count of 3
2 x 18	round up	no	decompose (sort of , done incorrectly)
		no	decompose
		no	double 18
		no	algorithm
10 x 4 *	step count	yes	
		yes	with dots
9 x 3 *	round up	no	step count
		no	step count (with dots)
5 x 6 *	double than multiply	no	step count (with dots)
		no	step count (with dots)

* Set 5 was for struggling students

Discussion/Analysis

Looking at the overall data, students used the anticipated strategy to solve the multiplication problem only 28% of the time. 72% of the time, the students used either the algorithm or another strategy. This shows that my hypothesis (guessing what strategy the students would use) was only correct 28% of the time. Students used step count (with either dots or numbers) and the algorithm for most problems.

The step count strategy was by far the favorite strategy among the students. Step count was used on 43% of all problems, whether that strategy was anticipated or not. Step count was an anticipated strategy for 14 problems, and was used 9 of those times. This means that the anticipated strategy was used 64% of the time when the anticipated strategy was step count. This far exceeds the percentage of students that used the anticipated strategy when the anticipated strategy was any other strategy. The anticipated strategy of "double than multiply" was used 0% of the time, "decompose" was used 17% of the time, "round up" was used 21% of the time, and "close to double" was used 25% of the time. If you look at the results table, you can see that many of these strategies were used when they were not anticipated.

What can be learned from this data is that it is very hard to correctly guess what strategy students will use on a particular multiplication problem. Students are unique and see things in different ways. Just because one person looks at a particular problem and immediately thinks to round up, another student may tend to use the "close to double" strategy. This is why it is important to not require students to solve particular problems in particular ways. Doing this may hinder their ability to look at the problem and come up with a unique way of solving it that works for them. Students should be given creative liberty with solving problems, as long as they are solving them correctly.

Something else that was noticed in the data was a reliance on using the algorithm. We specifically asked the students to not use the algorithm to solve the problems on the worksheet, but students still used the algorithm for 20% of the time. Many students told me while working that they preferred this method because it was faster. This is probably my fault, because the game I created asked students to solve the problems as quickly as possible so that they would beat their partner and get to keep their card. The students had gotten accustomed to the idea of using the algorithm as a quick way of solving problems, and they were not going to risk taking extra time and/or not getting the problem right when they are in a competition. If the game would have not been competitive, the students may have been able to slow down and truly think about a creative way to solve the problems. Adding the pressures of time meant that students had to think quickly and go back on methods that they know to be fast and reliable. Teachers should realize that this is an important thing to remember, no matter what subject or content. When you ask students to give you a quick answer (e.g. not giving them enough time to think about a question before asking for responses), they will not think as deeply about the problem. However, if you give the students adequate time to think about it without feeling like they need to give a fast answer (e.g. in order to win a game), they will be able to give you more creative, more accurate responses.

Consideration for Further Studies

This research was conducted using students at the end of their 3rd grade school year. I would be interested to know how the results would be different if this research was instead done at the beginning of 3rd grade, or even at any other grade level. I would love to be able to see the differences in math strategies that students use across all grade levels. With the introduction of Common Core, there is a larger emphasis on teaching students many strategies they can use to solve problems. What I would like to find out is if those strategies actually stick with the students, and if so, which ones. Is there a strong preference for one strategy over another? How many different strategies do students use? What happens when the standard algorithm is introduced much later in the curriculum?

Conclusions

There were three main things that I learned from my research that I feel are important reminders to teachers.

- Be very careful not to teach the standard algorithm too quickly because students will become heavily reliant on it. Give them a chance to explore number relationships first so that they will have a deeper knowledge of numbers.
- Step count is heavily relied on for all types of problems, yet is one of the most inefficient strategies.
- It is very hard to guess what strategy students will choose because every student thinks about the numbers in a different way.

With regards to teaching the standard algorithm, I believe that it is an essential part of mathematics, but needs careful consideration on how and when it should be taught. As soon as students learn the standard algorithm for any concept in mathematics, they become fixated on that one method of solving problems and lose their ability to think as deeply about numbers and number relationships. They no longer want to use any method other than the standard algorithm, even if it is not the most efficient. This makes sense, because wouldn't you be excited to learn a new way of solving a problem that is fast and accurate? We want our students to eventually get to this point, but we also don't want to limit the number of tools in their toolbox. If you teach the standard algorithm first, you are only giving them one tool or strategy to use in solving a particular problem. However, if you teach them multiple strategies and ways of thinking about it *before* you introduce the standard algorithm, they will have many tools in their toolbox that they can use to solve problems. For example, think about the problem 20 x 5. Yes, you could use the standard algorithm to solve this problem. You could write it vertically and solve. But is this the most efficient way to solve it? No. A student who is fluent in mathematics would realize that there are many, fast ways to think about solving this one. They could count by 20s. They could think about ways to get to 100. They could multiply the 5 and the 2 and then just add a 0. They could use many different strategies to think about this problem without actually writing anything down. A student that is able to solve this problem without writing a single thing is probably much more fluent than a student that relies on the standard algorithm for every problem. As educators, our goal should be to develop students that have strong number sense and great fluency in mathematics. We should not aim to have them memorize many algorithms without a strong sense of number relationships and why those algorithms actually work.

Appendix

- Lesson Plan
- Game Instructions
- Problems for Game Cards
- Recording Sheets
- Interview Questions
- PowerPoint Presentation from Kansas Association for Teachers of Mathematics (KATM) Conference 2015
- Handout from Kansas Association for Teachers of Mathematics (KATM) Conference 2015

Multiplication War Lesson Plan

Objective:

- Students will be aware of different strategies used in solving multiplication problems and reflect on why strategies are efficient.
- Students to aware of the connections between different multiplication strategies.

Standards:

3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

SMP 5: Use appropriate tools strategically

Engage:

The teacher will write the problem 5 x 7 = 35 on the board and ask the students to think about how they can prove that it is correct. After giving the students some time to think, the teacher will call on students to share their strategies for solving this problem. The teacher should emphasize creativity and let the students know that there are many ways that this problem could be proven.

Emphasize that we are excited to see a variety of thinking and strategies being used.

Guiding Questions:

- How do you know that the product of 5 and 7 is 35?
- How could you prove it to a new student?
- What strategy did you use to solve the problem?
- Why did you pick that strategy?

Explore:

The teacher will show the class an example game recording sheet and explain what the rules and expectations are for the activity. The teacher will model how to play the game with another student or a co-teacher.

Explain:

As a whole group, discuss strategies used. Ask students why those chose particular strategies.

Coming back together as a class, students will sit facing the whiteboard to discuss strategies used while playing the game. By picking a few of the problems students encountered while playing the game and writing them on the whiteboard, we can discuss specific strategies used in specific problems and have students share what they did. They can explain their strategies to the entire class and why they chose to use them.

Focus Problem: 4 x 19

Call on students to explain their strategies

What types of problems would this strategy work for?

Extension:

Have the students play a game of war by dividing the deck of cards in half and each placing 1 card face up. Whoever has the card with the highest product gets to keep it.

How To Play Multiplication War

Teacher Prep Work:

The students will be partnered up and each given a game recording sheet. Both partners should be given the same game recording sheet so that they will be working on the same problems at the same time. For groups of students that may be struggling with this concept, a recording sheet for struggling groups is also provided (set #5).

Making the Cards:

Use 3 x 5 notecards to create the game cards. Write one multiplication problem on each card.

Make 1 set of cards for each group of 2 students. Write each of these multiplication problems on separate note cards. Place each set of 15 cards into a Ziploc bag to keep them separate.

How to Play the Game:

- The partners will place the cards between the two of them
- One student will pull the card from the top and flip it face-up so that both students can see it
- The students will solve the multiplication problem on the card. Whoever shouts out the correct product first wins that round and gets to keep the card.
- For the next card, the other student will get to flip it face-up. They will alternate turns for each card.
- The students are to help each other figure out the correct answer if there is a discrepancy with the answers. If there is still confusion on what the correct answer is, they can ask the teacher.
- While playing, the students should be paying attention to what 3 problems are on their game recording sheet. When they see a card that has the same problem as one on their worksheet, they should pause the game and fill out that section on their recording sheet.
- The student should either draw a picture or explain in words how they solved the problem and then write the correct answer. Students should work on the same problem at the same time, but should do their own work as much as possible in an effort to see their own individual strategies used to solve the problem.
- Any single player cannot claim the cards, whose problems appear on the recording sheet. These cards will be set aside and will not be counted toward the student's score.
- Once both students have finished filling out that problem on the game recording sheet, they continue playing the game until the approach another card that has the same problem as one on their worksheet
- Once the entire deck has been played, students will count their cards. Whoever has the most cards wins the game.

Problems for Multiplication War Game Cards

Standard 15 Card Set

6 x 15
5 x 9
17 x 4
19 x 3
4 x 16
6 x 7
17 x 3
6 x 10
4 x 19
50 x 4
13 x 3
2 x 18
10 x 4
9 x 3
5 x 6

Low Level (Set 5)

10 x 4
9 x 3
20 x 5
5 x 6
18 x 2
4 x 5
9 x 5
10 x 6
3 x 8
7 x 5
6 x 4
14 x 2
11 x 2
19 x 2

Extra Set

8 x 9
19 x 2
11 x 5
14 x 4
10 x 6
3 x 15
18 x 4
3 x 12
4 x 9
10 x 9

Set #1 Name: ______________________________

Multiplication War

Problem	Explain in words and/or draw a picture	Answer
6 x 15 =		
5 x 9 =		
17 x 4 =		

Set #2 Name: ______________________________

Multiplication War

Problem	Explain in words and/or draw a picture	Answer
19 x 3 =		
4 x 16 =		
6 x 7 =		

Set #3 Name: ______________________________

Multiplication War

Problem	Explain in words and/or draw a picture	Answer
17 x 3 =		
6 x 10 =		
4 x 19 =		

Set #4 Name: ______________________________

Multiplication War

Problem	Explain in words and/or draw a picture	Answer
50 x 4 =		
13 x 3 =		
2 x 18 =		

Set #5

Name: ________________________________

Multiplication War

Problem	Explain in words and/or draw a picture	Answer
10 x 4 =		
9 x 3 =		
5 x 6 =		

Student: _________________ Problem: _________________

Multiplication War Interview Questions

How did you solve the problem?

Can you describe your strategy? What other numbers would this strategy work for?

Is there another way to solve it? Can you use manipulatives to show this strategy?

How is this strategy similar or different from this other strategy?

Compare 3 x 11 and 4 x 19, which is larger? 3 x 7 and 3 x 17?

Other Notes:

ENHANCING MATHEMATICAL FLUENCY IN THE ELEMENTARY CLASSROOM

Jessica Leichter & Chepina Rumsey

About Us

Jessica Leichter

- Senior at K-State
  - Graduating in December!
- Currently student teaching at Leawood Elementary School (USD 229) in 2nd grade
- Did research project for university honors program on mathematical fluency

Chepina Rumsey

- Assistant professor at K-State
- Teaches elementary mathematics methods courses and graduate level courses in mathematics education
- Taught 4th grade in Keene, New Hampshire

What do you see?

$9+7$

$5+8$

$7+4$

$6+8$

NON-ROUTINE PROBLEMS CHANGE THE FOCUS FROM ACCURACY → EFFICIENCY AND FLEXIBILITY

49+27

15+68

428+72

216+815

3 Components of Fluency

Fluency includes three ideas: efficiency, accuracy, and flexibility.

- Accuracy depends on several aspects of the problem-solving process, among them, careful recording, the knowledge of basic number combinations and other important number relationships, and concern for double-checking results.
- Efficiency implies that the students does not get bogged down in many steps or lose track of the logic of the strategy. An efficient strategy is one that the student can carry out easily, keeping track of sub-problems and making use of intermediate results to solve the problem.
- Flexibility requires the knowledge of more than one approach to solving a particular kind of problem. Students need to be flexible to be able to choose an appropriate strategy for the problem at hand and also to use one method to solve a problem and another method to double-check the results

Basic Fact Mastery

Figure 1. Phases of Basic Fact Mastery (Baroody, 2006)

Phase 1: Modeling and/or counting to find the answer

- Example: Solving 6 x 4 by drawing 6 groups of 4 dots and skip counting the dots .

Phase 2: Deriving answers using reasoning strategies based on known facts

- Example: Solving 6 x 4 by thinking 5 x 4 = 20 and adding one more group of 4

Phase 3: Mastery (efficient production of answers)

- Example: Knowing that 6 x 4 = 24.

Garza-Kling & Bay-Williams (in press). "Three Steps to Mastering Multiplication Facts." *Teaching Children Mathematics.*

Level 1: Direct Modeling

- Concrete manipulation of objects
- Start at 1 when counting objects
- Doesn't keep track of any number in their head

Level 2: Counting Strategies

- Students learn to keep one number constant and change the other to make more efficient.
- Students are able to keep track of sub-problems in order to help solve the problem

Level 3: Flexible Strategies, Number Facts

- Students use a procedure flexibly based on context
- Depending on operation and context, the student is able to manipulate one or both numbers in order to make an easier problem

*Traditional algorithms should be introduced as another strategy that students can include in their "toolbox" not as the end goal of developing strategies.

CCSS Standards: Fluency

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

CCSSM Expectations Related to Basic Facts

Grade 1 Standard 1.OA.6: "Add and subtract within 20, ***demonstrating fluency**** for addition and subtraction within 10. ***Use strategies*** such as counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums."

Grade 2 Standard 2.OA.2 : "***Fluently*** add and subtract within 20 using mental strategies (reference to 1.OA.6). By end of Grade 2, ***know from memory*** all sums of two one-digit numbers."

Grade 3 Standard 3.OA.7: "***Fluently*** multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, ***know from memory*** all products of two one-digit numbers.

*The use of the words "**fluency**" and "**know from memory**" illustrates that CCSSI recognizes the importance of allowing students to progress through Phase 2 before expecting automaticity with their facts (Phase 3).*

Ways To Build Fluency

- Number talks
- Games
- Apps
- Share multiple strategies
- Manipulatives
- Incorporate math facts into the daily routine
- Find any excuse to talk about number relationships!
- REPITITION!

Multiple strategies help students develop accuracy, flexibility, and efficiency

- Counting Strategies:
  - Counting on
  - Counting on from Larger (no based on order in problem)
- Derived Facts (making an easier problem):
  - Doubles (+1, -1, +2, -2)
  - Combinations of 10
  - Making 10 (decomposing numbers to find combinations of 10)
  - Associative property, Commutative Property
  - Adding ten
- Tools:
  - Number Lines,
  - 100's charts,
  - Base 10 blocks,
  - Place Value mats,
  - Cubes
  - 2 Color Counter
  - Cuisenaire Rods

Apps

- XtraMath.org
- Apps by Motion Math
  - Wings
- "Sushi Monster" by Scholastic
- "Math Flash Cards" by King's Apps
- Math Monsters- Bingo
- Math Slide

*Be critical of "math fluency" apps. Do they truly enforce accuracy, flexibility, and efficiency?

Games

- Shut the Box
- Cicada
- How Close to 100?
- Guess the Number
- Coin Trading with Base 10
- Strike It Out
- War

My Research

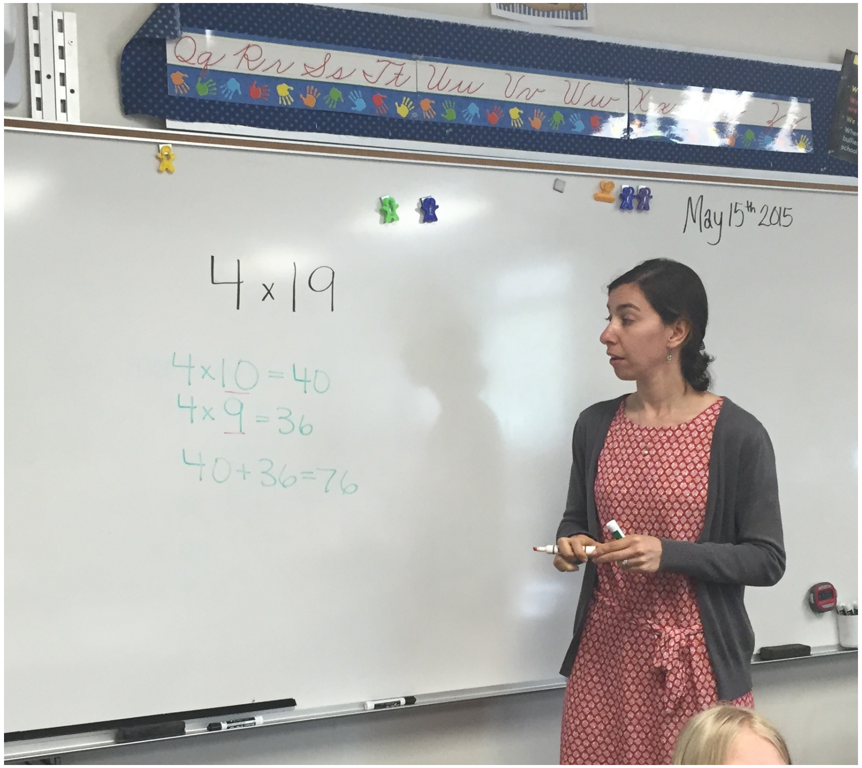
- The purpose of this research was to investigate mathematical fluency in a third grade classroom by using a game involving multiplication. We evaluated how students solved these problems and what strategies they used to better understand their level of knowledge and if they have reached mathematical fluency.

Questions I Had

- Are the strategies students choose very similar or are they inventive?
- Why do students choose the strategies they do?
- Are students more apt to work out a problem using arithmetic or use a strategy?
- How can teachers expand students' use of strategies?

Method

- Mini lesson on strategies
- Students played a modified game of "war" involving multiplication facts on notecards
- Certain cards had a star on them- these needed to be worked out on their recording sheets
- Data: observations, recording sheets, personal interviews



Set #1 Name: ____________________________

Multiplication War

Problem	Explain in words and/or draw a picture	Answer
6 x 15 =		
5 x 9 =		
17 x 4 =		

Hypothesis: Anticipated Strategies

- 6 x 15
  - double then multiply
- 5 x 9
  - step count
- 17 x 4
  - decompose
- 19 x 3
  - round up

Results

- Students used the anticipated strategy only 28% of the time
- Students used either the algorithm or another strategy 72% of the time
- Students mostly used step count (with either dots or numbers) or the algorithm for most problems
- Step count was used on 43% of all problems, whether that strategy was anticipated or not

Student: NT

Problem: 10 x 4

Multiplication War Interview Questions

How did you solve the problem?

Sees the zero, take it out, then add the zero back on the end

Can you describe your strategy? What other numbers would this strategy work for?

It will work for numbers ending in 0, like 10, 100.

When asked if 9x3 could use the same strategy, she said you could use dots, but not "zero strategy".

Is there another way to solve it? Can you use manipulatives to show this strategy?

Use dots and circles

How is this strategy similar or different from this other strategy?

Realized "taking off zero, putting it back" is more efficient

Compare 3 x 11 and 4 x 19, which is larger? 3 x 7 and 3 x 17?

reasoning based on calculation. (incorrect)

3x7 = 21

3x17 = 37

bc 3x1 is 3, then put the 7 on the end. (Misapplication of Bx10 strategy)

B is any number

Other Notes:

Student: AJ

Problem: 6 x 7

Multiplication War Interview Questions

How did you solve the problem?

6 x 6 = 36

6 is one away from 7, need 6 more so

36 + 6 = 42 so 7 x 6 = 42

Can you describe your strategy? What other numbers would this strategy work for?

I would also work for 11 x 12.

Is there another way to solve it? Can you use manipulatives to show this strategy?

6 x 4 = 24

6 x 5 = 30

~~6x7 = 42~~ 6 x 6 = 36

6 x 7 = 42

Using problems you know.

How is this strategy similar or different from this other strategy?

Compare 3 x 11 and 4 x 19, which is larger? 3 x 7 and 3 x 17?

bc both #s are larger "points"

bc

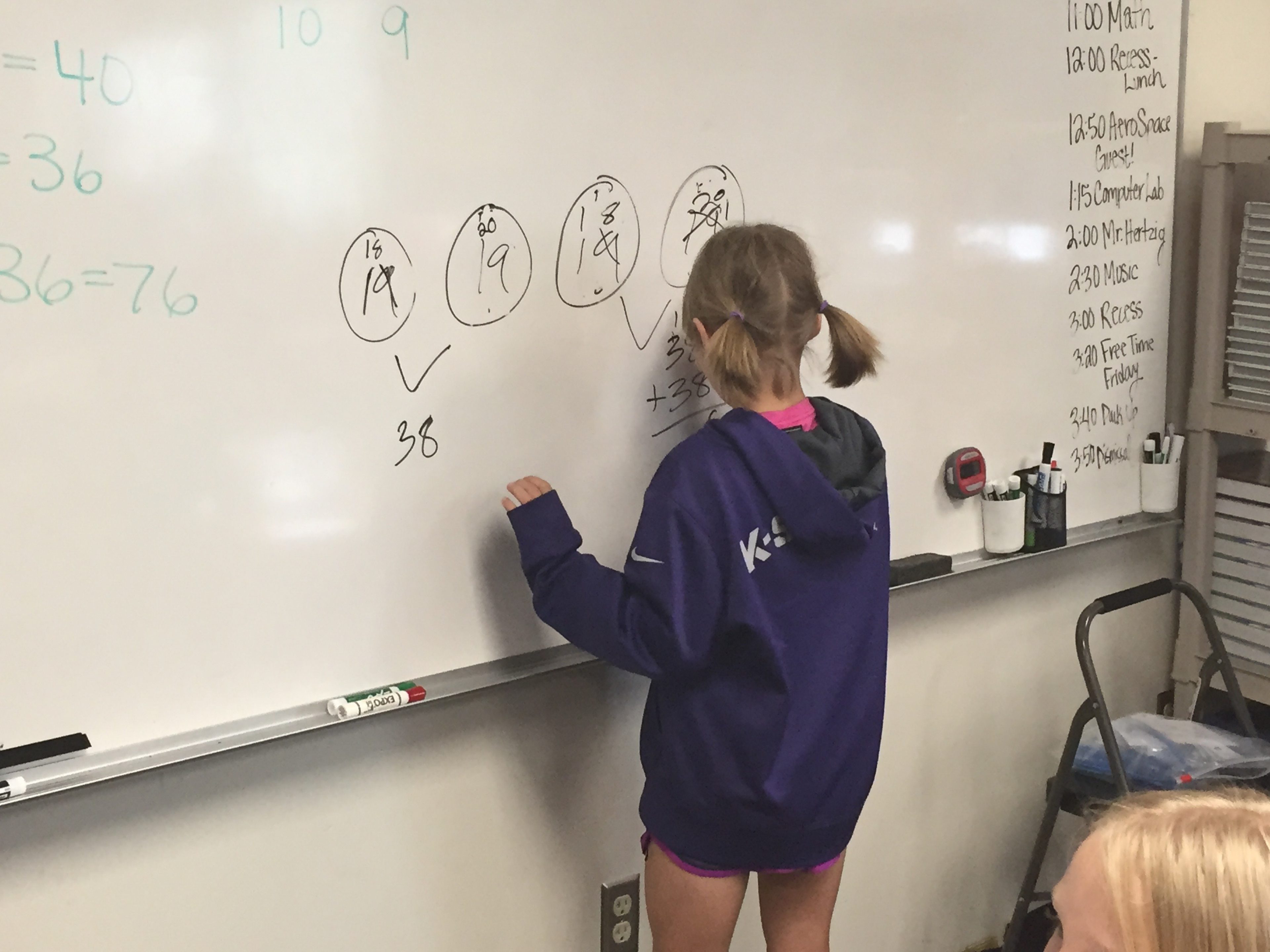
7 is 10 away from 17

Ignore 3

"if it was 3x7 and 2x17 I would have to solve"

One number is the same. If it was 2x17,

Other Notes:


10 9
=40
=36
36=76
38
11:00 Math
12:00 Recess-Lunch
12:50 AeroSpace Guest!
1:15 Computer Lab
2:00 Mr. Hertzig
2:30 Music
3:00 Recess
3:20 Free Time Friday

$\overset{18}{\cancel{19}}$ $\overset{20}{19}$ $\overset{18}{\cancel{19}}$ $\overset{20}{\cancel{19}}$

38

$$\begin{array}{r} {}^{1} \\ 38 \\ +38 \\ \hline 76 \end{array}$$

Discussion/Analysis

- It is very hard to guess what strategy students will use
- Reliance on using the algorithm, even when not <u>efficient</u>
- Focusing on speed hinders students' creativity
- Teaching multiple strategies gives students a toolbox to work with
  - In case they forget the algorithm
  - To use when it is most efficient/needed

Things To Think About

- What am I doing to assure that students are gaining fluency in all 3 areas (accuracy, flexibility, and efficiency)?
- What can I do in my classroom <u>each</u> day to build fluency?
- How fluent are my students right now? Where should I go from here?

THANK YOU FOR COMING!

Enhancing Mathematical Fluency in the Elementary Classroom

Presenters: Jessica Leichter & Chepina Rumsey
KATM Conference 2015

Ways to build your students' fluency

- Number talks
  - Formally/informally
- Games
  - Shut the Box
  - Cicada
  - How Close to 100?
  - War
  - Strike It Out
  - Guess the Number
  - Fouria/Coin Trading with Base 10
- Apps
  - Apps by Motion Math
  - "Sushi Monster" by Scholastic
  - Find more at: http://www.edutopia.org/blog/10-apps-for-math-fluency-monica-burns
- Share multiple strategies
  - Make 10, doubles, near doubles, associative/commutative property, counting on, etc.
- Tools/manipulatives
  - Number line, 100s charts, place value mats, base 10 blocks, cubes, 2 color counters, snap cubes, etc.
- Incorporate math facts into the daily routine
  - Call on students using math (Say 5+7 to call on student #12)
  - Have a math fact competition to see who gets to line up first
- Find any excuse to talk about number relationships
  - Share times when you see number relationships in real life (even if its not math time!)
  - Bring in examples from home of how you use math in everyday life
  - Encourage students to notice number relationships in their own lives

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What I Learned From My Research

- Be very careful not to teach the standard algorithm too quickly because students will become heavily reliant on it. Give them a chance to explore number relationships first so that they will have a deeper knowledge of numbers.
- Step count is heavily relied on as well for all types of problems, yet is one of the most inefficient strategies
- It is very hard to guess what strategy students will choose because every student thinks about the numbers in a different way

Articles on Fluency

Boaler, J. (2014). Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts. Youcubed at Stanford.

Gojak, L. (2012, November 1). Fluency: Simply Fast and Accurate? I Think Not!

Kansas State Department of Education. (2013). Fluency is More Than Mere Speed.

Matney, G. (2014). Early Mathematics Fluency with CCSSM. Teaching Children Mathematics, 21(1), 27-33.

Parrish, S. (2014). Number talks: Helping children build mental math and computation strategies, grades K-5. Sausalito, CA: Math Solutions.

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How Close to 100?

https://www.youcubed.org/task/how-to-close-100/

Number of Players: 2

Materials:

- two dice
- recording sheet (10x10 square grid)

Task Instructions:

- This game is played in partners. Two children share a blank 100 grid.
- The first partner rolls two number dice.
- The numbers that come up are the numbers the child uses to make an array on the 100 grid.
- They can put the array anywhere on the grid, but the goal is to fill up the grid to get it as full as possible.
- After the player draws the array on the grid, she writes in the number sentence that describes the grid
- The second player then rolls the dice, draws the number grid and records their number sentence.
- The game ends when both players have rolled the dice and cannot put any more arrays on the grid.
- How close to 100 can you get?
- Variation (arrays, multiplication, division, fractions, percentages, etc.)
- Each child can have their own number grid. Play moves forward to see who can get closest to 100.

How Close To 100?

Guess the Number

There are two players in this game:

- Player A – the person who *knows* the secret number and
- Player B – the person who is trying to *guess* the number

1. Player A thinks of a 2- or 3-digit number.
2. Player B guesses the number.
3. Player A tells Player B how many digits are correct and how many digits are in the correct place. These clues help Player B to make a new guess to try to figure out the mystery number!

Guess	Number of Digits Correct	Number of Places Correct

Rules

- Players alternate turns.
- On each turn, a player rolls the two dice and places that number of single cubes in the Level 1 column of his or her game board.
- Whenever possible, a player must trade 10 single cubes for one long rod and/or 10 long rods for one flat block.
- Blocks must be placed in the appropriately labeled column. No more than 10 blocks may be in any column at the end of a turn.
- The first player to get one flat block is the winner.

Level 100	Level 10	Level 1

Cicada Game

Cicada Fun Facts:

- Their predators is the "Cicada Wasp Killer" and are killed by a fungus as well.
- They have 5 eyes.
- Squirrels eat them.
- They have thin, straw like mouths to drink tree fluids.
- They emerge in cycles.
- Some broods only co-emerge every 221 years!

Materials:

- counters (two colors)
- Hundreds Chart

This is a two person game, one person is the **cicada**, and one person is the **predator**.

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- The players will be secretly deciding the number of years that will represent their cycle of emerging from underground.

Without telling each other, the players choose a number with these restrictions:

- Cicada chooses a number between 10-20
- Predator chooses a number between 2-10

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- On the hundreds chart, each player will mark the years that they will emerge, without switching the number! For example if the cicada chooses 10, then the cicada will put down a marker on 10, 20, 30, 40… etc.
- Then count how many times there are both a cicada and a predator on the same number/ year.

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WHO WINS??

- If there are less than 4 places of overlap, then the CICADA wins.
- If there are 4 or more places of overlap, then the PREDATOR wins.

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Strike it Out

Try this game: draw a number line from 1 to 20. The first player picks two numbers, crosses them out and circles either their sum or their difference. The second player crosses out the circled number and another number that's still left, and again circle the sum or the difference.

The winner is the person who stops their opponent from being able to move!