Relating Volatility and Jumps between two markets under Directional Change

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Shengnan Li

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Abstract

Directional change (DC) is a new concept in sampling financial market data. Instead of recording the transaction prices at fixed time intervals, as is done in time series, DC lets the data alone decide when to record a transaction. In DC, a data point is recorded when the price has risen or dropped against the current trend by a significant percentage, which is known as the threshold. The magnitude of the threshold is determined by the analyst. Previous studies on DC mainly focus on analysing single price sequences of one market. This thesis focuses on a new path; working on the DC comparative analysis between two markets. We propose a novel data-driven approach to combine the observed DC series of two markets into a single data sequence, which we call the DC combined sequence. This allows us to conduct a comparative analysis between two markets under DC. Based on this approach, we propose a novel indicator that measures the relative volatility between two markets. In addition, we define jumps under DC. Under this measure, we can pinpoint the size, direction, and quantity of DC jumps in a market. Lastly, under the DC comparative analysis, we build a new DC approach to identify co-jumps between two markets.

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Chapter 1. Introduction

1.1 Background

The research subject of this thesis is the development of relative volatility and co-jump measures relating two markets under the DC framework. We shall introduce the definition of the relative volatility, jumps, and co-jumps, and then proceed to study the behavior of the price changes through the empirical and data-driven approach of DC.

The concept of "directional change" (DC) was first published by Guillaume et al. (1997), where they presented an algorithm to sample the DC market data. DC is an alternative way to record the price movements compared with data sampling under a fixed time interval as in time series (TS); details of DC will be introduced in Chapter 2. In TS, transactions are sampled under a regular time interval. In contrast, DC samples the transactions based on the significant price changes, so the time stamp of the DC data is passively determined by the price changes. This passivity leads to the greater emphasis, in the data collected, of time periods where there are more significant events, which allows more potential analysis of these regions that would otherwise have to be actively studied through more extensive sampling in a traditional TS setting. One disadvantage of DC, considering the irregular time interval, is that TS data allows easy and direct real-time comparative analysis, e.g. observers can directly compare the returns of two markets at every 1-minute interval. Because the DC data sequence is not sampled at a regular timescale, there is no direct way to implement this type of analysis with DC data, i.e., it is not suitable without processing for real-time direct comparative analysis, especially in high-frequency data[1].

Under the DC comparative analysis, this research focuses on relating two markets in their volatility and co-jumps. In DC, researchers worked on measuring the volatility of the price movement in a single market (Tsang et al., (2017)). In Chapter 3, we work on a new path to measure the volatilities in two related markets (we named it relative volatility) through a data-driven approach. This study aims to develop an indicator to measure the relative volatility, which could be useful in real-time analysis.

In financial markets, jumps are events usually related to unexpected information; for example, surprising economic data or a major historical event (e.g., COVID-19) may lead to uncommon trading behaviors from the traders; and these trading transactions may cause price jumps. In time series, researchers consider jumps based on the asset pricing model; a jump is a different source of risk compared to the risk of continuous volatility (Lee and Mykland (2008)) such that the jump is identified through the module-based method (details see section 2.3.1). In DC, there is an absence of published references focusing on jumps. This research aims to establish the definition of a DC jump, which is then used to implement the back-testing of detecting jumps in DC; we will present the details of the DC jumps in Chapter 4.

Compared to past decades, along with more unexpected incidents (such as financial crises, natural disasters, geopolitical uncertainty, etc.), financial markets are more

[1] High-frequency data has been of interest since the late 1980s when the ability to collect data with the aid of new and improved technology arose (Dacorogna et al. (2001)).

fragile than before; the events of flash crashes across multiple assets have become more frequent, and this is a concern of researchers and related institutions[2]. The risk of co-jumps has been emphasized by the researchers in the TS analysis (Barndorff-Nielsen and Shephard (2006), Jacod and Todorov (2009), and Bollerslev et al. (2008)); for instance, observers have been focusing on identifying co-jumps and measuring their risks to the markets (for details see Section 2.3). Based on the work on DC jumps, we propose a definition of co-jumps in DC (named DC co-jumps) related to two markets and develop an indicator to detect co-jumps. This allows us to investigate the relationship between certain historical events and the presence of DC co-jumps (for details see Chapter 5).

1.2 Research motivations and objectives

The focus of this research is the analysis of the relationship between two markets. This research is conducted under the directional change (DC) framework (Tsang et al. (2017)); specifically, in terms of the relative volatility and co-jump characteristics (as introduced in Section 1.1). The motivation and ambitions arising from the usage and resulting extension of the DC framework include:

1. When performing comparative analysis, the classical time series method examines the volatility in two related markets by comparing the volatility of returns of the two markets during a pre-determined time interval. As previously discussed, in DC, researchers have focused on the measure of volatility in a single market. In this thesis we want to propose an approach to measure the volatilities in two related markets such

[2] For example, the Reserve Bank of Australia studied the flash crash (appreciation) of the Japanese Yen against several currencies (including the Australian dollar); *https://www.rba.gov.au/publications/smp/2019/feb/box-b-the-recent-japanese-yen-flash-event.html*

that the approach is better suited to be applied to high-frequency data or to the market micro-structure; in this research, the micro-structure indicates the tick data which the timescale is under the milliseconds. Would the approach allow better examination of the market micro-structure during periods of extreme fluctuation? In Chapter 3, we will present the DC approach of measuring relative volatility and discuss its benefits through historical studies.

2. In the financial markets, a jump is an event which is usually caused by unexpected information. Past empirical studies have proved that jumps have substantial impact on risk management and asset pricing (Liu et al. (2003) and Johannes (2004)). In time series analysis (TS), the jump is a different source of risk in addition to the risk of continuous volatility in the asset pricing model (details about jumps in TS will be introduced in Section 2.3). In DC, there is no research in the field of jumps. Therefore, this thesis aims to define jumps under DC; based on this, we detect jumps through a data-driven approach. This leads us to pose the question of how the presence of TS and DC jumps are related? We then investigate this question in the form of a comparison of the detected jumps from both approaches in Forex data. Fundamentally, detecting jumps in TS is different from detecting jumps in DC; in TS a jump is identified by a model-based method, while in DC, we detect jumps through a data-driven approach.

3. The identification of co-jumps has been a topic of interest over decades. Researchers emphasised that some macroeconomic news has a major impact on joint jumps spanning different assets. In portfolio risk management, it is important to accurately understand the resulting tail co-jumps and hedge against them. In TS, co-jump identification has been widely studied. Dungey and Hvozdyk (2012) introduced co-

jumps as the occurrence of contemporaneous discontinuities in two price series, although there is no formal test procedure for the exact timing of identifying them. In this thesis, we consider how to define DC co-jumps, and then develop an indicator to identify them. Based on this, we study whether jumps between two markets happen together. Is there a relationship between certain historical events and DC co-jumps? We will give the details about the study of DC co-jumps in Chapter 5. Fundamentally, the concept of co-jumps in DC is different to the concept in TS. Under the DC framework, considering two markets' price sequences, the co-jump is the event such that a jump in one market is followed by a jump in another market. A formal definition of the DC co-jump and how it may be detected is given in Chapter 5.

1.3 Thesis structure

The thesis structure is based on the objectives discussed in the previous section. It begins with an overview of the data analysis between time series and DC in Chapter 2, describing the previous studies researchers have done in financial market data analysis. It then explains the concept of DC, jumps and co-jumps. Chapter 3 introduces the methodology of mRV with the application of measuring mRV between $GBPUSD$ and $EURUSD$ during the Brexit referendum event. Chapter 4 introduce the definition of DC jump; we will show a comparative analysis of the detected DC jumps and TS jumps and discuss the relationship between the historical events and DC jumps. Chapter 5 introduce the approach of detecting DC co-jumps; we will study which historical events have more influence in causing DC co-jumps. In Chapter 6, we give a conclusion.

Chapter 2. Literature survey

This chapter will introduce the research literature concerning Directional Change (DC). Some past works about DC volatility will be discussed, which is our fundamental for establishing DC micro-market relative volatility (mRV). We first give a short overview of the early research in forex markets in time series analysis (in low frequency data) and then review the studies in high-frequence data under the analysis of TS and DC. We then introduce the concept of DC and the mechanism of DC data sampling. We shall give an overview about the DC volatility measurement for a single price sequence. After that, we will introduce the background of jumps and co-jumps in the financial markets.

2.1 Early research in forex markets in low frequency data

With the breakdown of the Bretton Woods system in 1971, researchers were attracted to the study of floating exchange rates using time series data (weekly and monthly), especially in the statistical analysis of the FX price changes. Boothe and Glassman (1987) stressed that the distribution of the exchange rate changes is essential for examining the uncertainty of the price movements (referred to as volatility). Early studies focused on finding a proper distribution to summarise the exchange rate changes in low-frequency data (i.e., weekly and daily). Westerfield (1977) indicated that the exchange rate changes were Paretian stable[3]. Rogalski and Vinson (1978) used the same data as Westerfield, and they suggested that the floating exchange rates were better described by the Student distribution. McFarland et al. (1982) examined the logarithmic

[3] Paretian stable refers to the fact that the exchange rates changes follow the stable distribution (Fama 1963).

daily exchange rates and concluded that the logarithmic daily exchange rates followed a stable Paretian distribution (also called a stable distribution). Boothe and Glassman (1987) proved that the exchange rate changes were not following a normal distribution and noted that the data was sharp leptokurtic and more fat-tailed than the normal distribution. Glassman (1987) compared the bid-ask spreads with the volatility and concluded that the size of the spread is related to the exchange rate volatility.

2.2 Overview of the data analysis between time series (TS) and DC

Since the late 1980s, high-frequency data has been of interest when the ability to collect data with the aid of new and improved technology arose (Dacorogna et al. (2001)). Nowadays, the advanced technology of big data collection and storage gives more opportunity for comprehensive analysis of financial data especially in high-frequency data. Frankel and Rose (1995) noted that while the theoretical coherence of the structural models at the time was attractive, their forecasting ability in practice was limited. Taylor (1995) concluded that further attempts to provide explanations of short-term exchange rate movements based solely on macroeconomic fundamentals may not prove successful which might account for the shift towards more purely financial models of exchange rate movements and heightened interest in market microstructure. Flood and Taylor (1996) demonstrated that macroeconomic models were not satisfactory in their goal of exchange rate determination. Beginning from 1990s, once the shortcomings of the macro approach became clear (Frankel and Rose, (1995); Taylor, (1995); Flood and Taylor (1996)) researchers have been more focused on the analysis of the behaviour of price movement through a micro-view; for instance Lyons (2001) who examined a lot of the assumptions made in the past from the perspective of the market microstructure. The microstructure approach examines the behaviour and interactions of individual agents in the market; for instance Lyons (1996) analysed microstructure data to determine how the informational content of trades was related to trading intensity, quote intensity and trading behaviour. The market participants have also become interested in high-frequency trading which has led to the development of many computational tools to assist in this sphere; big data being an important one (Wu et al. (2013); Han and Li, (2018)). The U.S. Securities Exchange Act Release No. 34-61358, 75 FR 3594, 3606 (January 21, 2010)[4] noted that estimates of high-frequency trading (HFT) typically exceed 50% of total volume in U.S.-listed equities and concluded that HFT is a dominant component of the current market structure and likely to affect all aspects of its performance. Observers can easily access the datasets in different time frames, e.g., the timescales of weekly, daily, and minutely sampling. In the past decade, the financial markets' tick data (the deal transactions) have become a popular research project as analysts and traders endeavour to discover any valuable information to capture and interpret more micro-behaviour. Hence, it is essential to have the ability to correctly understand and interpret market data. The classical method studies the price movements based on a time series. A time series is a series of data points sampled regularly in time order. One should pre-determine the size of the time interval (e.g., 30 minutes), and then record the data point at the end of each time interval. TS data is also used in technical analysis (Pring, (2014)): people have developed many tools to study the price movements like Relative Strength Index (RSI) (Wilder, (1978)), Bollinger Bands (Bollinger, (2001)), Moving Average Convergence Divergence (Appel, (1985)) and Stochastic Momentum Index (Blau, (1993)). However, as discussed in section 1.1, TS may not adequately summarise some situations when the markets have the significant volatility for short periods. This drove researchers to establish a new

[4] For details see the link: https://www.sec.gov/rules/concept/2010/34-61358fr.pdf

mechanism to record the market transactions. DC is a data driven approach for studying the price movements. It allows us to study the financial markets on a data-led timescale, which means DC lets the data dictate when to sample the data points. In other words, the timescale is passively defined by the significant price changes. Hence, DC data can give precise insights when monitoring the significant price movements especially in high-frequency data. Comparing with TS data, Chen and Tsang (2021) discussed how DC data is more suitable for tracking the market in order to detect important signals.

The concept of DC was first introduced by Guillaume et al in 1997 when they proposed a DC approach to examine the trend-following behaviour of the price changes. In fact, the technique of DC data sampling had been used to plot a Zig Zag pattern on the technical chart (Sklarew (1980)). The Zig Zag pattern is a useful technical chart pattern which is used to identify the price trends. Tsang (2010) formally defined the concept of directional change: the price movements are defined by a series of DC uptrends and downtrends (the formal DC definition will be introduced in Section 2). Glattfelder et al. (2011) illustrated the statistical discovery of 12 scale laws based on DC in high-frequency FX data. Tsang et al. (2015) defined the reversal points as extreme points, which are confirmed when the cumulative price changes reach a threshold. The threshold defines the size of what is termed to be a significant price change. Tsang et al. (2017) present a set of DC indicators capturing market information. Chronologically, DC records the extreme points, and this is then converted into a DC sequence. The previous studies in the DC method mainly focus on analysing single price sequences of one major market, which includes forecasting the price trend reversals, trading algorithm design, stock index trading strategies, using the DC scaling laws to build trading models, DC agent-based models, measuring regime changes under the DC approach, and technical pattern (‘Head and Shoulders Pattern’) recognition (Bakhach et al. (2016), Bakhach et al. (2018), Glolub et al. (2017), Ma et al. (2017), Dupuis and Olsen (2012), Petrov et al. (2018), Tsang and Chen (2018), Li and Tsang (2017)).

2.3 Directional Change

Directional change (DC) is a data-driven process of data sampling from financial markets. DC data is recorded as a series of alternate upward and downward trends. For any trend, the reversal point is confirmed when the price has changed beyond a threshold (a pre-determined price distance in terms of a percentage) from the last highest/lowest price of the current trend (for details see Appendix A in Chen and Tsang (2021)). The process of DC data sampling is based on the DC algorithm in equation (2.1) and (2.2) below (Tsang et al. (2017)). In time series analysis, the market data is collected on a pre-determined timescale. However, the mechanism of DC data sampling uses the significant price changes such that the market data is recorded when the price change has reached a certain threshold from the last peak/trough of the price. In practice, the analyst determines the threshold as a percentage. Hence, price changes are recorded as a series of alternate uptrends and downtrends, and the timestamp of each DC data point is determined dynamically. In an uptrend, a peak is determined as a DC extreme point (EP) when the current price P_t is lower than the last high price P_h by a fixed threshold (in percentage) θ:

$$P_t \leq P_h \times (1 - \theta). \tag{2.1}$$

In contrast, a downtrend is terminated by a DC extreme point when the current price P_t is higher than the last low price P_l by a fixed threshold:

$$P_t \geq P_l \times (1 + \theta), \tag{2.2}$$

where the size of the threshold θ is given by the analyst. We define the current price P_t as the DC confirmation point when the DC extreme point is determined. Figure 3.1 below is an example of a DC summary of the exchange rate of *EURUSD* into a sequence of extreme points. According to Tsang et al. (2017), a DC downtrend (uptrend) decomposes into two parts – a DC event and an overshoot event. The DC timescale, in Figure 3.1, illustrates a dynamic timescale such that the end of the current interval is determined when the price change has reached a threshold from the last highest or lowest price. Under the process of DC data sampling, for example, the last high price is kept updated when there is a higher high price until we determine a DC extreme point based on equation (2.1); in Figure 2.1, the last high price is the extreme point when the EP1 is confirmed.

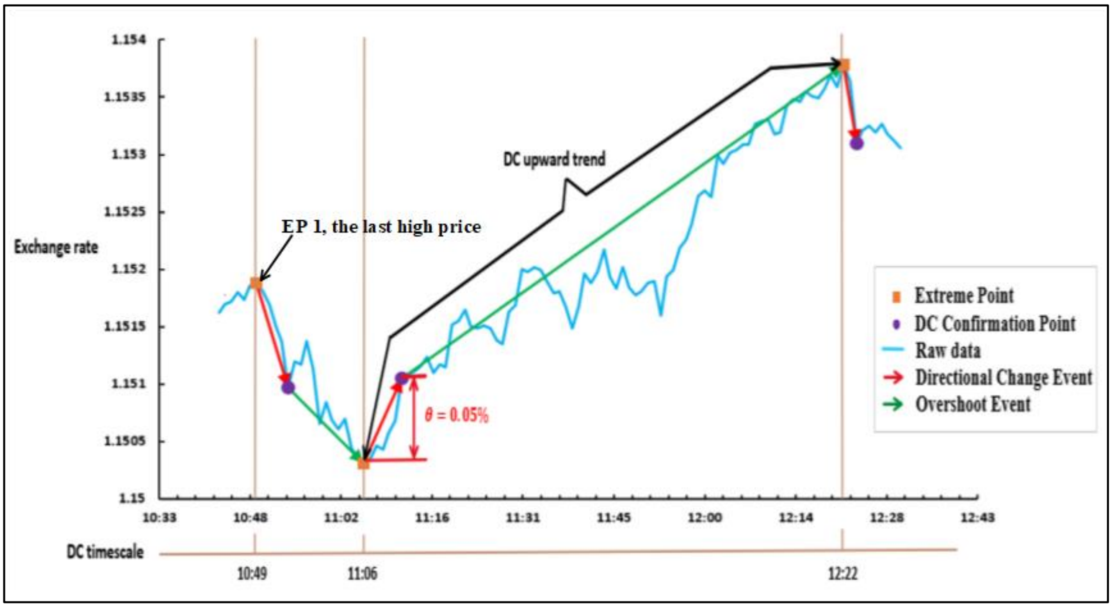

Figure 2.1 The price curve of *EURUSD* on 3rd May 2016. An example of a DC summary with a threshold (**θ**) of 0.05%. The three vertical brown lines (determined by

DC extreme points) separate the price curve into an uptrend and a downtrend. Under DC timescale, the time intervals indicate the periods of DC trends.

A DC extreme point is a couple which contains a timestamp $EP.t$ with a price $EP.p$:

$$EP \ = (EP.t, EP.p). \tag{2.3}$$

A DC sequence S_A^{θ} is a finite sequence which comprises the extreme points of the market A ordered by $EP.t$:

$$S_A^{\theta} \ = (EP_1, EP_2, \dots, EP_k \dots, EP_n), \tag{2.4}$$

where EP_k is a DC extreme point, θ is the threshold, and A is the market identify (e.g., market A).

Figure 2.2 plots a series of 3 DC trends formed by the 4 contiguous EPs from a DC sequence. As we see in figure 2.2, the DC trends are plotted like a zigzag pattern such that the directions of the adjacent DC trends are changing alternately.

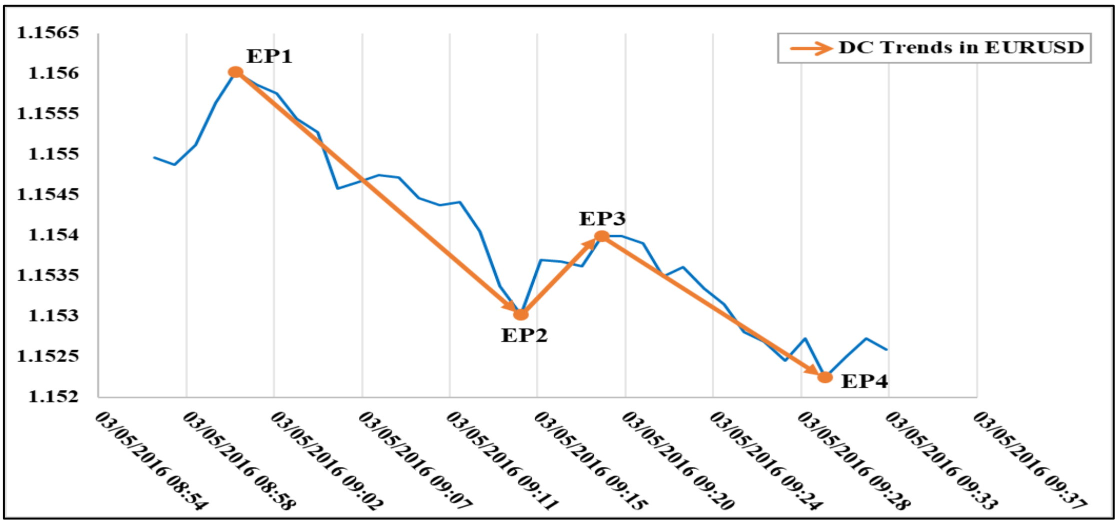

Figure 2.2 An example of DC trends in EURUSD using a threshold of 0.05%. The chart illustrates a series of 3 DC trends formed by 4 EPs.

Tsang et al. (2017) introduced the total price movement (TMV) to measure the price distance between the extreme points that begin and end a DC trend.

The TMV from extreme point EP_{i-1} to the next extreme point EP_i, denoted by TMV_i, is defined by the increase in proportional terms from $EP_{i-1}.p$ to $EP_i.p$ normalized by the threshold:

$$TMV_i = \frac{EP_i.p - EP_{i-1}.p}{EP_{i-1}.p \times \theta}, \tag{2.5}$$

where $EP_i.p$ is the price of the EP at the end of the i^{th} DC trend, and θ is the threshold defined by the analyst.

We can obtain the period of the i^{th} DC trend, denoted by T_i, as the time interval between $EP_i.t$ and $EP_{i-1}.t$.

The time-adjusted return of DC (we call this TR for short) is the ratio of the TMV_i to T_i. Tsang et al. (2017) suggested that TR is a new way to evaluate the return of the DC trend when one considers the time taken for a DC trend. In the comparative analysis, TR is an indicator to measure the speed of forming the DC trend when two DC trends have equal values in their TMVs.

The time-adjusted return of the DC trend (TR) measures the actual percentage of price change per time unit. For example, the time-adjusted return of the i^{th} DC trend, denoted by R_{DC_i}, is calculated by:

$$R_{DC_i} = \frac{|EP_i.p - EP_{i-1}.p|}{EP_{i-1}.p \times T_i} = \frac{|TMV_i| \times \theta}{T_i}, \tag{2.6}$$

where θ is the threshold defined by the analyst, T_i is the period between the $EP_i.t$ and $EP_{i-1}.t$, i.e. T_i = $EP_i.t - EP_{i-1}.t$.

Throughout this thesis, we use $|*|$ to denote the absolute value. We set the terminal time of the R_{DC_i} by $ET(R_{DC_i}) = EP_i.t$. This will be important for ordering the TRs of two markets to produce a combined TR sequence of the two markets (we will introduce this in Chapter 5).

DC measures the volatility of a single market based on the frequency of the observed EPs over a period (Guillaume et al. (1997)). Tsang (2017) discussed how the DC approach could measure market volatility. Given a period of T, the more DC trends observed, the indication is the more volatile the market. As explained in Figure 2.2, a DC trend is defined by connecting two adjacent EPs. Hence, the number of DC trends are quantified by the number of observed extreme points N_{DC}. Over the period T, the higher value of N_{DC} indicates higher volatility. The idea of DC instantaneous volatility proposed by Petrov et al. (2019) that the equation (2.7) is developed based on the theory of Brownian motion for the price returns. Specicifically, Petrov et al. (2019) discussed that the progresses of the directional change intrinsic time has similar properties to the random walk; based on equation (2.7), the volatility can be estimated for a trendless time series by counting the number of directional changes within the time interval:

$$\sigma_{DC} = \theta\sqrt{\frac{N_{DC}}{T}}, \tag{2.7}$$

where N_{DC} is the number of extreme points from a market over the period T and θ is the threshold which is utilised to obtain the market's DC sequence.

2.4 Jump and co-jump

A jump is a different source of risk compared to the risk of continuous volatility. Empirical studies proved that jumps have a substantial impact on risk management, option pricing and hedging strategy (Liu et al. (2003) and Johannes (2004)). In the financial markets, jumps are the market reactions to unexpected information or events (Lahaye et al. (2011)). The initial issue of studying jump risk was identifying jump events and analysing the detected jumps' behaviour. In the asset pricing model, a jump is considered as a discontinuous component. In time series, a number of researchers have worked on jump detection (Barndorff-Nielsen and Shephard (2004), Huang and Tauchen (2005), Andersen et al. (2007), Andersen et al. (2011), Lee and Mykland (2008), and Aït-Sahalia and Jacod (2009)). Barndorff-Nielsen and Shephard (2004) first proposed the technique to locate jumps at a daily frequency. Lee and Mykland (2008) built a statistical method to detect intraday jumps.

2.4.1 Jump identification in time series

In time series, the idea of detecting TS jumps based on the asset pricing model (Nielsen and Shephard (2004) and Lee and Mykland, (2008)); a jump (TSJ) is a different source of risk compared to the risk of continuous volatility in the asset pricing model. In the practice, following the works of Nielsen and Shephard (2004), a jump is a component of the realized variance (see equation (2.13)); Lee and Mykland (2008) proposed an approach to detecting jump (see equation (2.15)).

In the asset return process model, the continuous time log-price process, p_t, evolves as follows:

$$dp_t = \mu_t dt + \sigma_t dW_t, \qquad 0 \leq t \leq T \tag{2.8}$$

where T is the total number of days in the sample, μ_t is the drift rate, σ_t is the instantaneous volatility and dW_t is Brownian motion. The solution to equation (2.8) is generally called an Itô process. As σ_t is a stochastic volatility process with a sample path that is right continuous, it is unable to capture the discontinuous jump event. When we wish to include the jump phenomena, we expand equation (2.9) as follows:

$$dp_t = \mu_t dt + \sigma_t dW_t + k_t dq_t, \qquad 0 \leq t \leq T, \tag{2.9}$$

where q_t is the counting process, and k_t is the jump size when $dq_t = 1$.

The discrete-time returns are

$$r_t = p_t - p_{t-1}, \qquad t = 1, 2, \ldots \tag{2.10}$$

where the unit time interval is usually referred to as a ‘day’. Assume that there are $M + 1$ observations per day of high-frequency data, then the continuously compounded M intra-daily returns for day t are denoted by

$$r_{t,j} = p_{t,j} - p_{t,j-1}, \quad t = 1, 2, \ldots, T \tag{2.11}$$

where $p_{t,j}$ is the jth intraday log-price on day t and T gives total number of days sampled.

The daily realized variance is defined by:

$$RV_t = \sum_{j=1}^{M} r_{t,j}^2, \qquad t = 1,2, \ldots, T, \tag{2.12}$$

where j is the intraday interval. As emphasized by Barndorff-Nielsen and Shephard (2004), the realized variance is decomposed into two components with increasing sample frequency (the size of the intraday time interval tending to zero), $M \to \infty$:

$$RV_t \to \int_{t-1}^{t} \sigma_s{}^2 ds + \sum_{j=1}^{N} k_{t,j}^2, \qquad t = 1,2, \ldots, T, \tag{2.13}$$

where the first term is the integrated variance for the continuous component, and the second term is the jump component.

Barndorff-Nielsen and Shephard (2004) introduced the bipower variation to estimate the instantaneous volatility as:

$$BV_t \equiv \mu_1^{-2}\left(\frac{M}{M-1}\right)\sum_{j=2}^{M}|r_{t,j}||r_{t,j-1}|, \qquad t = 1,2,\dots,T, \tag{2.14}$$

where $\mu_1 = \sqrt{2/\pi}$ and M is the total number of intervals per day. As $M \to \infty$, we have:

$$BV_t \to_p \int_{t-1}^{t} \sigma_s{}^2\, ds, \qquad t = 1,2,\dots,T. \tag{2.15}$$

According to Lee and Mykland (2008), a jump is detected by the ratio:

$$Jump_{t,j} = \frac{|r_{t,j}|}{\sqrt{BV_{t,j}/M}}, \quad t = 1,2,\dots,T. \tag{2.16}$$

Lee and Mykland (2008) infer the presence of jumps from the distribution of the statistic's maximum over the sample size. Under the null hypothesis of no jump in the day t and the time interval j, the sample maximum of the absolute value of a standard normal converges to a Gumbel distribution. We reject the null hypothesis of no jump if:

$$Jump_{t,j} > G^{-1}(1-\alpha)S_n + C_n, \tag{2.17}$$

where $G^{-1}(1-\alpha)$ is the $1-\alpha$ quantile function of the standard Gumbel distribution, $C_n = (2\,log\,n)^{0.5} - \frac{log(\pi)+log(log\,n)}{2(2\,log\,n)^{0.5}}$ and $S_n = \frac{1}{(2\,log\,n)^{0.5}}$, n being the total number of observations (i.e., $M \times T$). According to Lee and Mykland (2008), given the significance level of $\alpha = 1\%$, the threshold for $\frac{|Jump_{t,j}|-C_n}{S_n}$ is β^* , with $\beta^* =$

$-log(-log(0.99)) = 4.6001$. Thus, if $\frac{|Jump_{t,j}|-C_n}{S_n} > 4.6001$, we reject the null hypothesis of no jump and establish the presence of a jump.

2.4.2 co-jumps

Barndorff-Nielsen and Shephard (2006), Jacod and Todorov (2009), and Bollerslev et al. (2008) define co-jumps using multivariate tests whereas Lahaye et al (2011) define co-jumps in a natural way using a univariate test with co-jumps as simultaneous significant jumps to permit straightforward estimates of co-jumps. There have even been alternative definitions of co-jumps using wavelets (Barunik and Vacha, 2018). A co-jump is defined by Lahaye et al (2011) in that they detect jumps happening simultaneously in two markets by using the product of the indicator functions of the jumps in the individual markets; the co-jump indicator function on a set of markets Mkt at a period t, j:

$$COJump_{t,j}^{Mkt} = \prod_{Mkt} I\left(\left|Jump_{t,j}^{m_i}\right|\right) \quad (2.18)$$

where $I(\cdot)$ is the indicator function for a positive argument and $Jump_{t,j}^{m_i}$ refers to significant jumps detected at period t, j on market m_i in the set Mkt. For example, given two markets' datasets, when jumps are determined from both two markets at period t, j, $COJump_{t,j}^{Mkt} = 1$; however, if there is a jump identified from one of two markets or there are no jumps at period t, j, then $COJump_{t,j}^{Mkt} = 0$. Lahaye et al (2011) detected co-jumps between *EURUSD* and *GBPUSD*, and between *EURUSD* and *JPYUSD*; the propotion of the determined TS co-jumps over the total observations are less than 1%.

Once adequate definitions of co-jumps were found, applications and implications of these definitions were sought. Barunik and Vacha (2018) investigated how co-jumps significantly influence correlations in currency markets. It was also found that the market conditions preceding jumps and co-jumps are associated with higher quote volume, greater illiquidity, greater jump-signed order flow (Piccotti, (2018)). The association of traditional time series co-jumps (defined as a jump of both assets within the same time interval) with macroeconomic news announcements was studied by Chatrath et al (2014). They conducted a co-jump regression analysis and concluded that positive surprises (difference between the actual value and the consensus value normalized by standard deviation) in U.S. macroeconomic announcements increases the probability of observing cojumps with a negative jump of the foreign currency (Euro, Sterling, Japanese Yen). In addition, it was found that a negative surprise in a U.S. announcement increases the probability of co-jumps with a positive jump exhibited by the foreign currency. Lahaye et al (2011) discussed the link between macroeconomic news and co-jumps from the back testing restuls based on a probit model; they concluded that there was a strong relation between news surprises and co-jumps. Also, through a regression analysis, Dungey and Hvozdyk (2012) observed that the probability of co-jumps presenting increased with the scheduled macroeconomic news. Bibinger and Winkelmann (2014) analysed on co-jumps in futures on German government bonds with short and long maturity; they concluded that the interest rate decision has a marjor impart on co-jumps presenting. Caporin et al. (2017) exaimed the relation between stocks co-jumps and news; they found that co-jumps associated with bad news increased the stock variances and the correlations, and the stock prices are more likely to drop. In contrast, the goods rise the stock variances and the correlations, and the stock prices are likely to increase.

Chapter 3. Relative Volatility

This chapter introduces a new approach in measuring relative volatility between two markets based on the directional change (DC) framework. DC is a data-driven approach for sampling financial market data such that the data is recorded when the price changes have reached a significant amplitude rather than recording data on a pre-determined timescale. Being able to measure relative volatility between two different assets helps analysts to monitor the relative strength of the volatility between two markets; and this could be an additional tool to better inform the role of risk management. In DC, the majority of the published references focus on the study of volatility measurement of a single market (Guillaume et al. (1997), Tsang (2017), and Petrov et al. (2019)) for instance. In measuring relative volatility of two markets, due to the varying timescale of the DC data, there is no direct way to measure the volatility of two markets simultaneously. Especially in the study of the high-frequency data, observers have to consider the pre-determined period to be used in order to collect the DC data of two markets to enable the measurement of the relative volatility. As discussed in Section 1.1, it is suspected that it would be preferable to let the data dictate the time interval based on the behaviour of the two markets' price changes. Hence, in terms of the contribution of this chapter, we propose the new concept of DC micro-market relative volatility (mRV) to evaluate relative volatility between two markets. Unlike the time series method, mRV dynamically redefines the timescale based on the frequency of the observed DC data between the two markets. As we shall show through the results of our studies (discussion in Section 3.5.3), it is useful for measuring the relative volatility in micro-market activities (high-frequency data) both in terms of providing more data during times of significant events and enabling the precise localisation of the measurement of volatility.

The remainder of this chapter is organised as follows. Section 3.2 introduces the concept of Directional Change and the volatility measurement in the DC framework. Section 3.3 presents the measure of DC relative volatility using a pre-determined period. Section 3.4 introduces the concept of DC micro-market relative volatility mRV and its measurement method. Section 3.5.1 contrasts the classical method (time series approach) with the DC method from the perspective of measuring relative volatility. Section 3.5.2 illustrates the back-testing of measuring relative volatility between *EURUSD* and *GBPUSD* over seven years from 2012 to 2018. Particularly, mRV detected that Sterling was extremely volatile in comparison to the Euro in the week of the Brexit referendum. Inter alia, mRV detected that *GBPUSD* was extremely volatile compared to *EURUSD* after the voting time of the Brexit referendum. In Section 3.5.3, we discuss the benefits of measuring mRV compared to the classical method. In addition, Section 3.5.4 proposes a scaling-law to evaluate the relationship between the average period of sub-sequence and threshold chosen by the analyst. In Section 3.6, we give our conclusions.

3.1 Introduction

Evaluating volatilities between different financial instruments is a primary idea in the application of risk management and trading strategy. The classical approach of measuring relative volatility is through comparing the variance of the price return on the regular timescale. It is capable of evaluating relative volatility if the objective dataset could better coincide with a period of relatively high homogeneity (like daily or weekly time interval). However, in high-frequency data, the general approach might not present an accurate result for evaluating relative volatility, and there are two main reasons: (1) on the pre-determined timescale it is hard to summarise the real behaviour in terms of micro-market activity because, for instance, the volume of the participants' transactions are not equal on the regular timescale. (2) the markets' reactions to a sudden event might not be synchronously recorded in the prices, in other words, there might be a time delay between the response of markets. For instance, in measuring the consistency of the co-jumps between two markets, one price jump of market A may be followed by a price jump from market B with a short time delay. Under the DC framework, we propose a new concept of DC micro-market relative volatility (mRV) in evaluating relative volatility. In mRV, measuring relative volatility does not require a pre-defined timescale since the mRV approach determines the timescale based on a data-driven process. Specifically, we build the DC relative sequence, which combines the DC sequences of two markets into a single sequence. In a DC relative sequence, the timescale is passively defined by the observation of the DC data.

As introduced in Section 2.1, the methodology of measuring relative volatility is different between the TS method and the DC method. Thus, there are some questions as follows: could the DC approch show similar results to the TS method in measuring relative volatility? How can observers benefit from using mRV compared with the TS method? In addition, to measure mRV, we build the DC relative sequence to combine the DC sequences of two markets into a single sequence. Can we find a new scaling law between the magnitude of the threshold and the timescale of the DC relative sequence? In other words, can we estimate the timescale of the DC relative sequence (in terms of an average value) given a certain threshold? We will answer these questions in the following sections.

3.2 Directional Changes

Directional change (DC) is a new framework in the data sampling of the financial market transactions for the analysis of the market behaviours. The process of DC data sampling is based on the DC algorithm in equation (2.1) and (2.2) below (Guillaume et al. (1997) and Tsang et al. (2015)). In time series analysis, the market data is collected under a pre-determined timescale. However, the mechanism of DC data sampling considers the significant price changes such that the market data is recorded when the price change has reached a certain threshold from the last peak/trough of the price. In practice, the analyst determines the threshold as a percentage. Hence, price changes are recorded as a series of alternate uptrends and downtrends, and the timestamp of each DC data point is determined dynamically.

3.2.1 DC volatility

DC measures the volatility of a single market based on the frequency of the observed EPs over a period (Guillaume et al. (1997)). Tsang (2017) discussed how the DC approach could measure market volatility. Given a period of T, the more DC trends observed; this provides an indication of greater market volatility. As explained in Figure 2.1 (in Section 2.3), a DC trend is defined by connecting two adjacent EPs. Hence, the number of DC trends are quantified by the number of observed extreme points N_{DC}. Over the period T, a higher value of N_{DC} indicates higher volatility. Petrov et al. (2019) presented the measure of instantaneous volatility such that the equation (2.7) is developed based on the theory of Brownian motion for the price returns (for deails of instantaneous volatility see Section 2.3).

It is worth reiterating that DC and time series (TS) sample data differently. Therefore, given the same raw tick data, DC and TS will generate different sample datasets. Although volatility measures under DC and TS both reflect the market, they cannot be compared directly.

3.3 DC relative volatility

DC relative volatility (DCRV) is a concept in comparing the intensity of one market's volatility relative to another market in a period T**.** The general method of evaluating relative volatility is through comparing the variances of the price returns between the two markets in a period T, which requires the same timescale of the two markets' price returns. For instance, analysts compare the variances of hourly price returns between market A and market B in a particular month. In DCRV, the relative volatility is measured by differencing the values of two markets' DC volatilities (σ_{DC}) in a period T; e.g. the measure of DCRV between market A and market B denoted $\sigma_{DC(A,B)}$, is given by:

$$\sigma_{DC(A,B)} = \sigma_{DC.A} - \sigma_{DC.B} = \theta \frac{\sqrt{N_{DC.A}} - \sqrt{N_{DC.B}}}{\sqrt{T}}, \tag{3.1}$$

where $\sigma_{DC.A}$ and $\sigma_{DC.B}$ are the DC volatilities of the markets A and B respectively, $N_{DC.A}$ and $N_{DC.B}$ are the number of extreme points of market A and market B over the period T and θ is the threshold which is applied to obtain the DC sequences of market A and market B.

Given the $\sigma_{DC(A,B)}$ over a period T:

i. If $\sigma_{DC(A,B)} > 0$, the volatility of market A is relatively higher than the volatility of market B.

ii. If $\sigma_{DC(A,B)} = 0$, the volatility of market A and market B are at the same level.

iii. If $\sigma_{DC(A,B)} < 0$, the volatility of market A is relatively lower than the volatility of market B.

3.4 DC micro-market relative volatility

Section 3.3 introduces the measure DCRV in a pre-determined period T evaluating the relative volatility depending on the length of the period. However, given a set of data, the DCRV may indicate different results in measuring relative volatility when the length of T is selected randomly. In the example below, Figure 3.1 shows a segment of the DC sequences of market A and market B. Given the three different lengths of the periods T_1, T_2, T_3, we obtain different numbers of EPs from the two markets. According to equation (3.1), the DCRV approach indicates three different results ($\frac{0.32\theta}{\sqrt{T_1}}$, $\frac{1.04\theta}{\sqrt{T_2}}$ and $\frac{0.83\theta}{\sqrt{T_3}}$) in measuring the relative volatility under the periods of T_1, T_2, T_3. Figure 3.1 raises the question of how should we select the length of the T for measuring the relative volatility.

DC takes a data-driven approach to sampling. Based on the same principle, it may be better to let the data pick T. That motivates us to find a data-driven measure of relative volatility. Also, the DCRV approach might be incapable of evaluating the event-based collapse at micro-level. Figure 3.2 below shows two differently arranged frequencies of the EPs from market A and market B in the same period T. In Scenario 1, there is a constant frequency of the observed EPs between the two markets in that, every two EPs of market B follows one EP of market A. In scenario 2, there is the same number of total EPs as in scenario 1. However, the frequency of the observed EPs is entirely different (six consecutive EPs of market B follow two EPs of market A, then two EPs of market B follow two EPs of market A). Although the two scenarios have differently arranged frequencies, the DCRV approach presents the same result because of the same number of EPs of the two scenarios (according to equation (3.1)).

The shortcoming described in the previous paragraph is addressed with the concept of DC micro-market relative volatility (mRV). This is a concept used to evaluate the relative volatility based on a data-driven process. In mRV, the period T is determined according to the observation of the extreme points of the two markets. It is important to note time is passively defined in mRV. A formal definition of mRV and how it may be measured is given in the next sections.

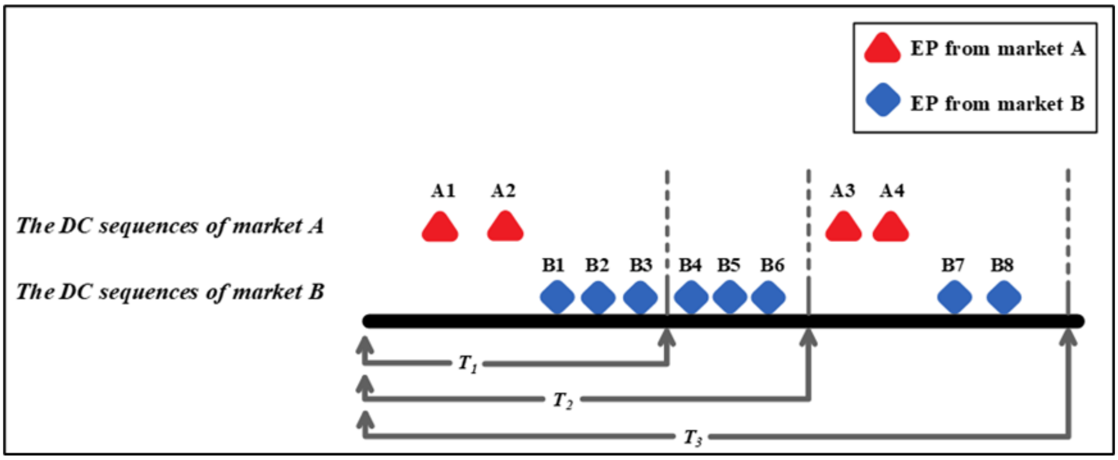

Figure 3.1 The DC sequences of market A and market B with the periods of $\boldsymbol{T_1, T_2, T_3}$. Under the three different lengths of the periods, the DCRV measurement shows different conclusions in evaluating relative volatility between the two markets.

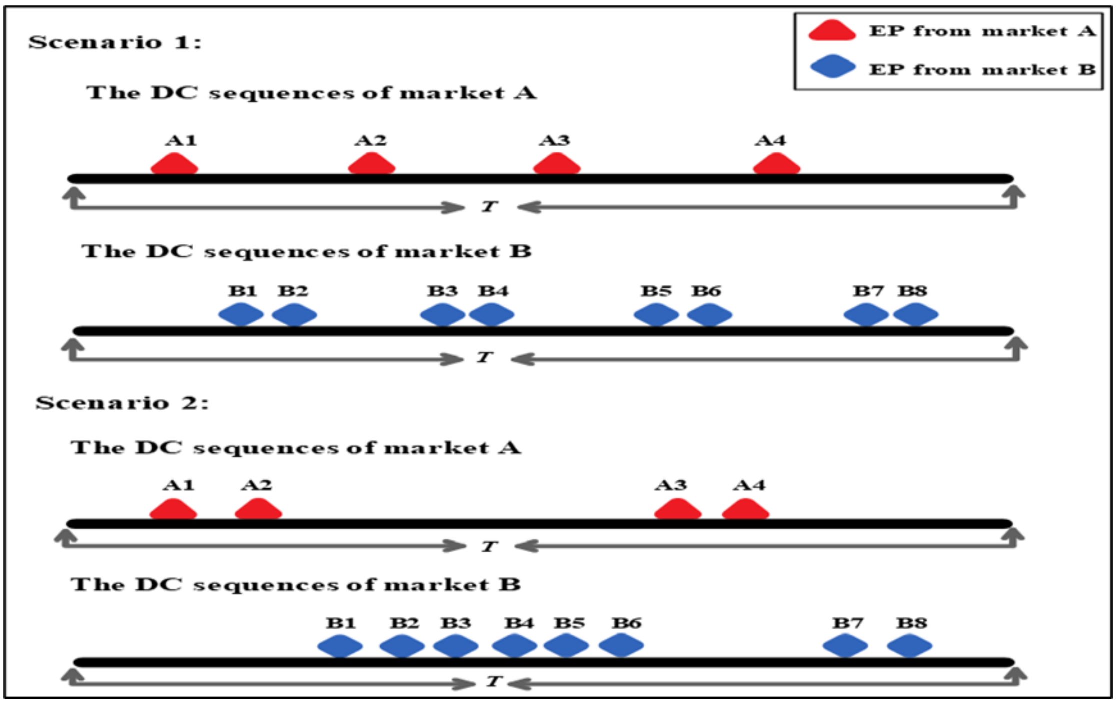

Figure 3.2 The same number of EPs from market A and market B in the same period *T*.

3.4.1 DC relative sequence (RS)

As discussed in the beginning of Section 3.4, a progressive data driven method is to dynamically determine the period T based on the observing EPs of the two markets. The DC relative sequence (RS) combines the two DC sequences into a new sequence in chronological order. In a RS, the termination of the current period depends on changes of the market identity between the current EP and the next EP. Figure 3.3 illustrates the DC relative sequences according to scenario 1 and scenario 2 as described in Figure 3.2. In scenario 1 of Figure 3.3, the T_1 is terminated when the identity of the EP.A4 is different from the identity of the EP.B3. In scenario 2 of Figure 3.3, the T_2 is terminated when the identity of the EP.A13 is different from the identity of the EP.B12 (we suppose that the EP.13 is from market A). Hence, in scenario 1 of Figure 3.3, the DC relative sequence is decomposed into the four sub-sequences of the time periods

$T_1, T_2, T_3,$ and T_4. Likewise, the DC relative sequence of scenario 2 is decomposed into the two sub-sequences given by T_1 and T_2.

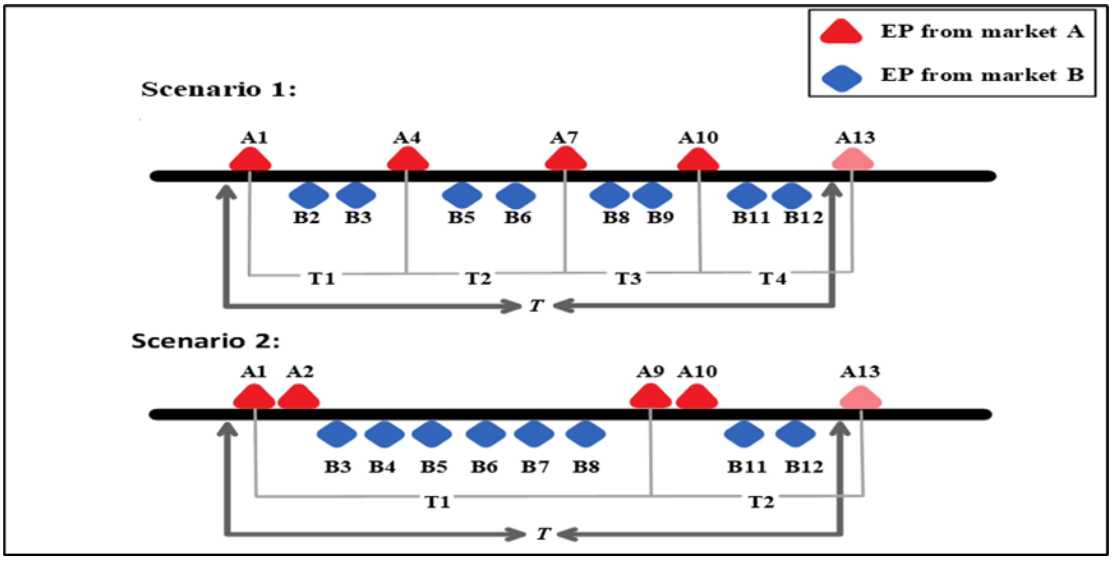

Figure 3.3 The decomposed periods of Figure 3.2. We assume that the EP.A13 is from market A for both scenario 1 and scenario 2.

3.4.2 Formal definitions of DC relative sequence

A DC combined sequence comprises all observed EPs from the two DC sequences of $\mathrm{S}_{\mathrm{A}}^{\theta}$ and $\mathrm{S}_{\mathrm{B}}^{\theta}$ ordered by the timestamp $EP.t$:

$$S_{S_A^\theta, S_B^\theta} = (EP_1, EP_2, \dots, EP_m), \tag{3.2}$$

where m equals the amount of the total number of EPs from both $\mathrm{S}_{\mathrm{A}}^{\theta}$ and $\mathrm{S}_{\mathrm{B}}^{\theta}$, and $EP_1, EP_2, \dots, EP_m$ are either from $\mathrm{S}_{\mathrm{A}}^{\theta}$ or $\mathrm{S}_{\mathrm{B}}^{\theta}$. The examples of scenario 1 and scenario 2 from Figure 3.2 are summarised as follows:

(1) Scenario 1 from Figure 3.2:

$$S_{S_A^\theta, S_B^\theta} = (A_1, B_2, B_3, A_4, B_5, B_6, A_7, B_8, B_9, A_{10}, B_{11}, B_{12}). \qquad \text{(E3.1)}$$

(2) Scenario 2 from Figure 3.2:

$$S_{S_A^\theta, S_B^\theta} = (A_1, A_2, B_3, B_4, B_5, B_6, B_7, B_8, A_9, A_{10}, B_{11}, B_{12}). \qquad \text{(E3.2)}$$

A **DC relative sequence** ($\boldsymbol{RS}$) is generated by a division process $\Gamma(S_{S_A^\theta, S_B^\theta})$ which divides a DC relative sequence into z sub-sequences according to the identity of the adjacent EPs:

$$RS_{S_A^\theta, S_B^\theta} = \Gamma(R_{S_A^\theta, S_B^\theta}) = \left(Y_1, Y_2, \ldots, Y_j, \ldots, Y_z\right), \qquad (3.3)$$

where Y_j is a sub-sequence of RS. All Y_j contain at least two EPs that one EP from S_A^θ and another from S_B^θ, thus the maximum value of z is $\frac{m}{2}$. Otherwise, at least one Y_j contains more than two EPs, so z $< \frac{m}{2}$. For every Y_j:

$$\forall \text{ j}: Y_j = (EP_{j,1}, EP_{j,2}, EP_{j,3}, \ldots, EP_{j,k-1}, EP_{j,k}). \qquad (3.4)$$

The termination of the current sub-sequence Y_j depends on the identity of the next EP. When the identity of the upcoming EP is not the same as the identity of the current EP, the length of the period of the current Y_j is determined by:

$$T(Y_j) = EP.t_{j+1,1} - EP.t_{j,1}. \qquad (3.5)$$

Given the DC sequences S_A^θ and S_B^θ of scenario 1 and scenario 2 in Figure 3.3, we obtain the DC relative sequences:

(1) Scenario 1 from Figure 3.3:

$$RS_{S_A^\theta, S_B^\theta} = ((A_1, B_2, B_3)_1, (A_4, B_5, B_6)_2, (A_7, B_8, B_9)_3, (A_{10}, B_{11}, B_{12})_4).$$

(E3.3)

(2) Scenario 2 from Figure 3.3:

$$RS_{S_A^\theta, S_B^\theta} = ((A_1, A_2, B_3, B_4, B_5, B_6, B_7, B_8)_1, (A_9, A_{10}, B_{11}, B_{12})_2). \quad \text{(E3.4)}$$

3.4.3 The measure of DC micro-market relative volatility (mRV)

The approach of DC micro-market relative volatility bases on equation (3.1), while the subject of the measurement is the sub-sequence Y of $RS_{S_A^\theta, S_B^\theta}$:

$$mRV_Y = \theta \frac{\sqrt{N_{DC.A}} - \sqrt{N_{DC.B}}}{\sqrt{T(Y)}}, \quad (3.6)$$

where, $T(Y)$ is defined in equation (3.5). We shall abuse the notation by using mRV as a measure as well as an abbreviation of the concept. Given the measure mRV_Y of the sub-sequence Y:

i. If $mRV_Y > 0$, the volatility of market A is relatively higher than the volatility of market B.
ii. If $mRV_Y = 0$, the volatility of market A and market B are at the same level.
iii. If $mRV_Y < 0$, the volatility of market A is relatively lower than the volatility of market B.

In scenario 1 of Figure 3.3, we measure the mRV in the first sub-sequence Y_1 of $RS_{S_A^\theta, S_B^\theta}$ through equation (3.6):

$$mRV_{T(Y_1)} = \frac{\theta(\sqrt{1}-\sqrt{2})}{\sqrt{T(Y_1)}} = \frac{-0.41\theta}{\sqrt{T(Y_1)}}. \tag{E3.1}$$

Given the DC relative sequence of equation (3.3), mRV measures each sub-sequence through equation (3.6):

$$mRV_{(RS_{S_A^\theta, S_B^\theta})} = (mRV_{(Y_1)}, mRV_{(Y_2)}, \dots, mRV_{(Y_j)}, \dots, mRV_{(Y_z)}), \tag{3.7}$$

where, $mRV_{(RS_{S_A^\theta, S_B^\theta})}$ is a sequence, and Y_j refers to the sub-sequence j.

In scenario 1 and scenario 2 of Figure 3.3, the mRV is measure by:

(1) Scenario 1 from Figure 3.3:

$$mRV_{(RS_{S_A^\theta, S_B^\theta})} = (mRV_{(Y_1)}, mRV_{(Y_2)}, mRV_{(Y_3)}, mRV_{(Y_4)}). \tag{E3.2}$$

(2) Scenario 2 from Figure 3.3:

$$mRV_{(RS_{S_A^\theta, S_B^\theta})} = (mRV_{(Y_1)}, mRV_{(Y_2)}). \tag{E3.3}$$

3.4.4 Discussion: the merits of using mRV in micro markets

When measuring mRV, the sub-sequence Y_j is the primary object. $T(Y_j)$ is a secondary object defined by the sub-sequence. DC is a data-driven approach of sampling the market data such that the DC data is only recorded when significant price changes are observed. Under the DC framework, the DC relative sequence is a combined sequence of two markets' sequences. We then divide the DC relative sequence into a number of sub-sequences based on the market identity of the adjacent extreme points (EPs). At the end of Section 3.4.2, the example of scenario 1 (equation (E3.3)) illustrates that B_3 is the last EP of the first sub-sequence because A_4 (the next EP) is from a different market compared to B_3. According to equation (3.5), the period T of the first sub-sequence (Y_1) is passively determined by $T(Y_1) = EP.t_{2,1} - EP.t_{1,1}$; in the example of E.3, we have $T(Y_1) = A_4.t - A_1.t$. Hence, the period T is intrinsically determined by the behaviour of the two markets' price changes, rather than being a fixed time interval pre-determined by the analyst. Based on the sub-sequence, we can precisely locate the timestamp when a significant mRV value is determined within the period T. For example, an unusual 'flash event' may produce a series of EPs from market A compared to one EP from market B within a sub-sequence. We can then simply measure the relative volatility of this special event by calculating the mRV of the sub-sequence.

3.4.5 Discussion: regarding threshold selection

Fundamentally, the DC data summarises the original price movement based on a pre-determined threshold. In practice, observers utilize the threshold to capture the significant price changes and filter out the unnecessary noise of the price movement. Hence, the magnitude of the threshold directly impacts the frequency of the EPs over a period. An extremely small threshold will cause every tick data point to be determined as an EP. On the other hand, an extremely large threshold will give the result of recording no DC data. So, what is the 'right' threshold for us to use? It is unlikely to find an 'optimal' threshold for sampling DC data in this research. In fact, there are no 'wrong' ways of determining the size of thresholds. It is actually the observer's prerogative to set the threshold to suit the individual observer's needs. High-frequency traders might prefer a smaller threshold to acquire the micro price changes, while institutions might be more focused on larger price movements. In addition, Glattfelder et al. (2011) show that the same statistical measures can be observed under different thresholds.

3.5 Experiment

In this section, we contrast the realised volatility (the classical time series method under the regular timescale) and mRV in the measurement of relative volatility. It is worth reiterating that DC and time series work on different datasets sampled from tick data, and therefore, volatility measures utilizing those frameworks cannot be compared directly. The aim of this experiment is to examine the consistency of measuring relative volatility between the two methods.

3.5.1 Comparing relative volatility between Time Series and DC

In this section, we compare the realised volatility (the classical time series method under the regular timescale) and mRV as a measure of relative volatility. The aim of this experiment is to examine the consistency of measuring relative volatility between the two methods.

In time series, we select four groups of the data under the regular time intervals $\Delta\mathrm{t}$ = {10 seconds, 1 min, 5 min, and 15 min}. The return at time t, R_t, is defined by:

$$R_t = \ln P_t - \ln P_{t-\Delta\mathrm{t}}, \tag{3.8}$$

where, $\ln P_t$ is the logarithmic price at the end of each time interval $\Delta\mathrm{t}$. Given the sequence of the returns over a period τ (e.g., a trading day or a trading week), the realised volatility is defined by the standard deviation (Alexander (2008)):

$$\sigma_\tau = \sqrt{\frac{\sum(R_t - \bar{R})^2}{n-1}}, \tag{3.9}$$

where n is the number of returns over a period τ, and $\bar{R}$ is the mean of the sequence of the returns. Given the standard deviation of market A and market B, we calculate the difference of $\sigma_{\tau,A}$ and $\sigma_{\tau,B}$ to evaluate the relative volatility between the market A and market B over a period τ:

$$Dsd_{\tau,A-B}^{\Delta t} = \sigma_{\tau,A} - \sigma_{\tau,B}, \tag{3.10}$$

where Δt is the initially selected time interval to obtain the logarithmic price.

Glattfelder et al. (2011) discovered 12 DC scaling laws in the market. For instance, the analytical relationship between the size of threshold and the average percentage change of a DC trend. The DC scaling law 10 gives the statistical property that the average period of a DC trend $\langle T_{tmv} \rangle$ is approximately equal to a function of the threshold θ:

$$\langle T_{tmv} \rangle = \left(\frac{\theta}{C_{t,tmv}}\right)^{E_{t,tmv}}, \tag{3.11}$$

where $E_{t,tmv}$ and $C_{t,tmv}$ are the scaling law parameters, $\langle . \rangle$ is the operator to calculate the mean, and θ is the threshold. Based on equation (3.11), we can estimate the average period of a DC trend $\langle T_{tmv} \rangle$ given a threshold θ, and vice versa (we present a basic summary of the 12 DC scaling laws in Appendix C). Hence, we obtain the four corresponding thresholds given the time intervals $\Delta \mathrm{t} = \{10 \text{ seconds}, 1 \text{ min}, 5 \text{ min}, \text{and } 15 \text{ min}\}$. A DC total movement defines a trend of the price movement between two adjacent extreme points (see Figure 2.1 in Section 2.2). According to the DC definition, a trend is terminated when the price changes have reached a certain threshold θ from the last peak/trough of the price. In DC, the peak/trough defines the extreme point (EP). Given a threshold and the scaling law 10 (equation (3.11)), we can estimate the average period of the trend and vice versa. Glattfelder et al. (2011) estimated the average values of the parameters $C_{t,tm}$ and $E_{t,tm}$ across 13 pairs of exchange rates, and obtained $C_{t,tm} = 0.00165$ and $E_{t,tm} = 2.02$. In this experiment, $\langle T_{tmv} \rangle$ is the Δt. Given Δts, using equation (3.11), we obtain the corresponding thresholds, θ = {0.005%, 0.013%, 0.028%, 0.048%}. Based on the four thresholds, we calculate the DC sequences of the market A and market B and generate the DC relative sequence $RS_{S_A^\theta, S_B^\theta}$ through equation (3.3). Then, we measure the mRV through equation (3.7). According to equation (3.10), we evaluate the relative volatility in the period τ of daily (*D*), weekly (*W*) and monthly (*M*) of $Dsd_{\tau,A-B}^{\Delta t}$. As introduced in equation (3.7), $mRV_{(RS_{S_A^\theta, S_B^\theta})}$ is a sequence. Hence, we calculate the mean value of $mRV_{(RS_{S_A^\theta, S_B^\theta})}$ over the period τ to match the value of $Dsd_{\tau,A-B}^{\Delta t}$. In the back-testing, we calculate the mean of daily $\langle mRV_{(RS_{S_A^\theta, S_B^\theta})} \rangle_D$, the mean of weekly $\langle mRV_{(RS_{S_A^\theta, S_B^\theta})} \rangle_W$, and the mean of monthly $\langle mRV_{(RS_{S_A^\theta, S_B^\theta})} \rangle_M$. The data source is from *Tickstory*[5] that gives direct access to the database of *Dukascopy*[6]. We select *EURUSD* as the major exchange rate comparing with five exchange rates. Table 3.1 summaries the two approaches in the measure of relative volatility.

5 *Tickstory* is a retailer of market data that their data source is from *Dukascopy*. https://www.tickstory.com/
6 Dukascopy Bank is a Swiss online bank which provides high quality market data in different types. https://www.dukascopy.com/swiss/english/home/

Table 3.1 The summary of the two approaches in the measurement of relative volatility. For the classical method, the equations (3.8)-(3.10) provide the definitions for the measure of the relative volatility in daily, weekly and monthly. The back-testing picked 24 hours tick-by-tick data on weekdays from Monday 00:00:00.000 to Friday 22:00:00.000.

	$Dsd_{\tau,A-B}^{\Delta t}$	$\langle mRV_{(RS_{S_A^\theta,S_B^\theta})}\rangle_\tau$
The raw data sampling	The sequences of the returns under Δt = {10 s, 1 min, 5 min, 15 min} over seven years from 2012 to 2018	The DC relative sequence under θ = {0.005%, 0.013%, 0.028%, 0.048%} over seven years in tick data from 2012 to 2018
The periods of the measurement	Daily: $Dsd_{D,A-B}^{\Delta t}$, Weekly: $Dsd_{W,A-B}^{\Delta t}$ Monthly: $Dsd_{M,A-B}^{\Delta t}$	Daily: $\langle mRV_{(RS_{S_A^\theta,S_B^\theta})}\rangle_D$, Weekly: $\langle mRV_{(RS_{S_A^\theta,S_B^\theta})}\rangle_W$ Monthly: $\langle mRV_{(RS_{S_A^\theta,S_B^\theta})}\rangle_M$
The measure of the pairs of exchange rates	$Dsd_{\tau,GBPUSD-EURUSD}^{\Delta t}$, $Dsd_{\tau,USDJPY-EURUSD}^{\Delta t}$, $Dsd_{\tau,AUDUSD-EURUSD}^{\Delta t}$, $Dsd_{\tau,USDCAD-EURUSD}^{\Delta t}$, $Dsd_{\tau,GBPJPY-EURUSD}^{\Delta t}$	$\langle mRV_{(RS_{S_{GBPUSD}^\theta,S_{EURUSD}^\theta})}\rangle_\tau$, $\langle mRV_{(RS_{S_{USDJPY}^\theta,S_{EURUSD}^\theta})}\rangle_\tau$, $\langle mRV_{(RS_{S_{AUDUSD}^\theta,S_{EURUSD}^\theta})}\rangle_\tau$, $\langle mRV_{(RS_{S_{USDCAD}^\theta,S_{EURUSD}^\theta})}\rangle_\tau$ $\langle mRV_{(RS_{S_{GBPJPY}^\theta,S_{EURUSD}^\theta})}\rangle_\tau$

In the seven year dataset, we obtain 1825 results for $Dsd_{D,A-B}^{\Delta t}$, 366 results for $Dsd_{W,A-B}^{\Delta t}$ and 84 results for $Dsd_{M,A-B}^{\Delta t}$. $\langle mRV_{(RS_{S_A^\theta,S_B^\theta})}\rangle_\tau$ also exhibited the same number of results. Given the results of the back-testing, we measure the correlation between the results of the two approaches. As the data have not been fitted to a Gaussian distribution, we evaluate the correlation through the Spearman rank-order correlation coefficient. The Spearman correlation tests the association of the ordinal relationship between $Dsd_{\tau,A-B}^{\Delta t}$ and $\langle mRV_{(RS_{S_A^\theta,S_B^\theta})}\rangle_\tau$. Table 3.2 is a summary of the results of the correlation coefficient.

Table 3.2 The results of the correlation coefficient. The function $\mathbf{Corr}(.)$ is the correlation test given the two sequences obtained by the approaches of $\boldsymbol{Dsd_{\tau,A-B}^{\Delta t}}$ and $\langle \boldsymbol{mRV}_{(RS_{S_A^\theta,S_B^\theta})}\rangle_{\boldsymbol{\tau}}$. In the first row, EU, GU, UJ, AU, UC, and GJ are the abbreviation of EURUSD, GBPUSD, USDJPY, AUDUSD, USDCAD, and GBPJPY, respectively. The last column indicates the average value of each row spanning the five pairs of exchange rates under the parameters of Δt and θ. All the correlation coefficients below satisfy the significance level of $\boldsymbol{p} < 0.05$.

	GU-EU	UJ-EU	AU-EU	UC-EU	GJ-EU	Average
Daily:						
$\mathrm{Corr}(Dsd_{D,A-B}^{10s}, \langle mRV_{(RS_{S_A^{0.005\%},S_B^{0.005\%}})}\rangle_D)$	0.830	0.835	0.634	0.766	0.782	0.769
$\mathrm{Corr}(Dsd_{D,A-B}^{1min}, \langle mRV_{(RS_{S_A^{0.013\%},S_B^{0.013\%}})}\rangle_D)$	0.869	0.921	0.713	0.794	0.862	0.832
$\mathrm{Corr}(Dsd_{D,A-B}^{5min}, \langle mRV_{(RS_{S_A^{0.028\%},S_B^{0.028\%}})}\rangle_D)$	0.839	0.917	0.733	0.784	0.847	0.824
$\mathrm{Corr}(Dsd_{D,A-B}^{15min}, \langle mRV_{(RS_{S_A^{0.048\%},S_B^{0.048\%}})}\rangle_D)$	0.780	0.858	0.657	0.703	0.778	0.755
Weekly:						
$\mathrm{Corr}(Dsd_{W,A-B}^{10s}, \langle mRV_{(RS_{S_A^{0.005\%},S_B^{0.005\%}})}\rangle_W)$	0.855	0.839	0.621	0.783	0.791	0.778
$\mathrm{Corr}(Dsd_{W,A-B}^{1min}, \langle mRV_{(RS_{S_A^{0.013\%},S_B^{0.013\%}})}\rangle_W)$	0.908	0.941	0.723	0.794	0.893	0.852
$\mathrm{Corr}(Dsd_{W,A-B}^{5min}, \langle mRV_{(RS_{S_A^{0.028\%},S_B^{0.028\%}})}\rangle_W)$	0.898	0.950	0.786	0.830	0.905	0.874
$\mathrm{Corr}(Dsd_{W,A-B}^{15min}, \langle mRV_{(RS_{S_A^{0.048\%},S_B^{0.048\%}})}\rangle_W)$	0.881	0.919	0.751	0.780	0.868	0.840
Monthly:						
$\mathrm{Corr}(Dsd_{M,A-B}^{10s}, \langle mRV_{(RS_{S_A^{0.005\%},S_B^{0.005\%}})}\rangle_M)$	0.952	0.845	0.615	0.797	0.792	0.800
$\mathrm{Corr}(Dsd_{M,A-B}^{1min}, \langle mRV_{(RS_{S_A^{0.013\%},S_B^{0.013\%}})}\rangle_M)$	0.938	0.955	0.757	0.789	0.898	0.867
$\mathrm{Corr}(Dsd_{M,A-B}^{5min}, \langle mRV_{(RS_{S_A^{0.028\%},S_B^{0.028\%}})}\rangle_M)$	0.887	0.966	0.846	0.867	0.943	0.902
$\mathrm{Corr}(Dsd_{M,A-B}^{15min}, \langle mRV_{(RS_{S_A^{0.048\%},S_B^{0.048\%}})}\rangle_M)$	0.839	0.949	0.833	0.886	0.925	0.886

Table 3.2 summarizes the results of the correlation coefficients between $Dsd_{\tau,A-B}^{\Delta t}$ and $\langle mRV_{(RS_{S_A^\theta,S_B^\theta})}\rangle_\tau$. The statistical tests report strong positive correlation in that all the correlation coefficients are over 0.6. The far-right column is the mean of each row, which indicates the average correlation coefficients across the five pairs of exchange rates under the time intervals Δt = {10 seconds, 1 min, 5 min, and 15 min} (with the four corresponding thresholds θs). In Figure 3.4 (1), the three dot-lines illustrate the values of the right end column over the periods of daily, weekly and monthly timescales. Figure 3.4 (1) indicates that the correlation coefficients are tightly bunched for Δt of 10s and 1 min, while the spread enlarges at timeframes of 5 min and 15 min. Figure 3.4 (2) shows the average correlation coefficients of each of the dot-lines and the average

correlation coefficients are 0.795, 0.836, and 0.864 for the daily, weekly, and monthly data. Overall, the results of the correlation test conclude that there exists positive correlation between $Dsd_{\tau,A-B}^{\Delta t}$ and $\langle mRV_{(RS_{S_A^\theta, S_B^\theta})} \rangle_\tau$ from 2012 to 2018.

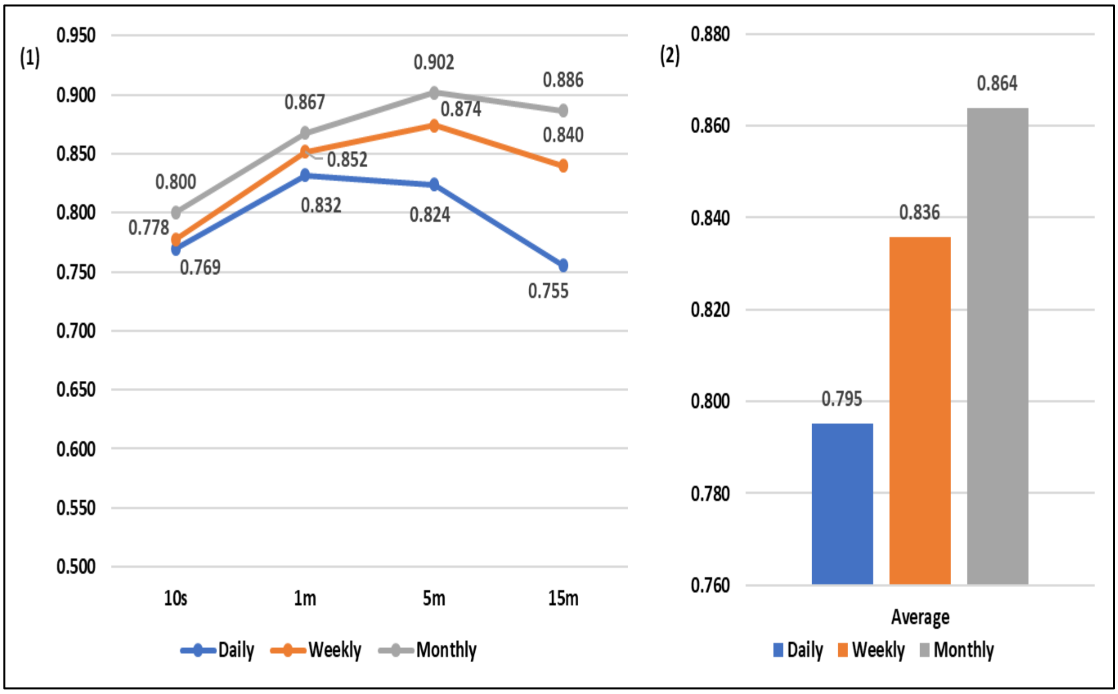

Figure 3.4 (1) the three dot-lines indicate the average values of the correlation under the pairs of parameters Δt and θ; (2) the three columns show the average values of each line from the left chart, which indicate the average correlation coefficients at frequencies of daily, weekly and monthly sampling.

3.5.2 The back-testing of *mRV* between Sterling and Euro

This section will discuss the application of measuring *mRV* between *GBPUSD* and *EURUSD*. The unexpected result of the Brexit referendum caused Sterling to fall -8.016% against the US dollar in 24/06/2016, which was the most significant single day drop since 2000[7]. In the same day, Euro crashed -2.65% against the US dollar. The goal of

[7] According to the data source from Reuters Eikon.

this experiment is to ask whether mRV is useful for measuring the relative volatility between the two markets. To answer that question, we have conducted two sets of experiments. First, we examine the average monthly mRV over a long historical period from 2012 to 2018 to view the relative volatility between Sterling and Euro in the long-term. Second, we test the mRV at the micro-level in that we monitor the mRV over each sub-sequence during the week of Brexit referendum.

Throughout the two experiments, we select two thresholds θs = {0.05%, 0.1%} to calculate the mRV. According to equation (3.6), the value of mRV could be very small if we select too low a threshold. Hence, we normalise the values of mRV by the threshold, $mRV = \frac{mRV}{\theta}$. We simplify the notation for the mean of monthly $\langle mRV_{(RS_{S^{0.05\%}_{GBPUSD}, S^{0.05\%}_{EURUSD}})} \rangle_M$ to $\langle mRV^{0.05\%}_{(RS)} \rangle_M$ in this section.

Figure 3.5 illustrates the mean of monthly $\langle mRV^{0.05\%}_{(RS)} \rangle_M$ under the threshold of 0.05% over seven years. From 01/2012 to 09/2014, the volatility of *EURUSD* was relatively higher compared to *GBPUSD* in that the $\langle mRV^{0.05\%}_{(RS)} \rangle_M$ was changing smoothly between -0.01 to 0 (except the months of 08/2013, 01/2014, and 02/2014, in which the values of the mRV were slightly positive). During the year of 2015, *EURUSD* was more highly volatile compared to *GBPUSD* after the quantitative easing (QE) announcement from the European Central Bank[8]. In the periods between 01/2016 and 06/2016, there was a sharp climb in the values obtained from -0.011 to 0.0374. After the month of Brexit referendum (06/2016), Sterling retained higher volatility compared to the Euro

[8] Details check https://www.ecb.europa.eu/press/pr/date/2015/html/pr150122_1.en.html

until the end of 2016. Under the threshold of 0.1%, the $\langle mRV_{(RS)}^{0.1\%} \rangle_M$ shows a consistent result (see Figure A1 in Appendix A).

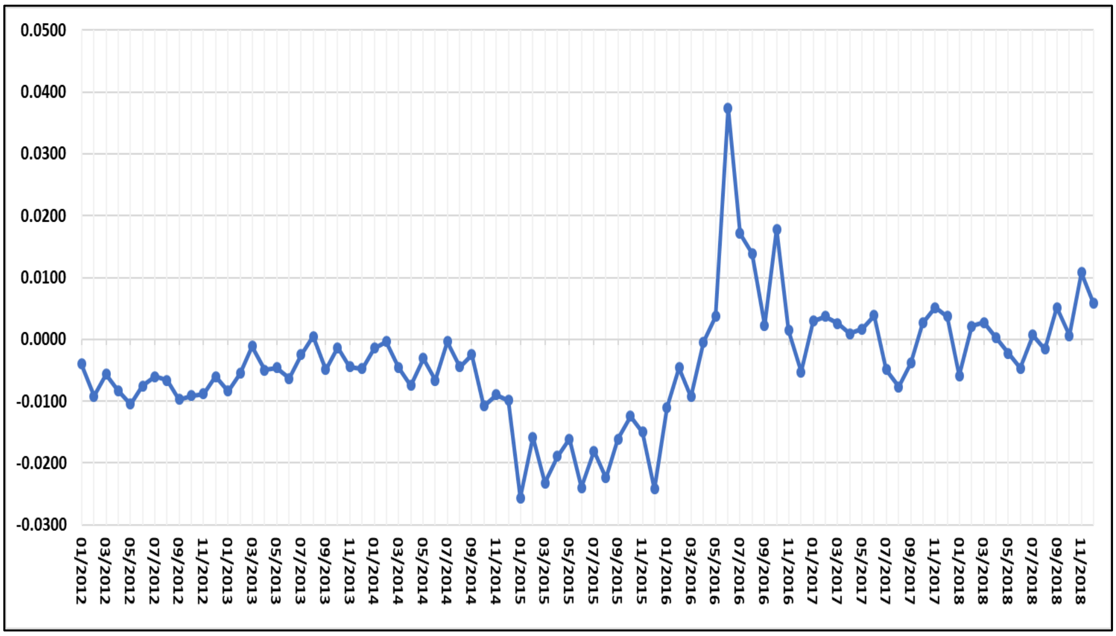

Figure 3.5 The mean of monthly $\langle \boldsymbol{mRV}_{(\boldsymbol{RS})}^{\boldsymbol{0.05\%}} \rangle_{\boldsymbol{M}}$ measures the monthly average $\boldsymbol{mRV}$ under the threshold of 0.05%. From 2012 to 2018, there were 84 data points. The values of $\boldsymbol{mRV}$ are normalised by θ.

In the second experiment, we evaluate the mRV in each sub-sequence under the thresholds of 0.05% and 0.1%. We select the DC relative sequences $RS_{S_{GBPUSD}^{0.05\%}, S_{EURUSD}^{0.05\%}}$ and $RS_{S_{GBPUSD}^{0.1\%}, S_{EURUSD}^{0.1\%}}$ from 16/06/2016 to 30/06/2016 such that the periods cross the five working days before and after the Brexit referendum day on 23/06/2016. Given the DC relative sequences, we calculate $mRV_{(RS)}^{0.05\%}$. Figure 3.6 plots the $mRV_{(RS)}^{0.05\%}$ of the 2200 sub-sequences under the threshold 0.05%. Note that the x-axis in Figure 3.6 is not physical time, but the indices of the sub-sequences; the y-axis is the mRV value. We highlight (in red colour) the sub-sequences in the period right after the voting of Brexit referendum until the end of next day from 22:00 06/23/2016 to 22:00 06/24/2016[9] (UTC) (24 hours after the vote of Brexit referendum). This corresponds to index 0 to 2200 in Figure 3.6. Hence, the 2200 sub-sequences are separated into three parts:

1) Part 1: from 00:00 16/06/2016 to 22:00 06/23/2016 (140 hours in total trading hours);
2) Part 2: from 22:00 06/23/2016 to 22:00 06/24/2016 (24 hours);
3) Part 3: from 00:00 06/27/2016 to 24:00 30/06/2016 (96 hours).

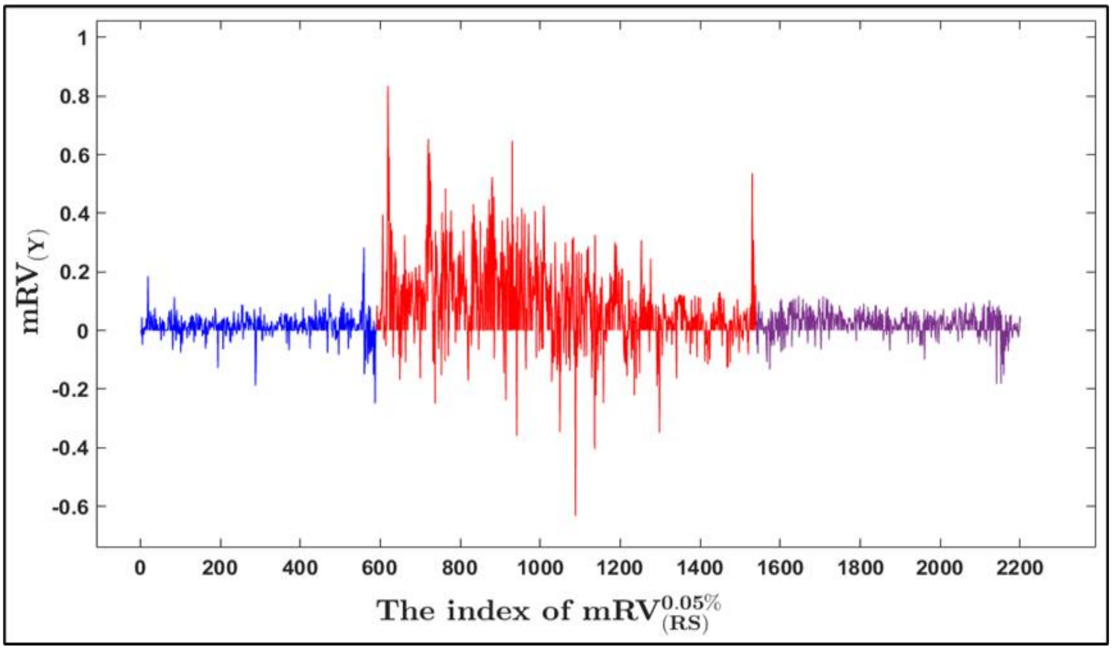

Figure 3.6 The sequence of $\boldsymbol{mRV^{0.05\%}_{(RS)}}$ over the periods from 16/06/2016 to 30/06/2016. We select the tick-by-tick data of *GBPUSD* and *EURUSD* to calculate the $\boldsymbol{mRV}$ of each sub-sequence. Figure 3.6 plots 2200 sub-sequences observed under the threshold of 0.05%. Note that the x-axis refers to the index of the sub-sequences. Part 1(blue line): from 00:00 16/06/2016 to 22:00 06/23/2016 (140 hours); Part 2 (red line):

[9] The voting ended at 22:00, which corresponds to index 590 (the period of the sub-sequence starts from 21:53:58 23/06/2016 to 22:00:06 23/06/2016) in Figure 3.6.

from 22:00 06/23/2016 to 22:00 06/24/2016 (24 hours); Part 3 (purple line): from 00:00 06/27/2016 to 24:00 30/06/2016 (96 hours).

Two observations stand out from the results shown in Figure 3.6:

Observation 1: *GBPUSD* is highly relatively volatile compared to *EURUSD* in Part 2. In the highlighted area of Figure 3.6 (the period of Part 2), there are enormous changes in mRV after the voting time. In Part 2, we observe the sub-sequence of the highest mRV reached the value 0.834 in the period T from 23:17:53 23/06/2016 to 23:18:27 23/06/2016. In this sub-sequence, there are 35 EPs of *GBPUSD* and 1 EP of *EURUSD* in 34 seconds. In contrast, the lowest value of mRV is -0.633 and there is 1 EP of *GBPUSD* and 4 EPs of *EURUSD* in the period T of 3 seconds (from 03:59:28 24/06/2016 to 03:59:31 24/06/2016). In table 3.3, we present the mean and median of the $mRV_{(RS)}^{0.05\%}$ in the three periods (from the second column to the fourth column). Visibly, the values of $\langle mRV_{(RS)}^{0.05\%} \rangle$ and $Median\left(mRV_{(RS)}^{0.05\%}\right)$ of Part 2 are higher than the values in Part 1 and Part3, which indicates the significant volatility of *GBPUSD* compared to *EURUSD* after the voting. This conclusion is further confirmed by the ratio test, as shown in the last two columns of Table 3.3. In the column of Part2/Part1, the ratios reach 5.736 and 4.743 under the mean and median values of $mRV_{(RS)}^{0.05\%}$. In the column of Part2/Part3, the ratios reach 3.723 and 2.591.

Table 3.3 The mean and median of the $\boldsymbol{mRV_{(RS)}^{0.05\%}}$. The operator $\boldsymbol{Median}(.)$ denotes the median of a sequence.

	Part 1	Part 2	Part 3	Part2/Part1	Part2/Part3
$\langle mRV_{(RS)}^{0.05\%} \rangle$	0.014	0.082	0.022	5.736	3.723
$Median\left(mRV_{(RS)}^{0.05\%}\right)$	0.011	0.053	0.02	4.743	2.591

Observation 2: *GBPUSD* and *EURUSD* are much more volatile in Part 2 than Part 1 and Part 3.

During the period of Part 2, we observe 949 sub-sequences out of the total 2200, which account for 43% of the total sub-sequences in 11 trading days. The period of Part 2 is 24 hours after the Brexit referendum, which means around 39 sub-sequences determined in each hour. Also, we observe 1251 sub-sequences in the periods of Part 1 and Part 3 (236 hours in total). Thus, there are approximately 5 sub-sequences in each hour over 236 hours. According to the definition of DC volatility (Section 3.2.2), in a period T, the higher value of N_{DC} (the number of EPs) indicates higher volatility. Hence, we evaluate the instantaneous volatility (σ_{DC}, equation (3.9)) of *GBPUSD* and *EURUSD* in Part 2 and obtained the values of 0.00598 and 0.00374, respectively. We also measure the daily σ_{DC} of Part 1 and Part 3 to compare with the σ_{DC} of Part 2. Figure 3.7 illustrates the daily instantaneous volatility from 16/06/2016 to 30/06/2016. For both *GBPUSD* and *EURUSD*, there is an increase in 23/06/2016, and a peak in 24/06/2016 (the period of Part 2). The σ_{DC} of *GBPUSD* and *EURUSD* declines after 24/06/2016.

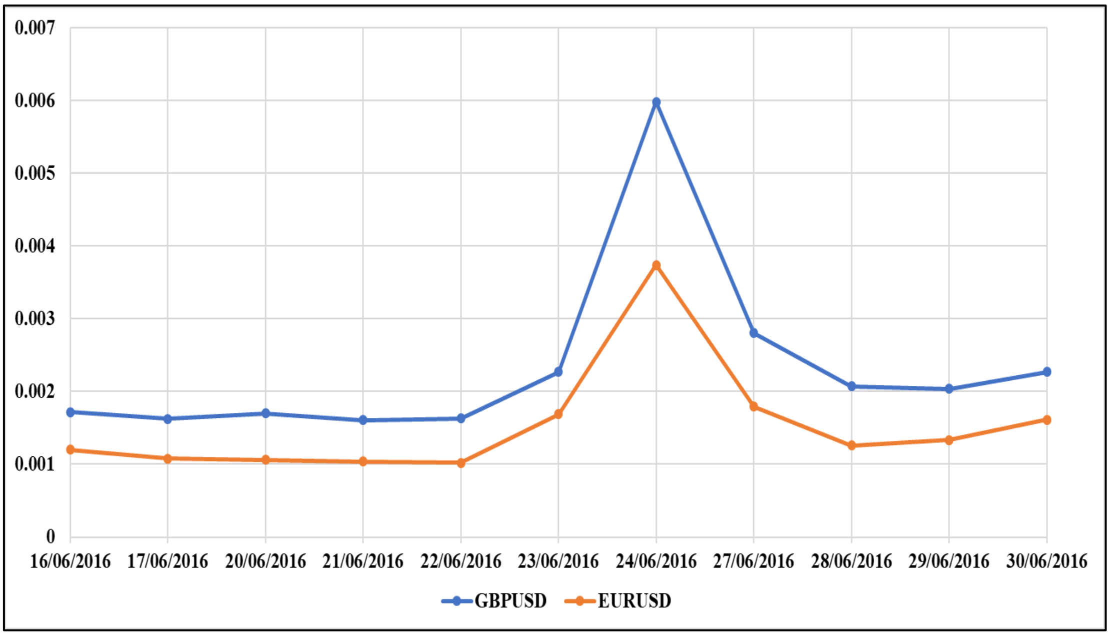

Figure 3.7 The daily instantaneous volatility $\boldsymbol{\sigma_{DC}}$ of *GBPUSD* and *EURUSD*. On 17/06/2016 (Friday), the trading hours were terminated at 22:00 (UTC). On 23/2016, we select the period from 00:00 to 22:00 (the period before the end of the voting). On 24/2016, the period was selected from 22:00 06/23/2016 to 22:00 06/24/2016 (the period of Part 2).

We summarise the testing results in table 3.4: the third column and the fourth column present the mean and median of σ_{DC} in Part 1 and Part 3; $Part2.\sigma_{DC}$ is the σ_{DC} of Part 2; the last two columns are the ratios $Part2.\sigma_{DC}/\langle Part1.\sigma_{DC}\rangle$ and $Part2.\sigma_{DC}/\langle Part3.\sigma_{DC}\rangle$. For both *GBPUSD* and *EURUSD*, the instantaneous volatility of Part 2 is much higher than Part 1 and Part 3. For *GBPUSD*, the ratios $Part2.\sigma_{DC}/\langle Part1.\sigma_{DC}\rangle$ and $Part2.\sigma_{DC}/\langle Part3.\sigma_{DC}\rangle$ are 3.4 and 2.61, respectively. For *EURUSD*, the ratios are 3.17 and 2.49, respectively. Obviously, in the period of Part 2, the volatility of *GBPUSD* and *EURUSD* was much higher than in the periods of Part 1 and Part 3. The results of evaluating instantaneous volatility prove the conclusion of observation 2.

Table 3.4 The measure of instantaneous volatility in the periods of three parts.

Name		Part 1	Part 3	$Part2.\sigma_{DC}$ $/\langle Part1.\sigma_{DC}\rangle$	$Part2.\sigma_{DC}$ $/\langle Part3.\sigma_{DC}\rangle$
GBPUSD					
	$\langle\sigma_{DC}\rangle$	0.00176	0.00230	3.40	2.61
	$Median(\sigma_{DC})$	0.00166	0.00217		
EURUSD					
	$\langle\sigma_{DC}\rangle$	0.00118	0.00150	3.17	2.49
	$Median(\sigma_{DC})$	0.00107	0.00147		

We repeat the second application under the threshold of 0.1%. The results are consistent with what we found in the second application in Section 3.5.2 (for details see Appendix B).

3.5.3 Benefits of measuring mRV

As discussed in Section 3.4.4, the mRV measure has been developed under the DC framework. DC is an alternative approach to record price movements. Instead of recording the transaction prices at fixed time intervals, as is done in time series, DC lets the data alone decide when to record the transaction. In practice, we measure the mRV of every observed sub-sequence. The sub-sequences are the result of the division process of a DC relative sequence (RS, see equation (3.3)). The period of a sub-sequence is passively determined by the observed extreme points of the two markets. Hence, we can precisely locate the time when we observe a significant value of mRV (for details see *Observation* 3 below). The precise time location of mRV allows the observation of significant values which may not be registered by Dsd (an example will be presented in *Observation* 4). Because the division process of a RS is not conducted using regular time intervals, the frequency of the sub-sequences varies over a given trading period, e.g. a trading day. The more observed EPs of the two markets there are, the more sub-sequences will likely be determined (we will discuss this point in *Observation* 5 below).

Observation 3: DC can precisely locate the exact times within which an extreme mRV occurred. This cannot be done under time series (TS).

As mentioned, at the beginning of section 3.5.3, using mRV can give a precise time location when there is a significant value of the relative volatility. In micro-market analysis, it is beneficial for analysts who need to monitor the relative volatility in high-frequency data. In contrast, the classical method Dsd cannot give the same precise timing because the measure of Dsd is based on sampling at fixed time intervals. So, the presence of a significant value can only be narrowed down to the particular fixed time interval in which it occurred in this case.

Fundamentally, since DC and TS are different frameworks for data sampling, there is no direct comparison between mRV and Dsd. To draw parallels with the mRV result in Figure 3.6, we calculated the Dsd between *GBPUSD* and *EURUSD* during the same time period (from 00:00 16/06/2016 to 24:00 30/06/2016). Based on equation (3.10), we sampled the TS data at 10 second time intervals (Δt = 10 seconds) and calculated the value of Dsd for every period of 10 minutes (τ = 10 minutes). Sampling at 10 second intervals allows the capture of patterns in high frequency data and then the period of 10 minutes for the calculation of Dsd permits the gathering of sufficient data points for an accurate calculated figure. In Figure 3.8 (2), we labelled the four significant Dsd values with their respective time intervals. Correspondingly, there were also four significant values of mRV. As shown in detail in table 3.5, for mRV, the periods of the four significant values were located within the time intervals associated with the significant values of Dsd. Specifically, the periods of the four sub-sequences are distinct and each is less than 1 minute.

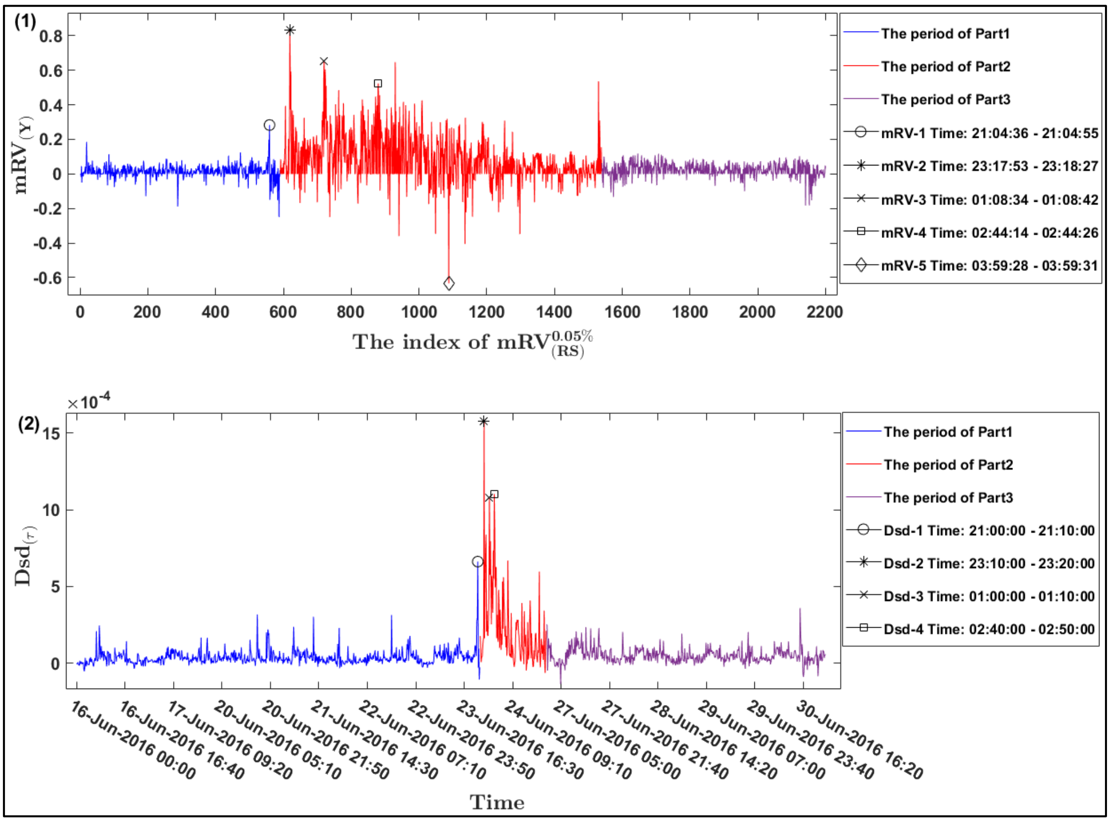

Figure 3.8 The measure of relative volatility in the periods from 16/06/2016 to 30/06/2016; Part 1(blue line): from 00:00 16/06/2016 to 22:00 06/23/2016 (140 hours); Part 2 (red line): from 22:00 06/23/2016 to 22:00 06/24/2016 (24 hours); Part 3 (purple line): from 00:00 06/27/2016 to 24:00 30/06/2016 (96 hours). (1) The sequence of $\boldsymbol{mRV^{0.05\%}_{(RS)}}$ between *GBPUSD* and *EURUSD*; $\boldsymbol{\theta}$ = 0.05%; the x-axis refers to the index of the sub-sequence; the y-axis refers to the value of $\boldsymbol{mRV}$. (2) The series of $\boldsymbol{Dsd^{\Delta t=10s}_{\tau=10min}}$ between *GBPUSD* and *EURUSD*; $\Delta\mathbf{t}$ = 10 seconds, $\boldsymbol{\tau}$ = 10 minutes; the x-axis refers to the timescale; the y-axis refers to the value of $\boldsymbol{Dsd}$.

For instance, as illustrated in Figure 3.9, the time interval of the highest Dsd (Dsd-2) was determined as being the 10 minute interval from 23:10:00 to 23:20:00. Whereas in contrast, we observed the sub-sequence of the highest mRV (mRV-2) was contained within the 34 second time interval that ran from 23:17:53 to 23:18:27, which was located within a small sub-interval of the time interval for Dsd.

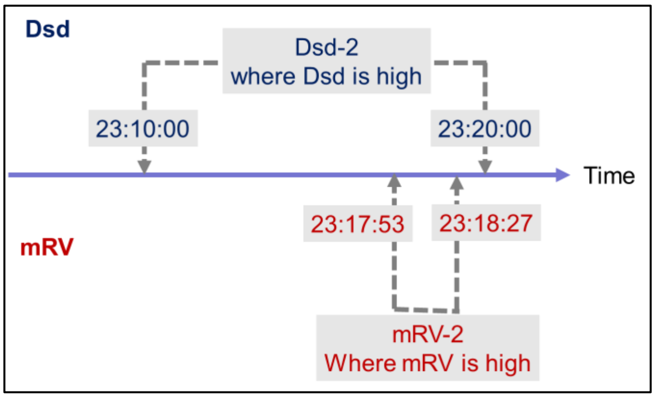

Figure 3.9 $\boldsymbol{mRV}$ shows a more precise period of high relative volatility between GBPUSD and EURUSD.

Table 3.5 The observations of relative volatility using the methods of $\boldsymbol{mRV}$ and $\boldsymbol{Dsd}$.

mRV (DC)			Dsd (TS)		
	Periods	Values		Periods	Values
mRV-1	21:04:36 – 21:04:55	0.2831	Dsd-1	21:00:00 – 21:10:00	0.0006624
mRV-2	23:17:53 – 23:18:27	0.8343	Dsd-2	23:10:00 – 23:20:00	0.001575
mRV-3	01:08:34 – 01:08:42	0.6531	Dsd-3	01:00:00 – 01:10:00	0.00108
mRV-4	02:44:14 – 02:44:26	0.5236	Dsd-4	02:40:00 – 02:50:00	0.001103
mRV-5	03:59:28 – 03:59:31	-0.6331			

Observation 4: Through mRV, DC enables us to observe changes in relative volatility that are not observable under Dsd in time series.

We observed a sub-sequence (which we labelled as mRV-5 in Figure 3.8 (1)) with the biggest negative mRV value from 03:59:28 to 03:59:31 24/06/2016. This sub-sequence only lasted for 3 seconds. The mRV-5 mentioned above records the lowest mRV value

(-0.6331) in the whole period observed in Figure 3.8 (1). Notice that we do not observe significant negative values in Dsd in Figure 3.8 (2). There are two possibilities why the significant negative value might not be reflected in the Dsd that we can take away from this case. Firstly, the 3 seconds of high relative volatility for *EURUSD* compared with *GBPUSD* (as indicated by mRV-5) would tend to be diminished by the rest of the recordings within the 10 minutes. Secondly, with a sampling period of 10 seconds, a 3 second spike might well not be even sampled in the first place. Thus, mRV enables us to observe changes in relative volatility between markets that cannot be observed by other means.

Observation 5: The frequency of determining sub-sequences depends on the intrinsic behaviour of the two markets' price changes.

As discussed at the beginning of this section, the period T of the sub-sequences obtained in order to calculate the values of mRV are passively determined by the observation of the extreme points of the two markets. Hence, the period T is intrinsically determined by the behaviour of the two markets' price changes, rather than being a fixed time interval pre-determined by the analysts. In Figure 3.8 (1), the majority of the sub-sequences are determined within the period of Part 2 (from 22:00 06/23/2016 to 22:00 06/24/2016 (24 hours)) as both two exchange rates were much more volatile in Part 2 (see *Observation* 2) compared to within the periods of Part 1 and Part 3. This illustrates how the approach facilitates the recording of more of the fine-grained behaviour during periods of high flux. In contrast, we cannot observe such a quantity of data in TS as the data was collected using a fixed time interval. Specifically, in table 3.6, there were 949 sub-sequences confirmed in Part 2, which accounted for 43%

of the total sub-sequences. However, during the same period, 144 Dsd values were calculated under TS, which only accounted for 9% of the total observations.

Table 3.6 The number of observations in the periods of three parts under the methods of $\boldsymbol{mRV}$ and $\boldsymbol{Dsd}$.

	Number of observations (Percentage of total, %)	
	mRV	Dsd
Part 1	590 (27%)	840 (54%)
Part 2	949 (43%)	144 (9%)
Part 3	661 (30%)	576 (37%)
Total	2200	1560

3.5.4 The relationship between the threshold and the average period of the sub-sequence

In Section 3.4.4, we discussed the merits of not requiring a pre-determined time interval for measuring mRV. The period T of the sub-sequence is passively determined by the observation of the extreme points of the two markets. However, how long is the period of a sub-sequence before being terminated in practice? Is there a relationship between the threshold's magnitude and the period of the sub-sequence? Hence can we obtain a degree of control over the period of a typical sub-sequence through intelligent selection of the threshold? We implemented back-testing to examine the relationship between the average period of the sub-sequence $\langle T(Y) \rangle$ and the size of the threshold θ. As discussed in Section 3.5.1, Glattfelder et al. (2011) developed 12 scaling laws under the DC framework. The DC scaling law 10 gives an estimation of the average period of a DC trend given a DC threshold. Following their work, we discovered a scaling law between the average period of a sub-sequence $\langle T(Y) \rangle$ and the size of the threshold θ.

As shown in table 3.7, we selected four pairs of exchange rates over four years (from 2015 to 2018). The experiment selected 100 thresholds to calculate the $\langle T(Y) \rangle$ over four years, ranging from 0.005% to 0.104% with the values increasing in increments of 0.001%. The raw data type is tick-by-tick. Table 3.7 summarises the details of the data sources for the back-testing.

Table 3.7 Specification of the back-testing. The back-testing utilises 24 hours of tick-by-tick data during the weekdays from Monday 00:00:00.000 to Friday 22:00:00.000.

Data Type	Tick-by-tick
Periods	24 hour weekdays, from 2015 to 2018
DC relative sequences	$RS_{S^{\theta}_{GBPUSD}, S^{\theta}_{EURUSD}}$, $RS_{S^{\theta}_{USDJPY}, S^{\theta}_{EURUSD}}$, $RS_{S^{\theta}_{AUDUSD}, S^{\theta}_{EURUSD}}$, $RS_{S^{\theta}_{USDCAD}, S^{\theta}_{EURUSD}}$,
Thresholds	100 thresholds from 0.005% to 0.104% with an increment of 0.001%

Following equation (3.11) in Section 3.5.1, we have a new 'period-threshold' scaling law between the average period of a sub-sequence $\langle T(Y) \rangle$ and the size of threshold θ:

$$\langle T(Y) \rangle = \left(\frac{\theta}{C_{T,\theta}}\right)^{E_{T,\theta}}, \tag{3.12}$$

where $\langle T(Y) \rangle$ indicates the average period of the sub-sequence related to a certain threshold θ, and $E_{T,\theta}$, $C_{T,\theta}$ are the parameters of the scaling law. Figure 3.10 illustrates the log-log chart of the $\langle T(Y) \rangle$ versus the DC threshold θ in the four pairs of exchange rates. Under logarithmic scaling, there are apparent linear relationships between $\langle T(Y) \rangle$ and θ crossing the four pairs of exchange rates. For example, the blue dot-line indicates the scaling law of *GBPUSD* and *EURUSD*.

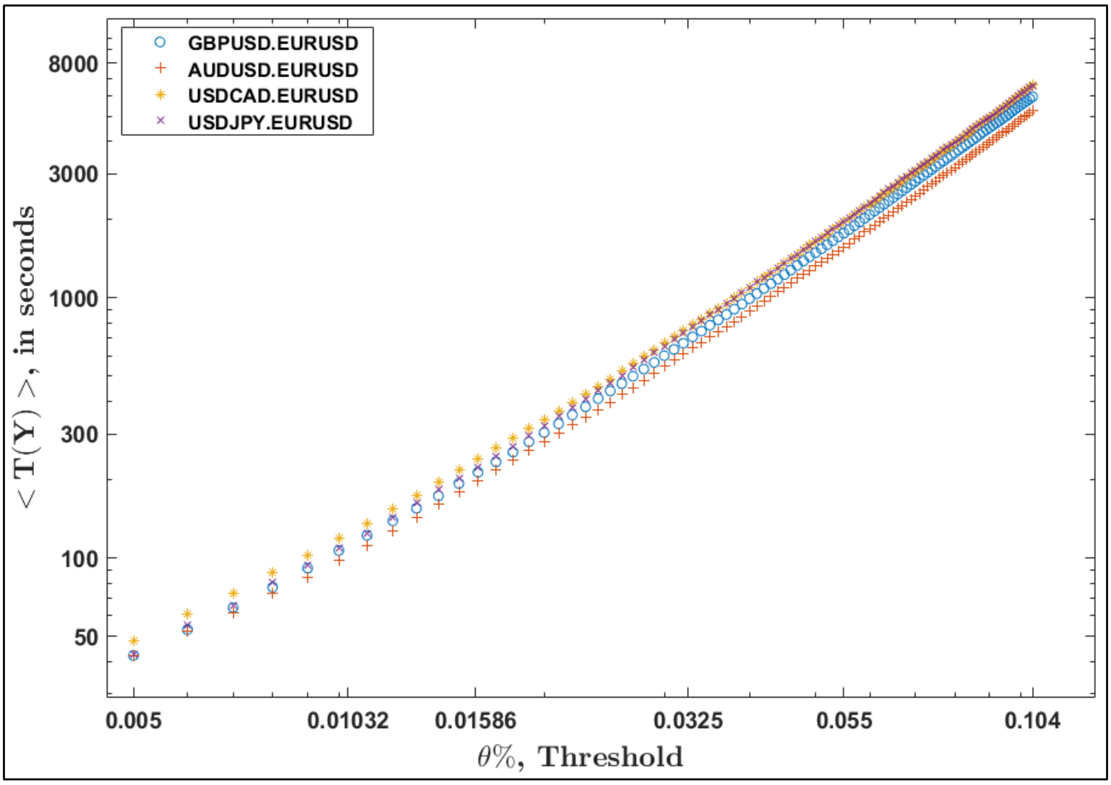

Figure 3.10 The scaling law of the average period of a sub-sequence related to the size of the DC threshold. On the horizontal axis, the thresholds are chosen from 0.005% to 0.104% with an incremental step of 0.001%. On the vertical axis, the unit of $\langle \boldsymbol{T}(\boldsymbol{Y}) \rangle$ is seconds. The estimated scaling law parameters are summarised in table 8.

Table 3.8 The 'period-threshold' scaling law: the parameters

DC relative sequence	$C_{T,\theta}$	$E_{T,\theta}$	R^2
$RS_{S^{\theta}_{GBPUSD}, S^{\theta}_{EURUSD}}$	6.80147E-06	1.711108	0.99665483
$RS_{S^{\theta}_{USDJPY}, S^{\theta}_{EURUSD}}$	6.92514E-06	1.739012	0.997322782
$RS_{S^{\theta}_{AUDUSD}, S^{\theta}_{EURUSD}}$	6.72848E-06	1.68019	0.996119037
$RS_{S^{\theta}_{USDCAD}, S^{\theta}_{EURUSD}}$	6.20971E-06	1.698149	0.996568535

Under the DC framework, the data-driven approach passively determines the time interval of the sub-sequence based on observed extreme points of the two markets. On the other hand, unlike time series which uses a fixed time interval, there is no explicit timeline for the termination of a sub-sequence. In other words, if there is no upcoming DC data, we can't terminate the current sub-sequence. The 'period-threshold' scaling law gives a relationship between $\langle T(Y)\rangle$ and θ. This gives us a basic estimate for the average period of the sub-sequence given the size of the threshold. However, in practice, there is no explicit guarantee between the average period and the actual period of a sub-sequence. For example, for the sub-sequence of $RS_{S^{\theta}_{GBPUSD},S^{\theta}_{EURUSD}}$, the $\langle T(Y)\rangle$ is approximately 1493 seconds (or 25 minutes) if the threshold is specified as 0.05%, but using the same size of the threshold, the $\langle T(Y)\rangle$ was 40 seconds in the 24 hours after the Brexit referendum. By changing the threshold, the 'period-threshold' scaling law, allows the analyst control of the typical time period when the market is behaving normally. In future work, we would like to investigate the effect of the threshold on the deviation of the time period from the average values given by the scaling law in order to obtain indications as to the accuracy of the results from the 'period-threshold' scaling law.

3.5.5 Discussion on Experiments

In Section 3.5.1, we calculated the relative volatility using the approaches of $Dsd^{\Delta t}_{\tau,A-B}$ and $\langle\sigma_{DC(RS_{S^{\theta}_A,S^{\theta}_B})}\rangle_{\tau}$. The Spearman correlation test indicated high correlation for the measure of relative volatility between the two approaches. The correlation coefficients reached average values of 0.795, 0.836 and 0.864 under the periods of daily, weekly and monthly windows. This means mRV agrees moderately with the relative volatility measure from the time series methodology. In Section 3.5.2, the results of monthly relative volatility indicated that *EURUSD* was relatively more volatile than *GBPUSD* from 2012 to 2015. Starting from 2016, *GPBUSD* was exceedingly more volatile than *EURUSD* after the unexpected result of the Brexit referendum. Throughout the long-term back-testing, we observed that the significant mRV changes corresponded to the major historical events during that period. The second application summarises two observations in high-frequency data. The first observation concluded that *GBPUSD* was far more relatively volatile then *EURUSD* right after the time of the Brexit vote. For the second observation, we noted a substantial number of sub-sequences in Part 2, which accounted for 43% of the total sub-sequences in 11 trading days. This observation indicates that *GBPUSD* and *EURUSD* were both more volatile in Part 2 compared to Part 1 and Part 3. In Section 3.5.3, compared with the time series method Dsd, we illustrated that DC can precisely locate the exact times within which an extreme mRV occurred (*Observation* 3). One weakness of the DC approach is that we don't know when the current sub-sequence will terminate. This is a disadvantage of the data driven approach; if there is no upcoming DC data, we can't terminate the current sub-sequence. This is only a problem during times with limited amounts of DC events. In Section 3.5.4, we proposed the 'period-threshold' scaling law to estimate the average period of a sub-sequence $\langle T(Y) \rangle$ given a certain threshold. In practice, the deviation between the average value $\langle T(Y) \rangle$ and the actual value $T(Y)$ could be significant especially during major events. Nevertheless, this new scaling law gives observers a basic guide to inform their influence on the average period of the sub-sequence when they select the size of the threshold.

3.6 Conclusions

Directional change is an alternative way of sampling the price changes to form a DC sequence based on a data-driven process. Under the DC framework, this study opens a new path in studying the relative volatility between two markets. The DCRV approach evaluates the relative volatility based on the pre-determined period T. We have shown (in Section 3.4) that the DCRV measure is sensitive to the size of T. Also, we introduce mRV, a data-driven measure of relative volatility. To develop mRV, we introduce the DC relative sequence (Section 3.4.2). It is a sequence which chronologically combines two markets' DC sequences. In practice, the termination of the current sub-sequence depends on the identity of the upcoming extreme point (EP). Hence, the period T is dynamically defined by the length of the sub-sequence. In Section 3.5.1, the correlation test proved that mRV has a similar conclusion to the time series method in measuring relative volatility. Also, the correlation test indicates a positive relationship between the correlation coefficient and the period τ in that the longer the selected period τ, the higher the correlation coefficient obtained. In Section 3.5.2, we executed back-testing in evaluating relative volatility between *GBPUSD* and *EURUSD*. In the long historical period from 2012 to 2018, significant changes in mRV corresponded to major historical events. We also tested the relative volatility in high-frequency data from 16/06/2016 to 30/06/2016 such that the periods span the five working days before and after the Brexit referendum day on 23/06/2016. Specifically, we separated the 11 trading days into three parts: (1) Part 1: from 00:00 16/06/2016 to 22:00 06/23/2016 (140 hours of total trading hours); (2) Part 2: from 22:00 06/23/2016 to 22:00 06/24/2016 (24 hours following the vote of Brexit referendum); (3) Part 3: from 00:00 06/27/2016 to 24:00 30/06/2016 (96 hours). The advantage of the data-driven process is that it was possible to locate the sub-sequences which showed the highest and the lowest mRV. In Part 2, we observed significant changes in mRV, which indicated the extreme volatility of *GBPUSD* versus *EURUSD*. In Observation 2, by comparing the number of sub-sequences between the three parts, we concluded that *GPBUSD* and *EURUSD* were both more volatile in Part 2 than Part 1 and Part 3.

To conclude, under the DC framework, we developed a new method to measure relative volatility by using mRV which can be narrowed down to a precise time location in times of extreme values of relative volatility. This cannot be done under time series (*Observation* 3, details see Section 3.5.3). We believe, for instance, that mRV can give an alternative approach to monitor in near real time the relative volatility in micro-market activities when analysts consider high-frequency data or tick data.

Chapter 4. Jumps

In recent decades, significant price turbulence has become more common in the financial markets. Past financial crises have emphasized the risk of instantaneous extreme price changes, so-called jumps. In fact, a substantial amount of the empirical research supported that the existence of jumps not only accompanied major financial crises but are also associated to different major news events, such as economic data, political crisis, natural disasters, etc. Being able to identify jumps could give a better understanding about the jumps' behaviours and monitor the response of jumps to new information. In this chapter, we propose a novel approach to identify jumps based on a data-driven approach; that of Directional Change (DC).

Compared with the data recording in time series, DC offers an alternative approach of sampling the price movement. In time series, a jump (TSJ) is a different source of risk compared to the risk of continuous volatility in the asset pricing model (Lee and Mykland, (2008)). The classical method identifies jumps through a model-based approach (for details see section 2.3). However, in DC, the jumps are identified by a data-driven approach. About the contribution of this chapter, we proposed the definition of jumps in DC (the data-driven approach) and implemented the back-testing of detecting DCJs from the selected datasets; we compared both the data-driven approach (DC) and model-based method (TS) for the ability to detect jumps, and the results indicate that the two approaches complement each other in identifying jumps in Forex. According to our back-testing, the results indicate that both approaches are effective in detecting jumps. The two approaches both found common jumps. Some TS jumps were not found as DC jumps and vice versa. Also, we examine the relationship between major economic events and the jumps under both methods. The outcomes demonstrated that some jumps followed the economic events. DCJ can give precise information about the behavior of jumps in terms of size, direction, and quantity. According to our back-testing, DC jumps offer the benefit of more fine-grained analysis in the monitoring of jump behavior in high-frequency data.

The remainder of this chapter is organised as follows. Section 4.1 introduces the motivation of the study of DC jumps. Section 4.2 presents the process of DC data summary. Section 4.3 introduces the time-adjusted return sequence (TR sequence). In section 4.4, we will introduce the definition of DC jump. The experimental design and results will be given in section 4.5 and section 4.6. The results of the experiment are discussed in section 4.7. In Section 4.8, we present our conclusion.

4.1 Introduction

In the financial markets, one of the core topics is to understand the 'true' value of the asset price. Some analysts believe that the efficient market hypothesis suggests that asset prices should react to all the relevant news (Bodie et al. (2013)). Others think that the asset prices may not totally coincide with the fundamentals (Levy and Post (2005)). From a practical perspective, researchers have been focussing on various factors to improve the asset price model. One subject of study is to understand how markets react to the information contained in news bulletins (Andersen et al. (2007); Jiang et al. (2011)). Some researchers emphasise that some sensitive news may cause price jumps that have an impact on risk management and asset pricing (Chatrath et al (2014); Jurdi (2020)). Erdemlioglu and Gradojevic (2019) conclude that there are two challenging issues for studying jump behaviours: (1) the difficulty of identifying jumps; (2) analysing the determinants of jumps.

In time series analysis (TS), the jump is a different source of risk in addition to the risk of continuous volatility in the asset pricing model. The classical method detects the jumps through the model-based approach. Barndorff-Nielsen and Shephard (2004) introduced bipower variation to estimate the instantaneous volatility. Jumps are determined through filtering out the instantaneous volatility from the realised variance. Barndorff-Nielsen and Shephard (2004) first presented their method to locate the jumps at a daily frequency. Lee and Mykland (2008) detected the jump arrival times and size in the intraday timeframe.

Detecting jumps is also important for researchers to measure the market reactions to different news events through evaluating the jump behaviour. The events can generally be separated into two categories: (1) scheduled events, such as the scheduled macroeconomic announcements; (2) unscheduled events, such as natural disasters. In general, the unscheduled events may have more impact on jumps presenting than the scheduled events as participants are highly sensitized to the uncertain risk. However, some scheduled events may cause extreme turbulence in the financial markets. For instance, the unexpected result of the Brexit referendum produced significant shocks spanning various assets.

As introduced in section 2.3, Barndorff-Nielsen and Shephard (2004) first presented their method to locate the jumps at a daily sampling frequency. Lee and Mykland (2008) detected the jump arrival times and size in the intraday timeframe. The classical method identifies jumps based on a fixed time interval such that jump behaviour may depend on the length of the pre-determined time interval chosen. Many jumps may present randomly as unscheduled events that can occur at any time. For example, a jump may start at any time point within a fixed time interval and in addition, a jump may exist spanning the boundary of the current time interval. Fundamentally, the jump behaviours are the reactions of the participants' trading actions. However, one can never know the participants' trading behaviours to infer the timing of the jumps' arrivals.

As discussed above, the method of identifying jumps is developed under the framework of time series. Under the DC framework, we introduce a data-driven approach to detect jumps, which is different to the classical method in that it does not rely on a pre-defined model. DC is a concept for sampling the financial market data (Guillaume et al. (1997)). Tsang et al., (2017) developed the DC indicators to summarise the features of the price movements. They recognized that some features observed by DC indicators may not be discovered in time series analysis. Encouraged by their works, the DC jump (DCJ) is defined based on the DC indicator, time-adjusted return (TR). In DC, we detect the presentence of the jump based on the TR of the DC trend. Specifically, the existence of a DCJ is judged by two factors of the DC trend: (1) significant price changes; (2) a short time period over which this occurs. These two requirements are quantified by the TR. The formal definition of the DCJ will introduce in section 4.4.

Theoretically, the jump in DC is a different concept compared to the jumps in TS. They cannot directly compare with each other. In DC, what is termed a DCJ is an event whereby the price has changed by a significant magnitude in a short period. Since the DC approach of identifying jumps is different to the time series method, there are some questions that follow: could we detect jumps under DC? Can we find the unique jumps under both methods? Or are the jumps detected under DC independent of the jumps detected under time series? Are DCJs associated to news events? Can we detect DC jumps during unscheduled events? And how can analysts benefit from the observed DCJs? We will answer these questions in the following sections.

4.2 The DC data summary

In DC, we judge the presence of jumps based on DC trends. Specifically, we consider two factors to determine a DC jump: (1) significant price changes; (2) a short period of the DC trend. Tsang et al. (2017) introduced the indicator of the time-adjusted return of DC (we call this TR for short). TR not only measures the magnitude of total price travel of the DC trend (the TMV), but also evaluates the periods of the price movement to complete the DC trend. Section 2.2 introduced a formal definition of the TMV and TR. Here, we would like to review the definitions in a practical way.

For a certain financial instrument, a buyer and a seller made a deal at a certain price which is then recorded as the raw transaction price with a confirmed timestamp. In quantitative analysis, the recorded raw price is also called tick data. Over the period of trading activities, we record a sequence of tick data in irregular time. Normally, analysts summarise the raw data on a regular time interval and the result is called time series data. Given a pre-determined time scale like an hourly time interval, we record the transaction prices at the end of every hour.

DC is an alternative way of data sampling. As introduced in section 2.2, DC records the reversal point when there is a significant opposite price change from the last downtrend/uptrend. In practice, a significant change is defined by the threshold on a percentage scale which is given by the researcher. Thus, DC summarises the original transaction data as a series of alternative uptrends and downtrends which we call the DC trends. The reversal point between two DC trends is defined as the extreme point (EP). An EP is a couple which comprises a transaction price ($EP.p$) and a timestamp ($EP.t$):

$$EP \ = (EP.t, EP.p). \tag{4.1}$$

Tsang et al. (2017) introduced useful DC indicators to be used in the analysis of price movements. We will introduce the indicators which will be used for DC jump detection in this chapter.

As shown in Figure 4.1 below, the upward DC trend is the connection from EP_1 to EP_2. The price distance travelled of the DC trend is measured by the total price movement (TMV) which is the percentage change normalized by the threshold θ. Hence, we can obtain the price distance from EP_1 to EP_2:

$$TMV = \frac{EP.p_2 - EP.p_1}{EP.p_1 \times \theta}, \tag{4.2}$$

Also, the time distance between EP_1 and EP_2 is given by $T = EP.t_2 - EP.t_1$. Tsang et al. (2017) proposed the time-adjusted return of the DC trend (TR), denoted by R_{DC}, to measure the price change over time. In the example of Figure 4.1, we can calculate the TR by:

$$R_{DC} = \frac{|EP.p_2 - EP.p_1|}{EP.p_1 * T} = \frac{|TMV| \times \theta}{T}. \tag{4.3}$$

Note, the terminal time of the R_{DC} above is defined by $ET(R_{DC_2}) = EP.t_2$. In practice, TR measures the 'speed' of forming the DC trend, which considers both the price change and the time taken. TR is the fundament for us to judge the presence of a DC jump as we need to consider both the significant price change and the (short) time period of the DC trend.

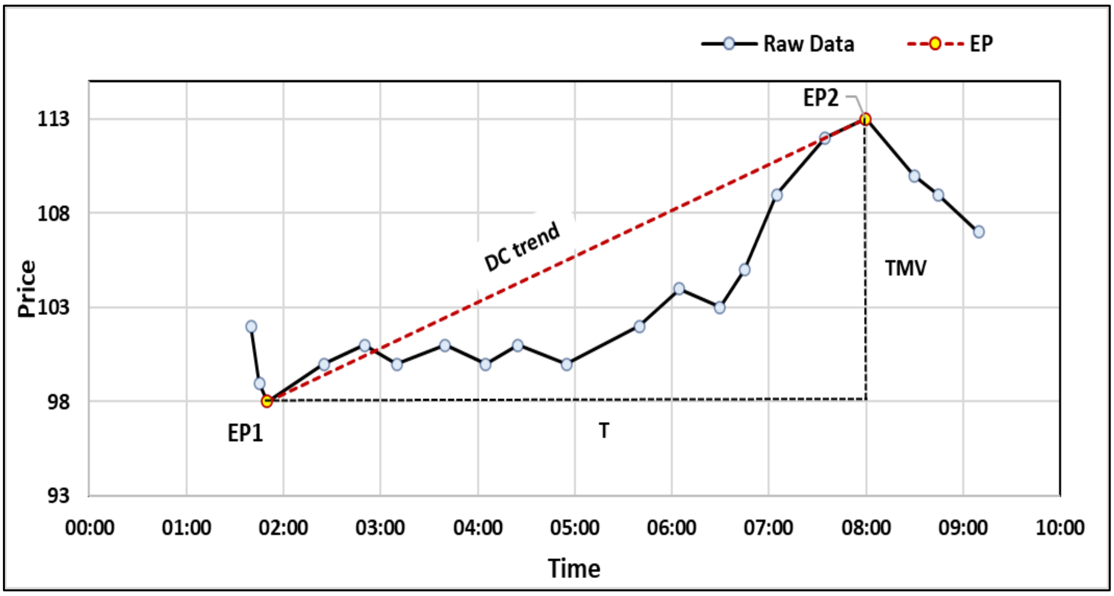

Figure 4.1 A hypothetical example of the data summary in DC.

4.3 The TR sequence

This section will introduce the TR sequence of a single market. In section 2.2, equation (2.4) defines a DC sequence which comprises a series of extreme points of the market A, i.e., $S_A^\theta = (EP_1, EP_2, \ldots, EP_n)$. In Figure 4.2, we plot the DC trends of market A by connecting the EPs. For each DC trend, we calculate the TR through equation (4.3); the

R_{DC_i} is the TR in i^{th} DC trend. Thus, given a DC sequence of a market, we can generate the TR sequence using equation (4.3).

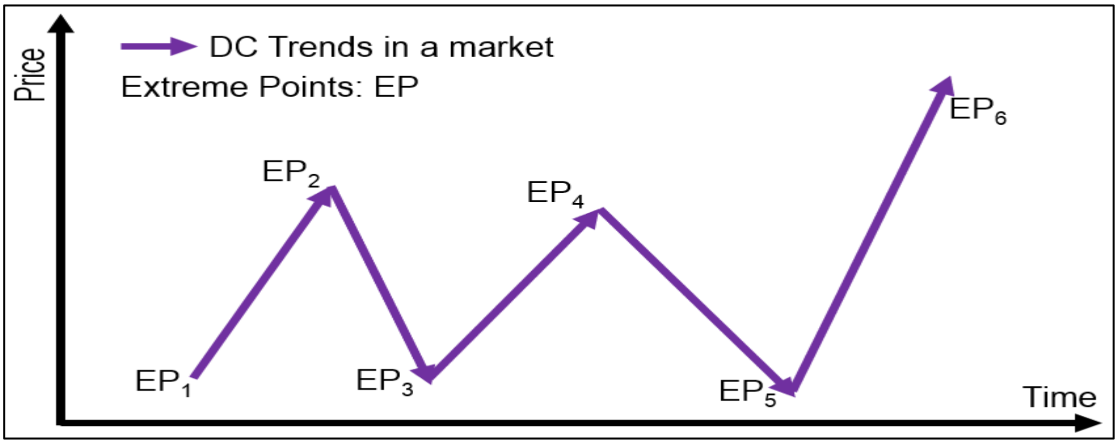

Figure 4.2 An example of DC trends in a market. The chart illustrates a series of 5 DC trends formed by the 6 EPs.

A TR sequence, denoted by S_{TR}, is a finite sequence of TRs:

$$\left(R_{DC_1}, R_{DC_2}, \dots, R_{DC_n}\right), \tag{4.4}$$

where R_{DC} is obtained through equation (4.3) and n equals the total number of DC trends from a DC dataset.

In Figure 4.3, there are four EPs (from EP_3 to EP_6). Given the four EPs, we form three DC trends, and obtain the time intervals of the three DC trends (T_4, T_5, and T_6). We calculate the TRs of the three DC trends (from TR_4 to TR_6). At the bottom of Figure 4.3, we present a segment of the TR sequence.

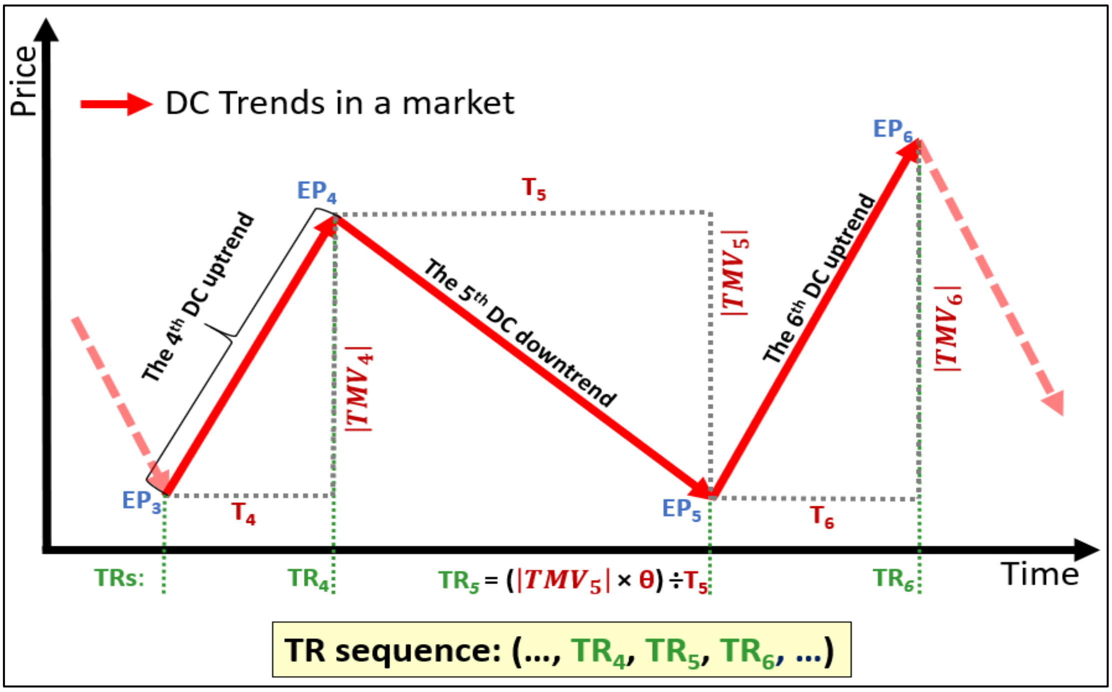

Figure 4.3 Features of a $\boldsymbol{TR}$ sequence: (1) Threshold, $\boldsymbol{\theta}$; (2) Extreme Point, $\boldsymbol{EP}$; (3) The period of the DC trend, $\boldsymbol{T}$; (4) The time-adjusted return, $\boldsymbol{TR}$; (5) the absolute value of $\boldsymbol{TMV}$, $|\boldsymbol{TMV}|$. For each DC trend, we generate the $\boldsymbol{TR}$, e.g., $\boldsymbol{TR_5} = (|\boldsymbol{TMV_5}| * \boldsymbol{\theta})/\boldsymbol{T_5}$. At the bottom of this chart, we present a segment of the $\boldsymbol{TR}$ sequence.

4.4 DC Jump ($\boldsymbol{DCJ}$)

As discussed in Section 4.1, a DC Jump ($\boldsymbol{DCJ}$) is an event such that the price has changed by a significant magnitude in a short period. In other words, DC judges the presence of a jump according to two requirements: (1) significant price changes; (2) a short time period over which this occurs. As introduced in Section 4.2, TR measures the time-adjusted return of the DC trend. Hence, we detect the existence of a DCJ based on the TR of the DC trend; the significance of a TR is judged with reference to the historical TRs, e.g., whether the TR is above 95% of the historical observations.

4.4.1 The empirical cumulative distribution function

We test the significance of the TR against the empirical cumulative distribution function. Given a chosen historical TR sequence, we sort the elements of the TR sequence in ascending order of magnitude $(TR_1, TR_2, \dots, TR_{n-1}, TR_n)$, then the empirical cumulative distribution function of the TRs is the function defined as:

$$F(x) = n^{-1} \sum_{i=1}^{n} 1_{\{TR_i \leq x\}} \tag{4.5}$$

where $1_{\{TR_i \leq x\}}$ is an indicator function[10]. Given the TR sequence we obtain the empirical cumulative distribution ($\boldsymbol{ECD}$) through equation (4.5).

4.4.2 The definition of *DCJ*

A DC Jump ($\boldsymbol{DCJ}$) is an event of the DC trend such that the price has changed by a significant magnitude in a short period. Under the DC framework, a DCJ is parameterised by the TR which measures the absolute return per time unit in the DC trend. Therefore, a DCJ is detected when we detect a significant magnitude for the TR in the DC trend. Given the ECD of the historical TR sequence, we quantify a significant result by s (in percentage terms); the DCJ is determined to have occurred when the TR of the DC trend is above s% of the ECD. On a practical level, the value of s is decided by the analysts. This motivates the following formal definition:

[10] Details about ECDF see van der Vaart (1998).

Definition: DCJ

A DCJ is an event of the DC trend from one extreme point EP_{i-1} to the next EP_i when the R_{DC_i} is greater than s% of the ECD of the historical S_{TR}.

For instance, given $s = 95\%$, a DCJ is determined when the R_{DC} is above 95% of the ECD of the historical S_{TR}.

4.4.3 The practice of detecting DCJ

We first collect the historical TR sequence from a dataset, then we form the ECD of the historical S_{TR} through equation (4.5). Given a certain value of s (e.g., 95%), we need to find the indices i such that R_{DC_i} is greater than s% of the ECD of the historical TR sequence. For instance, let s^* denote this value of R_{DC_i}, then a DCJ is determined when the R_{DC} of the DC trend is greater than s^*:

$$R_{DC} > s^*. \tag{4.6}$$

In figure 4.4, given $s = 95\%$, the DCJ is identified when the R_{DC} is above 95% of the historical S_{TR}, and we obtain $s^* \approx 0.40783\%$. The upward jump starts from the EP_2 (Timestamp: 14:59:22.552; Price: 1.112305) to the end of the EP_3 (Timestamp: 15:00:16.574; Price: 1.11719). Note, as the time provided for tick data is accurate to the millisecond, it is impossible to use the timestamp to plot the chart. Instead, we use the index of the tick data as the horizonal axis.

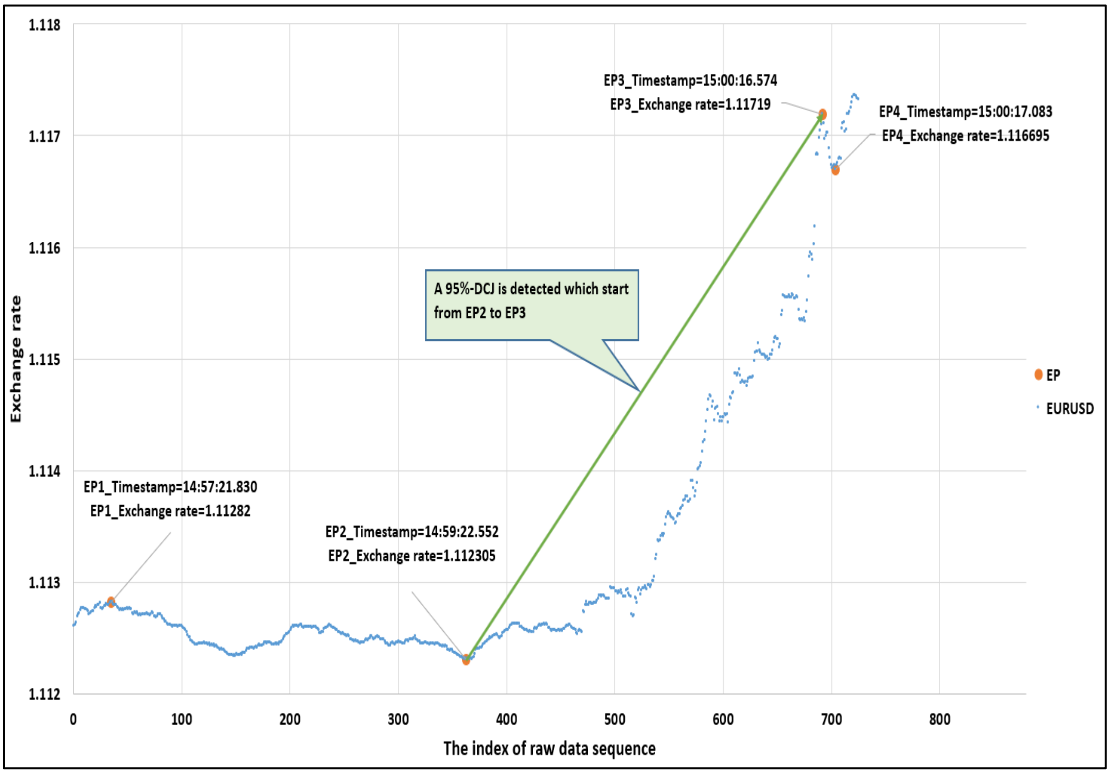

Figure 4.4 Given $\boldsymbol{s}$ = 95%, an upward $\boldsymbol{DCJ}$ is detected in the DC trend from EP2 to EP3, where the $\boldsymbol{R_{DC}}$ = 0.48828% and $|\boldsymbol{TMV}| = \mathbf{14.65432}$. The blue dot-points are the exchange rates of *EURUSD* (tick data) from 14:57:13 to 15:00:59 in 03/03/2020. The vertical axis is the exchange rate of *EURUSD*. The horizontal axis is the index of the data sequence of *EURUSD* in chronological order. The orange dot-points are the $\boldsymbol{EP}$s ($\boldsymbol{\theta} = \mathbf{0.03}$%).

4.5 Experimental design

4.5.1 The dataset

Table 4.1 summarises the data sources, original frequency, and the period. The samples incorporate three exchange rates including *EURUSD*, *GBPUSD*, *USDJPY*, *USDCAD* and *AUDUSD*. The exchange rate is the value of one country's currency relative to the currency of another country. The sample period is the 6 years from 2014 to 2019. We consider 24-hour trading tick data from Monday 00:00:00.000 to Friday 22:00:00.000 (UTC).

Table 4.1 Description of the raw datasets used in the experiment

Asset	Source	Frequency	Trading hours	Period
EURUSD *GBPUSD* *USDJPY* *USDCAD* *AUDUSD*	Dukascopy[11]	Tick-by-tick	24 hours a day	01/01/2014 – 31/12/2019 (six years)

4.5.2 The experimental design of DCJ detection

To implement the DCJ detection, we separate the dataset into two groups: the historical dataset and the testing dataset. In the first group, we select the historical dataset of an exchange rate (e.g. *EURUSD*) to acquire the s^*. Then, we detect the DCJs in the second dataset. We use the 'moving window' approach to implement the experimental process. Given the 6-year dataset, we determine the window size W = 260 trading days (the total number of trading days per year in average, according the timeframe of Datastream Eikon), and the unit of the window movement M = 1 trading day. The whole process is

[11] Dukascopy Bank is a Swiss online bank which provides high quality market data in different forms. https://www.dukascopy.com/swiss/english/home/

illustrated in Figure 4.5: (1) calculate the TRs in the past 260 trading days in the 1st window; (2) form the ECD of the observed TR sequence; (3) given the s = 0.99, we acquire the s^* of the historical ECD; (4) detect the DCJs in Day 261 based on the obtained s^*.

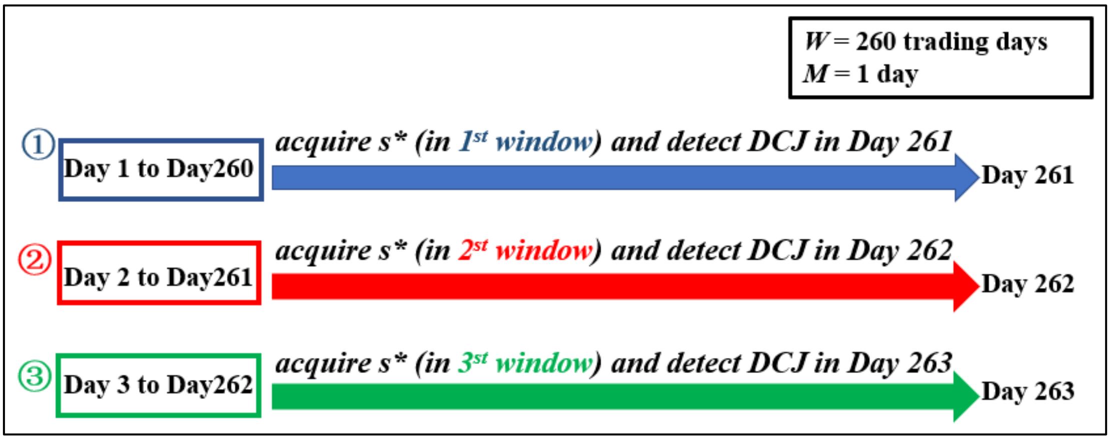

Figure 4.5 An example of the DCJ detection: (1) the window size W = 260 trading days; (2) the window movement M = 1 day. In the 1st window, we acquire the $\boldsymbol{s}^*$ of the historical ECD of the TR sequence, then detect the DCJ on Day 261.

Overall, the experiment aims to detect the DCJs of the three exchange rates in the years from 2015 to 2019. Following the illustration of Figure 4.5, the process keeps detecting the DCJs in the current trading day based on the s^* obtained by the previous window (the past 260 trading days).

4.5.3 The relationship between jumps and the scheduled economic events

Investigations of the relationship between jumps and the economic events have been conducted by many researchers. Andersen et al. (2003) discussed the influence of economic news on asset returns. Lee and Mykland (2008) observed that the jumps are related to the news; for individual stocks, they concluded that the jumps are not only associated with the regular company's news but also connected with unscheduled news. Lahaye et al. (2011) presented a detailed analysis of the relationship between jumps and scheduled economic events for different asset classes. They evaluated the possibility of the identified jumps being associated with the US economic data. Compared to the jumps in equities and bonds, Lahaye et al. (2011) concluded that the exchange rate jumps have a lower connection with these types of economic events. In time series (TS) analysis, the method of identifying TS jump has presented in Section 2.4.1.

We evaluate the relationship between the jumps and scheduled economic events. For each exchange rate, we selected the economic data of the two relevant countries (or economic area). For the U.S., the federal government and private institutions release a substantial quantity of economic data at regular intervals. Fleming and Remolona (1997) summarised 21 major macroeconomic announcements. Baumohl (2012) argued that there are more than 40 economic indicators (data) released every month. Because our aim is to examine the DCJs associated with economic events, we only select major macroeconomic data. Also, the exchange rate value is directly affected by the interest rate decisions from the central banks. Hence, we choose the major economic data based on the most pressing concerns of the central banks. According to the Federal Reserve (FED) monetary policy[12], the two most important economic goals of their policy are maximum employment and stable prices. Therefore, we focus on the economic data in terms of the three major aspects: GDP, consumer price, and employment. Table 4.2 gives information about the selected macroeconomic announcements. For *EURUSD*, *GBPUSD* and *USDJPY*, we consider the four countries' GDP announcements (or

[12] FED Monetary Policy: https://www.federalreserve.gov/monetarypolicy.htm

economic area), consumer price index and interest rate decisions as economic indicators. We also select the U.S. retail sales and nonfarm payroll which are the FED's most important economic indicators.

This section will answer two questions in the relationship between DCJs and major economic events:

(1) What percentage of the jumps were followed major economic events (MEEs)?

(2) How many MEEs were followed by jumps?

We answer the two questions above through the results of our back-testing in the following sections.

Table 4.2 Information regarding the selected economic events

Announcement	Variable name	Frequency
U.S. Fed Funds Interest Rate Decision	FED_IRD	8 times per year
The European Central Bank Interest Rate Decision	ECB_IRD	8 times per year
Bank of England Interest Rate Decision	BoE_IRD	8 times per year
Bank of Japan Interest Rate Decision	BoJ_IRD	8 times per year
Bank of Canada Interest Rate Decision	BoC_IRD	8 times per year
Bank of Australia Interest Rate Decision	BoA_IRD	Not regular frequency
GDP	CountryName_GDP	Quarterly
Consumer Price Index	CountryName_CPI	Monthly
US Retail Sales	US_RS	Monthly
US Nonfarm Payrolls	US_NFP	Monthly

Note, For AUDUSD, the number of CPIs is 24.

As discussed in Section 4.1, the unscheduled events may cause more jumps presenting because the participants are highly sensitive to uncertain risks. We will discuss the two case studies about the detected DCJs associated to unscheduled events in the following sections.

4.6 Results

This section presents the results of the experiments. We start with the data description of the detected DC jumps over the years from 2015 to 2019. Then, we will show the three examples of the detected DCJs and TSJs after the announcement of the scheduled events. The next section will present the identified DCJs associated to the scheduled events and unscheduled events. The final part will study the detected common jumps and the unique jumps between the DCJs and TSJs.

4.6.1 An overview of detected DCJs

Table 4.3 (a) below shows a statistical summary of the determined DCJs in the five exchange rates in the periods from 2015 to 2019. For the back-testing we selected the threshold $\theta = 0.1\%$ and $s = 0.99$. Note: (1) the first row N(DCs) is the total number of DC trends over 5 years from 2015 to 2019; (2) N(days) is the total number of trading days during the 5 years; (3) N(DCJ-days) counts the number of the days which include at least one DCJ, a DCJ day; (4) the fourth row is the probability of a day including at least one DCJ over the total trading days, P(DCJ-day) = (N(DCJ-days) / N(days)); (5) N(DCJs) is the total number of DCJs; (6) the sixth row is the number of DCJs per DCJ day, E(N(DCJs) | N(DCJ-days)) = N(DCJs)/ N(DCJ-days) days; (7) <DCJ-size> is the average DCJ size, which is measured by the average of the absolute TMV of the observed DCJs; (8) < DCJ-periods> is the average period of a DCJ.

Table 4.3 (a) The statistical summary of detected DCJs in *EURUSD*, *GBPUSD*, and *USDJPY* in the years from 2015 to 2019. Threshold θ = 0.1%; s = 0.99.

	EURUSD	*GBPUSD*	*USDJPY*	*USDCAD*	*AUDUSD*
N(DCs), (1)	32060	39413	33079	28315	45181
N(days), (2)	1303	1302	1298	1303	1302
N(DCJ-days), (3)	107	128	82	126	152
P(DCJ-day), (4)	8.2%	9.8%	6.32%	9.67%	11.67%
N(DCJs), (5)	336	841	589	292	538
E(N(DCJs)|N(DCJ-days)), (6)	3.1	6.6	7.2	2.3	3.5
<DCJ-size>, in aTMV (7)	2.1	2.5	2.4	2.2	2.4
<DCJ-period>, in seconds (8)	6.7	4.7	5.9	6.8	6.0

Except $AUDUSD$, The $P(DCJ$-day) indicates that the number of DCJ days are less than a tenth of the total trading days for the three exchange rates. The sixth row presents the number of DCJs per jump day. For the five currencies, the E(N(DCJs) | N(DCJ-days)) indicates that there were more than two DCJs within one day. This suggests that the DCJs were more likely to present within some specific days. As discussed at the beginning of Chapter 4, the jumps are mainly caused by the events. Hence, a higher value of the E(N(DCJs) | N(DCJ-days)) suggests that there were major events in the past 5 years, and these events caused more DCJs. The <DCJ-size> is the average DCJ size in the five year period, which is measured by the mean of the absolute TMV of the observed DCJs. The <DCJ-period> shows the average period (in seconds) of the DCJ in the past five years. In table 4.3 (a), the average DCJ sizes of the five exchange rates are greater than 2. In addition, except $GBPUSD$, the fiures of the <DCJ-period> are around 6 seconds.

We also repeaded the same test with additional three thresholds as shown below in table 4.3 (b), table 4.3 (c), and table 4.3 (d). The results of the three tables indicate the negative relashionship between the threshold and the N(DC), P(DCJ-day), and N(DCJs).

Also, crossing the four thresholds, we observed that the average TMV of a DCJ is greater than 2; the Ave(DCJ-period) is positive related to the level of the thresholds.

Table 4.3 (b) The statistical summary of detected DCJs in *EURUSD*, *GBPUSD*, and *USDJPY* in the years from 2015 to 2019. Threshold θ = 0.05%; s = 0.99.

	EURUSD	*GBPUSD*	*USDJPY*	*USDCAD*	*AUDUSD*
N(DCs), (1)	117977	142361	118433	104955	165467
N(days), (2)	1303	1302	1298	1303	1302
N(DCJ-days), (3)	221	288	183	286	335
P(DCJ-day), (4)	16.96%	22.10%	14.04%	21.95%	25.71%
N(DCJs), (5)	1150	2693	1769	1059	1833
E(N(DCJs)|N(DCJ-days)), (6)	5.2	9.4	9.7	3.7	5.5
Ave(DCJ-size), in aTMV (7)	2.3	2.7	2.7	2.6	2.6
Ave(DCJ-period), in seconds	2.4	2.1	2.3	3.1	2.5

Table 4.3 (c) The statistical summary of detected DCJs in *EURUSD*, *GBPUSD*, and *USDJPY* in the years from 2015 to 2019. Threshold θ = 0.075%; s = 0.99.

	EURUSD	*GBPUSD*	*USDJPY*	*USDCAD*	*AUDUSD*
N(DCs), (1)	54965	66872	55918	49286	78419
N(days), (2)	1303	1302	1298	1303	1302
N(DCJ-days), (3)	155	178	123	183	207
P(DCJ-day), (4)	11.90%	13.66%	9.44%	14.04%	15.89%
N(DCJs), (5)	557	1346	932	490	901
E(N(DCJs)|N(DCJ-days)), (6)	3.6	7.6	7.6	2.7	4.4
Ave(DCJ-size), in aTMV (7)	2.3	2.5	2.4	2.3	2.5
Ave(DCJ-period), in seconds	4.4	3.2	3.7	4.6	3.9

Table 4.3 (d) The statistical summary of detected DCJs in *EURUSD*, *GBPUSD*, and *USDJPY* in the years from 2015 to 2019. Threshold θ = 0.125%; s = 0.99.

	EURUSD	*GBPUSD*	*USDJPY*	*USDCAD*	*AUDUSD*
N(DCs), (1)	20298	24985	21163	18217	29337
N(days), (2)	1303	1302	1298	1303	1302
N(DCJ-days), (3)	72	96	61	87	113
P(DCJ-day), (4)	5.54%	7.39%	4.73%	6.69%	8.68%
N(DCJs), (5)	207	590	405	183	370
E(N(DCJs)|N(DCJ-days)), (6)	2.9	6.1	6.6	2.1	3.3
Ave(DCJ-size), in aTMV (7)	2.1	2.6	2.3	2.2	2.4
Ave(DCJ-period), in seconds	10.9	7.4	10.0	11.4	9.0

It is important to note that the DC is a data driven approach to detect jumps. Under the DC framework, we judge the DC jump presenting based on the historical data (for details see Section 4.4). In the practise, the process of detecting DC jumps has shown in Section 4.5.2.

4.6.2 Examples of the detected DCJs and TSJs

This section presents three scenarios where the TSJs overlapped the DCJs. We will elaborate on three examples of the detected jumps from the point of view of the two different approaches. Fundamentally, the TS jump is identified based on the time interval. we can't make a judgement of a TSJ presenting before the end of the current time interval. In this study, the time interval is 15 mins; the parameters used based on the recommendations from Lee and Mykland (2008). The details of detecting TSJs have been discussed in Section 2.4.1. In the three scenarios below, based on the definition of TSJ, we judge the presence of a TSJ at the end of the blue region (the end of the 15 minutes interval).

Scenario 1: Two DCJs were found at the beginning of the TSJ (15 minutes)

Scenario 1 shows the detected TSJ and DCJs after the US_NFP announcement at 12:30 on 2015/08/07. In table 4.4 (a), the classical method identified a TSJ in the period from 12:30 to 12:45 (a 15 minute time interval) with a jump size of -0.54%. In table 4.4 (b), DC determined two consecutive DCJs at the beginning of the TSJ time interval. The first of the two, DCJ1, was identified from 12:30:01.400 to 12:30:04.200 followed by DCJ2 from 12:30:04.200 to 12:30:11.900. The periods of DCJ1 and DCJ2 were 2.8 seconds and 7.6 seconds, respectively. In figure 4.6, the long blue bar indicates the TSJ of the 15 minute time interval (horizontal axis) with jump size –0.54% (vertical axis). The black and red bars show DCJ1 and DCJ2 at the beginning of the blue bar. As is clear from the chart, the TSJ gives less information about the TSJ location. In contrast, DC gives precise information about the DCJs in terms of location, direction, and the size. In TS, the model-based approach determined the presence of the jump at the end of the 15 minutes (2015/08/07/ 12:45). However, DC identified the two jumps after confirming the two DC trends (see the DCJ definition in section 4.4.2). Therefore, the data-driven approach (DC) identified the jumps earlier than the model-based approach (TS).

Table 4.4(a) A *EURUSD* TSJ was detected within the 15 minute time interval after the US Nonfarm Payrolls announcement. The significance level α=0.01.

	TSJ_StartTime	TSJ_EndTime	JumpSize	Period
TSJ	2015/08/07 12:30	2015/08/07/ 12:45	-0.54%	15 minutes

Table 4.4 (b) Two DCJs were determined within the same 15min interval. Threshold θ = 0.1%; s = 0.99.

	DCJ_StartTime	DCJ_EndTime	JumpSize	Period (seconds)
DCJ1	2015/08/07 12:30:01.400	2015/08/07 12:30:04.200	0.21%	2.817
DCJ2	2015/08/07 12:30:04.200	2015/08/07 12:30:11.900	-0.59%	7.666

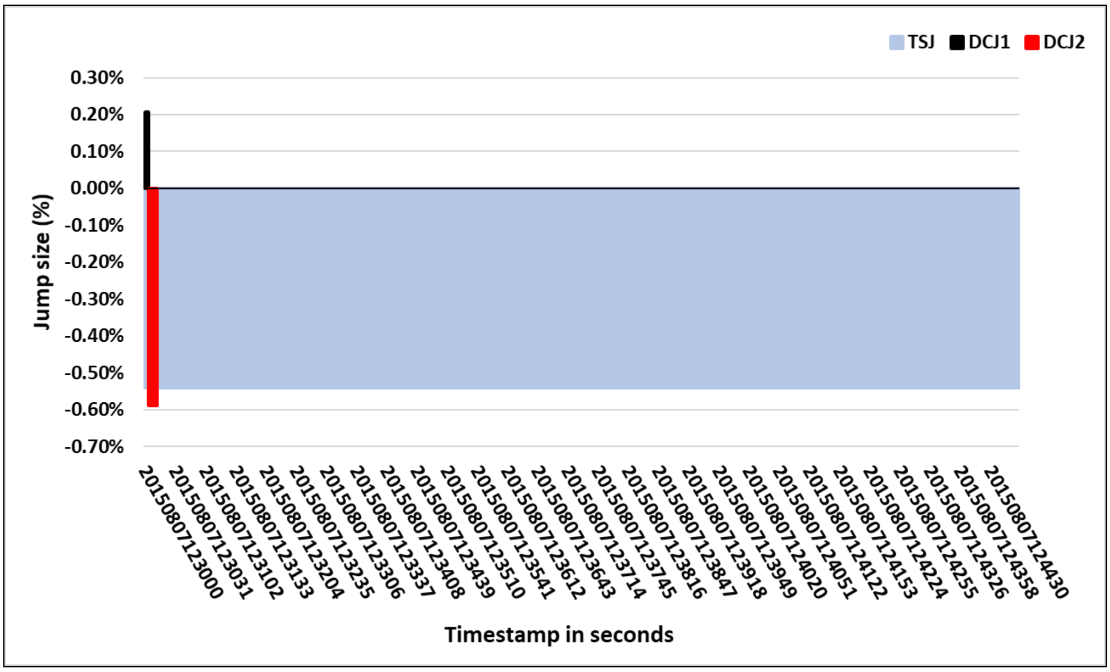

Figure 4.6 The period and the size of the TSJ and the DCJs. The horizontal axis is the timestamp in seconds. The vertical axis is the jump size (%).

Scenario 2: Five DCJs were identified at the beginning of the TSJ (15 minutes)

As shown in table 4.5 (a), the TSJ was identified within the 15 minutes after the ECB_IRD was released at 11:45 on 2019/09/12. At the start of TSJ, DC detected 5 consecutive jumps (table 4.5 (b)). In figure 4.7, the large upward DCJ1 (0.49%) was followed by the downward DCJ2 with a greater size (-0.51%). Then, there were three sequential DCJs of small size. On the other hand, TS only confirmed a downward TSJ, and the jump size was -0.52%.

Table 4.5 (a) A *EURUSD* TSJ was detected within the 15 minute interval after the EU interest rate decision announcement. The significance level α=0.01.

	TSJ_StartTime	TSJ_EndTime	JumpSize	Period
TSJ	2019/09/12 11:45	2019/09/12 12:00	-0.52%	15 minutes

Table 4.5 (b) Five DCJs were determined within the same 15min interval. Threshold θ = 0.1%; s = 0.99.

	DCJ_StartTime	DCJ_EndTime	JumpSize	Period (seconds)
DCJ1	2019/09/12 11:44:34.500	2019/09/12 11:45:15.300	0.49%	40.849
DCJ2	2019/09/12 11:45:15.300	2019/09/12 11:46:14.000	-0.51%	58.65
DCJ3	2019/09/12 11:46:14.000	2019/09/12 11:46:19.700	0.12%	5.755
DCJ4	2019/09/12 11:46:19.700	2019/09/12 11:46:28.400	-0.20%	8.682
DCJ5	2019/09/12 11:46:28.400	2019/09/12 11:46:48.600	0.18%	20.221

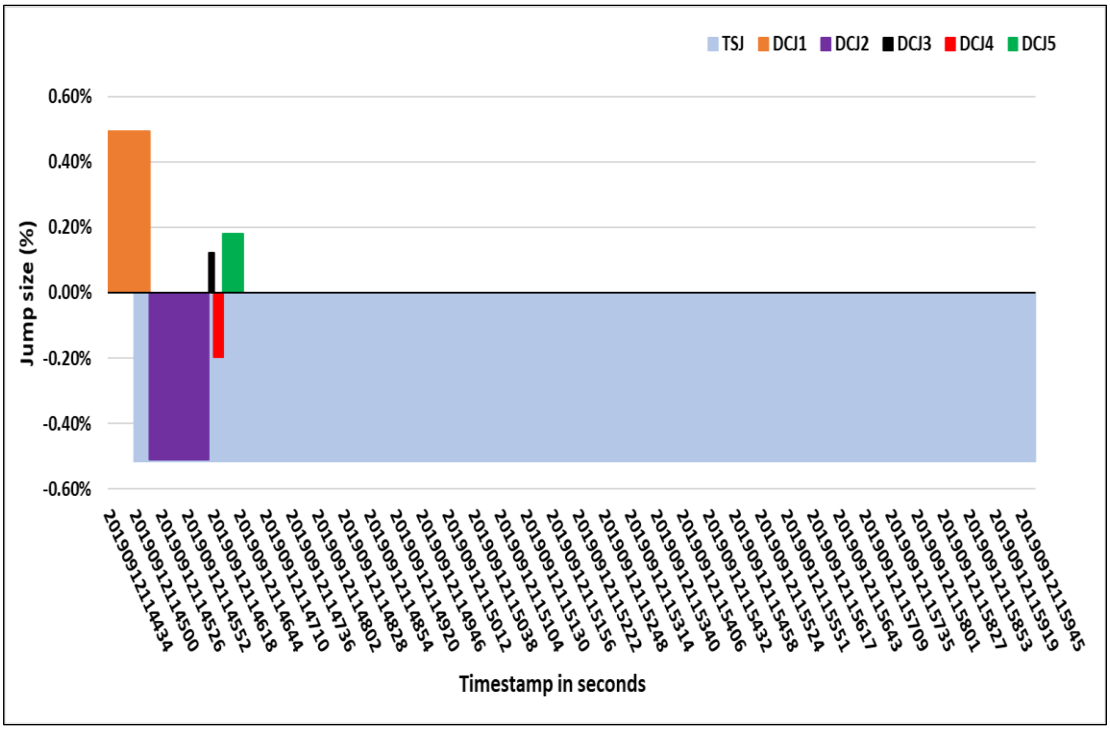

Figure 4.7 The period and the size of the TSJ and the five DCJs. The horizontal axis is the timestamp in seconds. The vertical axis is the jump size (%).

Scenario 3: Four DCJs were found within the TSJ during the ECB Press Conference (15 minutes)

As shown in table 4.6 (a), the TSJ was identified within the 15 minute time interval from 13:30 to 13:45 on 2015/12/03. For DC, we found 4 DCJs within the 15 minutes. As illustrated in figure 4.8, the TSJ size was significant at 2.06%. In DC, there were 3 upward DCJs and one downward DCJ. Scenario 3 shows a different case compared with the previous two examples in that all the four DCJs were not presenting at the beginning of the TSJ. This suggests that jumps may present at the anytime within the TSJ (15 minute time interval).

Table 4.6 (a) A *EURUSD* TSJ was detected within the 15 minute interval. The significance level α=0.01.

	TSJ_StartTime	TSJ_EndTime	JumpSize	Period
TSJ	2015/12/03 13:30	2015/12/03 13:45	2.06%	15 minutes

Table 4.6 (b) Four DCJs were determined within the same 15 minutes interval. Threshold θ = 0.1%; s = 0.99.

	DCJ_StartTime	DCJ_EndTime	JumpSize	Period (seconds)
DCJ1	2015/12/03 13:33:15.500	2015/12/03 13:33:35.900	0.83%	20.327
DCJ2	2015/12/03 11:33:47.700	2015/12/03 13:33:53.200	0.23%	5.549
DCJ3	2015/12/03 11:33:53.200	2015/12/03 13:33:57.900	-0.20%	4.68
DCJ4	2015/12/03 11:35:07.100	2015/12/03 13:35:11.200	0.15%	4.084

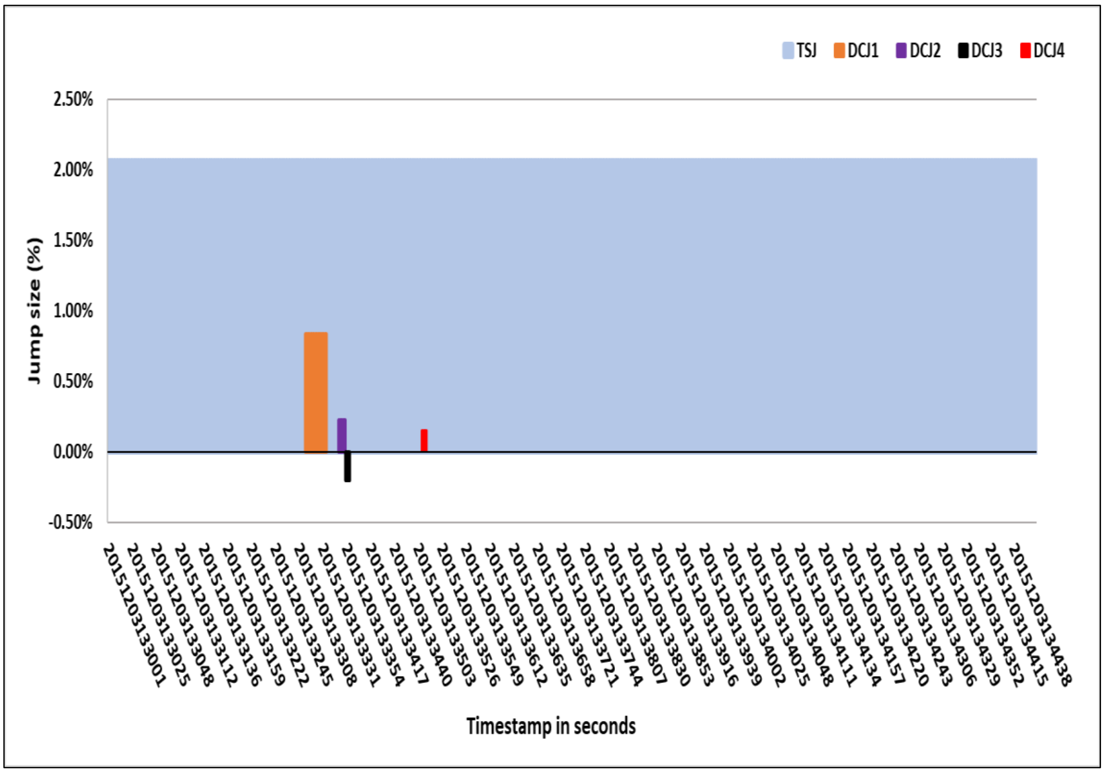

Figure 4.8 The period and the size of the TSJ and the four DCJs. The horizontal axis is the timestamp in seconds. The vertical axis is the jump size (%).

4.6.3 Does DC find jumps that follow events?

This section will show the results of DCJs related to the scheduled events and unscheduled events. The scheduled events are the major economic events which have been listed in Section 4.5.3. The unscheduled events focus on the detected DCJs during the 'flash crash' of Sterling and Yen.

4.6.3.1 Do the jumps follow major economic events (MEEs)?

To answer the first question, we evaluate the events caused DCJs of *EURUSD*, *GBPUSD*, *USDJPY*, *USDCAD*, and *AUDUSD* by studying the time frame consisting of the following 30 minutes after the economic data announcements. Table 4.7 (a) below shows the details of the DCJs identified within the 30 minutes following the economic announcements. Note: (1) N(DCJ-MEE) is the total number of DCJs following the MEEs; (2) N(DCJ) is the total DCJs detected over the five year period; (3) P(MME | DCJ) indicates the value of N(DCJ-MEE) over N(DCJ) as a proportion.

For *EURUSD*, there were a total of 138 DCJs corresponding to the MEEs, which accounted for 41.07% of the total detected DCJs. However, the P(MME | DCJ) of *GBPUSD* is 11.41%, which is much less than the results of *EURUSD*. For *USDJPY*, 20.2% of the total detected DCJs were corresponding to the MEEs. In addition, the results suggest that the DCJs were more associated with the interest rate decisions than other economic indicators except US_NFP. For *EURUSD*, there were 91 DCJs corresponding to the interest rate decisions, which account for 66% $\left(\frac{91}{138}\right)$ of the N(DCJ-MEE). For *GBPUSD* and *USDJPY*, there were 60 DCJs (62.5%) and 74 (62.7%) corresponding to the interest rate decisions, respectively.

Also, the results indicate that Euro exchange rate is more sensitive to the US MEEs than Sterling and Yen. We detected 102 *EURUSD* DCJs associated with U.S. MEEs, while, for *GBPUSD* and *USDJPY*, there were 49 DCJs and 71 DCJs related to US MEEs, respectively. Why is the Euro exchange rate more closely associated with the U.S. MEEs? There appears to be a stronger linkage between the Eurozone and US markets, which is indicated by the following; the Euro is more heavily weighted in the value of US dollar than Sterling. The ICE (Intercontinental Exchange) US dollar index[13] is a geometric average of six currencies weighted against the US dollar. Euro is the top-weighted currency which accounts for 57.6% of the US dollar index. Sterling and Yen account for 11.9% and 13.6% of the total index weight, which are around one-fifth of the weight of Euro. The Federal Reserve Board regularly releases the broad dollar index that is constructed using the currencies of the most important U.S. trading partners by volume of bilateral trade. According to the newest update of the trade weights (2019), the Euro zone contributes 20.086% of the total trade, while the contributions of the UK and Japan are 5.416% and 6.377%, respectively.[14] Therefore, in terms of the weight of the broad dollar index, the Euro weighting is around 3.7-fold that of Sterling and 3.1-fold that of Yen. Overall, the results of table 4.7 (a) lead to two stand out observations for *EURUSD*, *GBPUSD* and *USDJPY*:

Observation 1: the jumps of *EURUSD*, *USDCAD* are more sensitive to the MMEs

The P(MME | DCJ) figure for *EURUSD* shows 41.07% of the total determined DCJs follow MEEs. The P(MME | DCJ) figure for *USDCAD* shows 64.38% of the total determined DCJs follow MEEs. This result suggests that *EURUSD* and *USDCAD* jumps are firmly associated with the MEEs studied. As discussed in the previous section, there are more than 40 economic indicators published every month in the U.S. (Baumohl (2012)). This back-testing only selects a

[13] ICE U.S. Dollar Index Contracts:
https://www.theice.com/publicdocs/futures_us/ICE_Dollar_Index_FAQ.pdf
[14] U.S. total trade weights: https://www.federalreserve.gov/releases/h10/weights/default.htm

restricted sample of 8 MEEs to test the economic event caused DCJs. Hence, around 41% and 64% DCJs are related to these MEEs; indicating that *EURUSD* and *USDCAD* are highly sensitive to the MEEs.

Observation 2: For *GBPUSD*, *USDJPY* and *AUDUSD*, some jumps follow the MMEs

For *GBPUSD*, the P(MEE | DCJ) indicates that 11.41% of the total DCJs are related to the MEEs. For *USDJPY*, we observed that 20.2% of the total DCJs are associated to the MEEs. For *AUDUSD*, the P(MEE | DCJ) indicates that 28.25% of the total DCJs are related to the MEEs. For Sterling, Yen, and Australian dollar, the P(MEE | DCJ) figures does not indicate a solid relationship between DCJs and MEEs like that observed for the Euro and Canadian dollar. This suggests that the majority of jumps for *GBPUSD*, *USDJPY* and *AUDUSD* do not follow the major economic events.

Table 4.7 (a) The detected $\boldsymbol{DCJ}$s associated with the economic events. The event-caused $\boldsymbol{DCJ}$s were those identified within the 30 minutes after the economic data announcement. Threshold θ = 0.1%; s = 0.99**.**

	EURUSD	GBPUSD	USDJPY	USDCAD	AUDUSD
FED_ IRD	55	19	26	22	36
ECB_IRD	36	-	-	-	-
BoE_IRD	-	41	-	-	-
BoJ-IRD	-	-	48	-	-
BoC_IRD	-	-	-	90	-
BoA_IRD	-	-	-	-	54
EU_GDP	0	-	-	-	-
EU_CPI	0	-	-	-	-
UK_GDP	-	2	-	-	-
UK_CPI	-	4	-	-	-
Japan_GDP	-	-	0	-	-
Japan_CPI	-	-	0	-	-
Canada_GDP	-	-	-	3	-
Canada_CPI	-	-	-	27	-
Australia_GDP	-	-	-	-	13
Australia_CPI	-	-	-	-	15
US_GDP	4	1	1	2	1
US_NFP	40	24	39	41	28
US_RS	3	2	2	2	1
US_CPI	0	3	3	1	4
N(DCJ-MEE)	138	96	119	188	152
N(DCJ)	336	841	589	292	538
P(MME | DCJ)	41.07%	11.41%	20.20%	64.38%	28.25%

We also selected additional three thresholds, 0.05%, 0.075%, 0.125%, to repeat the same test; the results are summarised in table 4.7 (b), table 4.7 (c) and table 4.7 (d). The results of the the three tables indicate the same conclusions of table 4.7 (a): (1) For *EURUSD*, under the threshilds of 0.05%, 0.075%, 0.125%, the figures of the P(MME | DCJ) are 34.28%, 39.32%, and 39.61%, respectively; (2) For *GBPUSD* and *AUDUSD*, under the three thresholds, the figures of the P(MME | DCJ) are shown around 10% and 30%; (3) For *USDCAD*, we observed the the figures of the P(MME | DCJ) are over 50% under the three thresholds.

Table 4.7 (b) The detected $\boldsymbol{DCJ}$s associated with the economic events. The event-caused $\boldsymbol{DCJ}$s were those identified within the 30 minutes after the economic data announcement. Threshold θ = 0.05%; s = 0.99**.**

	EURUSD	GBPUSD	USDJPY	USDCAD	AUDUSD
FED_ IRD	153	43	79	80	122
ECB_IRD	123	-	-	-	-
BoE_IRD	-	109	-	-	-
BoJ-IRD	-	0	170	-	-
BoC_IRD	-	-	-	224	-
BoA_IRD	-	-	-	-	136
EU_GDP	0	-	-	-	-
EU_CPI	1	-	-	-	-
UK_GDP	-	6	-	-	-
UK_CPI	-	21	-	-	-
Japan_GDP	-	-	0	-	-
Japan_CPI	-	-	0	-	-
Canada_GDP	-	-	-	17	-
Canada_CPI	-	-	-	98	-
Australia_GDP	-	-	-	-	42
Australia_CPI	-	-	-	-	56
US_GDP	4	1	5	4	6
US_NFP	107	64	88	143	99
US_RS	6	15	12	5	16
US_CPI	16	7	10	17	21
N(DCJ-MEE)	410	266	364	588	498
N(DCJ)	1196	2771	1825	1059	1833
P(MME | DCJ)	34.28%	9.60%	19.95%	55.52%	27.17%

Table 4.7 (c) The detected $\boldsymbol{DCJ}$s associated with the economic events. The event-caused $\boldsymbol{DCJ}$s were those identified within the 30 minutes after the economic data announcement. Threshold θ = 0.075%; s = 0.99.

	EURUSD	GBPUSD	USDJPY	USDCAD	AUDUSD
FED_ IRD	73	30	46	47	53
ECB_IRD	61	-	-	-	-
BoE_IRD	-	67	-	-	-
BoJ-IRD	-	-	79	-	-
BoC_IRD	-	-	-	128	-
BoA_IRD	-	-	-	-	78
EU_GDP	0	-	-	-	-
EU_CPI	0	-	-	-	-
UK_GDP	-	2	-	-	-
UK_CPI	-	7	-	-	-
Japan_GDP	-	-	0	-	-
Japan_CPI	-	-	0	-	-
Canada_GDP	-	-	-	4	-
Canada_CPI	-	-	-	38	-
Australia_GDP	-	-	-	-	19
Australia_CPI	-	-	-	-	32
US_GDP	4	2	4	4	3
US_NFP	70	37	54	66	50
US_RS	4	7	4	3	8
US_CPI	7	6	10	3	16
N(DCJ-MEE)	219	158	197	293	259
N(DCJ)	557	1346	932	490	901
P(MME | DCJ)	39.32%	11.74%	21.14%	59.80%	28.75%

Table 4.7 (d) The detected $\boldsymbol{DCJ}$s associated with the economic events. The event-caused $\boldsymbol{DCJ}$s were those identified within the 30 minutes after the economic data announcement. Threshold θ = 0.125%; s = 0.99**.**

	EURUSD	GBPUSD	USDJPY	USDCAD	AUDUSD
FED_ IRD	34	14	16	23	34
ECB_IRD	21	-	-	-	-
BoE_IRD	-	25	-	-	-
BoJ-IRD	-	-	32	-	-
BoC_IRD	-	-	-	53	-
BoA_IRD	-	-	-	-	34
EU_GDP	0	-	-	-	-
EU_CPI	0	-	-	-	-
UK_GDP	-	2	-	-	-
UK_CPI	-	3	-	-	-
Japan_GDP	-	-	0	-	-
Japan_CPI	-	-	0	-	-
Canada_GDP	-	-	-	2	-
Canada_CPI	-	-	-	17	-
Australia_GDP	-	-	-	-	10
Australia_CPI	-	-	-	-	10
US_GDP	2	0	0	0	1
US_NFP	22	14	17	28	25
US_RS	3	1	2	1	1
US_CPI	0	2	3	1	3
N(DCJ-MEE)	82	61	70	125	118
N(DCJ)	207	590	405	183	370
P(MME | DCJ)	39.61%	10.34%	17.28%	68.31%	31.89%

4.6.3.2. How many MEEs were followed by jumps?

In the previous section, the P(MEE | DCJ) figure specifies the proportion of DCJs following the MEEs. In this section, we focus on the likelihood that a MEE announcement causes at least one DCJ, denoted by P(DCJ | MME). In other words, given the total number of MEEs, we would like to know how many MEEs are followed by DCJs. The P(DCJ | MME) gives a useful perspective to evaluate the relationship between the DCJs and the MEEs. For instance, if a number of unexpected MEE data points cause many DCJs, all of which will contribute to the P(MEE | DCJ) figure, which will then be high. However, in the same situation, the P(DCJ | MME) may not exhibit a high value since if other MEEs do not involve the presentation of data significantly different from market expectations; these events with many DCJs only have one unit of contribution in this case, the same as for an MEE followed by a single DCJ.

In table 4.8 (a) below, the N(MEE) is the total number of the selected MMEs over the five years. The N(MEE-DCJ) counts the number of the MEEs which are followed by at least one DCJ. P(DCJ | MME) indicates N(MEE-DCJ) as a percentage of the N(MME). Hence, P(DCJ | MME) indicates the proportion of MEEs which were followed by at least one DCJ. The N(MEE-DCJ) summaries the number of the MEEs which are followed by at least one DCJ. This indicates that there may be more than one DCJ associated to a specific MEE. Hence, we would like to know how many DCJs are associated to one MEE on average, which is represented by Ave(N(DCJ) per MEE) = $\frac{\mathrm{N}(DCJ-\mathrm{MEE})}{\mathrm{N}(\mathrm{MEE}-DCJ)}$ in table 4.8 (a) below.

Table 4.8 (a) The summary of the MEE caused jumps. Threshold θ = 0.1%; s = 0.99.

	EURUSD	GBPUSD	USDJPY	USDCAD	AUDUSD
FED_ IRD	25	9	12	7	14
ECB_IRD	10	-	-	-	-
BoE_IRD	-	15	-	-	-
BoJ_IRD	-	-	7	-	-
BoC_IRD	-	-	-	28	-
BoA_IRD	-	-	-	-	14
EU_GDP	0	-	-	-	-
EU_CPI	0	-	-	-	-
UK_GDP	-	2	-	-	-
UK_CPI	-	4	-	-	-
Japan_GDP	-	-	0	-	-
Japan_CPI	-	-	0	-	-
Canada_GDP	-	-	-	2	-
Canada_CPI	-	-	-	19	-
Australia_GDP	-	-	-	-	10
Australia_CPI	-	-	-	-	8
US_GDP	4	1	1	1	1
US_NFP	23	13	20	17	11
US_RS	2	1	2	2	1
US_CPI	0	2	2	0	1
N(MEE-DCJ)	64	47	44	76	60
N(MEE)	360	360	360	360	339
P(DCJ | MEE)	18%	13%	12.2%	21.11%	17.70%
Ave(N(DCJ)	2.1	2	2.7	2.5	2.5

Observation 3: Some MEEs were followed by jumps

For *EURUSD*, *GBPUSD*, *USDJPY*, and *AUDUSD*, the P(DCJ | MEE) figures were less than 20%; the P(DCJ | MEE) of *USDCAD* is 21.11%. For the five exchange rates, the jumps were mostly associated to the IRD and US_NFP; this may imply that the exchange rates were more sensitive to the interest rate decisions and the US employment figures.

Observation 4: One MEE is followed by two DCJs on average

The N(MEE-DCJ) figures for *EURUSD*, *GBPUSD*, *USDJPY*, *USDCAD* and *AUDUSD* are 64, 47, 44, 76 and 60, respectively. Following these confirmed MEEs, the Ave(N(DCJ) per MEE) figures indicate that at least two DCJs are associated to one MEE on average.

We also selected additional currencies under three thresholds, 0.05%, 0.075%, 0.125%, to repeat the same test in table 4.8 (b), table 4.8 (c) and table 4.8 (d) below. In table 4.8 (b), under the threshold of 0.05%, the P(DCJ | MEE)s of *EURUSD*, *GBPUSD*, and *USDJPY* are around 20%; while, we observed a higher figure of the P(DCJ | MEE) in *USDCAD* (40.28%) and *AUDUSD* (35.10%). For the results in table 4.8 (c), all the figures of the the P(DCJ | MEE) are less than 30%. For the results in table 4.8 (d), all the figures of the the P(DCJ | MEE) are less than 20%. To conclude, for the results of tables (a, b, c, d), the figures of P(DCJ | MEE) indicate that some MEEs followed by jumps; also, the figures of the Ave(N(DCJ) per MEE) are higher under the thresholds of 0.05% and 0.075%.

Table 4.8 (b) The summary of the MEE caused jumps. Threshold θ = 0.05%; s = 0.99.

	EURUSD	GBPUSD	USDJPY	USDCAD	AUDUSD
FED_ IRD	29	14	18	19	22
ECB_IRD	9	-	-	-	-
BoE_IRD	-	25	-	-	-
BoJ_IRD	-	-	13	-	-
BoC_IRD	-	-	-	33	-
BoA_IRD	-	-	-	-	21
EU_GDP	0	-	-	-	-
EU_CPI	1	-	-	-	-
UK_GDP	-	4	-	-	-
UK_CPI	-	14	-	-	-
Japan_GDP	-	-	0	-	-
Japan_CPI	-	-	0	-	-
Canada_GDP	-	-	-	9	-
Canada_CPI	-	-	-	35	-
Australia_GDP	-	-	-	-	14
Australia_CPI	-	-	-	-	15
US_GDP	4	1	3	3	4
US_NFP	30	18	28	40	31
US_RS	5	5	8	2	7
US_CPI	6	3	4	4	5
N(MEE-DCJ)	84	84	74	145	119
N(MEE)	360	360	360	360	339
P(DCJ | MEE)	23.33%	23.33%	20.56%	40.28%	35.10%
Ave(N(DCJ)	4.9	3.2	4.9	4.1	4.2

Table 4.8 (c) The summary of the MEE caused jumps. Threshold θ = 0.075%; s = 0.99.

	EURUSD	GBPUSD	USDJPY	USDCAD	AUDUSD
FED_ IRD	24	11	13	14	19
ECB_IRD	10	-	-	-	-
BoE_IRD	-	20	-	-	-
BoJ_IRD	-	-	10	-	-
BoC_IRD	-	-	-	30	-
BoA_IRD	-	-	-	-	19
EU_GDP	0	-	-	-	-
EU_CPI	0	-	-	-	-
UK_GDP	-	2	-	-	-
UK_CPI	-	7	-	-	-
Japan_GDP	-	-	0	-	-
Japan_CPI	-	-	0	-	-
Canada_GDP	-	-	-	3	-
Canada_CPI	-	-	-	23	-
Australia_GDP	-	-	-	-	11
Australia_CPI	-	-	-	-	12
US_GDP	3	2	3	3	2
US_NFP	34	17	23	26	16
US_RS	3	2	4	1	4
US_CPI	3	2	5	2	5
N(MEE-DCJ)	77	63	58	102	88
N(MEE)	360	360	360	360	339
P(DCJ | MEE)	21.39%	17.50%	16.11%	28.33%	25.96%
Ave(N(DCJ)	2.8	2.5	3.4	2.9	2.9

Table 4.8(d) The summary of the MEE caused jumps. Threshold θ = 0.125%; s = 0.99.

	EURUSD	GBPUSD	USDJPY	USDCAD	AUDUSD
FED_ IRD	20	8	9	8	13
ECB_IRD	7	-	-	-	-
BoE_IRD	-	9	-	-	-
BoJ_IRD	-	-	6	-	-
BoC_IRD	-	-	-	20	-
BoA_IRD	-	-	-	-	13
EU_GDP	0	-	-	-	-
EU_CPI	0	-	-	-	-
UK_GDP	-	2	-	-	-
UK_CPI	-	3	-	-	-
Japan_GDP	-	-	0	-	-
Japan_CPI	-	-	0	-	-
Canada_GDP	-	-	-	2	-
Canada_CPI	-	-	-	12	-
Australia_GDP	-	-	-	-	7
Australia_CPI	-	-	-	-	5
US_GDP	2	0	0	0	1
US_NFP	13	11	12	14	11
US_RS	2	0	2	1	1
US_CPI	0	1	2	1	1
N(MEE-DCJ)	44	34	31	58	52
N(MEE)	360	360	360	360	339
P(DCJ | MEE)	12.22%	9.44%	8.61%	16.11%	15.34%
Ave(N(DCJ)	1.9	1.8	2.3	2.2	2.3

4.6.3.3 The DCJs associated to the unscheduled events

In the previous section, we discussed the relationship between DCJs and MEEs where we observed some DCJs associated to the scheduled economic events. As discussed in section 4.1, unscheduled events may cause more impact on jumps presenting than the scheduled events as participants are highly sensitized to the uncertain risks. This section will discuss the case study of the association of the jumps to unscheduled events for Sterling. Note, the case study of the effect of an unscheduled event for Japanese Yen will be presented in Appendix E.

The unscheduled event of the sterling flash crash started near midnight of 6^{th} of October 2016 (UTC). The sterling depreciated by around 9% versus the dollar in this unscheduled event. This event had caused concern for the Bank of England and Bank for International Settlements (BIS).

An analysis report about the sterling ‘flash event’ was published by the Markets Committee from BIS in January 2017 (Jackson et al. (2017)).

The BIS report summarised three stages of the sterling fall of 7 October:

> **Stage 1:** *GBPUSD* fell from 1.2600 to 1.2494 during eight seconds from (23:07:03 to 23:07:11 UTC), and the sales of sterling was significantly imbalanced in terms of order flow.
>
> **Stage 2:** *GBPUSD* started accelerating in its decline with the mid-price reaching 1.24 from 23:07:15 (UTC). The report highlighted that there were a series of significant price gaps in the price changes in the second stage (see Figure 4.9).
>
> **Stage 3:** The market started to rise after breaking the 1.200 barrier (around 23:09:29 UTC).

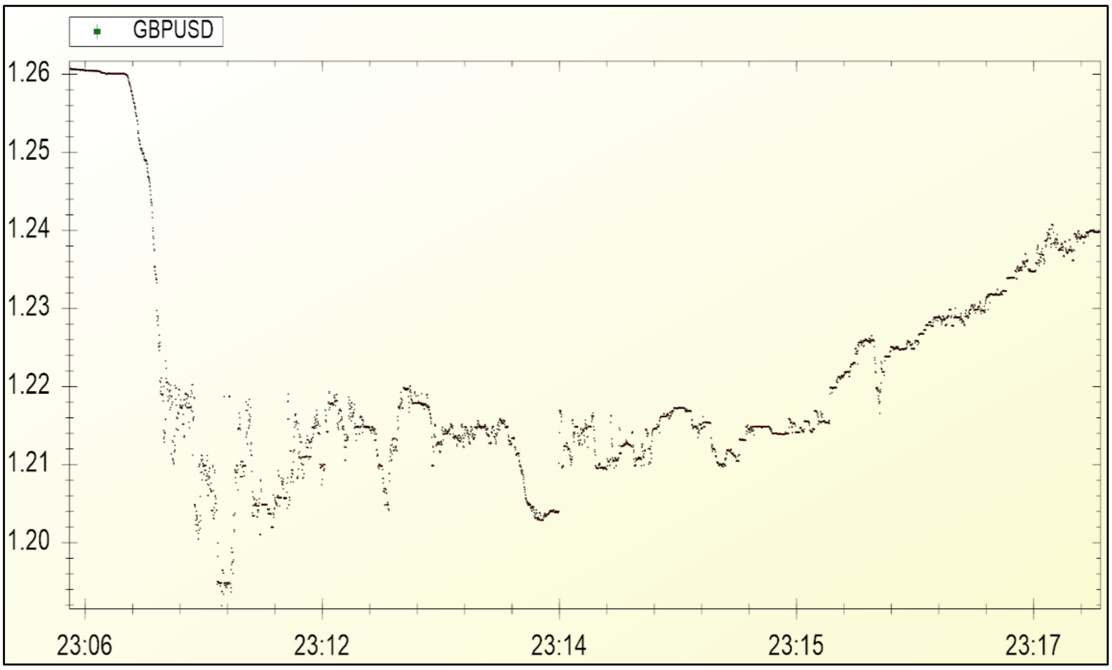

Figure 4.9 The exchange rate of *GBPUSD* on 06/10/2016 (UTC). The vertical axis is the exchange rate. The horizontal axis is the time in UTC. Source: tickstory.com; Dukascopy Bank.

The Markets Committee of BIS discussed the factors triggering the sterling crash. First, the event occurred during the illiquid period around midnight, and some regional bank holidays such as in China. Second, there was a large trade that attempted to lower the price during the illiquid period. Third, the market dealers hedged the options deals by selling sterling. Fourth, a lot of stop loss orders were automatically triggered during this big drop. In addition, under the circumstances of illiquidity, there was a big spread in the bid-ask price on the order books, which directly amplified the significant price changes. In the end, the report was concerned at the limitations of the policymakers' abilities in terms of prediction of and reaction to, the flash event. Also, they suggested to build the capacity for better data collection to aid further research.

As discussed in section 4.1, the data driven approach in DC offers the benefit of more fine-grained analysis in the monitoring of jump behavior in high-frequency data. Thus, we are interested to study the jump behavior in this flash event. The experiment aims to detect DCJs in the 20 minutes from 23:00:00.000 to 23:20:00.000 (UTC) on 6th of October 2016.

Table 4.9 presents the statistical summary of the observed DCJs: (1) the selected period for detection of the DCJs from 23:00:00.000 to 23:20:00.000 on 06/10/2016 (UTC); (2) the total DC trends confirmed in the periods; (3) N(DCJ) is the total DCJs detected in the 5 minutes; (4) <DCJ-size> is the average DCJ size, which is measured by the average of the absolute TMV of the total observed DCJs; (5) <DCJ-period> is the average period (in seconds) of a DCJ.

Table 4.9 The summary of detected $\boldsymbol{DCJ}$s during the Sterling flash event on 06/10/2016. Threshold θ = 0.1%; s = 0.99.

Asset	GBPUSD
Periods, (1)	20 minutes (UTC) (23:00:00 to 23:20:00)
N(DC), (2)	276
N(DCJ), (3)	209
<DCJ-size>, (4)	3.5
<DCJ-period>, (5)	1.4

From 23:00:00 to 23:20:00, we identified 209 DCJs from the 276 DC trends. The average of the absolute TMV was 3.5, and the average period of a DCJ was 1.4 seconds. According to the BIS report, starting from 23:07:15, there were a series of significant price gaps in the price changes in the second stage. Based on our results, the first DCJ was identified from 23:07:16.200 at the price of 1.22906, which coincides with the

circumstances outlined in stage 2 in the report. In addition, we observed four significant consecutive DCJs where their TMVs were 20 or larger (see table 4.10). Therefore, the 20 times the 0.1% threshold indicates the four consecutive DCJs reached the size of a 2% or larger change, and the period of each DCJ was 6.6 seconds on average. The four significant DCJs detected at the bottom of this big fall in sterling, the termination of the last of which coincides with the beginning of stage 3 from the BIS report;

Table 4.10 The four significant consecutive $\boldsymbol{DCJ}$s

Start time	End time	Absolute TMV	Periods (seconds)
23:09:20.100	23:09:20.200	20.6	0.05
23:09:20.200	23:09:21.200	20.4	1.001
23:09:21.200	23:09:31.600	20.2	10.459
23:09:31.600	23:09:46.500	20.1	14.929
	Average	20.4	6.6

4.6.4 Did DC and TS find the same jumps?

As introduced in section 4.1, the time series jump (TSJ) is an additional source of risk alongside the traditional risk of continuous volatility. The TSJ is considered as a component of the asset pricing model. Barndorff-Nielsen and Shephard (2004) first presented the method to locate the TSJs at the level of daily sampling frequency. Lee and Mykland (2008) detected the jump arrival times and their sizes in the intraday timeframe. The classical method identifies jumps within a fixed time interval where the size of the jump depends on the length of the time interval. Technically, the TS method detects the jumps through the model-based approach; researchers confirm the presence of a TSJ if there is a significant value of the realised return relative to the continuous volatility.

Jumps in DC and TS are completely different concepts. As noted above, in TS, a jump is an additional source of risk to the risk of continuous volatility in the asset pricing model. The TSJ is identified based on the model-based approach. In DC, a jump is an event of the DC trend such that the price has changed by a significant magnitude in a short period. We determine the existence of a DCJ when there is a significant magnitude of the time-adjusted return (TR) in the DC trend.

This section will present the results of jumps detected by the two different methods. The experiment selects *EURUSD*, *GBPUSD*, *USDJPY*, *USDCAD* and *AUDUSD* for analysing the detection of DCJs and TSJs in the period from 2015 to 2019. In DC, the DCJ significance is s = 0.99. In TS, the data timeframe is 15 minutes and the significance level α=0.01. The moving window size is 260 trading days (around 1 year), and the window movement is 1 trading day. The idea of detecting TS jumps based on the asset pricing model (Nielsen and Shephard (2004) and Lee and Mykland, (2008)); a jump (TSJ) is a different source of risk compared to the risk of continuous volatility in the asset pricing model (for details of TS method see Section 2.4.1). In TS, our back-testing follows the approach of Lee and Mykland (2008), and the algorithm parameters are set according to their suggestions.

Table 4.11 (a) below summarizes the details of the detected DCJs and TSJs over the five years. N(DCJ) and N(TSJ) are the numbers of jumps identified by the two approaches. N(DCJ-in-TSJ) represents how many DCJs were found within the 15 minute time interval associated to a TSJ. P(DCJ-TSJ) indicates N(DCJ-in-TSJ) as a proportion of the total N(DCJ), and hence describes the percentage of the DCJs that coincide with a TSJ. N(TSJ-overlapping-DCJ) is the number of TSJs that overlap with

DCJs. P(TSJ-overlapping-DCJ) is N(TSJ-overlapping-DCJ) as a proportion of the total number of TSJs, N(TSJ).

Table 4.11 (a) The summary of the $\boldsymbol{DCJ}$s and TSJs detected over the five years from 2015 to 2019. DCJs: Threshold θ = 0.1%; s = 0.99. TSJs: The significance level α=0.01.

	EURUSD	GBPUSD	USDJPY	USDCAD	AUDUSD
N(DCJ)	336	841	589	292	538
N(TSJ)	441	444	440	416	124
N(DCJ-in-TSJ)	219	474	387	145	181
P(DCJ-TSJ)	65.18%	56.36%	65.70%	49.66%	33.64%
N(TSJ-overlapping-DCJ)	54	61	50	59	30
P(TSJ-overlapping-DCJ)	12.24%	13.74%	11.36%	14.18%	24.19%

Table 4.11 (a) leads to two observations:

Observation 5: Jumps found in both DC and TS; DCJs and TSJs exhibit a degree of a mutual validation in terms of jump detection.

For *EURUSD*, *GBPUSD*, *USDJPY* and *USDCAD*, the P(DCJ-TSJ) figure indicates over half of DCJs coincided with TSJs (note, 49.66% for *USDCAD*). For *AUDUSD*, the figure of P(DCJ-TSJ) is 33.64%. The results suggest that the jumps found under DC were also frequently associated with jumps found by TS analysis. In other words, DCJs and TSJs are not totally independent methods of observation.

Observation 6: DC and TS both found unique jumps

The P(TSJ-overlapping-DCJ) figure indicates that many TSJs are not associated with DCJs. For *EURUSD*, *GBPUSD*, *USDJPY*, *USDCAD* and *AUDUSD* that all the figures of the P(TSJ-overlapping-DCJ) are less than 30%. Hence, we can conclude that both DC and TS identified a considerable number of unique jumps.

We also selected additional three thresholds, 0.05%, 0.075%, 0.125%, to repeat the same test as shown the results in table 4.11 (b, c, d). For *EURUSD*, *GBPUSD*, *USDJPY* and *USDCAD*, the P(DCJ-TSJ) figure indicates over half (or around 50%) of DCJs coincided with TSJs. However, for *AUDUSD*, the figures of the P(DCJ-TSJ) are around 30%. Also, for the results of table 4.11 (b, c, d), the figures of the P(TSJ-overlapping-DCJ) are less than 30% in *EURUSD*, *GBPUSD*, *USDJPY*, *USDCAD*. For *AUDUSD*, the figures of the P(TSJ-overlapping-DCJ) are over 30% under the thresholds of 0.05% and 0.075% (20.97% under the threshold of 0.125%).

Table 4.11 (b). The summary of the $\boldsymbol{DCJ}$s and TSJs detected over the five years from 2015 to 2019. DCJs: Threshold θ = 0.05%; s = 0.99. TSJs: The significance level α=0.01.

	EURUSD	GBPUSD	USDJPY	USDCAD	AUDUSD
N(DCJ)	1196	2771	1825	1059	1833
N(TSJ)	441	444	440	416	124
N(DCJ-in-TSJ)	687	1261	1082	464	495
P(DCJ-TSJ)	57.44%	45.51%	59.29%	43.81%	27.00%
N(TSJ-overlapping-DCJ)	98	101	73	122	49
P(TSJ-overlapping-DCJ)	22.22%	22.75%	16.59%	29.33%	39.52%

Table 4.11 (c). The summary of the $\boldsymbol{DCJ}$s and TSJs detected over the five years from 2015 to 2019. DCJs: Threshold θ = 0.075%; s = 0.99. TSJs: The significance level α=0.01.

N(DCJ)	557	1346	932	490	901
N(TSJ)	441	444	440	416	124
N(DCJ-in-TSJ)	350	700	595	231	290
P(DCJ-TSJ)	62.84%	52.01%	63.84%	47.14%	32.19%
N(TSJ-overlapping-DCJ)	69	72	62	86	43
P(TSJ-overlapping-DCJ)	15.65%	16.22%	14.09%	20.67%	34.68%

Table 4.11 (d). The summary of the $\boldsymbol{DCJ}$s and TSJs detected over the five years from 2015 to 2019. DCJs: Threshold θ = 0.125%; s = 0.99. TSJs: The significance level α=0.01.

N(DCJ)	207	590	405	183	370
N(TSJ)	441	444	440	416	124
N(DCJ-in-TSJ)	140	343	259	94	127
P(DCJ-TSJ)	67.63%	58.14%	63.95%	51.37%	34.32%
N(TSJ-overlapping-DCJ)	45	51	40	43	26
P(TSJ-overlapping-DCJ)	10.20%	11.49%	9.09%	10.34%	20.97%

4.7 Discussion

This section will discuss the major claims made in this chapter. The major discussion of this chapter, in Section 4.7.2, elaborates on the core value of DCJs for monitoring the markets' behaviours in jumps. Specifically, analysts can obtain the detailed jumps' behaviours from detected DCJs during scheduled and unscheduled events, which will be discussed in Section 4.7.3 and Section 4.7.4. Lastly, we will study the relationship between the observed TSJs and DCJs.

4.7.1 Jumps are found under DC

We executed back-testing for detecting DCJs in the three exchange rates over 5 years. There were more jumps determined in Sterling than Euro and Yen. The $P(DCJ\text{-day})$ indicates that *GBPUSD* exhibited more jump days than did *EURUSD* and *USDJPY*. Also, the $E(\mathrm{N}(DCJ\text{s}) \mid N(DCJ\text{-days}))$ enabled us to conclude that we detected more jumps in Sterling and Yen than Euro in a jump day. This suggests that there were major events which cause a large number of DCJs in *GBPUSD* and *USDJPY*. As discussed in section 4.1, the unscheduled events (the uncertain risk) may have more impact on jumps than scheduled events. In fact, the two cases studies of unscheduled events support our conclusion that the jumps are highly sensitive to the unscheduled events (we will discuss the two cases studies in the following sections).

4.7.2 Discussions: DCJs provide valuable information about jumps

This section discusses the benefits of DCJs in analysing the jumps' behaviour. As discussed at the beginning of section 2.3, the TS method identifies jumps through a model-based approach. TSJ is a different source of risk compared to the risk of continuous volatility and the existence of jumps may affect asset pricing and risk management. Hence, a jump is a discontinuous component in the asset pricing model. In practice, researchers judge the presence of jumps based on a fixed time interval. In DC, a jump (DCJ) is an event of the DC trend such that the price has changed by a significant magnitude in a short period. In other words, a DCJ covers the period from one extreme point EP_{i-1} to the next EP_i. Thus, the location of the DCJ is precisely delimited by the two adjacent extreme points (EP_{i-1} and EP_i). Fundamentally, the DCJ identification is sensitive to the price and time especially during flash events. Hence, DCJ offers the benefit of more fine-grained analysis in the monitoring of jump behaviour in high-frequency data. We will discuss the details in following.

Statement 1: DCJ specifies the precise time of the jump

The DC approach precisely locates the timing of the jump process. DC gives the start (EP_{i-1}) and the end (EP_i) of the DCJ. In TS, the classical method judges the presence of the TSJ within a fixed time interval. Traditional TSJ cannot give the precise location of within the time interval. In scenario 3 (section 4.6.2), a TSJ was identified during the ECB Press Conference (within the 15 minute time interval from 13:30 to 13:45 on 2015/12/03). However, in the same period of 15 minutes, we detected four DCJs at different time locations.

Statement 2: DCJ gives the direction of every identified jump

The DCJ gives the direction of every identified jump, e.g., upward jump or downward jump. The TSJ direction depends on the price return of the fixed time interval. If the return over the 15 minutes is negative, the jump direction is also negative. However, in practice, there may be one upward jump followed by one larger downward jump within the 15 minutes. In this case, the TS method would not identify the first upward jump. In scenario 2, DC detected 5 consecutive jumps within the 15 minutes after the ECB_IRD was released at 11:45 on 2019/09/12. In table 4.5 (b), there were three upward DCJs and two downward DCJs.

Statement 3: DCJ analysis retains the magnitude of the individual jumps

DCJs give the magnitude of every identified jump. We can measure the jump size by calculating the percentage change of the DC trend. The TSJ size is the difference of the realised return and the instantaneous volatility. If there are two jumps within a time interval, we cannot know the exact sizes of these two jumps. In scenario 1, the classical method identified a TSJ in the period from 12:30 to 12:45 (15 minute time interval) with a jump size of -0.54%. However, DC determined two consecutive DCJs at the beginning of the TSJ time interval. In table 4.4 (b), the size of the DCJ1 was 0.21% (from 12:30:01.400 to 12:30:04.200) followed by the DCJ2 with a jump size of -0.59% (from 12:30:04.200 to 12:30:11.900).

4.7.3 Do DCJs follow scheduled events?

In section 4.6.3.1, the back-testing results demonstrate that *EURUSD* was highly sensitive to the MEEs (Observation 1). However, most of *GBPUSD* DCJs were not associated with MEEs (Observation 2). Also, we observed that DCJs were more sensitive to IRD as the monetary policy directly influences the value of the country's currency. In section 4.6.3.2, the P(DCJ | MME) represents the likelihood that a MEE announcement causes a DCJ. The P(DCJ | MME) figures show lower values for both EURUSD (18%) and GBPUSD (13%). This suggests that many MEEs were not causing jumps (Observation 3). In addition, there were at least two DCJs associated to one MEE on average (Observation 4).

4.7.4 Are DCJs associated with unscheduled events?

In section 4.6.3.3, we observed that jumps were highly sensitive to the unscheduled events based on two cases studies. Firstly, the background of the two cases was that there was no advanced notification of the happenings according to the hindsight studies from the reports by the authorities. Secondly, in the two cases there exist the significant price changes within a very short period. Thirdly, the flash price shocks of the two events were both caused by a lack of liquidity and unusual trading behaviours. These properties surrounding unscheduled events often do not exist in the vicinity of scheduled events. Because, the scheduled events, such as MEEs, are normally released in normal trading hours where there are usually no liquidity issues. In addition, the participants should have awareness of planned events. Hence, the market participants may have already considered the scheduled events in their trading strategies.

In the Sterling flash crash, we detected 209 DCJs in 20 minutes from 23:00:00.000 to 23:20:00.000 in 06/10/2016 (UTC). The average DCJ size was 3.5 which was larger than the average DCJ size from 2015 to 2019 (the Ave(DCJ-size) of 5 years was 2.5). In addition, we observed four consecutive DCJs at the bottom of this big crash in Sterling (from 23:09:20.200 to 23:09:46.500), which coincided with stage 3 of this event (Sterling reached the lowest price and started to rally) from the BIS report. The Ave(DCJ-size) of the four DCJs was 20.4, which indicates the price changes of each DCJ was 2.04% with the average period of 6.6 seconds.

In the Yen flash event, we observed 24 DCJs in 5 minutes from 22:35:00.000 to 22:40:00.000 in 02/09/2019 (UTC). In this unscheduled event, the average DCJ size was 3.3 which was significantly larger than the average DCJ size of the detected DCJs over the 5 years (the Ave(DCJ-size) of 5 years was 2.4).

4.7.5 The discussion of the detected DCJs and TSJs

In section 4.6.4, we evaluated the relationship between DCJs and TSJs. In TS, a jump is a different source of risk compared to the risk of continuous volatility. Fundamentally, The TS jump (TSJ) and DCJ is a different concept. In DC, a DCJ is an event in that the price has changed by a significant magnitude in a short period. In section 2.3, we introduced a well known TS method of detecting intraday jumps. Based on the TS method, we implemented back-testing to detect jumps in the same periods from 2015 to 2019. We analysed the relationship between the DCJs and TSJs in section 4.6.4. From table 4.11, for *EURUSD*, *GBPUSD*, and *USDJPY,* we determined 219, 474, and 387 DCJs within the TSJs. This indicates that over half of the DCJs were located within the TSJs (see the P(DCJ-TSJ) figures in table 4.11). The results show that 12.24% of the *EURUSD* TSJs, 13.74% of the *GBPUSD* TSJs, and 11.36% of the *USDJPY* TSJs overlapped with the DCJs. Therefore, the results recognise that there were common jumps between TSJs and DCJs in the five years (Observation 5). On the other hand, around 35% of *EURUSD* DCJs, 44% of *GBPUSD* DCJs, and 34.3% of *USDJPY* DCJs were not associated with TSJs. This suggested that DC and TS both identified a number of unique jumps (Observation 6).

Statement 4: DCJs and TSJs complement each other

The DCJs complement the TSJs in terms of jump detection. The results indicated that the two approaches found common jumps (Observation 5) and also there were unique jumps between DCJs and TSJs (Observation 6). DCJ is more sensitive to the data rather than limited by a pre-determined timescale. This allows the DC approach to detect the jumps during the random flash events which usually cause significant price changes within a very short period (in seconds). Also, the TSJ had been the subject of many studies tracking news events and thus improving the asset pricing model. Therefore, both TSJs and DCJs offer a viable solution to analyse jumps' behaviour but with a different underpinning approach.

4.8 Conclusion

In this chapter, we propose a new approach, under the DC framework, to detect jumps in the FX markets. This DC approach is different from the classical time series method. DC is an alternative way to sample the market transactions that we can naturally refer to as a data-driven approach. In our experiment, DCJs were sensitive to news events especially during unscheduled events. Also, we studied the observed DCJs and TSJs; as discussed in Statement 4 (in Section 4.7.5), they complement each other in the aspect of jump detection by providing both unique information and mutual confirmation. We concluded that the DC approach and TS method found both common jumps and unique jumps. This confirmed that DCJs complement the TSJs in detecting jumps especially in high-frequency data.

In section 4.4, we defined the concept of the DCJ based on a significant magnitude of the TR. Specifically, TR measures the price changes over time such that a greater TR value indicates greater price changes in a shorter period. In addition, the approach given can be used in a practical manner to detect the DCJs in the financial markets. Our back-testing results indicate that the relationship between MEEs and DCJs depends on the currency. Euro is more associated with MEEs than Sterling and Yen. Compared with the DCJs associated with MEEs, the DCJs were highly sensitive to the unscheduled events as we observed the DCJs in larger quantities and with a higher jump size on average. This conclusion supported our initial insight that the market participants are more sensitive to uncertain risks.

The concept of the jump is different between DC and TS. TSJ is the discontinuous component of the asset pricing model. DCJ is an event of the DC trend such that the price has changed by a significant magnitude in a short time period. Hence, the two approaches cannot replace each other. In section 4.6.2, we presented the three scenarios that both DC and TS found jumps in the same periods after the news announcements. The three examples illustrated that the summary statistics of DCJs can give detailed information about their size, direction, and frequency. Hence, DCJs offer an alternative tool for studying the jump behaviour especially in high-frequency data (tick data). In section 4.6.4, we studied the identified DCJs and TSJs. The results showed that the jumps found under DC were also frequently associated with jumps found by TS method (Observation 5). Also, the two approaches both identified unique jumps (Observation 6). TS detects jumps based on a fixed time interval such that we cannot know the amount and locations of the jumps within this time interval. In contrast, DC identifies jumps for every observed DC trend, which gives precise jump information in terms of the timing, magnitude, and direction.

Overall, we have proposed a new approach to detect jumps under the DC framework. We presented the finding that DCJs were sensitive to news events which was supported by the back-testing. Also, DCJs offer the benefit of more fine-grained analysis in the monitoring of jump behavior in high-frequency data. As per the concern from BIS report, the policymakers need to build the capability of better data collection to more optimally react to future flash events. As noted in Statement 4, we believe, using DCJs and TSJs can give us a better insight into studying the jumps' behaviour. This could support further research in the improvement of risk monitoring.

Chapter 5. Co-jumps

In Chapter 1, we discussed the drawback of DC in the study of real-time comparative analysis. In time series (TS), analysts can easily observe the price returns of two markets during the same time interval, e.g. monitoring the price returns of two financial instruments at every 1-minute interval. This parallel comparative analysis (in TS) for the same fixed time intervals is not applicable in DC due to the irregular time scale of the DC data. This chapter aims to introduce a method, under the DC framework, for re-sampling the observed DC data of two markets into a single data sequence which is the foundation of studying co-jumps in DC. In Chapter 3, we introduced how to develop a DC relative sequence given two single DC sequences. Based on the DC relative sequence, we developed the measure of mRV to test the relative volatility in micro-market activities. The experimental results demonstrated that observers can precisely locate the timestamp when a significant mRV value is determined. In Chapter 4, we built a DC approach to identify DC jumps (DCJs) for the financial markets. In Section 4.6.2, compared with TS jumps, we showed that DCJs can give precise jump information in terms of the timing, magnitude, and direction.

As introduced in Section 2.3, there are co-jumps in the FX markets. Historical major events were shown to cause the common jump of two exchange rates especially for the highly correlated currencies e.g., the co-jumps between *EURUSD* and *GBPUSD*. Observers need tools to identify co-jumps before they can be studied and the underlying causes ascertained. Following the work of the previous two chapters, we aim to find a solution to identify co-jumps under the DC framework. The contributions of this chapter: (1) In DC, we first define the co-jumps called DC co-jumps. (2) Based on the definition, we develop a new method to identify the DC co-jumps. (3) We studied the co-jumps between *EURUSD* and *GBPUSD*, and the co-jumps between *EURUSD* and *JPYUSD*. Our study indicates that co-jumps are highly sensitive to the US economic data; based on the identified co-jumps related to the economic data, over 90% of co-jumps in these major currency pairs are associated with US economic data. Also, the DC co-jumps inherit the merits of the DC jump which offers precise information on the timing, magnitudes, and the direction of the DC co-jumps. By employing this new method, we want to understand the relationship between the events and co-jumps (more details will be introduced in section 5.7). Especially, we want to track the presence of co-jumps during the global COVID-19 pandemic.

This chapter is organised as follows: in Section 5.1, we will provide the motivation for the study in the DC framework of co-jumps; Section 5.2 gives a review of the relevant concepts in DC for developing the framework; Section 5.3 contains an exposition of how to combine the two TR sequences of two markets ($CTRS$) into a combined sequence. Given the $CTRS$, Section 5.4 discusses how to generate the TR sub-sequences which are the foundation for defining the DC co-jump; Section 5.5 presents the definition of a DC co-jump; details about our retrospective studies will be given in section 5.6 and section 5.7; the results will be discussed in section 5.8; Section 5.9 concludes this chapter.

5.1 Introduction

The risk of co-jumps has been emphasized in risk management over the past 10 years. In time series (TS) analysis, Dungey and Hvozdyk (2012) concluded that the co-jump is the occurrence of a joint contemporaneous discontinuity in two price series. However, they emphasized that there is no precise quantification in timing co-jumps. As discussed in Chapter 4, jumps are associated with significant events. These events are generally categorised into scheduled events and unscheduled events (details about the events shall be explained in Section 5.6). Bollerslev et al. (2008) detected co-jumps in over 50 stocks traded on the New York Stock Exchange which indicates that a similar pattern might be expected in the forex markets. They observed that the co-jumps usually present in the morning associated with the release of macroeconomic news. Gnabo et al. (2014) argued that worldwide news events may cause various degrees of co-jumps which depend on the significance of the received news. They stressed that it is important to accurately understand the tail risk of the co-jumps in the financial markets. In the classical method, Jacod and Todorov (2009) identified co-jumps under the daily approach. Gnabo et al. (2014) tested co-jumps under the intraday frequency, with the approach being the bivariate extension (two price series) of Lee and Mykland (2008).

The classical method identifies co-jumps through a model-based approach. In this report, we present a new tool to detect co-jumps based on the data-driven approach or DC approach. Fundamentally, the concept of co-jump is different between the time series analysis and DC. Under the DC framework, the co-jump is the event that a jump in market A is followed by a jump in market B. In DC, we judge the presence of the co-jump based on the TR sub-sequence (we will introduce the TR sub-sequence in section 5.4). In Chapter 3 (DC relative volatility), we introduced how to generate the combined DC sequence of two markets and the DC relative sequence. In Chapter 4 (DC jumps), we introduced the TR sequence of a single market. The existence of a DC jump is based on the TR (the time-adjusted return) of the DC trend; the significance of a TR is judged with reference to the historical TR sequence. In this chapter, we propose the combined TR sequence of two markets ($CTRS$) and the partitioned $CTRS$ ($PTRS$). The DC approach of identifying co-jumps is the bivariate extension of Chapter 4 where we judge the co-jumps of two assets' prices based on the TR sub-sequence. In practice, the co-jump detection is through an indicator function which will be presented in section 5.5.

Overall, under the DC framework, this chapter aims to develop a new method in detecting co-jumps that we will call DC co-jumps. Thus, it could give a new tool to evaluate the market reactions from flash market shocks crossing two exchange rates in the FX markets. Being able to detect DC co-jumps, we want to explore how frequently do co-jumps happen over the long term. Also, in terms of the co-jumps, we want to know whether a country's economic data affects another country's currency. In March of 2020, the COVID-19 pandemic caused significant economic damage to society in Europe. Did the pandemic cause co-jumps in Euro and Sterling? In the following sections, we will present the approach of identifying DC co-jumps and answer these questions as part of the studies.

5.2 The review of DC data

DC is a data-driven process of data sampling in the financial markets. DC data is recorded as a series of alternative upward and downward trends. For any trend, the reversal point is confirmed when the price has changed beyond a threshold (a pre-determined price distance in percentage terms) from the last highest/lowest price of the current trend (for details see Appendix A in Chen and Tsang (2021)). This highest/lowest price is identified as the DC extreme point (EP, the reversal point). A DC trend is an upward (or a downward) price movement between any two adjacent EPs. In Section 2.3, we introduced DC sequence is a finite sequence which comprises the consecutive extreme points ordered by their timestamps (see equation (2.4)). Given a

DC sequence, we can calculate the sequence of the time-adjusted return of the DC trend based on the equation (2.6) (details will be introduced in the next section).

5.3 The TR sequence and the combined TR sequence of two markets

In Section 2.3 equation (2.4) defines a DC sequence which comprises a series of extreme points of a market, i.e., $S_A^\theta = (EP_1, EP_2, \ldots, EP_n)$. For each DC trend, we calculate the TR through equation (2.6). Thus, given a DC sequence of a market, we can generate the TR sequence.

An TR sequence, denoted by S_{TR}, is a finite sequence of TRs:

$$\left(R_{DC_1}, R_{DC_2}, \ldots, R_{DC_n}\right), \tag{5.1}$$

where R_{DC} is obtained through equation (2.6) and n equals the total number of DC trends from a DC dataset.

Based on the TR sequence, we develop the combined TR sequence of two markets which comprises all observed TRs of two different markets.

The combined TR sequence ($CTRS$) of two TR sequences for market A and market B, denoted by $CR^{A,B}$, comprises all the R_{DC}s of the two markets ordered by $ET(R_{DC})$:

$$CR^{A,B} = \left(R_{DC_1}^{M}, R_{DC_2}^{M}, \ldots, R_{DC_w}^{M}\right) \tag{5.2}$$

such that $ET\left(R_{DC_1}^{M}\right) < ET\left(R_{DC_2}^{M}\right) < \ldots < ET\left(R_{DC_w}^{M}\right)$.

where w equals the total number of R_{DC}s from both market A and market B, and M denotes the market identity of the R_{DC_i} (e.g., market A or market B).

For example, given the TR sequences of market A and market B, we can have the $CTRS$ for market A and market B as shown below:

$$CR^{A,B} = ({R_{DC}}_1^A, {R_{DC}}_2^B, {R_{DC}}_3^A, {R_{DC}}_4^B, {R_{DC}}_5^A, {R_{DC}}_6^B, {R_{DC}}_7^B, {R_{DC}}_8^A, {R_{DC}}_9^A, {R_{DC}}_{10}^B, {R_{DC}}_{11}^A, {R_{DC}}_{12}^B). \tag{E5.1}$$

5.4 The Partitioned *CTRS*

In section 5.3 we discussed how to combine the two TR sequences into a single sequence, i.e., $CTRS$. In this section, we shall introduce how to partition the contiguous $CTRS$ into successive TR sub-sequences. For each TR sub-sequence, there are a series of TRs from market A followed by a series of TRs from market B, or vice versa.

Given a $CTRS$, a partitioned $CTRS$ ($PTRS$) is the partition of the $CTRS$ into contiguous TR sub-sequences based on the market identity. Given a combined time-adjusted return of markets A and B ($CR^{A,B}$), A $PTRS$, denoted by $RS_{CR}^{A,B}$, is generated by a partition process $\Gamma(CR^{A,B})$ of $CR^{A,B}$ into z contiguous TR sub-sequences based on changes in market identity (M) of the adjacent TRs. Each TR sub-sequence comprises a series of R_{DC}^A's followed by a series of R_{DC}^B's or vice versa.

$$RS_{CR}^{A,B} = \Gamma(CR^{A,B}) = \left(U_1^{A,B}, U_2^{A,B}, \dots, U_j^{A,B}, \dots, U_z^{A,B}\right), \tag{5.3}$$

where $U_j^{A,B}$ is a TR sub-sequence. All $U_j^{A,B}$s contain at least two R_{DC} $'$ s, that is, at least one R_{DC} from market A and at least one from market B, thus the maximum value of z is $\frac{w}{2}$. Otherwise, at least one $U_j^{A,B}$ contains more than two R_{DC}'s, so s $< \frac{w}{2}$. For every $U_j^{A,B}$:

$$\forall \text{ j: } U_j^{A,B} = \left(R_{DC}{}_{j,1}^{A}, R_{DC}{}_{j,2}^{A}, \dots, R_{DC}{}_{j,g-1}^{B}, R_{DC}{}_{j,g}^{B}\right)_j. \tag{5.4}$$

Note, we suppose the R_{DC} of market A is first identified. For instance, we partition the $CR^{A,B}$ (E5.1) by:

$$\Gamma(CR^{A,B}) = ((R_{DC}{}_1^A, R_{DC}{}_2^B)_1, (R_{DC}{}_3^A, R_{DC}{}_4^B)_2, (R_{DC}{}_5^A, R_{DC}{}_6^B, R_{DC}{}_7^B)_3,$$
$$(R_{DC}{}_8^A, R_{DC}{}_9^A, R_{DC}{}_{10}^B)_4, (R_{DC}{}_{11}^A, R_{DC}{}_{12}^B)_5). \tag{E5.2}$$

For the example of E5.2, we terminate the first TR sub-sequence when $R_{DC}{}_3^A$ no longer belongs to the same market identity as $R_{DC}{}_2^B$. Thus, we have that $U_1^{A,B} = (R_{DC}{}_1^A, R_{DC}{}_2^B)_1$ and $U_4^{A,B} = (R_{DC}{}_8^A, R_{DC}{}_9^A, R_{DC}{}_{10}^B)_4$. In Figure 5.1, we determine $U_1^{A,B}$ when the market identity of the next $ET(R_{DC}{}_3^A)$ (market A) is not the same with the market identity of the current $ET(R_{DC}{}_2^B)$ (market B).

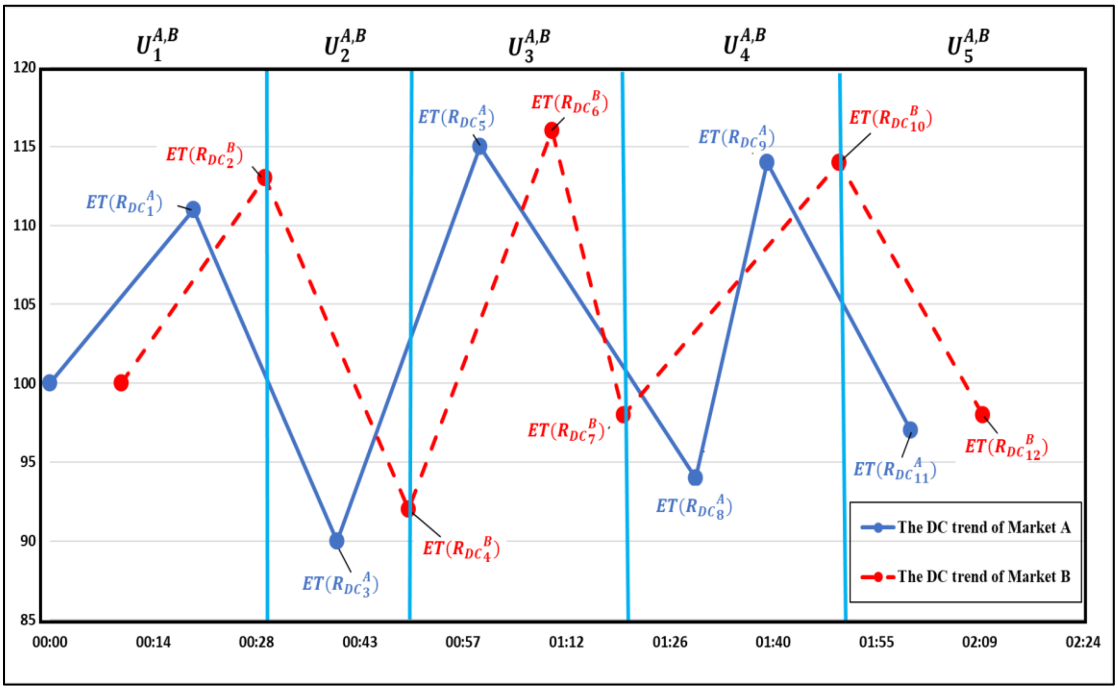

Figure 5.1 The example the two $\boldsymbol{TR}$ sequences of market A and market B. The $\boldsymbol{ET(R_{DC_i})}$ is the terminal time of the DC trend $\boldsymbol{i}$, i.e., $\boldsymbol{ET(R_{DC_i}) = EP_i.t}$ (for details about $\boldsymbol{ET(R_{DC_i})}$ see equation (2.6) in Section 2.3).

5.5 Co-Jumps in DC

As discussed in section 5.1, a co-jump is the event that the DCJ of market A (DCJ^A) is followed by the DCJ of market B (DCJ^B). Therefore, we say in this case that the DCJ^B co-jumps with DCJ^A. In directional change, we define the existence of a co-jump based on the TR sub-sequence.

Under the $PTRS$, we judge the presence of a co-jump based on the TR sub-sequence $U_j^{A,B}$. For any $U_j^{A,B}$, we divide this $U_j^{A,B}$ into two subsets according to the market identity M of the elements R_{DC}. Given a $U_j^{A,B}$, we obtain U_j^A and U_j^B based on the market identity M:

$$U_j^A = \left(R_{DC}{}_1^A, R_{DC}{}_2^A, \dots, R_{DC}{}_l^A\right)_j,\ \ U_j^B = \left(R_{DC}{}_1^B, R_{DC}{}_2^B, \dots, R_{DC}{}_u^B\right)_j, \tag{5.5}$$

where U_j^A and U_j^B are ordered by $ET(R_{DC})$, and $l + u = k$ (k is the length of the combined sub-sequence $U_j^{A,B}$).

Under the TR sub-sequence $U_j^{A,B}$, we say that U_j^B co-jumps with U_j^A, denoted by $CoJ_{U_j}^{A,B}$, if and only if (1) there first exists (at least) one TR in U_j^A that is bigger than s_A^* and (2) there then exists (at least) one TR in U_j^B that is bigger than s_B^*.

$$\exists\ R_{DC}{}^A \in U_j^A, R_{DC}{}^B \in U_j^B : (R_{DC}{}^A > s_A^*) \wedge (R_{DC}{}^B > s_B^*).$$

Note that, $CoJ_{U_j}^{A,B}$ defines that U_j^B (experiencing a jump second) co-jumps with U_j^A (that jumps first) in the combined sequence $U_j^{A,B}$. In other words, $CoJ_{U_j}^{B,A}$ doesn't necessarily correspond to $CoJ_{U_j}^{A,B}$.

5.5.1 The indicator for the detection of $CoJ_{U_j}^{A,B}$

Given s_A^* and s_B^*, we count the number of DCJs in U_j^A and U_j^B using the functions $N(U_j^A)$ and $N(U_j^B)$:

$$N(U_j^A, s_A^*) = \textstyle\sum_{i=1}^{l} 1_{\{R_{DC}{}_i^A > s_A^*\}},$$
$$N(U_j^B, s_B^*) = \textstyle\sum_{i=1}^{u} 1_{\{R_{DC}{}_i^B > s_B^*\}}, \tag{5.6}$$

where s_A^* (and s_B^*) is the significant value[15] used to judge the presence of the DCJ. According to the definition of the DCJ, we identify the DCJ if the R_{DC} is greater than s_A^* (for details see the definition of DC jump in Chapter 4).

We define the indicator function to detect whether U_j^B co-jumps with U_j^A in the $U_j^{A,B}$, denoted by $ICJ(U_j^{A,B})$.

$$ICJ(U_j^{A,B}) = \begin{cases} 1, & N(U_j^A, s_A^*)N(U_j^B, s_B^*) \geq 1 \\ 0, & otherwise \end{cases}, \tag{5.7}$$

where $ICJ(U_j^{A,B}) = 1$ indicates that a $CoJ_{U_j}^{A,B}$ is confirmed in the $U_j^{A,B}$, otherwise $ICJ(U_j^{A,B}) = 0$. We will give examples of the detection of DC co-jumps in Section 5.7.1.

5.6 Retrospective studies

In section 5.1, we introduced co-jumps and how they are related to major events. These events are generally comprised of two categories: (1) the scheduled events; (2) the unscheduled events. In the financial markets, most scheduled events are the major economic events (MEEs). The unscheduled events generally refer to the unexpected events or the sudden incidents, e.g., the COVID-19 pandemic. As introduced in the published references (details have been discussed in section 2.3 and section 5.1), in recent years, researchers stressed the risk of co-jumps spanning multiple markets under the time series (TS) framework. Fundamentally, the concept of the co-jumps in DC is

[15] s^* is the significant value of R_{DC} corresponding to an observer defined significant percentile of the historical R_{DC}'s. For details see Section 4.3.3 in Chapter 4.

different to the concept in TS. Under the DC framework, considering two markets' price sequences; the co-jump is the event such that a jump in one market is followed by a jump in another market. In Chapter 4, we demonstrated that the DC method offer precise information on jumps. This merit is also inherited by the DC co-jumps (we will give examples in section 5.7.1). Overall, there are three major motivations for us to study the DC co-jumps: (1) Can the DC method give precise information of co-jumps in practice? (2) Which MEEs have more influence on co-jumps? Or can we alert participants to avoid the risk of co-jumps for certain MEEs? The following sections will work on these questions.

5.6.1 Data sets

This section presents the data used in the back-testing of co-jump detection through the DC approach. Our study focuses on the jump behavior of two pairs of exchange rates. We select *EURUSD* as the major exchange rate pairing with *GBPUSD*, *USDJPY, USDCAD and AUDUSD*. As shown in table 5.1, the data type is selected as tick-by-tick (raw transactions) over 24 hours from 2015 to 2019. Throughout the application, we select two common arbitrary thresholds, θ = {0.05%, 0.1%}, and two significance levels, s = {95%, 99%}.

Table 5.1 Description of the datasets used in the experiment. Note, EU, GU, UJ, CA and AU are the abbreviation of *EURUSD*, *GBPUSD*, *USDJPY*, *USDCAD* and *AUDUSD*, respectively.

Asset	Source	Frequency	Trading hours[16]	Period
EU-GU *EU-UJ* *EU-CA* *EU-AU*	Dukascopy[17]	Tick-by-tick	24 hours a day	01/01/2015 – 31/12/2019 (five years)

[16] The trading hours on Friday from 00:00:00 to 22:00:00.
[17] Dukascopy Bank is a Swiss online bank which provides different types of high-quality market data. https://www.dukascopy.com/swiss/english/home/

5.6.2 The processes of detecting DC co-jumps

Based on the methodology of Section 5.5, in the paragraph that follows A represents EU and B represents GU; we detect the presence of a $CoJ_U^{A,B}$ in the TR sub-sequence $U^{A,B}$. The TR sub-sequence is the sub-sequence of a partitioned $CTRS$ ($PTRS$). The combined TR sequence ($CTRS$) comprises all the R_{DC}s of the two markets ordered by $ET(R_{DC})$. The details of generating $CTRS$ and $PTRS$ are given in the examples E5.1 and E5.2. In DC, we study whether U^B co-jumps with U^A in $U^{A,B}$. In other words, a $CoJ_U^{A,B}$ is identified when there first exists (at least) one jump in U^A followed by (at least) one jump in U^B. Given a TR sub-sequence $U^{A,B}$, the process of $CoJ_U^{A,B}$ detection follows through the use of the indicator function $ICJ(U^{A,B})$. Based on equation (5.7), a $CoJ_U^{A,B}$ is detected when $ICJ(U^{A,B})$ = 1, otherwise $ICJ(U^{A,B})$ = 0. Table 5.2 (a, b, c, d) summaries the processed DC data which will be used to obtain DC co-jumps; we selected four pairs of currencies under the thresholds of 0.05%, 0.075%, 0.1% and 0.125%.

For instance, to detect $CoJ_U^{EU,GU}$, we first set θ = 0.05% and s = 95%. Table 5.2 (a) summarizes the details of the obtained DC trends and jumps in *EURUSD* and *GBPUSD*. From 2015 to 2019, there were 268162 DC trends where 121888 and 146274 DC trends are from *EURUSD* and *GBPUSD*, respectively. Also, we identified 15553 jumps in total where 5963 and 9590 jumps are from *EURUSD* and *GBPUSD*, respectively.

Table 5.2 (a) The summary of the obtained DC trends and jumps in *EURUSD* and *GBPUSD*. Note, N(*) is the counting function, s = 0.95

Threshold	0.05%	0.075%	0.1%	0.125%
N(DC trends) in total	268162	121837	71471	45283
N(DC trends) for *EURUSD*	121888	54965	32059	20298
N(DC trends) for *GBPUSD*	146274	66872	39412	24985
N(Jumps) in total	15553	7399	4503	2969
N(Jumps) for *EURUSD*	5963	2712	1604	1014
N(Jumps) for *GBPUSD*	9590	4687	2899	1955

Table 5.2 (b) The summary of the obtained DC trends and jumps in *EURUSD* and *USDJPY*. Note, N(*) is the counting function, s = 0.95

Threshold	0.05%	0.075%	0.1%	0.125%
N(DC trends) in total	236410	110883	64031	41461
N(DC trends) for *EURUSD*	117977	54965	31494	20298
N(DC trends) for *USDJPY*	118433	55918	32537	21163
N(Jumps) in total	11999	5792	3398	2237
N(Jumps) for *EURUSD*	5767	2712	1572	1014
N(Jumps) for *USDJPY*	6232	3080	1826	1223

Table 5.2 (c) The summary of the obtained DC trends and jumps in *EURUSD* and *USDCAD*. Note, N(*) is the counting function, s = 0.95

Threshold	0.05%	0.075%	0.1%	0.125%
N(DC trends) in total	222932	104251	59809	38515
N(DC trends) for *EURUSD*	117977	54965	31494	20298
N(DC trends) for *USDCAD*	104955	49286	28315	18217
N(Jumps) in total	10952	5150	2952	1918
N(Jumps) for *EURUSD*	5767	2712	1572	1014
N(Jumps) for *USDCAD*	5185	2438	1380	904

Table 5.2 (d) The summary of the obtained DC trends and jumps in *EURUSD* and *AUDUSD*. Note, N(*) is the counting function, s = 0.95

Threshold	0.05%	0.075%	0.1%	0.125%
N(DC trends) in total	283444	133384	76675	49635
N(DC trends) for *EURUSD*	117977	54965	31494	20298
N(DC trends) for *AUDUSD*	165467	78419	45181	29337
N(Jumps) in total	14238	6771	3960	2584
N(Jumps) for *EURUSD*	5767	2712	1572	1014
N(Jumps) for *AUDUSD*	8471	4059	2388	1570

Given the TR sequences of $EURUSD$ and $GBPUSD$, we generate the $CTRS$ ($CR^{EU,GU}$), and then partition the $CTRS$ ($\Gamma(CR^{EU,GU})$) to obtain the $PTRS_{CR}^{EU,GU}$ (see the definitions of (5.6) and (5.7)). Given the $PTRS_{CR}^{EU,GU}$, we judge the presence of a $CoJ_{U}^{EU,GU}$ in each TR sub-sequence $U^{EU,GU}$ through the usage of the indicator function $ICJ(U^{EU,GU})$.

5.7 Results

5.7.1 The examples of identified DC co-jumps

This section gives two examples of identified DC co-jumps. The detection process has been explained in Section 5.6.2. Both examples summarise precise information on timing, magnitudes, and the direction of the detected DC co-jumps.

Under the threshold $\theta = 0.1\%$ and $s = 0.95$, we detected a DC co-jump between *EURUSD* and *GBPUSD*. As introduced in Section 2.2 (Chapter 2), a DC trend, denoted by DCT, is a connection of two adjacent EPs. An EP is a couple which comprises a timestamp ($EP.t$) with a price ($EP.p$), i.e., $EP = (EP.t, EP.p)$. Thus, we write a DCT as a pair of EPs, e.g., $DCT = (EP_1, EP_2)$.

DC Co-jump Example 1:

Example 1 illustrates the process of detecting a DC co-jump. For a DC co-jump, we shall focus on two trends shown in Table 5.3 that we refer to as DCT_1 and DCT_2:

$$DCT_1 = ((18{:}00{:}04.300, 1.1986), (18{:}00{:}13.800, 1.20344)),$$

$$DCT_2 = ((18{:}00{:}05.500, 1.35785), (18{:}00{:}14.300, 1.36578)).$$

Table 5.3 The example of detected DC co-jump between *EURUSD* and *GBPUSD* in the $\boldsymbol{TR}$ sub-sequence $\boldsymbol{U_j^{EU,GU}}$. The parameters for detecting the DC co-jump: threshold $\boldsymbol{\theta} = 0.1\%$ and $\boldsymbol{s} = 0.95$.

$U^{EU,GU}$	Start time	End time
	20/09/2017 18:00:04.300	20/09/2017 18:00:14.300

DCJ_EU (DCT_1)

Start time	Start price	End time	End price	Period (seconds)	TMV	$R_{DC_1}^{EU}$
18:00:04.300	1.1986	18:00:13.800	1.20344	9.6	4.04	0.042%

DCJ_GU (DCT_2)

Start time	Start price	End time	End price	Period (seconds)	TMV	${R_{DC}}_2^{GU}$
18:00:05.500	1.35785	18:00:14.300	1.36578	8.8	5.84	0.07%

We detected a DC co-jump in the following TR subsequence (equation (5.4)) $U^{EU,GU} = ({R_{DC}}_1^{EU}, {R_{DC}}_2^{GU})$ where ${R_{DC}}_1^{EU}$ and ${R_{DC}}_2^{GU}$ are the time-adjusted returns of the trend DCT_1 and DCT_2. In this TR subsequence, there is only one trend in the *EURUSD* and one trend in the *GBPUSD*. According to equation (2.5), the TMV of the trend DCT_1 is $TMV_1 = \frac{1.20344-1.1986}{1.1086 \times 0.001} = 4.04$. The Period T for DCT_1 from 18:00:04.300 to 18:00:13.800 is 9.6 seconds. Therefore, the time-adjusted return in T1 is ${R_{DC}}_1^{EU} = \frac{|TMV_1| \times \theta}{T_1} = 0.042\%$. The ${R_{DC}}_1^{EU}$ is greater than 95% ($s = 0.95$) of the preceding DC trends in the current moving window of 260 days for *EURUSD*. Hence, DCT_1 is a DC jump. Similarly, we calculated the time-adjusted return for DCT_2. ${R_{DC}}_2^{GU} = 0.07\%$ which is bigger than 95% of the time-adjusted returns in the preceding trends in the current moving window for *GBPUSD*; DCT_2 is also a DC jump. Therefore, a DC co-jump ($CoJ_U^{EU,GU}$) is confirmed that U^{GU} co-jumps with U^{EU} in the TR subsequence $U^{EU,GU}$.

In fact, this DC co-jump was detected just a few seconds after the announcement of FED_IRD (U.S. Fed Funds Interest Rate Decision) on 20/09/2017. In Figure 5.2, *GBPUSD* co-jumps with *EURUSD* in the period of the TR sub-sequence from 18:00:04.300 to 18:00:14.300. The jump sizes (the TMV) of *EURUSD* and *GBPUSD* are 4.04 and 5.84, respectively.

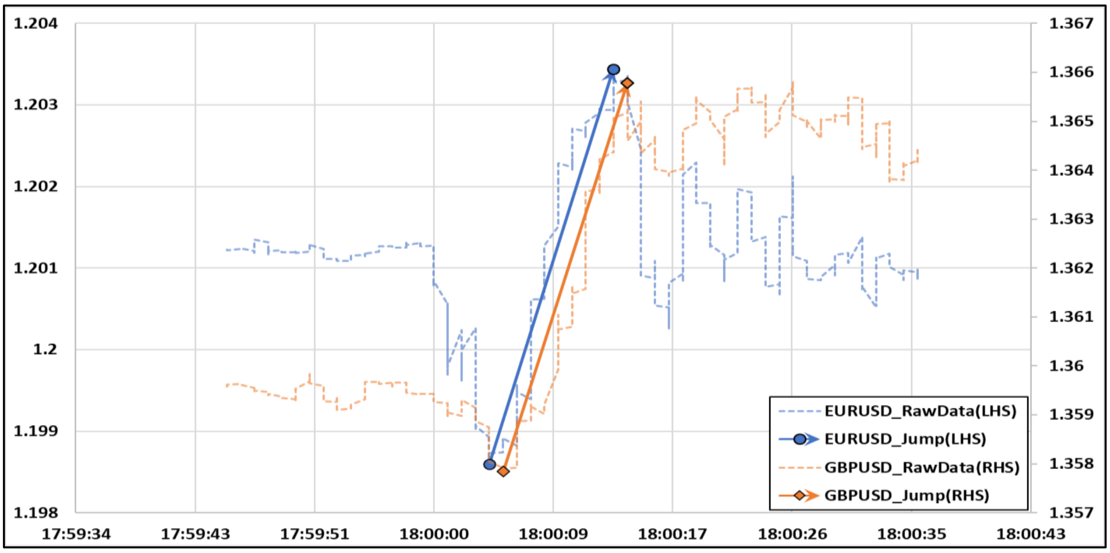

Figure 5.2 The identified DC co-jump in the $\boldsymbol{U^{EU,GU}}$ on 20/09/2017. The period of the $\boldsymbol{TR}$ sub-sequence is about 10 seconds from 18:00:04.300 to 18:00:14.300. The jump sizes (the TMV) of EURUSD_Jump and GBPUSD_Jump are 4.04 and 5.84, respectively.

DC Co-jump Example 2:

In the previous example, we presented a DC co-jump between *EURUSD* and *GBPUSD* that the direction of EURUSD_Jump and GBPUSD_Jump is identical. In this example, we illustrate a DC co-jump while the direction of the two jumps is opposite. In general, *EURUSD* and *USDJPY* had the opposite relationship according to the data source of Eikon. In the second example, we observed a DC co-jump between *EURUSD* and *USDJPY* that there was one DC jump from *EURUSD* followed by another DC Jump from *USDJPY*. As shown in table 5.4 below, there are two DC trends in the $U^{EU,UJ}$. We obtained the ${R_{DC_1}}^{EU} = 0.01\%$ and ${R_{DC_2}}^{UJ} = 0.01\%$ followed by the same computing progress of the first example. Given the s = 0.95, both DCT_1 and DCT_2 are greater than 95% of the time-adjected returns in current moving window of 260 days for *EURUSD* and $USDJPY$. Thus, both DCT_1 and DCT_2 are jumps. Therefore, we noted that U^{UJ} co-jumps with U^{EU} in the TR sub-sequence $U^{EU,UJ}$.

Table 5.4 The example of detected DC co-jump between *EURUSD* and *USDJPY* in the $\boldsymbol{TR}$ sub-sequence $\boldsymbol{U^{EU,UJ}}$. The parameters for detecting the DC co-jump: threshold $\boldsymbol{\theta} = 0.05\%$ and $\boldsymbol{s} = 0.95$.

$U^{EU,UJ}$	Start time	End time
	05/02/2016 13:30:59.700	05/02/2016 13:31:30.100

DCJ_EU (DCT_1)

Start time	Start price	End time	End price	Period (seconds)	TMV	$R_{DC_1}^{EU}$
13:30:59.700	1.12201	13:31:29.500	1.1185	29.8	-6.26	0.01%

DCJ_UJ (DCT_2)

Start time	Start price	End time	End price	Period (seconds)	TMV	$R_{DC_2}^{UJ}$
13:31:00.700	116.6795	13:31:30.100	117.044	29.4	6.24	0.01%

As illustrated in Figure 5.3, the direction of the two jumps is opposing. The jump size of $EURUSD$ and $USDJPY$ are -6.26 and 6.24, respectively. This DC co-jump is observed within the 2 minutes following the data release of the US Nonfarm Payrolls at 13:30 (UTC) on 05/02/2016 and the period of the $U^{EU,UJ}$ is from 13:30:59.700 to 13:31:30.100.

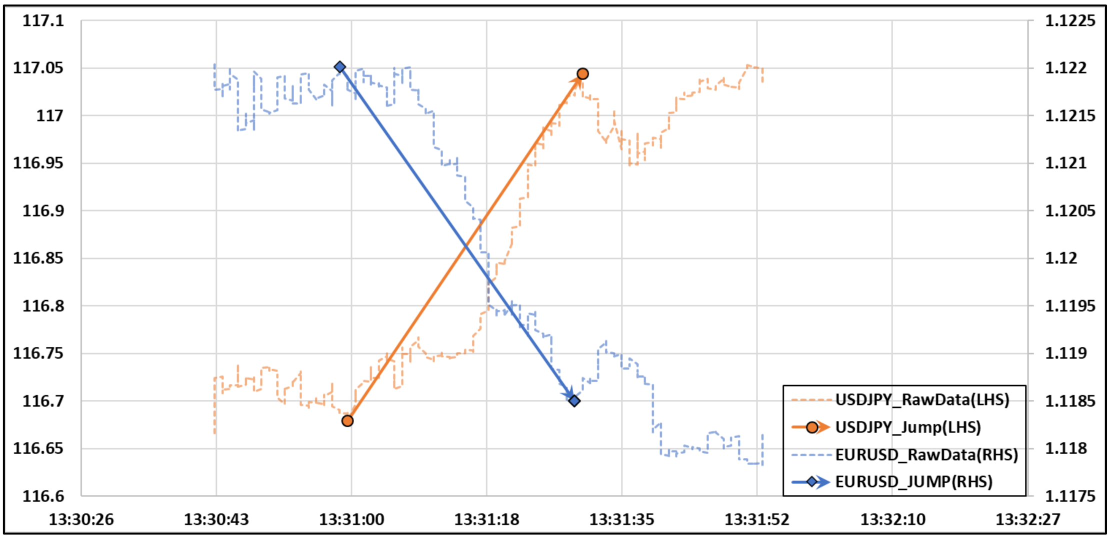

Figure 5.3 The identified DC co-jump in the $\boldsymbol{U^{EU,UJ}}$ on 05/02/2016. The period of the $\boldsymbol{TR}$ sub-sequence is about 30 seconds from 13:30:59.700 to 13:31:30.100. The jump sizes of EURUSD_Jump and USDJPY_Jump are -6.26 and 6.24, respectively.

5.7.2 A co-jump is not a common event

Under the thresholds of 0.05%, 0.075%, 0.1% and 0.125%, table 5.5 (a, b, c, d) below summarises the details of the detected DC co-jumps of four pairs of exchange rates from 2015 to 2019. In row 1 of table 5.5 (a), N(SS) is the number of TR sub-sequences over the five year period. N(CoJ) is the number of confirmed DC co-jumps (in row 2). In row 3, P**(**N(Coj) **|** N(SS)**)** is the proportion of detected DC co-jumps over total TR sub-sequences (N(SS)), i.e., P**(**N(Coj) **|** N(SS)**)** = $\frac{\mathrm{N(Coj)}}{\mathrm{N(SS)}}$. According to the results from table 5.5 (a, b, c, d), we observed that a co-jump is not a common event. Under the four thresholds, the figures of the P**(**N(Coj) **|** N(SS)**)** are less than 2%.

Table 5.5 (a) The summary of the DC co-jumps between *EURUSD* and *GBPUSD*. The parameters of detecting DC co-jump: $\boldsymbol{s} = 0.95$. Periods: from 2015 to 2019.

		0.05%	0.075%	0.1%	0.125%
1	N(SS) (1)	68040	31010	18253	11582
2	N(CoJ) (2)	615	451	281	207
3	P(N(Coj) | N(SS)) (5)	0.90%	1.454%	1.54%	1.787%

Table 5.5 (b) The summary of the DC co-jumps between *EURUSD* and *USDJPY*. The parameters of detecting DC co-jump: $\boldsymbol{s} = 0.95$. Periods: from 2015 to 2019.

		0.05%	0.075%	0.1%	0.125%
1	N(SS) (1)	58292	27512	16011	10390
2	N(CoJ) (2)	813	421	253	158
3	P(N(Coj) | N(SS)) (5)	1.395%	1.530%	1.580%	1.521%

Table 5.5 (c) The summary of the DC co-jumps between *EURUSD* and *USDCAD*. The parameters of detecting DC co-jump: $\boldsymbol{s} = 0.95$. Periods: from 2015 to 2019.

		0.05%	0.075%	0.1%	0.125%
1	N(SS) (1)	58239	27658	15915	10296
2	N(CoJ) (2)	667	366	227	143
3	P(N(Coj) | N(SS)) (5)	1.145%	1.323%	1.426%	1.389%

Table 5.5 (d) The summary of the DC co-jumps between *EURUSD* and *AUDUSD*. The parameters of detecting DC co-jump: $\boldsymbol{s} = 0.95$. Periods: from 2015 to 2019.

		0.05%	0.075%	0.1%	0.125%
1	N(SS) (1)	73236	34606	20013	12978
2	N(CoJ) (2)	955	472	333	227
3	P(N(Coj) | N(SS)) (5)	1.304%	1.364%	1.664%	1.749%

***Observation 1*:** A co-jump is not a common event

Observation 1 is shown by studying the frequency of DC co-jumps over the number of TR sub-sequences in 5 years. For the four pairs of the exchange rates, the figures of P(N(Coj) **|** N(SS)) are less than 2% under the four thresholds; this indicates that a co-jump is not a common event.

In this section, we confirmed that the frequency of a co-jump presenting is very low compared with the total number of TR sub-sequences.

5.7.3 Introducing major economic events (MEEs)

As discussed in section 2.3.2, co-jumps are usually associating with the major economic events (MEEs). In Chapter 4, we detected jumps after the announcements of the economic data. Also, we observed that different economic data have various degrees of the influence on jumps. In other word, jumps are highly sensitive to some certain MEEs in comparison to others. In this chapter, the presence of a co-jump may be associated with the MEEs from at least three countries. We focus on the $CoJ_U^{EU,GU}$ (the DC co-jumps of *EURUSD* with *GBPUSD*), $CoJ_U^{EU,UJ}$ (the DC co-jumps of *EURUSD* with *USDJPY*), the $CoJ_U^{EU,CA}$ (the DC co-jumps of *EURUSD* with *USDCAD*) and $CoJ_U^{EU,AU}$ (the DC co-jumps of *EURUSD* with *AUDUSD*). Because the US dollar is the comparative currency, the presence of a DC co-jump could be the result of the MEEs from one of the three countries. This section intends to study the relationship between DC co-jumps and the three countries' MEEs.

Note, the domestic MEEs indicate those MEEs from the relevant currency's country, e.g., the domestic MEEs of Sterling (*GBPUSD*) are the UK MEEs. The US MEEs (denoted by MEE_{US}) are the MEEs from the US. Also, the non-US MEEs are the MEEs from the domestic countries. For example, the MEEs of a DC co-jump ($CoJ_U^{EU,GU}$) refer to three countries: the US, the UK, and Euro zone. Hence, for the $CoJ_U^{EU,GU}$, we define the domestic MEEs as the MEEs from both the UK and Europe (denoted by $\text{MEE}_{\text{non-US}}$), and the MEE_{US} indicates the US MEEs.

The back-testing selected the same MEEs as used in section 4.5.1 to track the identified $CoJ_U^{EU,GU}$, $CoJ_U^{EU,UJ}$, $CoJ_U^{EU,CA}$, and $CoJ_U^{EU,AU}$ during the 30 minutes following the MEE announcements. The details about the selected MEEs are described in table 5.6.

Table 5.6 Information regarding the selected major economic events.

Announcement	Variable name	Frequency	# of MEEs over 5 years
US Fed Funds Interest Rate Decision	US_IRD	8 times per year	40
The European Central Bank Interest Rate Decision	EU_IRD	8 times per year	40
Bank of England Interest Rate Decision	GB_IRD	8 times per year	40
Bank of Japan Interest Rate Decision	JP_IRD	8 times per year	40
Bank of Canada Interest Rate Decision	BoC_IRD	8 times per year	40
Bank of Australia Interest Rate Decision	BoA_IRD	Not regular frequency	55
GDP	US_GDP, UK_GDP, EU_GDP, JPY_GDP, CA_GDP, AU_GDP	Quarterly	20 per country
Consumer Price Index	US_CPI, UK_CPI, EU_CPI,JPY_CPI,CA _CPI,	Monthly	60 per country
US Retail Sales	US_RS	Monthly	60
US Nonfarm Payrolls	US_NFP	Monthly	60

Note, For AUDUSD, the number of CPIs is 24.

In all, there are 240 US MEEs (8 FED_IRD, 4 US_GDP, 12 US_CPI, US_RS and US NFP per year for five years, i.e. (8+4+12+12+12)x5=240). The non-US MEEs are the MEEs from the domestic countries. In the study of DC co-jumps between *EURUSD* and *GBPUSD*, the non-US MEEs are the MEEs from the UK and Europe. Thus, there are 240 non-US MEEs (8 ECB_IRD, 8 BoE_IRD, 4 EU_GDP, 4 UK_GDP,12 EU_CPI, and 12 UK_CPI per year for five year, i.e. (8+8+4+4+12+12)x5=240).

5.7.4 The impact of US MEEs on DC co-jumps

We will give a detailed study of the impact of US MEEs on DC co-jumps between *EURUSD* and *GBPUSD*; then the other three pairs will be discussed. In Section 5.7.3, we noted that the presence of a co-jump should be affected by at least three countries' MEEs. Thus, we expect that the MEEs of Euro zone, the UK, and the US will have an impact on the presence of DC co-jumps, but we expect that the degree of the impacts on the co-jumps from the three countries' MEEs is different. The US dollar is the comparative currency to both Euro and Sterling, i.e., *EURUSD* and *GBPUSD*. For instance, a US MEE may cause a jump in US dollar, and this may trigger a jump in both *EURUSD* and *GBPUSD* simultaneously. However, in *EURUSD*, Euro is not directly comparative to Sterling. Also, in *GBPUSD*, Sterling is not directly comparative to Euro. So, for example, an EU MEE may cause a jump in Euro, and this may trigger a jump in *EURUSD*; but it may not cause a jump in *GBPUSD*.

Specifically, our questions about the MEEs' impact on co-jumps are listed below:

1. The US MEEs may trigger a DC co-jump in *EURUSD* and *GBPUSD*.
2. The domestic MEEs (Europe and the UK) may or may not trigger a DC co-jump in *EURUSD* and *GBPUSD*.

To investigate the issues raised above, this section will study the relationship between the MEEs (from Europe, the UK, and the US) and the DC co-jumps. This could answer the following questions. Do DC co-jumps follow both US and domestic MEEs? How often do DC co-jumps happen after US and domestic MEEs?

In this study, we continue to use the observations of the previous sections:

i. We focus on DC co-jumps between *EURUSD* and *GBPUSD* in the 2015-2019 dataset, as described in Section 5.6.1.
ii. We focus on the US and the domestic MEEs listed in Table 5.6.

We define that a DC co-jump follows a MEE if the co-jump happens within 30 minutes after the economic data announcement (the MEE). Table 5.7 (a) summarizes the results of the DC co-jumps associated with the MEEs. The data indicated that most of the co-jumps are associated with the US MEEs; we will give a statistical summary in table 5.7 (b).

Table 5.7 (a) The detected DC co-jumps associated with the MEEs. The parameters of detecting DC co-jump: threshold $\boldsymbol{\theta} = 0.05\%$ and $\boldsymbol{s} = 0.95$. Periods: 2015 to 2019.The DC co-jumps were those identified within the 30 minutes after the economic data announcement. Note, EU, GU, and UJ are the abbreviations of *EURUSD* and *GBPUSD*.

Index	The name of MEEs	$CoJ_U^{EU,GU}$
1	US_IRD	94
2	ECB_IRD	12
3	BoE_IRD	1
4	EU_GDP	0
5	EU_CPI	0
6	UK_GDP	0
7	UK_CPI	0
8	US_GDP	1
9	US_NFP	78
10	US_RS	6
11	US_CPI	8

Table 5.7 (b) The statistical summary of table 5.7 (a). Threshold $\boldsymbol{\theta} = 0.05\%$ and $\boldsymbol{s} = 0.95$.

Index	The measurement	$CoJ_U^{EU,GU}$
1	N(CoJ-MEE_{US})	187
2	N(CoJ-$MEE_{non\text{-}US}$)	13
3	N(CoJ-MEE)	200
4	N(CoJ)	615
5	P(CoJ-MEE_{US} | CoJ-MEE)	93.5%
6	P(CoJ-$MEE_{non\text{-}US}$ | CoJ-MEE)	6.5%
7	P(MEE | CoJ)	32.52%

Note: (1) Row 1: N(CoJ-MEE_{US}) is the total number of DC co-jumps following the US MEEs; (2) Row 2: N(CoJ-$MEE_{non\text{-}US}$) is the total number of DC co-jumps following the non-US MEEs (the two domestic MEEs); (3) Row 3: N(CoJ-MEE) is the total number of DC co-jumps following the MEEs; (4) Row 4: N(CoJ) is the total DC co-jumps detected over the five year period; (5) Row 5: P(CoJ-MEE_{US} | CoJ-MEE) is the proportion of N(CoJ-MEE_{US}) over N(CoJ-MEE), i.e., P(CoJ-MEE_{US} | CoJ-MEE) = $\frac{\mathrm{N(CoJ-MEE_{US})}}{\mathrm{N(CoJ-MEE)}}$; (6) Row 7: P(MEE | CoJ) indicates the value of N(CoJ-MEE) over N(CoJ) as a proportion, i.e., P(MEE | CoJ) = $\frac{\mathrm{N(CoJ-MEE)}}{\mathrm{N(CoJ)}}$.

Table 5.7 (b) presents the statistical summary of the relationship between co-jumps and the MEEs based on the data from table 5.7 (a). We found the most significant number of DC co-jumps to follow the US MEEs.

Following are our observations from table 5.7 (b):

1. We detected 615 DC co-Jumps over 5 years from 2015 to 2019 (N(CoJ), row 4 of table 5.7 (b)).
2. 200 out of 615 DC co-jumps associated with MEEs (N(CoJ-MEE) in row 3 of table 5.7 (b)), which accounted for 32.52% (P(MEE | CoJ) = $\frac{200}{615}$ = 32.52%, in row 7 of table 5.7 (b)), roughly one third, of the total detected DC co-jumps.
3. From the 200 CoJ-MEEs, 187 DC co-jumps followed the US MEEs (N(CoJ-MEE_{US}), row 1 of table 5.7 (b)), i.e., 93.5% of the CoJ-MEEs (P(CoJ-MEE_{US} | CoJ-MEE), row 19). 13 DC co-jumps follow the domestic MEEs (N(CoJ-$MEE_{non\text{-}US}$), row 16), i.e., 6.5% of the CoJ-MEEs (row 6 of table 5.7 (b)).
4. From the 187 CoJ-MEE_{US}, the numbers of DC co-jumps associated with US_IRD and US_ NFP are 94 and 78, respectively. Together these make up just under 92% ($\frac{172}{187}$) of CoJ-MEE_{US}.
5. In the same period, there were 240 US MEEs and 240 domestic MEEs (the sum of the MEEs from the UK and Europe, as explained in Section 5.7.3.).
6. From the 200 CoJ-MEEs, we observed that 78% ($\frac{187}{240}$) DC co-jumps followed the US MEEs, and 5.4% ($\frac{13}{240}$) DC co-jumps were found after the domestic MEEs.

7. Therefore, DC co-jumps followed the US MEEs substantially more than they followed the domestic MEEs in both absolute numbers (point 3) and percentage wise (point 6).

In summary, we found that US MEEs have a significant impact on the DC co-jumps. This is supported by point 3 above that 93.5% of the CoJ-MEEs follow the US MEEs. While the impact of the domestic MEEs is very small that 6.5% (in row 6 of table 5.7 (b)) of the CoJ-MEEs follow the domestic MEEs (the EU and UK). Importantly, of the US MEEs, we discovered that US the interest rate decision announcements (US_IRD) and non-farm payroll (US_NFP) numbers impact significantly on the presence of DC co-jumps (point 4). This suggests that Forex traders should be aware of the risk of co-jumps between *EURUSD* and *GBPUSD* following the release of US economic data especially interest rate decisions and non-farm payroll number announcements.

We also repeaded the same test for four pairs of currencies under the thresholds of 0.05%, 0.075%, 0.1% and 0.125%; the results are shown below in the table 5.8 (a, b, c, d). The four tables indicate the following conclusions: (1) the majority of the DC co-jumps associated with the US MEEs that the figures of the P(CoJ-MEEUS | CoJ-MEE) are over 90%; (2) few DC co-jumps followed to the the domestic MEEs that the figures of the P(CoJ-MEE$_{\text{non-US}}$ | CoJ-MEE) are less than 5%; (3) for the co-jumps $CoJ_U^{EU,CA}$ and $CoJ_U^{EU,AU}$, there is a solid relationship between the MEEs and the co-jumps presenting that the figures of the P(MEE | CoJ) are over 50%; (4) for the co-jumps $CoJ_U^{EU,GU}$ and $CoJ_U^{EU,UJ}$, we observed the figures of the P(MEE | CoJ) are around 30% and 40%, respectively. Overall, under the thresholds of 0.05%, 0.075%, 0.1% and 0.125%, we observed the similar conclusions with the observations from table 5.7.

Table 5.8 (a) The detected DC co-jumps associated with the MEEs. The parameters of detecting DC co-jumps: threshold $\boldsymbol{\theta} = 0.05\%$ and $\boldsymbol{s} = 0.95$. Periods: 2015 to 2019.The DC co-jumps were those identified within the 30 minutes after economic data announcements. Note, EU, GU, UJ, CA, and AU are the abbreviations of *EURUSD*, *GBPUSD*, *USDJPY*, *USDCAD* and *AUDUSD*.

Index	MEEs	$CoJ_U^{EU,GU}$	$CoJ_U^{EU,UJ}$	$CoJ_U^{EU,CA}$	$CoJ_U^{EU,AU}$
1	US_IRD	94	147	170	209
2	EU_IRD	12	3	5	8
3	UK_IRD	1	-	-	-
4	JP_IRD	-	2	-	-
5	CA_IRD	-	-	2	-
6	AU_IRD	-	-	-	0
7	EU_GDP	0	0	0	0
8	EU_CPI	0	0	0	0
9	UK_GDP	0	-	-	-
10	UK_CPI	0	-	-	-
11	JP _GDP	-	0	-	-
12	JP _CPI	-	0	-	-
13	CA _GDP	-	-	3	-
14	CA _CPI	-	-	6	-
15	AU _GDP	-	-	-	0
16	AU _CPI	-	-	-	0
17	US_GDP	1	6	8	9
18	US_NFP	78	123	129	181
19	US_RS	6	16	24	27
20	US_CPI	8	19	18	22
The statistical summary					
1	N(CoJ-MEE$_{US}$)	187	311	349	448
2	N(CoJ-MEE$_{non\text{-}US}$)	13	5	16	8
3	N(CoJ-MEE)	200	316	365	456
4	N(CoJ)	615	94	95	96
5	P(CoJ-MEE$_{US}$ | CoJ-MEE)	93.5%	98.42%	95.62%	98.25%
6	P(CoJ-MEE$_{non\text{-}US}$ | CoJ-MEE)	6.5%	1.58%	4.38%	1.75%
7	P(MEE | CoJ)	32.52%	38.87%	54.72%	47.75%

Table 5.8 (b) The detected DC co-jumps associated with the MEEs. The parameters of detecting DC co-jumps: threshold $\boldsymbol{\theta} = 0.075\%$ and $\boldsymbol{s} = 0.95$. Periods: 2015 to 2019.The DC co-jumps were those identified within the 30 minutes after economic data announcements. Note, EU, GU, UJ, CA, and AU are the abbreviations of *EURUSD*, *GBPUSD*, *USDJPY*, *USDCAD* and *AUDUSD*.

Index	MEEs	$CoJ_U^{EU,GU}$	$CoJ_U^{EU,UJ}$	$CoJ_U^{EU,CA}$	$CoJ_U^{EU,AU}$
1	US_IRD	68	71	101	101
2	EU_IRD	11	3	6	6
3	UK_IRD	0	-	-	-
4	JP_IRD	-	1	-	-
5	CA_IRD	-	-	0	-
6	AU_IRD	-	-	-	0
7	EU_GDP	0	0	0	0
8	EU_CPI	0	0	0	0
9	UK_GDP	0	-	-	-
10	UK_CPI	0	-	-	-
11	JP _GDP	-	0	-	-
12	JP _CPI	-	1	-	-
13	CA _GDP	-	-	0	-
14	CA _CPI	-	-	3	-
15	AU _GDP	-	-	-	0
16	AU _CPI	-	-	-	0
17	US_GDP	1	5	5	5
18	US_NFP	65	82	79	109
19	US_RS	6	10	8	14
20	US_CPI	6	8	6	13

The statistical summary.

1	N(CoJ-MEE$_{US}$)	146	176	199	242
2	N(CoJ-MEE$_{non\text{-}US}$)	11	5	9	6
3	N(CoJ-MEE)	157	181	208	248
4	N(CoJ)	93	94	95	96
5	P(CoJ-MEE$_{US}$ | CoJ-MEE)	92.99%	97.24%	95.67%	97.58%
6	P(CoJ-MEE$_{non\text{-}US}$ | CoJ-MEE)	7.01%	2.76%	4.33%	2.42%
7	P(MEE | CoJ)	34.81%	42.99%	56.83%	52.54%

Table 5.8 (c) The detected DC co-jumps associated with the MEEs. The parameters of detecting DC co-jumps: threshold $\boldsymbol{\theta} = 0.1\%$ and $\boldsymbol{s} = 0.95$. Periods: 2015 to 2019.The DC co-jumps were those identified within the 30 minutes after economic data announcements. Note, EU, GU, UJ, CA, and AU are the abbreviations of *EURUSD*, *GBPUSD*, *USDJPY*, *USDCAD* and *AUDUSD*.

Index	MEEs	$CoJ_U^{EU,GU}$	$CoJ_U^{EU,UJ}$	$CoJ_U^{EU,CA}$	$CoJ_U^{EU,AU}$
1	US_IRD	51	51	66	84
2	EU_IRD	7	2	3	1
3	UK_IRD	0	-	-	-
4	JP_IRD	-	2	-	-
5	CA_IRD	-	-	0	-
6	AU_IRD	-	-	-	0
7	EU_GDP	0	0	0	0
8	EU_CPI	0	0	0	0
9	UK_GDP	0	-	-	-
10	UK_CPI	0	-	-	-
11	JP _GDP	-	0	-	-
12	JP _CPI	-	0	-	-
13	CA _GDP	-	-	1	-
14	CA _CPI	-	-	3	-
15	AU _GDP	-	-	-	0
16	AU _CPI	-	-	-	0
17	US_GDP	1	3	4	6
18	US_NFP	40	43	48	62
19	US_RS	4	2	7	7
20	US_CPI	2	4	5	8

The statistical summary.

1	N(CoJ-MEE$_{US}$)	98	103	130	167
2	N(CoJ-MEE$_{non-US}$)	7	4	7	1
3	N(CoJ-MEE)	105	107	137	168
4	N(CoJ)	93	94	95	96
5	P(CoJ-MEE$_{US}$ | CoJ-MEE)	93.33%	96.26%	94.89%	99.40%
6	P(CoJ-MEE$_{non-US}$ | CoJ-MEE)	6.67%	3.74%	5.11%	0.60%
7	P(MEE | CoJ)	37.77%	42.29%	60.35%	50.45%

Table 5.8 (d) The detected DC co-jumps associated with the MEEs. The parameters of detecting DC co-jumps: threshold $\boldsymbol{\theta} = 0.125\%$ and $\boldsymbol{s} = 0.95$. Periods: 2015 to 2019.The DC co-jumps were those identified within the 30 minutes after economic data announcements. Note, EU, GU, UJ, CA, and AU are the abbreviations of *EURUSD*, *GBPUSD*, *USDJPY*, *USDCAD* and *AUDUSD*.

Index	MEEs	$CoJ_U^{EU,GU}$	$CoJ_U^{EU,UJ}$	$CoJ_U^{EU,CA}$	$CoJ_U^{EU,AU}$
1	US_IRD	39	37	44	65
2	EU_IRD	4	2	3	2
3	UK_IRD	0	-	-	-
4	JP_IRD	-	2	-	-
5	CA_IRD	-	-	0	-
6	AU_IRD	-	-	-	0
7	EU_GDP	0	0	0	0
8	EU_CPI	0	0	1	0
9	UK_GDP	0	-	-	-
10	UK_CPI	0	-	-	-
11	JP _GDP	-	0	-	-
12	JP _CPI	-	0	-	-
13	CA _GDP	-	-	1	-
14	CA _CPI	-	-	0	-
15	AU _GDP	-	-	-	0
16	AU _CPI	-	-	-	0
17	US_GDP	1	1	2	4
18	US_NFP	29	25	32	41
19	US_RS	1	2	4	2
20	US_CPI	2	4	2	4
The statistical summary.					
1	N(CoJ-MEE$_{US}$)	72	69	84	116
2	N(CoJ-MEE$_{non-US}$)	4	4	5	2
3	N(CoJ-MEE)	76	73	89	118
4	N(CoJ)	93	94	95	96
5	P(CoJ-MEE$_{US}$ | CoJ-MEE)	94.74%	94.52%	94.38%	98.31%
6	P(CoJ-MEE$_{non-US}$ | CoJ-MEE)	5.26%	5.48%	5.62%	1.69%
7	P(MEE | CoJ)	36.71%	46.20%	62.24%	51.98%

5.7.5 The case studies of the relationship between the major historical events and co-jumps

In the previous section, we analysed the influence of the economic data on co-jumps. In Section 5.6, we introduced that the presence of co-jumps may relate to non-economic events. Particularly, some key historical events may cause substantial co-jumps. This section presents two examples to explore the relationship between the major historical events and co-jumps in terms of posing the question whether major outlying events may have a significant impact on co-jumps? We study the Brexit Referendum which is a scheduled event with an unexpected result and the COVID-19 pandemic which is an entirely unexpected event with far reaching consequences. We find a strong association between these outlying events and the presence of co-jumps.

The two case studies are: (1) the effects of the Brexit Referendum covering a time period in 2016; (2) the COVID-19 pandemic during the year of 2020 in Europe.

Case study 1:

Brexit is the UK's departure from the European Union (EU). On 23th of June 2016, the UK held a referendum on its membership of the EU[18]. The result of the referendum caused more than 10% drop in *GBPUSD* on 24th of June 2016. Also, *EURUSD* had more than 4% fall in the same day.[19] In this study, we want to know the impact of the Brexit Referendum result on the co-jumps between *EURUSD* and *GBPUSD*. Figure 5.4 summarizes the daily number of DC co-jumps in 2016. Obviously, most of the co-jumps were detected after the day of the Brexit referendum. In table 5.9, we observed 141 DC co-jumps in 2016. While 86 DC co-jumps are confirmed on 24th of June which accounts for 61% (in row 3 of table 5.8) of the total DC co-jumps of the year.

[18] For details about Brexit see: https://www.government.nl/topics/brexit/question-and-answer/what-is-brexit
[19] According to the data from Refinitiv Eikon.

Table 5.9 The summary of the detected DC co-jumps between *EURUSD* and *GBPUSD* in 2016. The parameters of detecting DC co-jump: threshold $\boldsymbol{\theta}$ = 0.1% and $\boldsymbol{s}$ = 0.95. There were 86 DC co-jumps on 24th of June.

1	N(CoJ)	141
2	N(CoJ-Brexit)	86
3	P(N(CoJ-Brexit) | N(CoJ-Brexit))	61%

Note: (1) N(CoJ) is the total DC co-jumps detected in 2016; (2) N(CoJ-Brexit) is the number of DC co-jumps detected on 24th of June; (3) P(N(CoJ-Brexit) | N(CoJ)) is the proportion of N(CoJ-Brexit) over N(CoJ), i.e., P(N(CoJ-Brexit) | N(CoJ)) = $\frac{\mathrm{N(CoJ-Brexit)}}{\mathrm{N(CoJ)}}$.

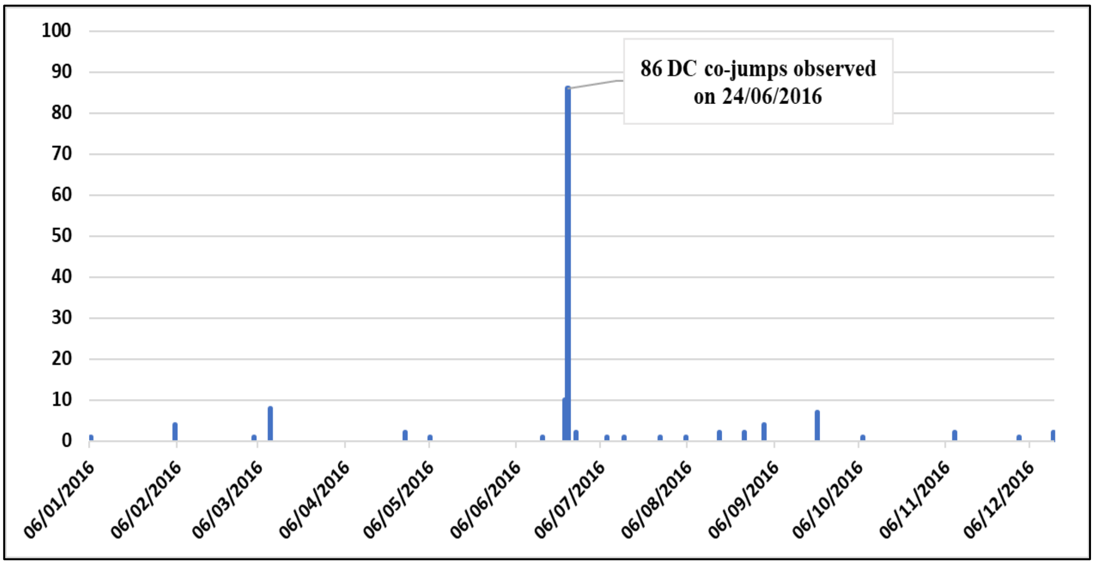

Figure 5.4 The daily number of DC co-jumps between *EURUSD* and *GBPUSD* in 2016. We observed 141 DC co-jumps in 2016. Parameters: $\boldsymbol{\theta}$ = 0.1% and $\boldsymbol{s}$ = 95%.

Case study 2:

In the second case study, we tracked the daily number of DC co-jumps between *EURUSD* and *GBPUSD* during the COVID-19 pandemic in 2020. On 13th of March 2020, the World Health Organization (WHO) stated that Europe was the epicenter of the coronavirus pandemic.[20] This unscheduled event caused turmoil in the FX markets especially in Euro and Sterling in March 2020. As shown in table 5.10, there were 72

[20] For the references about WHO's comments on the coronavirus pandemic in Europe see: (1) *https://www.bbc.co.uk/news/world-europe-51876784; (2) https://www.reuters.com/article/us-health-coronavirus-who/europe-is-epicenter-of-coronavirus-pandemic-who-idUSKBN2102Q0.*

DC co-jumps detected in March, which account for 77% (in row 3 of table 5.9) of the total number of the DC co-jumps within the year. Figure 5.5 illustrates the daily number of DC co-jumps. Obviously, most of the DC co-jumps were determined during March. Especially, we observed the largest number of DC co-jumps on 19/03/2020 (26 DC co-jumps detected) when more than 250 million people were in lockdown in Europe[21]. After March 2020, we observed 8 DC co-jumps on 10/06/2020 after the announcement of the US Interest Rate Decision[22].

Table 5.10 The summary of the detected DC co-jumps between *EURUSD* and *GBPUSD* in 2020. The parameters of detecting DC co-jump: threshold $\boldsymbol{\theta} = 0.05\%$ and $\boldsymbol{s} = 0.95$.

1	N(CoJ)	93
2	N(CoJ-March)	72
3	P(N(CoJ- March) | N(CoJ))	77%

Note: (1) N(CoJ) is the total DC co-jumps detected in 2020; (2) N(CoJ-March) is the number of DC co-jumps detected in March; (3) P(N(CoJ-March) | N(CoJ)) is the proportion of N(CoJ) over N(CoJ-March), i.e., P(N(CoJ-March) | N(CoJ)) = $\frac{\mathrm{N(CoJ-March)}}{\mathrm{N(CoJ)}}$.

[21]For the references about the lockdown of Europe in March 2020: https://www.theguardian.com/world/2020/mar/18/coronavirus-lockdown-eu-belgium-germany-adopt-measures.
[22] The DC co-jumps were those identified within the 30 minutes after the economic data announcement.

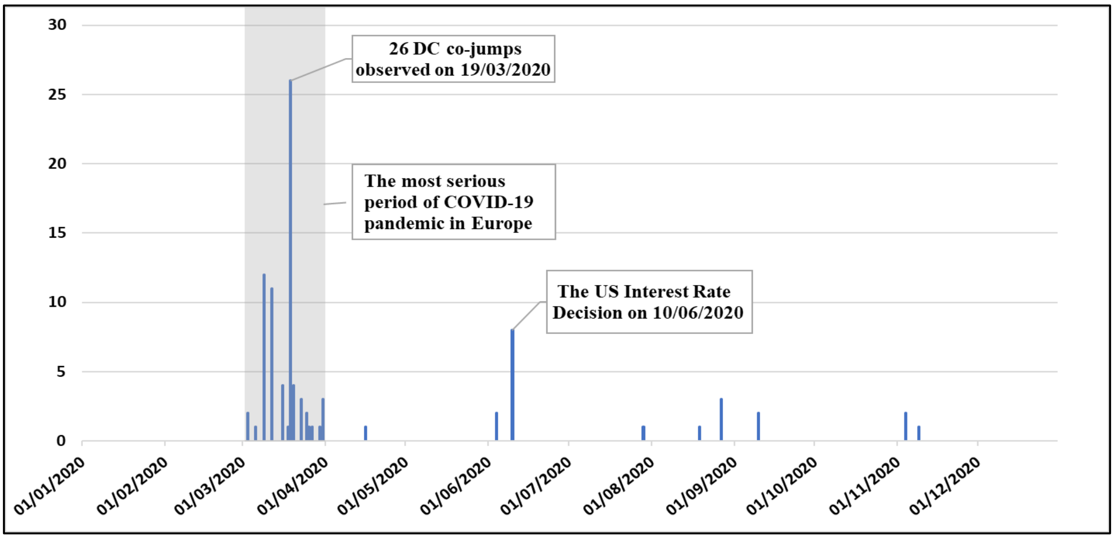

Figure 5.5 The daily number of DC co-jumps between *EURUSD* and *GBPUSD* in 2020. There were 93 DC co-jumps identified in total. Parameters: $\boldsymbol{\theta}$ = 0.05% and $\boldsymbol{s}$ = 95%.

To summarise, we observed that major historical events could cause a significant impact on co-jumps. The results of our back-testing indicate that major adverse news can cause many co-jumps in a single day. For instance, the amount of the co-jumps related to the Brexit referendum accounts for 61% of the total co-jumps for the year. This suggests that co-jumps may be more sensitive to key events than the economic data. This further reflects the unusual behaviours of the trading actions from the Forex traders when they encounter unexpected events.

5.7.6 Did DC and TS find the same co-jumps?

In Chapter 4 (Section 4.6.4), we presented the results of co-jumps detected by the two different methods. In this section, we will give a summary of identified co-jumps based on the TS method and DC approach. The back-testing selected *EURUSD* as the major exchange rate pairing with *GBPUSD*, *USDJPY*, *USDCAD*, and *AUDUSD*; the periods of the dataset used between 2015 to 2019 (for details see table 5.1). For the TS method, we selected the method proposed by Lahaye et al (2011) that they detect jumps happening simultaneously in two markets by using the product of the indicator functions of the jumps in the individual markets (for details see equation (2.18) in Section 2.4.2).

Table 5.11 (a, b, c, d) summaries the details of the DC co-jumps and TS co-jumps over five years; specifically, we selected four pairs of currencies, under the thresholds of 0.05%, 0.075%, 0.1% and 0.125%. Note, in row 1 and row 2 of table 5.11 (a), N(DC-CoJ) and N(TS-CoJ) are the number of co-jumps identified by the two approaches; in row 3, N(DC-CoJ in TS-CoJ) represents how many DC co-jumps were found within the 15 minute time interval associated to a TS co-jumps; in row 4, P(DC-CoJ in TS-CoJ) indicates N(DC-CoJ in TS-CoJ) as a proportion of the total N(DC-CoJ), ($\mathrm{P(DC-CoJ\ in\ TS-CoJ)} = \frac{\mathrm{N(DC-CoJ\ in\ TS-CoJ)}}{\mathrm{N(DC-CoJ)}}$)); thus P(DC-CoJ in TS-CoJ) describes the percentage of the DC co-jumps that coincide with a TS co-jump; in row 5, N(TS-CoJ overlapping DC-CoJ) is the number of TS co-jumps that overlap with DC co-jumps; in row 6, P(TS-CoJ overlapping DC-CoJ) is N(TS-CoJ overlapping DC-CoJ) as a proportion of the total number of TS co-jumps, ($\mathrm{P(TS-CoJ\ overlapping\ DC-CoJ)} = \frac{\mathrm{N(TS-CoJ\ overlapping\ DC-CoJ)}}{\mathrm{N(TS-CoJ)}}$).

Table 5.11 (a) The summary of the DC co-jumps and the TS co-jumps. The parameters of detecting DC co-jump: threshold $\boldsymbol{\theta} = 0.05\%$ and $\boldsymbol{s} = 0.95$. The parameters of detecting TS co-jump: the significance level α = 0.01 and the time interval = 15 min. Note, EU, GU, UJ, CA, and AU are the abbreviations of *EURUSD*, *GBPUSD*, *USDJPY*, *USDCAD* and *AUDUSD*.

		EU with *GU*	*EU* with UJ	*EU* with *CA*	*EU* with *AU*
1	N(DC-CoJ)	880	813	667	955
2	N(TS-CoJ)	111	140	87	37
3	N(DC-CoJ in TS-CoJ)	208	157	133	147
4	P(DC-CoJ in TS-CoJ)	23.64%	19.31%	19.94%	15.39%
5	N(TS-CoJ overlapping)	39	43	36	25
6	P(TS-CoJ overlapping)	35.14%	30.71%	41.38%	67.57%

Table 5.11 (b) The summary of the DC co-jumps and the TS co-jumps. The threshold $\boldsymbol{\theta} = 0.075\%$.

		EU with *GU*	*EU* with UJ	*EU* with *CA*	*EU* with *AU*
1	N(DC-CoJ)	451	421	366	472
2	N(TS-CoJ)	111	140	87	37
3	N(DC-CoJ in TS-CoJ)	99	79	69	80
4	P(DC-CoJ in TS-CoJ)	21.95%	18.76%	18.85%	16.95%
5	N(TS-CoJ overlapping)	30	33	21	22
6	P(TS-CoJ overlapping)	27.02%	23.57%	24.14%	59.46%

Table 5.11 (c) The summary of the DC co-jumps and the TS co-jumps. The threshold $\boldsymbol{\theta} = 0.1\%$.

		EU with *GU*	*EU* with UJ	*EU* with *CA*	*EU* with *AU*
1	N(DC-CoJ)	278	253	227	333
2	N(TS-CoJ)	111	140	87	37
3	N(DC-CoJ in TS-CoJ)	58	44	43	50
4	P(DC-CoJ in TS-CoJ)	20.86%	17.39%	18.94%	15.02%
5	N(TS-CoJ overlapping)	19	22	18	18
6	P(TS-CoJ overlapping)	17.12%	15.71%	20.69%	48.65%

Table 5.11 (d) The summary of the DC co-jumps and the TS co-jumps. The threshold $\boldsymbol{\theta} = 0.125\%$.

		EU with *GU*	*EU* with UJ	*EU* with *CA*	*EU* with *AU*
1	N(DC-CoJ)	207	158	143	227
2	N(TS-CoJ)	111	140	87	37
3	N(DC-CoJ in TS-CoJ)	30	25	20	32
4	P(DC-CoJ in TS-CoJ)	14.49%	15.82%	13.99%	14.10%
5	N(TS-CoJ overlapping)	12	13	9	14
6	P(TS-CoJ overlapping)	10.81%	9.29%	10.34%	37.84%

According to table 5.11, we observed that co-jumps found in both DC and TS method. Under the four thresholds, the figures of the P(DC-CoJ in TS-CoJ) are observed between 15% and 20% crossing the four pairs of the currencies. There is a negative relationship between the P(TS-CoJ overlapping DC-CoJ) figures and the thresholds. For the first three pairs of the currencies (*EU* with *GU*, *EU with UJ* and *EU* with *CA*), under the thresholds of 0.05% and 0.75%, the figures of the P(TS-CoJ overlapping DC-

CoJ) are over 20%; while, for the thresholds of 0.1% and 0.125%, the figures of the P(TS-CoJ overlapping DC-CoJ) are less than 20% (for *EU* with *CA*, we saw 20.69% in table 5.11 (c)); note, under the threshold of 0.125% (table 5.11 (d)), there is an obvious decrease of the P(TS-CoJ overlapping DC-CoJ) that the figures are around 10%. In addition, for the co-jumps between *EURUSD* and *AUDUSD*, the P(TS-CoJ overlapping DC-CoJ) is over 50% under the thresholds of 0.05% and 0.75%.

Overall, table 5.11 indicates that both DC and TS approaches found the common co-jumps; also, the results conclude that the two methods detected a considerable number of unique co-jumps.

5.7.7 The examples of the co-jumps detected by both two methods

In the previous section, we gave a summary of identified TS co-jumps and DC co-jumps over five years. This section selected two examples of co-jumps detected by both two methods. In the two examples below, we found that the periods of the two DC co-jumps within the periods of the two TS co-jumps.

Co-jumps Example 1:

As showed in table 5.12 (a), on 2015/10/28, a TS co-jump is determined within 15 minutes from 18:00:00 to 18:15:00; for *EURUSD* and *GBPUSD*, the two jumps size are -1.05% and -0.32%, respectively. In table 5.12 (b), the DC approach also identified a DC co-jump on the same date from 18:00:04 to 18:00:25 (in 21 seconds); the jumps size of *EURUSD* and *GBPUSD* were -0.5% and -0.42%, respectively.

Table 5.12 (a) A TS co-jump was identified within 15 minute interval. The significance level α = 0.01. Note: EU and GU are *EURUSD* and *GBPUSD*.

StartTime	EndTime	EU_JumpSize	GU_JumpSize	Period
2015/10/28 18:00:00	2015/10/28 18:15:00	-1.05%	-0.32%	15 mins

Table 5.12 (b) A TS co-jump was determined within the same 15 minute interval. Threshold $\boldsymbol{\theta} = 0.05\%$ and $\boldsymbol{s} = 0.99$.

StartTime	EndTime	EU_JumpSize	GU_JumpSize	Period
2015/10/28 18:00:04	2015/10/28 18:00:25	-0.50%	-0.42%	21 secs

Co-jumps Example 2:

The example 2 presents the co-jumps between *EURUSD* and *USDJPY*; in this example, the directions of the co-jumps are opposite. In table 5.13 (a), we observed a TS co-jump on 2015/06/01 within the period from 14:00:00 to 14:15:00. For *EURUSD*, we identified a downward jump with the jump size of -0.51%; while, for *GBPUSD*, there was an upward jump with the jump size of 0.31%. At the beginning of the TS co-jump time interval, we detected a DC co-jump. As shown in table 5.13 (b), for *EURUSD* and *GBPUSD*, the two jumps size are 0.14% and -0.09% within the period of 3 seconds from 14:00:02 to 14:00:05.

Table 5.13 (a) A TS co-jump was identified within 15 minute interval. The significance level α = 0.01. Note: EU and GU are *EURUSD* and *GBPUSD*.

StartTime	EndTime	EU_JumpSize	GU_JumpSize	Period
2015/06/01 14:00:00	2015/06/01 14:15:00	-0.51%	0.31%	15 mins

Table 5.13 (b) A TS co-jump was determined within the same 15 minute interval. Threshold $\boldsymbol{\theta} = 0.05\%$ and $\boldsymbol{s} = 0.99$.

StartTime	EndTime	EU_JumpSize	GU_JumpSize	Period
2015/06/01 14:00:02	2015/06/01 14:00:05	0.14%	-0.09%	3 secs

5.8 Discussion

This section will discuss the observations from our studies in the previous section. The study object that we focus on is the DC co-jumps of $CoJ_U^{EU,GU}$ (the DC co-jumps between *EURUSD* and *GBPUSD*) and $CoJ_U^{EU,UJ}$ (the DC co-jumps between *EURUSD* and *USDJPY*). We start to discuss the information the observer can obtain from the DC co-jumps. After that, the major discussion is the relationship between the events (the MEEs and the major historical events) and the DC co-jumps.

5.8.1 Can the DC method give precise information of co-jumps in practice?

As illustrated in Section 5.7.1, the two examples shown us that the DC co-jumps can give precise information in terms of timing, magnitudes, and the directions. For instance, in table 5.3, a $CoJ_U^{EU,GU}$ was identified in the period of 10 seconds from 18:00:04.300 to 18:00:14.300 in that there was an upward jump of EU followed by an upward of GU. Separately, the period of EU jump (9.6 seconds) was almost 1 second longer than the period of GU (8.8 seconds). Also, the jump size of GU (5.84 TMV) was greater than the jump size of EU (4.04 TMV). Therefore, DC co-jumps can give deep insight to the observers for studying the behaviour of co-jumps.

5.8.2 Discussion: the relationship between co-jumps and MEEs

We studied the relationship between co-jumps and MEEs in Section 5.7.4. The results indicated that the DC co-jumps are highly sensitive to the US MEEs. For $CoJ_U^{EU,GU}$, from the determined DC co-jumps associated with MEEs (CoJ-MEEs), 93.5% of the CoJ-MEEs follow the US MEEs. In particular, we found that the US interest rate decision announcements (US_IRD) and non-farm payroll (US_NFP) numbers impact significantly on the presence of DC co-jumps. This suggests a cautionary note to the Forex traders to avoid the risk of co-jumps after US_IRD and US_NFP.

5.8.3 Do historical events cause DC co-jumps?

In Section 5.7.5, we studied the relationship between major historical events and co-jumps through two cases studies. Although the historical events from the two examples are not related to the economic data, the results indicated that the historical events may have even more influence on co-jumps compared with the MEEs. For instance, we tracked the daily number of DC co-jumps between *EURUSD* and *GBPUSD* during the COVID-19 pandemic in 2020. The results showed that 77% of the total number of the DC co-jumps were identified in March. This indicates the very uncommon trading activities from the traders during this major historical event.

5.8.4 The relationship between DC co-jumps and TS co-jumps

In Section 5.7.6 we gave the summary of the jointed co-jumps determined by the two approaches. The results revealed that around 20% of the DC co-jumps detected within the periods of the identified TS co-jumps. On the other hand, over 30% of the TS co-jumps were overlapped by DC co-jumps. In addition, Section 5.7.7 presented two examples that the detected DC co-jumps within the periods of the TS co-jumps. We observed that the DC co-jumps were presented at the beginning of the time interval of TS co-jumps. DC co-jumps gave the precise timing of the start and end of the co-jumps, and the periods of the DC co-jumps are less than 30 seconds. Thus, we believe DC approach can be an alternative tool for the analysts for locating the time of the co-jumps.

5.9 Conclusion

In Chapter 4, we proposed a new approach for detecting jumps in the DC approach. The benefits of the DC approach give us a new tool for fine-grained analysis in monitoring jumps' behaviour. In this chapter, we extended the study to the identification of the DC co-jumps. In DC, this is the first method proposed for the detection of co-jumps. Unlike time series data, the challenge of detecting DC co-jumps are the irregular timescales of the DC data in that observers cannot simultaneously judge jumps of two markets within the same time interval. Instead, we re-sample the two TR sequences of two markets into a single sequence, i.e., $CTRS$. In section 5.4, we introduced the partitioned $CTRS$ ($PTRS$) which partitions the $CTRS$ into contiguous TR sub-sequences. In section 5.5, we presented the definition of the DC co-jump in $U^{A,B}$ that we name $CoJ_U^{A,B}$. In section 5.5.1, we developed the indicator $ICJ(U^{A,B}$) to detect the $CoJ_U^{A,B}$. Therefore, we built the approach for the identification of DC co-jumps of two markets under the DC framework.

Our studies suggested that the DC method is beneficial for offering detailed information concerning detected co-jumps as shown by the examples in Section 5.7.1. From the statistical analysis of the five-year historical data, we observed that the co-jump is not a common event. For the back-testing of *EURUSD* and *GBPUSD*, using the threshold of 0.1%, there were 18253 TR sub-sequences over the five years while 1.54% of the total that we identified DC co-jumps. In addition, the results confirmed the existence of the relationship between the events and the DC co-jumps. Specifically, the US MEEs were the main factor affecting the presence of DC co-jumps, while the domestic MEEs had a very low influence on the presence of DC co-jumps. This study may give some suggestions to the Forex participants to understand which MEEs may have high likelihood to cause co-jumps between *EURUSD* and *GBPUSD*. For instance, in practice, our study can give a pre-alert for participants to avoid the risks of co-jumps after the announcements of the US Interest Rate Decision and the US Nonfarm Payrolls. Also, we observed that the unexpected outcomes of the major events caused a high proportion of the co-jumps. This may imply that there were unusual trading reactions to these unanticipated events from the markets' participants. These major historical events may cause multi-market shocks e.g., the identified DC co-jumps between Sterling and Euro. We think that DC co-jumps could be used as an indicator to monitor unusual trading behaviour spanning major exchange rates. Although the current stage of our study only works on two markets, future research could focus on identifying DC co-jumps crossing multiple markets.

Chapter 6. Conclusion

6.1 Summary of work completed

In the field of DC, this thesis presents a new path for the comparative analysis of two sequences in Forex. Compared with existing work, our new fundamental approach focuses on the real-time comparative analysis of two markets. This allows observers to implement comparative analysis using the irregular timescale of DC data sequences.

6.1.1 The DC measure of relative volatility (mRV)

Under the DC framework, this thesis exposes a new path for the study of the relative volatility between two markets. In this thesis, we introduce mRV, a data-driven measure of relative volatility that does not come with a predetermined time interval of measurement. In order to develop mRV, we develop the DC relative sequence (Section 3.4.2). It is a sequence which chronologically combines two markets' DC sequences, such that the termination of the current sub-sequence depends on the identity of the upcoming extreme point (EP). In this way, the period T is dynamically defined by the length of the sub-sequence. In Section 3.5.1, in low-frequency data, the correlation test proved that mRV has a high level of correlation to the time series method in measuring relative volatility. In addition, the correlation test indicates a positive relationship between the correlation coefficient and the period τ in that the longer the selected period τ, the higher the correlation coefficient between the two methods of measurement. In summary, using the DC framework, we have developed a new method to measure relative volatility by using mRV which allows measurements to be narrowed down to a precise time location in times of extreme values of relative volatility. This is something that cannot be done under the time series framework

(*Observation* 3, details see Section 3.5.3), thus demonstrating the benefit of using our approach in high-frequency data. We believe, for instance, that mRV can give an alternative measure to allow the monitoring in near real time of the relative volatility in micro-market activities when analysts consider high-frequency data or tick data. This demonstrates that time series measures of relative volatility and our approach can provide complementary methods for the analysis of financial data.

6.1.2 DC jumps (DCJs)

We proposed a new approach, under the DC framework, to detect jumps in the FX markets. This DC approach differs from the classical time series method which uses a model-based method to identify a jump in a time interval. Under DC, the alternative way to sample the market transactions that we can naturally refer to as a data-driven approach (the DC method) is that we judge the presence of a DCJ based on a DC trend. In the experiment, DCJs were found to be sensitive to news events especially those that were unscheduled. In addition, we studied the observed DCJs and TSJs, and demonstrated that they complement each other in terms of the jumps detected. It was identified that both DC approach and TS method readily found both common jumps and unique jumps. This confirms that DCJs are a useful complement to the TSJs for the identification of jumps in high-frequency data especially. The concept of the DCJ is based on a significant magnitude of the TR, which measures the price changes over time such that a greater TR value indicates greater price changes in a shorter period. The approach we give can be used in a practical manner to detect the DCJs in the financial markets and the back-testing results indicated a relationship between major economic events (MEEs) and DCJs.

6.1.3 DC co-jumps

The study of DC co-jumps is an extension of Chapter 4 which provides a method to detect a DCJ in a single market. In DC, the co-jump is the event that a jump in market A is followed by a jump in market B; we detect a DC co-jump based on the TR sub-sequence. The studies we performed suggested that the DC framework is beneficial in providing detailed information on detected co-jumps as indicated by the examples in Section 5.7.1. From statistical analysis of five-year historical data, it is clear that co-jumps are not common events. For the back-testing of *EURUSD* and *GBPUSD*, using the threshold of 0.1%, there were 18253 TR sub-sequences over the five years comprising 1.54% of the total in which we identified DC co-jumps. Furthermore, the results confirmed the existence of a relationship between major economic events and the presence of DC co-jumps. Specifically, the US MEEs seemed to be most closely associated with the presence of DC co-jumps, while the domestic MEEs were found to have a very low influence on the presence of DC co-jumps.

6.2 Summary of contributions

Contribution 1: Relative Volatility (mRV)

Under the DC framework, this is the first approach which allows us to compare the volatility of one market to another in micro-market structure. Further to this, we can pinpoint the timing of the changes in relative volatility between two markets. In DC comparative analysis, we first proposed a data-driven approach to combine two DC sequences on irregular timescales into a single combined sequence. This is the foundation for establishing DC micro-market relative volatility (mRV). In practice, mRV is capable of evaluating the relative volatility in high-frequency data, especially in tick data (the records of raw transactions). It is important to note, time is passively defined in mRV. The termination of the current sub-sequence depends on the market identity (market A or market B) of the upcoming EP. In the application of mRV to Sterling and Euro, the advantage of the data-driven process is that it allowed the precise location of the DC sub-sequences which showed the highest and the lowest mRV during the event of the Brexit referendum. Using the regular timescale as well, the correlation test proved that mRV provides similar conclusions to the time series method in measuring relative volatility. Therefore, we believe that the invention of mRV has provided a complementary tool to the traditional time series approach to evaluate the relative volatility.

Contribution 2: Jumps in DC

Under the DC framework, we proposed the definition of jumps in DC. The DC jump (DCJ) is an event whereby the price has changed by a significant magnitude in a short period. Compared to the classical method, we detect jumps based on the time-adjusted return of the DC trends. In other words, this new approach of identifying jumps does not require a pre-determined time interval. DCJs can give more information about the behavior of jumps in terms of size, direction, and quantity. Lastly, TSJ and DCJ are two different concepts that are not directly comparable. According to the back-testing, some TSJs were not identified as DCJs and vice versa. Thus, it can be said that the two methods complement each other and each is capable of providing unique information.

Contribution 3: Co-jumps in DC

Under the DC framework, we propose a definition of co-jumps in DC in two markets, which we refer to as DC co-jumps; based on this, we introduced an indicator to detect DC co-jumps. We found from studying 5 years of historical data that although co-jumps in general were rare, there existed a strong association of the DC co-jumps with US major economic events, especially those containing unexpected data. Based on the co-jumps we identified as related to the economic data, over 90% of co-jumps in these major currency pairs are associated with US economic data. This analysis provides a cautionary note for traders that market behaviour is likely to be abnormal following announcements of important US economic data containing unexpected information. Furthermore, the DC co-jump inherits the merits of the DC jump in that they offer precise information on the timing, magnitudes, and the direction of the DC co-jumps.

6.3 Future Work

In terms of potential directions for future work, our current work suggests that it would be a potentially productive area of research to investigate the correlation of mRV in the appropriate exchange rates (which is related to the presence of shocks/jumps) with the specific shocks to the US market quantified by Verdelhan (2018) and Mueller et al. (2017). Further research would be to look at the mRV for various bilateral exchange rates compared to US and global shock terms from similar models. Positive associations would both reinforce our thinking that mRV is a useful measure for different types of shocks and also possibly give some more insight into how our data driven approach associates with the market structure. This could further increase its utility as an instrument for decisions regarding pricing investments. This is further highlighted by the factor models in Mueller et al. (2017) that also use global shocks priced locally, which suggests an association of exchange rate co-movement and the global and local shock terms. The real-time nature of the mRV as a measure indicates that it could be potentially very useful in the real time analysis events of major historical significance, such as flash crashes.

In Chapter 4, DCJs can give a better insight into studying the jumps' behavior especially in monitoring flash events both in real time and providing analysis after the fact. We think the DCJ could give a real-time alert of the price crash through monitoring the price changes under our approach; especially, in high-frequecy trading, traders need more tools to measure the risk of their opening positions. Hence, we would like to work on the DCJs in terms of the application to real time risk monitoring in future research.

Under the framework of DC comparative analysis, this thesis proposed new studies in relative volatility and co-jumps. This is the beginnings of work on DC comparative analysis. For instance, in DC co-jumps, further research may focus on the co-jumps of multi-markets (more than two markets) which may lead to new tools to monitor the systematic risk spanning multiple financial markets. Another idea is working on DC co-movement; fundamentally, DC makes it easy to track the price movements by determining DC trends; the idea is could we find a similar 'pattern' of the DC trends between two markets? Thus, through recognising similar patterns in real-time we could quantify the similarity of the DC co-movements of two markets.

Appendix A. The mean of monthly $\langle \boldsymbol{mRV^{0.1\%}_{(RS)}} \rangle_{\boldsymbol{M}}$

To show that the results in this section are relatively insensitive to the choice of DC threshold, we repeated the same experiment (the first application in Section 5.2) under the threshold 0.1%. Figure A1 shows the mean of monthly $\langle mRV^{0.1\%}_{(RS)} \rangle_M$ over seven years from 2012 to 2018.

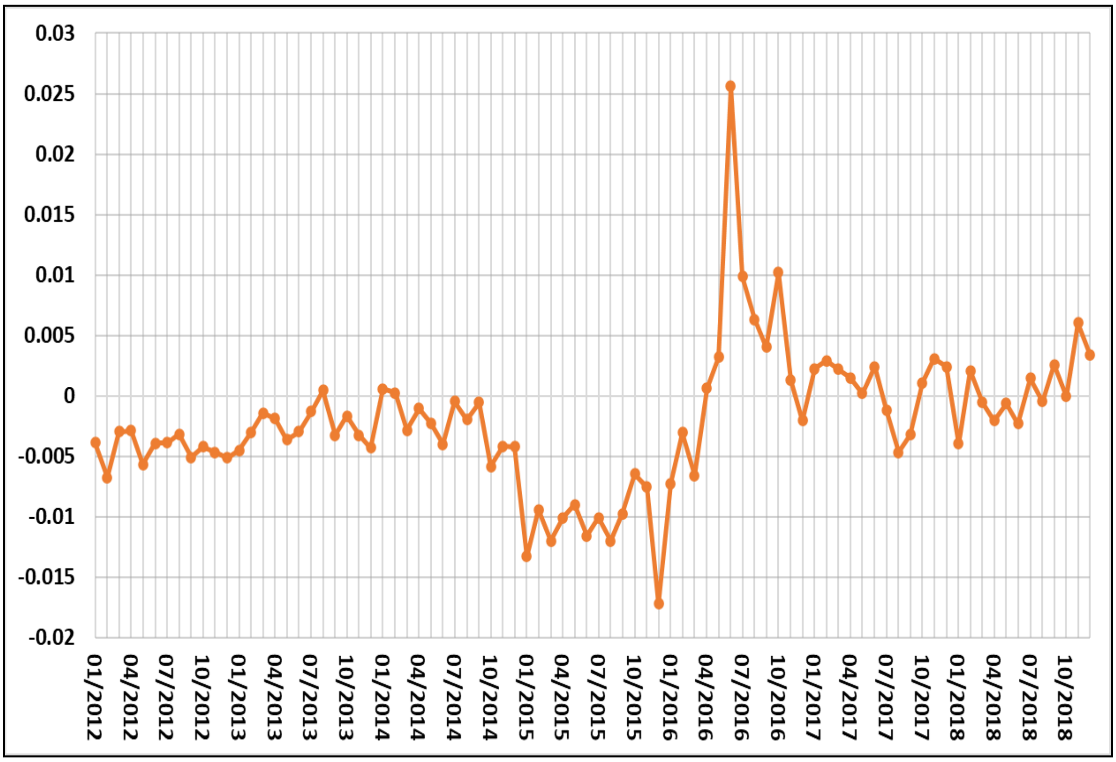

Figure A1. The mean of monthly $\langle \boldsymbol{mRV^{0.1\%}_{(RS)}} \rangle_{\boldsymbol{M}}$ measures the monthly average $\boldsymbol{mRV}$ under the threshold of 0.1%. From 2012 to 2018, there are 84 data points. The values of $\boldsymbol{mRV}$ are normalised by θ.

Appendix B. Evaluating mRV in the sub-sequences under the threshold 0.1%

Under the threshold 0.1%, there are a total of 718 sub-sequences observed. Figure B1 illustrates the values of mRV in the same time period (from 16/06/2016 to 30/06/2016). Over the periods of the three parts, we detect the amount of 180 (Part 1), 353 (Part 2), and 185 (Part 3) sub-sequences, respectively. Table B1 presents the mean and median of the $mRV_{(RS)}^{0.1\%}$ in the periods of the three parts.

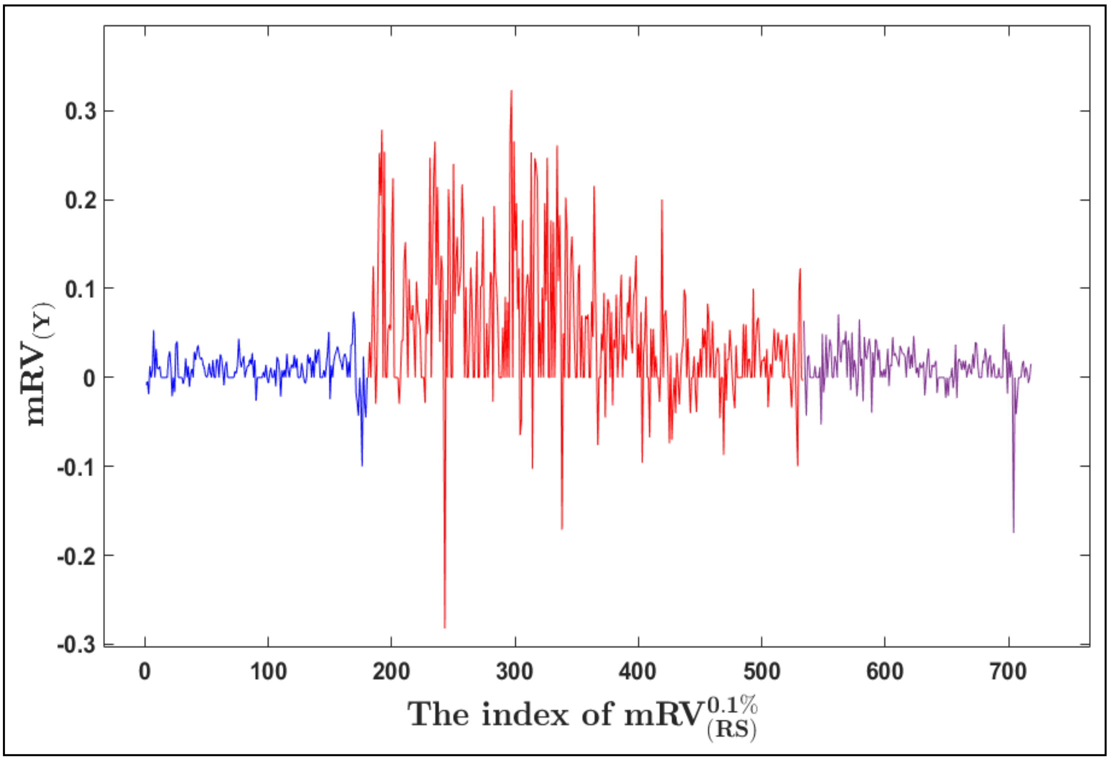

Figure B1. The sequence of $\boldsymbol{mRV_{(RS)}^{0.1\%}}$ in the periods from 16/06/2016 to 30/06/2016.

Figure B1 plots 718 sub-sequences observed under the threshold of 0.1%. Part 1(blue line): from 00:00 16/06/2016 to 22:00 06/23/2016 (140 hours); Part 2 (red line): from 22:00 06/23/2016 to 22:00 06/24/2016 (24 hours); Part 3 (purple line): from 00:00 06/27/2016 to 24:00 30/06/2016 (96 hours).

Table B1. the mean and median of the $\boldsymbol{mRV_{(RS)}^{0.1\%}}$. The operator $\boldsymbol{Median}(.)$ denotes the median of a sequence.

	Part 1	Part 2	Part 3	Part2/Part1	Part2/Part3
$\langle mRV_{(RS)}^{0.1\%} \rangle$	0.009	0.051	0.011	5.979	4.475
$Median\left(mRV_{(RS)}^{0.1\%}\right)$	0.008	0.032	0.011	3.959	2.818

Appendix C. About the 12 DC scaling law

The idea of the DC scaling law was proposed by Glattfelder et al. (2011) that they discovered 12 DC scaling laws in the market. In Section 3.5.1, we used the DC scaling law 10 to obtain the thresholds given the regular time intervals. Specifically, the DC scaling law discovered the relationship between different data properties of the forex markets. Given a market data property, one can estimate the corresponding data property. For instance, the DC scaling law 10 gives the statistical property that the average period of a DC trend $\langle Ttmv \rangle$ is approximately equal to a function of the threshold θ (for details see equation (3.16)). Table C1 below present an overview of the 12 DC scaling laws, more details see the literature.

Table C1. The summaries of 12 DC scaling laws from the literature.

DC scaling laws 1	The relationship between the number of DC events and the size of the threshold.
DC scaling laws 2	The relationship between the average yearly number of price moves and the related size of the price change.
DC scaling laws 3	The relationship between the maximum price move (defined by the vertical distance between the high and low price levels) and the size of that time interval.
DC scaling laws 4	The relationship between the average time interval and the related size of the price change.
DC scaling laws 5	The relationship between the average time interval of a DC event and the related size of the threshold.
DC scaling laws 6	The relationship between the average time interval of an overshoot event and the related size of the threshold.
DC scaling laws 7	The relationship between the average size of an overshoot event (the absolute value) and the related size of the threshold.
DC scaling laws 8	The relationship between the average size of a DC event (the absolute value) and the related size of the threshold.
DC scaling laws 9	The relationship between the total price move and the related size of the threshold.
DC scaling laws 10	The relationship between the average time period of a DC trend and the related size of the threshold.
DC scaling laws 11	The relationship between the number of tick counts during a DC trend and the related size of the threshold.
DC scaling laws 12	The relationship between the cumulative total price move (coastline) and the related size of the threshold.

Appendix D. The DCJs associated to the unscheduled events in Yen

The motivation of this study is derived from a published statement on monetary policy from the Reserve Bank of Australia [23](RBA, the central bank of Australia) in February 2019. The Japanese Yen exhibited a flash crash (appreciation) against several currencies. This flash crash event was the focus of many of the market participants and made the headlines in much of the financial media. Han and Westelius (2019) catalogued the details of the flash crash of the Yen. On 2nd of January 2019, the yen appreciated more than 3% against the dollar at 10:35 PM (UTC) in eight minutes. Many financial analysts discussed the reasons for the Yen crash. Financial Time and Bloomberg both reported that the initial trigger of the Yen crash was the negative sales outlook of Apple Inc. Nevertheless, there was no direct connection between the news concerning Apple and the Yen. RBA gave three key factors which are likely to have contributed to the brief but rapid deterioration:

(1) The impact of carry trade reversal. Carry trades get funding from low yield currencies (such as the Yen) to take a long position in a high yield currency (such as the US dollar or Australia dollar). However, when the long positions register a loss and trigger the ‘stop loss’, the carry trade will be reversed and increase the demand for the funding currency (the Yen).

(2) The seasonably low liquidity at the time of the day and year. The time of the flash crash was following the close of US markets but before the opening of the

[23] RBA - The Recent Japanese Yen Flash Event: *https://www.rba.gov.au/publications/smp/2019/feb/box-b-the-recent-japanese-yen-flash-event.html*

Asian markets; in the middle of this time period. Also, on 2nd January, the liquidity was likely low due to the new year vacation.

(3) Algorithmic trading may have amplified the flash event as ‘swing trading’ opens positions to chase the momentum of the price move.

Han and Westelius (2019) found that *USDJPY* increased 4% in one minute starting at 10:35 PM (UTC). RBA reported that the Yen appreciated 3% against the US dollar within 30 seconds, and in the absence of any major news. Thus, we implemented back-testing to detect DCJs in *USDJPY* in 5 minutes from 10:35 to 10:40 PM (UTC). The experiment selected the threshold $\theta = 0.1\%$ and $s = 0.99$. Table E1 summarises the details of the observed DCJs: (1) the selected periods for detecting DCJs from 22:35:00.000 to 22:40:00.000 on 02/09/2019 (UTC); (2) the total DC trends confirmed in those periods; (3) N(DCJ) is the total DCJs detected in the 5 minutes; (4) <DCJ-size> is the average DCJ size, which is measured by taking the average of the absolute TMV of the total observed DCJs; (5) <DCJ-periods> is the average period (in seconds) of a DCJ.

Table E1. The summary of detected $\boldsymbol{DCJ}$s during the Yen flash event on 02/09/2019. Threshold = 0.1% and s = 0.99.

Asset	USDJPY
Periods, (1)	5 minutes (UTC) (22:35:00 to 22:40:00)
N(DC), (2)	24
N(DCJ), (3)	24
<DCJ-size>, (4)	3.3
<DCJ-period>, (5)	10.2

Starting from 10:35, we observed 24 DC trends in 5 minutes. The results recognised that all of the 24 trends were confirmed as containing DCJs. The mean of the absolute TMV was 3.3, which indicates a significant jump size on average during the periods. Also, the average period of the DCJs was 10.2 seconds.

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