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Scalar Minimax Filter-based Phase Tracking for Continuous-Variable Quantum Key Distribution

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Abstract

In local local oscillator (LLO)-based continuous-variable quantum key distribution (CV-QKD), a difference between the linewidth values of two free-running lasers at Alice and Bob induces a phase drift noise. This work proposes a novel minimax filter-based phase tracking that aims to minimize the phase drift considering maximum residual phase error to achieve optimal phase estimation. Simulation results show that the minimax filter offers a lower phase estimation mean square error (MSE) value compared to the Kalman filter when worst-case phase drift error due to high linewidth difference or high measurement noise values are considered.

1 Introduction

Quantum key distribution (QKD) is a secure cryptographic key exchange between two remote authenticated participants, Alice and Bob, over an insecure quantum channel [1,2]. This scheme uses quantum mechanics laws to decode the quantum states of light. A system that uses single-dimension detection of the quantum signal is known as discrete-variable QKD (DV-QKD), while a multiple-dimension detection is known as continuous-variable QKD (CV-QKD). CV-QKD provides a relatively higher noise-resilient and cost-effective solution to support higher key rates for limited distances [1]. However, coherent detection of CV-QKD signals at Bob requires Bob's awareness about Alice's local oscillator (LO). Transmitting the LO exposes the key exchange to severe intensity attenuation and adversary attacks [1,3]. Alternatively, Bob uses a local LO (LLO) laser with a centre frequency and linewidth matched to Alice's LO. In practice, a difference between the linewidth values causes a fast-phase drift at Bob [4–6].

Several techniques were proposed to eliminate the phase drift at Bob. The work in [4] presented two different self-coherent schemes. The first technique is based on a delay-line interferometer, in which Bob uses an LLO that has the same delay line technique that Alice used to generate phase-coherent LO pulses. The second technique is based on modulation displacement. Both techniques used a single optical wavefront to derive the quantum information signal and the reference signal pairs. In the second technique, Alice modulates and sends quantum quadratures with fixed displacement. Bob uses this modulation displacement to estimate the phase drift. The proposed techniques with relatively high complexity do not eliminate the phase noise completely [1,6].

The experiment in [3] compensated the fast and slow phase-drift using the pilot-tone-assisted phase compensation method for the LLO-based CV-QKD system to decrease the level and excess noise and increase the asymptotic secure key rate to 7.04 Mbps over a transmission distance of 25 km. Similarly, the empirical study in [1] aimed to increase the secure key rate of the LLO-based CV-QKD system to exceed 20 Mb/s over a target transmission distance of 15 Km. The phase noise variance is numerically optimized and reduced to 0.01.

The studies in [5,6] used the Kalman filter to track the phase shift for the LLO-based CV-QKD system. For this purpose, the study in [5] proposed a scalar Kalman filter to estimate the fast-phase drift at Bob. However, the study in [6] showed that the scalar Kalman filter does not eliminate the phase drift completely, particularly when a slow-phase drift noise occurs due to a difference between the optical frequency of the reference and LO signals. Therefore, the vector Kalman filter was proposed in [6] that considers both fast phase shift and slow phase shift. The performance of the proposed method is constrained by the measurement noise.

The previous studies showed that the available techniques, such as the Kalman filter, do not eliminate the phase error completely [5] and their performance is limited by the measurement noise [6]. Therefore, this work proposes a novel scalar minimax filter-based phase tracking method to minimize the fast-phase drift. The proposed method considers maximum residual phase error to achieve optimal phase estimation. The performance of the minimax filter-based system is established and compared to a benchmark scalar Kalman filter when the optical frequency of the reference and LO signals is accurately tuned and matched.

The rest of this paper is organized as follows: the LLO-CVQKD system is described in Section II. Phase drift and noise model are discussed in Section III. The phase estimation and correction method is explained in Section IV. Results and discussion are provided in Section V. Finally, conclusions are drawn in Section VI.

2 System Description

A schematic diagram of an LLO-CVQKD system is illustrated in Fig. 1. At Alice, a laser L_A with a center frequency f_A generates a sequence of pulses at repetition rate f_{rep}. A beam splitter (BS) separates the sequence of pulses into signal pulses and zero-phase constant intensity phase reference pulses $|E_R|^2 = x_{RA}^2 + p_{RA}^2$ [1]. Amplitude modulator (AM) and phase modulator (PM) modulate signal pulses to produce zero-mean Gaussian modulated quadrature $|x_{SA} + ip_{SA}\rangle$ with a variance V_A. A polarizing beam combiner (PBC) interleaves the modulated signal pulses with the phase reference pulses and transmit the combination.

Bob splits the received pulses into signal and reference pulses using a polarizing beam splitter (PBS). Bob uses a heterodyne detector that consists of 90° hybrid and two balanced detectors (BD) to detect the received quadratures of the reference signal x_{RB} and p_{RB}. A phase misalignment of θ between the received and transmitter reference signals results due to spectral linewidth drift of $f_{line} = f_A + f_B$ between Alice's laser L_A and Bob's LLO L_B. The phase drift of the received signal can be calculated by

$$\theta = \tan^{-1}\left(\frac{p_{RA}}{x_{RA}}\right). \tag{1}$$

The resultant phase θ is then sent to a DSP unit for analysis, estimation and correction. The following section presents the analysis of phase drift and noise model.

3 Phase drift and noise model

The spectral linewidth drift between two free-running lasers cause a phase drift that can be modeled as a Wiener process [5, 6]:

$$\theta(t+1) = \theta(t) + w(t), \tag{2}$$

where $\theta(t)$ is the phase drift between the reference and the corresponding local oscillator pulses as given in (1) and $w(t)$ is a zero-mean Gaussian noise process due to a spectral linewidth drift of f_{line} and repetition frequency of f_{rep}. The variance of $w(t)$ is given as [1, 5]:

$$\sigma_w = \frac{2\pi f_{line}}{f_{rep}}. \tag{3}$$

Quantum uncertainty and imperfect detection adds further phase noise [1]. Additional term $\Delta(t)$ is added to (2) to model residual estimation phase error due to quantum uncertainty and phase error that was not completely eliminated in the previous iteration, $e(t) = \theta(t) - \widehat{\theta(t)}$. Hence, (2) is re-written as:

$$\theta(t+1) = \theta(t) + w(t) + \Delta(t), \tag{4}$$

where, $\Delta(t)$ is given by [7, 8]

$$\Delta(t) = L((\theta(t) - \widehat{\theta(t)}) + n(t)), \tag{5}$$

where $n(t)$ is a zero-mean Gaussian noise with variance of $\sigma_n = \sigma_w$, $\widehat{\theta(t)}$ is the estimated phase and L is a design parameter that characterizes the severity of the estimation error.

Phase measurement is corrupted by zero-mean Gaussian noise $v(t)$, with variance σ_v, induced from shot noise and total noise of heterodyne detection and given by:

$$\phi(t) = \theta(t) + v(t). \tag{6}$$

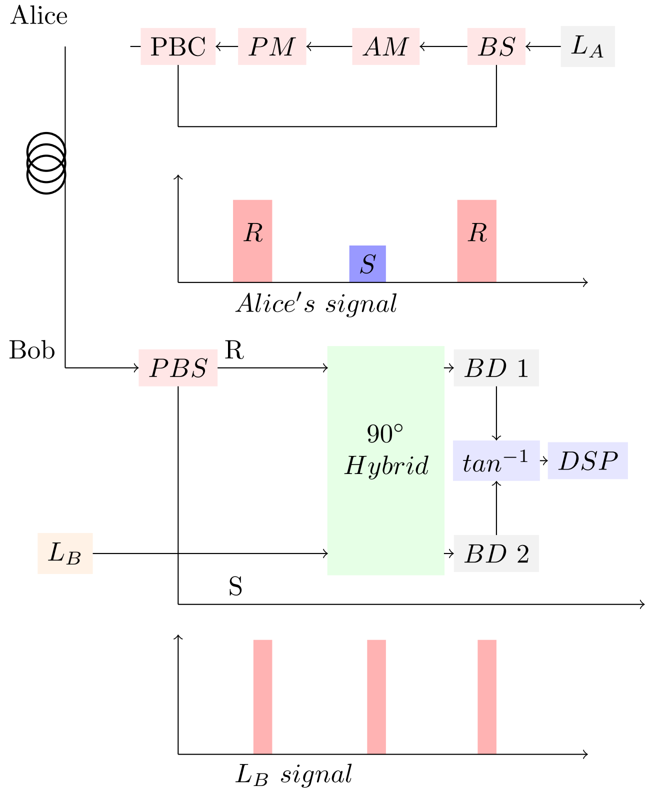

Figure 1: A diagram of local local oscillator continuous-variable quantum key distribution system

For a given measured phase $\phi(t)$, the estimator predicts the next phase by:

$$\widehat{\theta(t+1)} = \widehat{\theta(t)} + K(\phi(t) - \widehat{\theta(t)}), \tag{7}$$

where K is a design parameter that aims to minimize the estimation error.

From (4) and (7), it can be shown that the estimation error has the following form:

$$e(t+1) = (1 + L - K)e(t) + w(t) + Ln(t) - Kv(t). \tag{8}$$

Having presented phase drift and noise model, the next section presents filters design to estimate the phase.

4 Phase estimation and correction

In the literature, a vector minimax filter is used to track multiple parameters, such as position coordination. However, in this work, we derive a novel scalar minimax filter to track a single parameter which is the phase drift. A scalar minimax filter is a recursive optimization algorithm employed to estimate the phase drift between the reference and the corresponding local oscillator pulses. The minimax filter aims to minimize the estimation error by minimizing K considering a worst-case phase drift error scenario, and hence, maximum L. Therefore, the estimation error in (8) is divided into the following two parts:

$$e_K(t+1) = (1 + L - K)e_K(t) + w(t) - Kv(t), \tag{9}$$

and

$$e_L(t+1) = (1 + L - K)e_L(t) + Ln(t), \tag{10}$$

The optimized filter gains K* and L* are calculated by minimizing $e_K(t)$ and maximizing $e_L(t)$. Hence, the optimal solution yields:

$$K^* = \frac{\sigma_e^2\sigma_n}{\sigma_e^2\sigma_n + \sigma_v\sigma_n - \sigma_v\sigma_e^2}, \tag{11}$$

$$L^* = \frac{\sigma_e^2\sigma_v}{\sigma_e^2\sigma_n + \sigma_v\sigma_n - \sigma_v\sigma_e^2}, \tag{12}$$

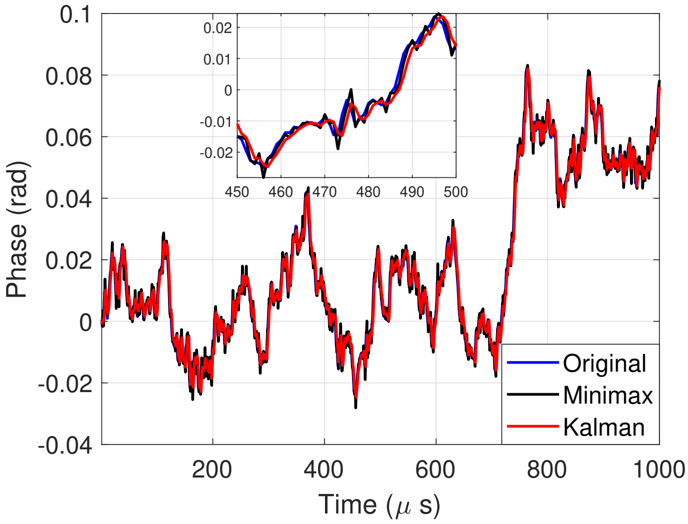

Figure 2: The original varying phase and the estimated phase using the Kalman and minimax filters.

where σ_e^2 is the variance of the estimation error in (8) and is given by:

$$\sigma_e^2 = (1 + L - K)^2\sigma_e^2 + \sigma_w^2 + L^2\sigma_n^2 - K^2\sigma_v^2. \tag{13}$$

The Kalman filter aims to minimize the estimation error by minimizing K, and it does not consider the worst-case phase drift error scenario modeled by L. Therefore, the Kalman filter gain K^* is given by [9]

$$K^* = \frac{\sigma_e^2\sigma_v^2}{\sigma_e^2 + \sigma_v^2}, \tag{14}$$

where

$$\sigma_e^2 = \frac{\sigma_e^2\sigma_v^2}{\sigma_e^2 + \sigma_v^2} + \sigma_w^2, \tag{15}$$

assuming independent process noise $n(t)$ and measurements noise $v(t)$ sources.

The performance of the minimax filter is studied in the following section.

5 Results and Discussion

A simulation is used to study the estimation error performance of the proposed minimax filter with the Kalman filter as the benchmark. The measurement noise variance is set to $\sigma_v = 6 \times 10^{-6}$ [6], repetition frequency value of 500 MHz and the laser linewidth of 100 kHz unless otherwise stated.

The estimated phase using the Kalman and minimax filters and the original varying phase is illustrated in Fig. 2. The figure shows that the minimax filter tracks the small-scale fluctuation of the phase with higher accuracy than the Kalman filter.

Phase estimation mean square error (MSE) for a range of filters iterations and two values of laser linewidth, 100 MHz and 100 Hz, is depicted in Fig. 3. The figure shows that a phase estimation MSE values of the minimax and Kalman filters are 1.6×10^{-6} and 3×10^{-4}, respectively, when lasers with a narrow linewidth values of 100 Hz are used. The estimation MSE performance of the minimax and Kalman filters degrades to 1.5×10^{-3} and 2.5×10^{-3}, respectively, when the lasers have larger linewidth values of 100 MHz. This is expected as the phase error in (3) increases linearly with the linewidth value of the lasers. The figure also shows that both filters have stable performance over different number of iterations.

The impact of laser's linewidth value on the phase estimation MSE, for the repetition frequency values of 500 MHz and 10 MHz, is demonstrated in Fig. 4. The figure shows that the phase estimation MSE is comparable for the minimax and Kalman filters and it is less than 0.003 when laser's linewidth value is below 2×10^4 Hz. The figure also shows slower increase in the phase estimation MSE at high repetition frequency value. However, increasing the repetition frequency value is limited by the spectral responsivity of the BD that does not exceed 500 MHz [1].

Phase estimation MSE performance for a range of measurement noise variance σ_v values between 0 and 1×10^{-3} is provided in Fig. 5. The figure shows that both filters have comparable performance at low measurement noise variance values (below 0.4). At a high measurement noise variance value, the minimax filter has better phase estimation MSE

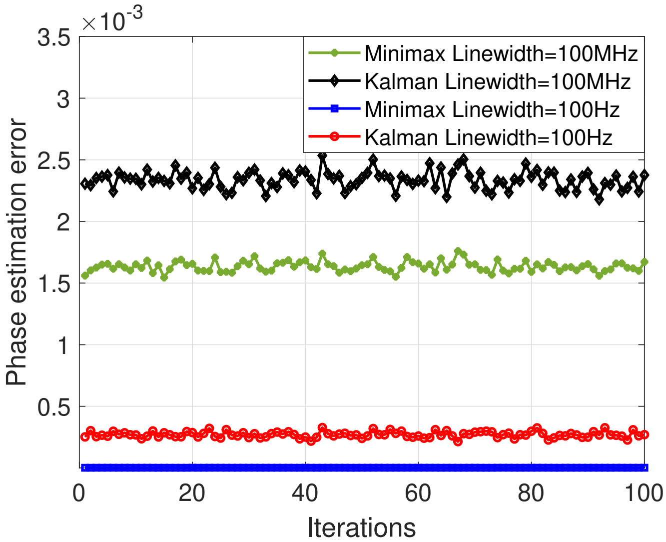

Figure 3: The phase estimation error for a range of filters iterations and two values of laser linewidth, 100 MHz and 100 Hz.

performance. For example, at a measurement noise variance value of 1×10^{-3}, the minimax filter achieves an MSE value of 0.005, outperforming the Kalman filter by 133%. This is expected as the minimax filter gain L considers maximum measurement noise variance σ_v in (12), whereas the Kalman gain K considers minimum measurement noise variance σ_v in (14). Hence, the minimax filter is more resilient to high measurement noise than the Kalman filter at at high measurement noise values.

6 Conclusions

This paper proposed using the minimax filter to track phase drifts in local local oscillator based continuous-variable quantum key distribution system. The proposed technique aimed to eliminate the phase drift noise by considering the residual phase error that was not completely eliminated in the previous iterations. Benchmarking the minimax performance against the Kalman filter showed that the minimax filter offers a higher resilience to measurement noise error. The results showed that the minimax filter can achieve a phase tracking accuracy higher than the Kalman filter by up to 133%.

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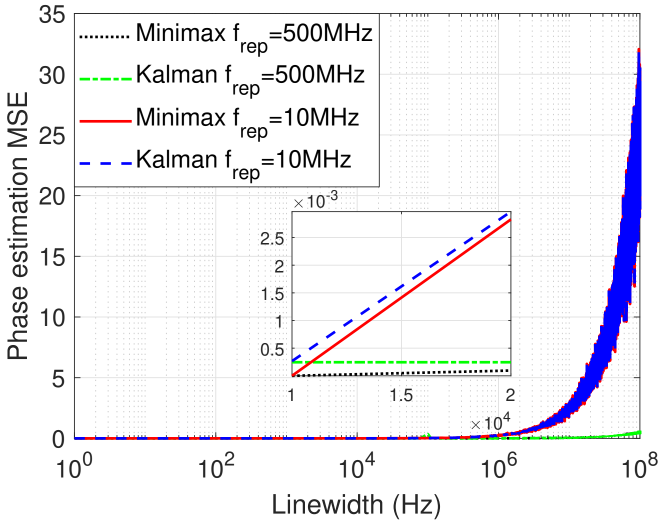

Figure 4: The phase estimation error as a function of laser's the linewidth value when the repetition frequency values are 500 MHz and 10 MHz.

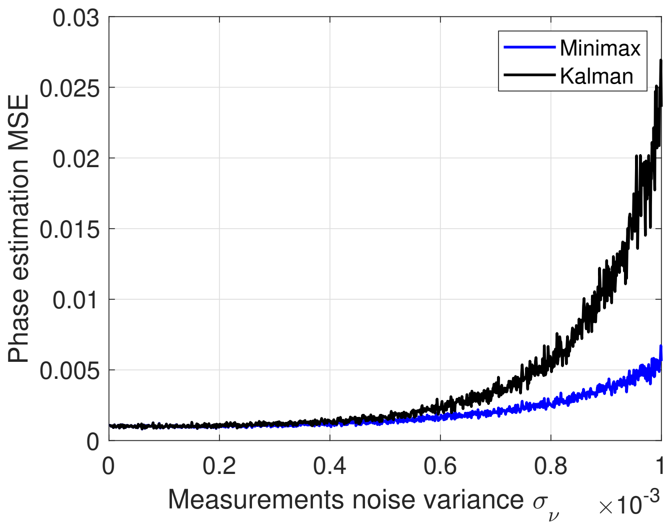

Figure 5: Phase estimation error performance for a range of measurement noise variance σ_v values between 0 and 6×10^{-3}.

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