Exploring the figured worlds of teachers and pupils in the primary mathematics classroom: Five illustrative cases

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Abstract

The primary mathematics classroom can be a difficult and confusing place to be, generating feelings of puzzlement, or even anxiety, sometimes resulting in a reluctance to engage. This study sought to explore the overarching research question: How can reviewing the nature of learning in contrasting primary mathematics classrooms through the lens of a figured world illuminate significant and influential differences in pupils' experiences.

This study focuses on five distinct episodes in five different year 2 primary mathematics classrooms. Stimulated recall using video recordings, eliciting both teacher and pupil views, explore the learning from both of these perspectives. This approach builds on Mason's (2002) discipline of noticing. Adopting a figured worldview of the mathematics classroom enables fresh insights from the pupil's perspective.

By developing an innovative analytical framework based on Holland, Lachicotte, Skinner & Cain's (2001) figured world and their notions of; identity, dialogue and cultural artefacts, Belenky, Clinchy, Goldberger & Tarule's (1986) ways of coming to know and Sfard's (1998) metaphors for learning, I was able to offer a new way of viewing the contrasting learning environments of these mathematics classrooms.

These five illustrative cases demonstrate distinct pedagogical approaches and their impact on pupils' mathematical identities. The implications of this research suggest that differently figured worlds could generate particular kinds of learning experiences. So pedagogically, teachers need to pay attention to aspects of learning beyond just the curricular imperatives, including the use of artefacts and other cultural elements associated with a mathematics environment. Pupils need opportunities to behaviour like thinking mathematicians for them to take on an identity of a confident mathematician.

Acknowledgements

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List of Abbreviations

ARG	Assessment Reform Group
DBS	Disclosure and Barring Service
DfE	Department for Education
DfES	Department for Education and Skills
MTE	Mathematics Teacher Exchange
NC	National Curriculum
NCETM	National Centre for the Excellence in the Teaching of Mathematics
Ofsted	Office for Standards in Education, Children's Services and Skills
PISA	Programme for International Student Assessment
QCA	Qualifications and Curriculum Authority
RQ	Research Question
SAT	Statutory Assessment Test
TIMSS	Trends in Mathematics and Science Study
ZPD	Zone of Proximal Development

Chapter 1

Introduction

1.0 The importance of the mathematics learning environment

A significant number of pupils in primary classrooms today find mathematics a challenging and difficult subject (Lee and Johnston-Wilder, 2017; Boaler, 2016). There has been a range of factors explored to account for this particular phenomenon, including;

- the belief that mathematics ability is fixed (Dweck, 2013),
- the view that mathematics as a subject itself is difficult (Mason, (2002)
- the belief that styles of teaching mathematics can add to making it a challenging subject (Jaworski, 2012, Ofsted 2008).

I have tried to understand these challenges throughout my teaching profession through the changing roles I have held as a teacher to a teacher educator and researcher. During my experience as an Intervention Primary Mathematics Teacher, I worked with individual pupils whose teachers had assessed them as being significantly behind the expected level in mathematics for their age. An element of my intervention strategy was to video record the mathematics sessions between the pupil and myself. I used these videos as an opportunity to revisit episodes in more detail, providing time and space to reflect and ponder on the details of the interactions. The video provided details that can often be missed in the moment of the lesson. From working closely with these particular pupils, I deduced there was more to understand why these pupils found mathematics challenging. To reflect on how the pupil expressed their ideas and reacted to the resources, I felt my epistemological understanding as a teacher was adjusting due to my changing pedagogical approaches. By allowing the pupils a greater opportunity for their views to be shared, I became more aware of the power of giving pupils openings to express their mathematical ideas.

Key Stage 1 was chosen for this study because pupils start more formal education as they transition from reception to key stage 1. Key stage 1 introduces a more formalised teaching approach, and the National Curriculum assessment tests are introduced at the end of key stage 1. Therefore, more specifically, year 2 was the focus of this study. I wanted to ascertain what the teacher appears to privilege in learning at this time. I also feel it is an essential developmental time when most pupils have established the social conventions specific to the mathematics classroom.

The cultural nature of the mathematics classroom creates a unique learning environment in which pupils and teachers come together. Unique because, unlike other subject areas, it is considered the norm to test pupils to establish their ability groups, view the subject as one of right and wrong, and use specific mathematical resources to reinforce these fixed-ability views. My personal experience has demonstrated how the specific cultural nature of the mathematics classroom can impact pupils' identity. As a pupil, I acted out the role afforded me, first based on poor recall of facts, but later based on problem-solving skills which strengthened my mathematical identity. Later, I took the role of primary teacher, creating the culture in which identities were formed. The many facets of teaching made it a complex place to be. However, my role as a mathematics intervention specialist allowed me time to focus on how pupils interacted with the subject, developing my understanding of the cultural nature of the mathematics classroom. Now, as a teacher educator, I support students in exploring the role of the mathematics teacher. However, I witnessed the views of the students towards the subject of mathematics are based on their individual mathematical identities. The emotional experience in learners can range from enthusiasm and excitement towards the subject to fear and anxiety. According to Ofsted (2008), and more recently, Jaworski (2012), mathematics teachers' pedagogical approaches in classrooms today rely on didactic teaching, with learning focused on remembering facts and strategies, objectifying the subject. However, the research literature indicates that such pedagogical approaches, even when successful, result in short-lived learning. This short-lived learning can be due to the over-reliance and overload on the short term memory leading to anxiety associated with this approach and can de-motivate pupils from continuing to study mathematics in the future (Hernandez-Martinez and Williams, 2013).

As a former teacher and now working with trainee teachers, I have taught and observed all the subjects in the primary national curriculum in various schools. I have found the didactic approach mentioned above, reliant on remembering facts, unique to the mathematics classroom. Sfard (1998) associates this process of rote learning facts as an acquisition approach to teaching. I experienced this acquisition approach in my first mathematics lessons as a child, trying to remember the halves and quarters of numbers but not being sure why. This approach initially made the subject alien to me. My later experience of an enquiry and investigative approach gave reason and understanding to my learning. My latter experience fits with what Sfard (1998) describes as a participatory approach, which she saw as associated more with the other curriculum subjects. This approach made the learning active and relevant by introducing opportunities for reasoning and offering a subjective view. This participatory approach used resources for everyone to support mathematical investigations and reasoning. In a classroom based on a didactic approach, resources were specifically for remedial support, suggesting cultural boundaries associated with how pupils used them.

The use of an investigative approach in primary school gave me, as a child, a purpose and context for my mathematics lessons. In this case, my peers and I were learning mathematical skills and approaches to solve contextual enquiries, such as measuring the height of the trees on the school grounds. If we planted trees, we could calculate how tall they might grow. I still remember how this involved trigonometry and applying my knowledge of angles.

However, merging technical mathematical language and the social norms of the pupils' broader experience can potentially introduce confusion. An example of this is illustrated by the following word problem, a type of question often used in mathematics lessons:

> *'If John has 12 sweets and shares them equally with his friend, how many sweets will they both have?'*

I am sure the expected answer is 6. However, according to John, it might depend on how many sweets he wants to share with his friend. In a question that initially sounds straightforward with a clear answer, the child's social world introduces uncertainty and confusion. The interpretation of individual words can be ambiguous depending on the pupil's prior experiences and understanding. To share might mean to offer just one sweet, equal might suggest the same size or taste. The insertion of the word 'equally' is key to this question, but how often are children asked to share their sweets' equally'? Equal is a mathematical term that needs to be understood and applied to different scenarios. For example, 6 plus 6 equal 12 uses the word equals to represent 'the same'. Therefore, 6 plus 6 is the same as 12, combining amounts to create a total. However, 12 shared equally between 2 would equal 6. The example in this scenario involves dividing, sharing or grouping an amount into sub-groups. It is not just the mathematics that has to be learnt. The language and social conventions of the mathematics classroom need to be contextualised and understood if pupils are to be successful with their mathematics. To achieve this, first, teachers need to know what pupils are thinking. Dialogue is essential to allow pupils to share their thoughts.

Similarly, to using resources, the way dialogue is used can create cultural boundaries. Asymmetric conversations between teachers and pupils or dialogic discussions can create different environments to work in (Alexander, 2008). Classrooms I have observed dominated by asymmetric discussions in favour of the teacher tended to produce pupils working in isolation. In contrast, classrooms predominantly using dialogic conversation appeared to be more engaged and immersed in a mathematical discussion. Nonetheless, my observations of pupils' limited understanding of language or social conventions can lead to them experiencing feelings of failure. Pupils can appear to be unable to engage in their particular classroom's mathematical discourse, suggesting it is a result of their mathematical ability.

The association of failure and confusion can then lead to helplessness and mathematics anxiety if not addressed (Hannula *et al,* 2004. Lee and Johnston-Wilder, 2017). The beliefs created around a subject at a sensitive stage of pupils' identity development will probably, according to Boaler (2016), continue into adulthood and stay with the learner unless addressed. This process could create a cyclical effect reseeding that anxiety.

There appear to be many aspects that may account for mathematics anxiety in the research literature. I felt a closer look at the possible influences contributing to the identity formation of pupils within the specific environment of the mathematics classroom would be helpful. It could provide valuable insight, which in turn could inform practice. To investigate in detail, I have focused on figured worlds as explored by Holland, Lachicotte, Skinner & Cain (2001). I describe figured worlds as being akin to actors in a play. Each character takes on their role, their role being formed from the social and cultural cues within that environment.

Dialogue and cultural artefacts are used to guide the actors to create their identities for that moment in time. By exploring the figured worlds of the individual participants in the mathematics classrooms, I felt it could offer insight into the many perspectives present. I wanted to hear from the teacher and the pupils how they viewed a moment within the mathematics lesson. Figured worlds are not set but transient moments and could offer insight into the main influences in forming mathematical identities. Boaler and Greeno's (2000) research viewed the mathematics classrooms through a figured world lens but only looked at the teachers' views in secondary school. Wickstrom (2017) also framed her work specifically on mathematical modelling through the lens of the figured world of the primary school. A closer look at how the teacher and pupils' worlds draw together might help give a fresh look at what is being acted out in primary mathematics classrooms today. I am interested in how the different pedagogical approaches create different learning environments that teachers and pupils share.

According to Holland *et al* (2001), the 'figured world' is the socially created environment in which the teacher and pupils act out their roles, contributing to and influencing the formation of their mathematical identity. Some environments enable pupils to be thinking agents that form the identity of an active mathematician (Wenger, 1998), while others learn to follow the procedural approaches promoted by the teacher, reducing learning to following instructions and steps (Boaler 2000). In an attempt to clarify this multi-faceted environment, my research seeks to understand better the differing figured worlds of the key stage one primary mathematics classroom.

1.1 Research Questions

This research project will consider how perceiving the classroom as a figured world could illuminate differences in the nature of learning in mathematics, answering the 'so what' question. If this enquiry is to make a difference, then the study, according to Gerald & Birkenstein (2010), will need to be able to answer two key questions: 'Who cares?' And 'So what?' Because of this study, more will be known about the figured worlds of the mathematics classroom, which could inform policy and practice, thus of interest to teachers and policymakers, responding to the 'who cares' question.

Three research questions have emerged:

Research Question 1

How can viewing the mathematics classroom through a figured world lens provide access to pupils' experiences and bring a fresh approach to exploring the nature of learning?

Research Question 2

How can creating and using a unique analytical framework drawing together Belenky *et al's* (1986) theories of coming to know and Sfard's (1998) metaphors for learning be applied through a figured world framing, bringing to the fore the multiple realities from both the teacher and pupils' perspectives?

Research Question 3

How does the way of coming to know privileged by the teacher influence the pupil's relationship with the subject of mathematics?

1.2 The focus of the study

Five informative episodes from five different mathematics lessons forming a collection of illustrative cases were the focus of this study. I selected an episode from each lesson that raised questions or confusion. In this study, I am using the term 'perturbation' to describe the disturbance of flow when pupils are engaged in mathematical thinking. The perturbation was a point in the lesson when the pupils were stuck or unsure of what to do. I wanted to take a much closer look at these particular episodes through multiple viewpoints to help understand more about the figured worlds of the teacher and pupils at these points of disruption. Each episode itself was an incident or point in the lesson that caused the pupils' disquiet when they appeared challenged or confused. The episode was captured on video as part of the whole lesson and was subsequently used for stimulated recall, thus avoiding reliance on memory alone. The pupils' responses within the episode and subsequent video footage were shared with the teacher. A social constructivist approach supported the figured world theoretical framework by observing the interplay between the individuals and the environment.

1.3 What the chapters aim to do

The following section will briefly outline the purpose of each chapter. I will start with the review of the literature where I set out the theoretical underpinning of my research and explore the conceptual framework in which it sits. The subsequent section on methodology covers my position as the researcher, areas of data collection and management, including ethical considerations. This section concludes with the analytical framework used to analyse the data. The findings from this research are then presented, followed by the discussion chapter. Finally, in my conclusion, I present the key findings, addressing the research questions and outlining my contribution to knowledge.
This includes the methodological approach I took and the implications for practice.

1.3.1 Chapter 2: Literature review

Chapter 2, the literature review, critiques existing studies on mathematical learning and orientates the theoretical framework I have used for this research. It locates the study within the research literature while also pondering my experiences as first a teacher and now a teacher educator. I adopt a figured world framing by paying particular attention to a social constructivist lens to view the complex relationships between the teacher, pupils and environment (Op 't Eynde, 2004).

The scene is set by first reviewing the educational context in which primary mathematics classrooms operate today. I have done this by looking at the political and social history of the mathematics curriculum since the introduction of the first formalised national curriculum in 1988. I then focus on the more recent aims and influences of the 2014 National Curriculum. Mathematics is often viewed as an objectified discipline and a subject that presents pupils with specific challenges. The theoretical framework of figured worlds, as used by Boaler and Greeno (2000), and Wickstrom (2017) has helped me as a researcher understand the socially interactive processes that emerged in the five classrooms. Specifically, focusing on the development of pupils' identity within the mathematics environment, the degree of mathematical thinking behaviours afforded pupils through dialogue and the use of other cultural artefacts could be observed and noted.

Teachers can encourage pupils' to think mathematically by allowing them to take responsibility and make their own decisions. If a pupil has the opportunity to think mathematically, they can have some control over how they engage in the learning process. The teachers can use cultural artefacts, for example, manipulative resources, worksheets and the practice of setting pupils by ability, in the mathematics classroom to create a particular working environment. The teacher uses these artefacts to establish an environment by signalling a specific mathematical identity. For example, the less able pupils are often given more simplistic worksheets, removing challenge or opportunities to engage in mathematical, being guided step by step by the teacher. This practice reinforces a poor mathematical identity by removing opportunities for mathematical thinking and restricting possibilities.

1.3.2 Chapter 3: Methodology

Chapter 3 presents the theoretical framework for the study where I adopt an interpretivist methodology. I begin by outlining the conceptual framework within which my study sits, including my position as a researcher. I have designed the study to explore the figured worlds of the teacher and pupils in a year 2 primary mathematics classroom. I have used the term 'illustrative case studies' because I will be looking at five separate lessons in five different schools, creating a collection of five cases. The features of each case are then brought together for analysis. They are not 'case studies' because I will only be looking at one specific episode from each school. I wanted depth of understanding rather than a general overview of events. I will then look at them together as a collection of cases. The mathematical thinking being afforded pupils was of interest to me and how it can impact their mathematical identity at that moment in time. Teachers often incorporate activities that appear to allow for opportunities for pupils to engage in mathematical thinking, for example, talk partners, problem-solving or using games. Pupils need autonomy over their activities to practise mathematical thinking. Dialogue needs to move beyond explanations to dialogic interactions where pupils can explore their learning and ideas. Activities need to be open-ended to allow for individual exploration and multiple possibilities.

The purpose of focusing on the perturbation was to raise questions about the disquiet from the pupils and teacher. The methodology was designed to look at one small episode in detail so the focus can be on what individual things impact learning and, in turn, the mathematical identity of the pupils at that moment. Following the lesson, I edited the footage of the perturbation and shared it with the teacher and pupils to comment on separately.

A phased approach has been taken toward the analytical framework for this study and is primarily based on Holland *et al's*, (2001) figured world, drawing on the themes of identity, dialogue and cultural artefacts. The methodological approach provided time and opportunity for the teacher to consider the excerpt in detail, employing reflective and reflexive opportunities through applying Mason's (2002) discipline of noticing. My methodological approach builds on Mason's (2002) discipline of noticing by introducing stimulated recall to assist memory, and the view of the pupils to offer greater insight for the teachers' reflective and reflexive process. Noticing in this study means breaking down the observations into clear stages to ensure nothing is missed. Principally starting with 'what can we see?' removing as much tacit knowledge as possible and any analysis of the moment observed. The reflective process established what was happening during the episode before moving on to the reflexive action of discussing the implications and how change might happen. Teachers could use this methodological approach to learn more about their pupils learning. For this to work, teachers would need to be granted time and space.

I used video footage from the lesson and video footage of subsequent discussions with the pupils to review the lesson developing Mason's (2002) approach, creating a collective reflection. I then developed themes based on Belenky, Clinchy, Goldberger & Tarule. (1986) ways of coming to know, and Sfard's (1998) metaphors for learning. Ethical considerations have been adhered to. All participants that took part in the study were anonymised and were free to withdraw consent at any time, without giving a reason. Data has been kept securely on devices that are password protected.

1.3.3 Chapter 4: Findings

Chapter 4 synthesises the outcomes from an analysis of multiple viewpoints from the individual schools and the lessons each teacher planned and taught on the research day. This chapter sets out the teachers' activities to achieve their aims in the lesson, referred to as cases. The teacher and I, as the researcher, engaged in a reflective discussion based on the observations, developing a reflexive practice by simply recalling a short episode from the lesson. The teachers' initial responses to the lesson and episode were recorded, and the pupils' review of the episode explained their perspectives. The teacher took time to observe the details of the video and consider the pupils' views. We then engaged in further discussion. I then took an inductive approach to form a holistic view of the data, resulting in emerging themes, for example, pedagogical approaches and forms of dialogue used during the perturbation. A deductive approach utilising the blended figured world framework was applied, focusing specifically on identity, dialogue, and cultural practices. This approach, which builds on Mason's (2002) discipline of noticing, addresses RQ 1: How can viewing the mathematics classroom through a figured world lens provide access to pupils' experiences and bring a fresh approach to exploring the nature of learning?

1.3.4 Chapter 5: Discussion

The discussion chapter addresses the research questions and focuses on the organisation of the learning environment. The learning environment includes social-cultural practices and interpretations of defining specific roles within the classroom concerning the teacher and pupil's identities. The unique analytical framework blends Sfard's (1998) metaphors of acquisition and participation with Belenky *et al's* (1986) received, procedural and connected learning models. The figured world themes of dialogue and cultural artefacts helped connect the environment's social and cultural contexts. An indication of what teachers privileged in their pedagogical approaches and how they impact pupils' identity within these unique mathematics environments emerged. This approach helped to address RQ2: How can creating and using a unique analytical framework drawing together Belenky *et al's* (1986) theories of coming to know and Sfard's (1998) metaphors for learning be applied through a figured world framing, bringing to the fore the multiple realities from both the teacher and pupils' perspectives?

I took a specific methodological approach to enable teachers to develop a new informed view of the mathematics environment. I am employing reflective and reflexive practice through Mason's (2002) discipline of noticing, including the multiple viewpoints of the classroom. The chapter compares the findings with similar research by Boaler and Greeno (2000) on secondary schools.

1.3.5 Chapter 6: Conclusion

Chapter 6 discusses the broader implications of the findings. I conclude by outlining my original contribution to knowledge and its implications for policy and practice, achieved through the insights I have gained into the figured worlds of the mathematics classroom. I have observed a range of pedagogical approaches adopted by teachers and concluded that the mathematics learning environment is complex to understand and occupy. The socio-cultural influences suggest more than just a focus on the transmission of mathematical knowledge is needed to disentangle what is happening within this environment. Viewing significant incidents or perturbations from the mathematics lessons through multiple lenses helped provide rich data for analysis.

Applying my analytical framework based on Holland *et al's* (2001) figured world view enabled me to focus on the complex ideas of mathematical understanding and how cultural boundaries play a part. The methodological approach taken for this research, alongside the analytical framework I created and applied, enabled new and original interpretations of the social environment of the primary mathematics classroom. This approach included the way teachers used artefacts within those cultural boundaries.

Finally, I conclude with my reflections on the process of undertaking this research study, RQ3: How does the way of coming to know privileged by the teacher influence the pupil's relationship with the subject of mathematics?

Chapter 2

Literature Review

2.0 Introduction

In this chapter, I will explore aspects of the social and cultural environment of the mathematics classroom. I will do this to enable me to take a fresh look at how interactions between teachers and pupils are characterised across a range of primary classrooms. I want to hear from pupils their interpretations of their mathematical experiences. I will also ascertain if there is a disparity between the teacher and pupils' views. This will be achieved by critiquing the writings of a group of theorists while drawing on the central theory for this study which is 'figured worlds'. The term 'figured worlds' will be used according to Urrieta (2007) and Holland *et al.* (2001). They recognise classrooms as socially created and culturally imbued spaces, where particular kinds of dialogue and cultural artefacts afford particular identities. It is where teachers and learners interact to make sense of mathematical ideas. The relationship between the teacher and pupils within the mathematics environment intrigues me, particularly how their unique mathematical identities influence each other. More specifically Holland *et al* (2001) define the figured world as:

> *'a socially and culturally constructed realm of interpretation in which particular characters and actors are recognised, significance is assigned to certain acts, and particular outcomes are valued over others.' (2001:52)*

This definition highlights the influence social and cultural aspects of a classroom can have on pupils, depending on what the teacher values. For example, in what can be described as a traditional style mathematics classroom where didactic teaching is common, the teacher instructs pupils how to perform mathematical algorithms (Boaler, 2002). The pupils are then assessed on how well they can reproduce that information, creating a figured world in which they passively follow what the teacher has instructed. Contrastingly, Wickstrom (2017) introduced modelling tasks to challenge the figured world of a traditional approach with the teacher employing real world activities by setting challenges for the pupils to explore and investigate for themselves. This approach creates a figured world where pupils are active participants in creating their understanding. The teacher, in this instance, puts value on the pupil's ability to reflect and apply knowledge to new situations. To me, Holland *et al's* (2001) definition suggest that the mathematics classroom is viewed as a performance with the production dictating the roles and characteristics of the actors' parts. All figured worlds are unique to the individual and the specific environment formed, reflecting the individual experiences and encounters. The main feature of figured world's theory I will be focusing on is the identity that pupils develop within the differing figured worlds of the mathematics classroom and how the use of dialogue and cultural artefacts afford or limit opportunities for mathematical thinking (Bonotto, 2013).

I have drawn my theoretical framework from critiquing a body of literature on learning and how it can be construed, including the writings of various theorists. The theoretical underpinning will be based primarily on Vygotsky's (1978) social constructivist theories and builds on Holland *et al's* (2001) figured worlds. The work of Sfard (1998, 2008) and Belenky *et al* (1986) are also drawn on to build a theoretical framework utilising their work on metaphors for learning and ways pupils come to know. The methodological approach I am taking builds on Mason's (2002) discipline of noticing.

As a teacher and teacher educator, I considered the introduction of a government-led curriculum a pivotal moment in the history of mathematics teaching. To provide a context and historical justification for this study, I will briefly consider the divergent mix of historical, social and political hegemonies. This approach will include looking back to the start of the formalisation of the English mathematics curriculum in 1988 (DfE, 1988), the introduction of the National Numeracy Strategy (NNS) in 1999 (DfEE, 1999) and then considering the significant reforms in the 2014 mathematics curriculum (DfE, 2013). This chapter concludes by synthesising the theoretical framework designed from a critical engagement with existing studies.

2.1 Reviewing the educational context and the focus on productivity and results

First, I will present a brief exploration of the historical context of the mathematics curriculum to give an overview of the subjective requirements placed on teachers in the form of a national curriculum. These external obligations assigned to teachers will inform my study in relation to their figured worlds and the learning environments created, thereby enabling deeper analysis. Successive governments develop educational models and subject content as part of broader political systems and ideologies, reflecting each successive government's aims and underpinning political theories.

In 1988, the first formal national curriculum was introduced with mathematics as one of the core subjects along with English and science. Ball (1990) identified flaws in the compilation of the new curriculum: for example, within specific subjects, topics were left out if they had not appeared in the writer's own private schooling. There was a strong emphasis on calculation and solving formal algorithms within the mathematics curriculum, with less emphasis on reasoning and conjecture. It was how the nature and concepts of the subject were interpreted that determined the content of the curriculum. This reflects what Bourdieu (1986) describes as cultural capital, drawing on personal experiences of education and knowledge, reflecting the individual's figured world. At the same time, a set of curriculum measures was introduced to assess academic investment in terms of education through testing. This introduction led to the privileging of specific ways of knowing and learning. Robeyns (2006) theories of justice describes this as the human capital model and a narrowing of the curriculum, limiting the teachers' freedom to interpret, thus impacting their figured worlds and that of their pupils. The most powerful constructs within the mathematics curriculum appear to be the attainment targets and programmes of study linking to specific knowledge. Mathematics became less about reasoning and exploration, as that was difficult to test, and a greater emphasis was placed on the result (Haladyna, 2006). The human capital model described by Robeyns (2006) views investment in education as a beneficial economic endeavour, providing an appropriately skilled workforce for the economic good of the country's productivity (Davis, 2003). This model requires specific outcomes based on accountability and testing regimes, narrowing the curriculum and further limiting the teachers' freedoms, objectifying the subject (Salmon, 1995: 55).

In 1999, the introduction of the National Numeracy Strategy saw a move by the government and their education advisors to become more involved in how the curriculum was being taught. This was again in a move to improve standards. The National Numeracy Strategy (1999) was heralding interactive, whole-class teaching as good practice. Moyles, Hargreaves, Merry, Paterson and Esarte-Sarries (2003) research suggested teachers were not wholly on board with what was being promoted as ‘interactive whole class teaching’. Little support had been offered to help teachers understand the theoretical underpinning or the features of this approach. The strategy offered a clear curriculum guide but introduced a not-so-clear pedagogical approach.

The human capital or traditional approach promoted appears to be less about 'pedagogical' methods and more about 'education'. This approach provides very different contexts and opportunities for creating different figured worlds. Leach and Moon (2008) examine the difference between the terms ‘education', and 'pedagogy', suggesting education is the acquisition of a body of previously established knowledge, and pedagogy supports a more social process. In the mathematics classroom, this can be seen as the difference between learning facts to be reproduced and exploring possibilities to create greater understanding. Leach and Moon (2008) argue that the term education closes down intellectual curiosity by presenting a narrow curriculum to be obtained. Simon (1981) further suggests that pedagogy advocates a social process that introduces intellectual curiosity and theory. Simon's (1981) work entitled 'Why no pedagogy in England?' suggested that when a traditional educational approach is taken, there is a lack of scientific or theoretical reasoning. In contrast, pedagogical methods introduce teaching and learning as a discipline in its own right based on theoretical rhetoric. The introduction of multiple pedagogical possibilities within a class of pupils raises challenges for the teacher and opportunities to be inclusive, thus raising the teaching profile beyond just delivering a curriculum.

Maximising the educational opportunities for wider society means developing a system that introduces broader possibilities for individual figured worlds, suggesting a connected approach, as described by Belenky *et al.* (1986) through ecologies of participation (section 2.6.2). Freire (1972), Bernstein (1972), and Bourdieu (1973), along with Bruner (1996) more recently, all considered schools to disadvantage social groups that do not conform to the social expectations of a society's ruling class. These theorists support the idea that when a society requires a specific return on monies invested in education (as reflected in Robeyns (2006) human capital model), this approach does not leave room for social justice and human rights models. They state that where figured worlds are created through the expectations of a narrow group of society, most pupils will find it difficult to belong or conform to those expectations, and many beneficial and unlooked-for talents will be lost to society. Rousseau (1712-78) challenged the ideas of conforming and just looking to the past for knowledge. If knowledge is to be extended and developed, it needs to be challenged and interrogated, which can only be done through understanding (Russell, 1946). Holland *et al's* (2001) figured world approach suggests that understanding and interrogating knowledge from an individual's perspective would enrich that process. The introduction of a national curriculum based on scholastic investment and measured by certification, according to Bourdieu (1986), missed the value of individual cultural capital and potential, suggesting a fixed intelligence (Dweck, 2013). Through the National Curriculum (1988, 2014) and accountability measures, politics has influenced the minutiae of the mathematics classroom in previously impossible ways. The National Curriculum has determined the goals set for the pupil and impacted differentially on the pupil's attitudes towards the subject (Tall, 2013).

In this short section, I have touched on a wide range of complex elements that have impacted teaching and learning within the mathematics classroom. The teacher’s role is to intertwine both theoretical approaches and social and political influences, all of which impact their epistemology, in turn influencing the classroom environment. Figure 1 attempts to illustrate the main elements that influence the figured worlds of both teachers and pupils. I have also included the key theorists I have drawn on. In the next section, I will take a closer look at the changes the present-day National Curriculum has had on the figured worlds of mathematics classrooms to inform the study further.

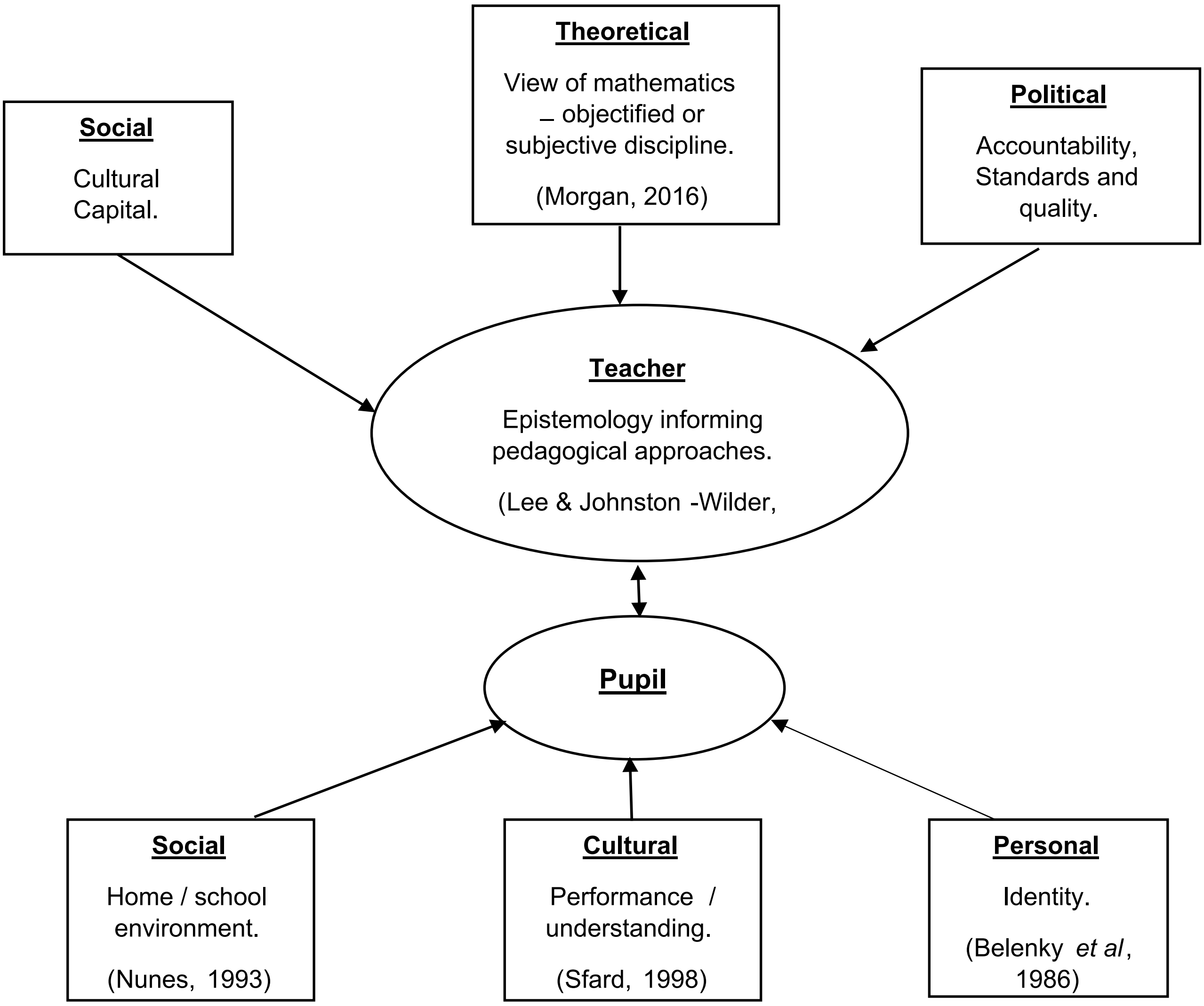


Figure 1: Influences discussed so far that contribute to the figured worlds of the Teacher and Pupil.

2.1.1 The introduction of the 2014 National Curriculum (NC)

The reform of the National Curriculum in 2014 had two significant influences on mathematics teaching at KS1: It further embedded the hegemony of teachers testing pupils to prepare for and undertake the end of key stage curriculum assessments (Ofsted, 2012). It also advocated the teaching for 'mastery' of the content of the curriculum. The NC reflects the government's interest in adopting approaches from successful Asian countries, which emphasise reasoning and conjecture, promoting a figured world of mathematical thinking behaviours through dialogue (DfE, 2013). This has created an interesting combination for teachers to negotiate.

The British Education Reform Act of 1988 introduced Key Stage assessments for years 2 and 6 in primary school in the core subjects, which included mathematics. The introduction of this test suggested the motivation for this change reflected on a Hirschian (1987) approach, following a core knowledge curriculum with a set text for each year group within a school (Hirsch, 1987). This approach became the driving force for the subject rather than the discipline itself (Pickering 1995). The push for accountability within the schooling system appears to have given preference to success in the test, over understanding the subject and creating lifelong learners and mathematicians. This pressure can skew teachers' concept of knowledge towards a definitive truth, encouraging the implementation of didactic styles of pedagogy in the race to meet goals and reach the targets of the test paper (Bell, 1994. Nussbaum 2010). The National Curriculum of 2014 reinforces the emphasis on speed and accuracy with the statement that - requires pupils to have 'the ability to recall and apply knowledge rapidly and accurately.' (NC, 2013, p. 3).

In 2005, the Mathematics Association commented:

> *'The current assessment system, backed by the accountability structure, encourages a mode of preparation for tests and examinations which focuses solely on the standard questions that appears on papers…This leads to the exclusion of more interesting and challenging problems and applications at all levels. These are the very things that are of importance to employers and higher education, because they stimulate interest and encourage independent thinking'*
>
> Mansell (2007: 61)

Mansell (2007), in this statement, implies that a curriculum based on testing and exams narrows the content and purpose of a subject. This in turn, disadvantages pupils when they want to apply their knowledge and skills to a broader context. This view supports Robeyns (2006) human capital model. The 2014 National Curriculum provided a clear indication to schools of the direction the government wanted the school curriculum to go. The focus was on improving performance by providing clear steps to acquiring essential knowledge for the assessment (DfE, 2013).

However, at the same time, attempting to reflect the curricula of the world's most successful systems, Michael Gove, the education secretary from 2010 to 2014, heralded the change to a performance-based curriculum and a figured world where value is placed on testing and performance, influencing pupils' mathematical identity. The wording suggested a stronger alliance with a didactic approach, which measures academic investment, creating a particular figured world. However, there was also a focus on competing on the world stage and viewing how successful countries approached mathematics teaching. The approaches of those more successful countries contrasted with the didactic approach promoted by Gove (2013).

In response to the education secretary, the Department for Education saw a shift in aims for the mathematics curriculum towards a 'Mastery approach'. This approach has been appearing in mathematics classrooms to greater and lesser extents and could influence the classrooms where my research will be taking place. The Mastery approach has been introduced to imitate the success of East Asian countries such as Singapore and China. In the 'Programme for International Student Assessment' (PISA) (2014) and the 'Trends in International Mathematics and Science Study' (TIMSS) (2000) reports, the new curriculum focused on the traits of this so-called 'mastery' approach (Harris and Jones, 2018). The mastery approach is not a recent phenomenon but can be traced back to Bloom (1971). Many different approaches include 'Mastery' in their titles, but Marton and Booth's research in 1997 explored patterns of variation. This research has led to what Marton (2015) now describes as Variation Theory. Variation Theory appears to cover the main features of what the Department for Education labelled a 'Mastery' approach. It is not a cover-all theory but focuses on the object of learning, specifically mathematical content. The theory aims for discernment, with a particular interest in the pupil experience, suggesting a focus on pupils' figured worlds. Pupils see the differences within their explorations before looking for what is the same. This discovery leads to conjecture, a vital feature of Variation Theory and is echoed throughout the mastery approach by authors such as Kullberg, Runesson Kempe and Marton (2017) and Askew (2015). Conjecture is also an essential feature of enquiry based learning approaches (Artique and Blomhøj, 2013). Therefore, the performance-based curriculum introduced in 2014 and the government advocating a Mastery approach based on successful countries introduced tension creating a dichotomy.

In a move to encourage the practice of a 'Mastery' approach, The National Centre for the Excellence in the teaching of Mathematics (NCETM), in conjunction with the Department for Education (2013), conducted a Mathematics teacher exchange (MTE). These particular bodies set up the exchange to learn from East Asian practice. The exchange programme involved schools from England and Shanghai, with the English schools learning from the Shanghai institutions. The findings from this collaboration have been presented and discussed in the report by Boylan, Wolstenholme, Demack, Maxwell, Jay, Adams and Reaney (2019). Their report summarises the similarities and differences between the two countries approaches to what is being described as the' Mastery' mathematics approach. According to Boylan *et al* (2019:19), a mastery approach suggests a figured world where pupils' views are valued, making them active learners exploring possibilities as described by Watson and Mason in 2006, Alexander in 2008, and later Askew in 2015. The report identifies two features that cross over between the Chinese interpretation of Mastery and the English version; regular use of formative assessment (Guskey 1997) and the belief that every child can succeed. An area that differs in approach between the two countries are the lesson's introduction. In England, mathematics lessons start with differentiated lesson objectives, the teacher models the learning, and the pupils' practise the activities. In China, lessons begin with a whole class challenge, and all pupils start with the same challenge, reflecting the National Numeracy Strategy of 1999. In addition, in England, grouping and teaching ability sets are prevalent. In China, they take a whole class approach.

The report highlights greater uniformity across schools in China than in England, so data was more limited in England (Ofsted 2011). Ofsted (2011) also reported a great diversity in teaching approaches across England, thus making it difficult to establish that a clear interpretation of the Mastery approach is being used, if at all. The literature available on Mastery tends to focus on how to implement the approach instead of researching what is happening in schools. However, Lord undertook a project in 2020 working with teachers, looking at achieving Mastery in more depth. Teachers were asked to adopt a Mastery approach to their problem-solving lessons. The findings showed that teachers paid greater attention to the learning behaviours of pupils and identified the importance of communication skills when implementing features of the approach. There appears to be a gap in the literature looking at what is happening within schools regarding the Mastery approach, particularly in primary schools. This gap could be because of its recent introduction and the uncertainty about it. For my study, I might find the figured worlds of the teachers and pupils influenced by the introduction of this approach. I could see a version of it acted out in the classrooms I will be conducting my research. However, the picture is not clear how the approach is being implemented in English schools. The Boylan *et al* report (2019) also suggests there is little consistency or understanding of the mastery approach. I will focus for this study on the features being enacted within the classroom and not the suggestion of a Mastery approach.

The aims of the National Curriculum include aspirations for fluency, conceptual understanding, reasoning, conjecture, generalisation, justification and applying mathematics to a variety of routine and non-routine problems. All these aspirations correspond with the underpinning rhetoric of a mastery approach. In my study, I am interested in finding out how, if at all, these National Curriculum aims have influenced teachers' epistemological stance and classroom practice, which in turn could impact pupils learning, and what this might look like in the classroom.

2.2 Specific challenges within a mathematics classroom

My study takes place within the English educational system, but it takes place explicitly within primary mathematics lessons. Therefore, in this section, I will explore the subject specific context and the associated challenges that will inform and impact the figured worlds of both the teacher and the pupils within this study. The link between challenge and the subject of mathematics has always intrigued me. Many view the subject as difficult and the notion of 'challenge' as off-putting. However, challenge and overcoming those challenges is what many find motivating and engaging about the subject.

Historically the subject of mathematics has been viewed as an objectified discipline or a subject that has fixed rules controlled by others. This view creates strong 'attitudes towards' and 'beliefs about' what mathematics is, leaving little room for subjectivity (Morgan 2016). This study will focus on the pedagogical approaches associated with mathematics instead of the discipline itself. Both aspects generate great debate about the opportunities or place for agency within teaching, learning or contributing to development within the discipline. Suppose teachers believe mathematics is a closed discipline in which everything is known that can be known. In that case, their pedagogical approaches will differ from teachers who believe pupils themselves have something of value to contribute to mathematics. Teachers are more likely to choose from another group of pedagogical approaches, particularly those that develop and encourage enquire approaches, placing pupils in the community of those who participate in mathematical thinking (Lee and Johnston-Wilder 2017). Therefore, I consider the teachers' view of mathematics as being either a subjective or objective subject would influence the pedagogical approaches they use in the mathematics classroom.

It is suggested by Morgan (2016) that mathematics is an agentless community and one of objective knowledge. I am interested in how teachers afford pupils the opportunity to think mathematically and how that can impact their figured world; therefore, in the next section, I will consider how pedagogical approaches can encourage mathematical thinking. Then I will explore the opinions associated with viewing mathematics as a complex subject and one that can lead to a figured world where helplessness features (Lee and Johnston-Wilder, 2017).

2.2.1 The nature of mathematical thinking within the primary mathematics classroom

The opportunities afforded pupils to think mathematically, and how it affects their figured worlds has interested me as both a teacher and teacher educator. The pedagogical approaches adopted by teachers appear to me to shape the types of mathematical thinking pupils engage in. To view mathematics as a subject of exploration and conjecture opens up the potential for risk-taking, which can be positive if part of a figured world that values challenge and errors. The traditional idea of education, specifically the mathematics classroom, is one based on memorising facts and algorithms, either from a pedagogical perspective or by viewing mathematics as an objectified discipline (Morgan, 2016). Unfortunately, if pupils have experienced humiliation and public embarrassment in the mathematics classroom through not being able to recall facts or algorithms, they are less likely to want to take risks. Limiting opportunities in this way is not considered conducive to promoting mathematical thinking (Rowland, 1999).

Here I will try and define my understanding of mathematical thinking by drawing on Bruner's (1996) theories. He uses the term agency to describe the ability of pupils to act of their own will. He merges agency and collaboration, suggesting a social constructivist view. Bruner (1996:93) states: *'the agentive mind is not only active in nature but seeks out dialogue and discourse with other active minds.'* In this statement, he proposes that to be agentive is not to just work in isolation but to be proactive and work in collaboration with others, seeking out others' thoughts and ideas to affirm or challenge. Bruner (1996) refers to the metaphor of narration to explore this relationship. To be a narrator is to share one's thoughts and ideas with each other, creating understanding through stories. To create these stories, one person negotiates their views and points of view with another, bringing their figured worlds together. Narrators have to fashion their work so others can interpret them, thus demonstrating mathematical thinking. Pupils exploring mathematical thinking in the classroom involve enacting and negotiating power relationships. Wood (2014) describes how pupils can do this in a play setting, experiencing shifting power structures, relationships, conflict, negotiation, resistance and subversion. Wood (2014) discusses child-centred approaches to learning, which emphasise self-regulation and control. All of these actions will influence their figured worlds.

This approach raises important considerations for the mathematics teacher and pivots on their theoretical stance and views on mathematics as an objectified discipline. The teacher may consider exploring multiple realities a waste of time in an already busy curriculum (Ernest, 1991). I consider meaning-making through mathematical thinking to be a key player in challenging mathematics as a difficult subject. Challenges can become the motivation for pupils' learning, overcoming challenges for themselves, can motivate and engage pupils.

For this study, I will view mathematical thinking in the context of the figured world of the mathematics classroom where agents, in this case, pupils and the teacher, come together as thinking mathematicians to practise their collaboration skills, testing out their theories while consciously meaning-making.

2.2.2 Mathematics - a difficult subject.

Reflecting on my early childhood experiences, I took a passive role within my mathematical figured world. I found it a difficult place to be and associated it with emotions of failure and rejection based on my inability to remember my timetables for the weekly test. I concluded that mathematics was too difficult for me to learn. This section will explore a range of reasons mathematics could be considered difficult, from the teacher's epistemic stance based on historical context, to pupils' emotional reactions to specific approaches.

Part of the historical context for this research is that mathematics is a difficult subject, a view shared by many. This view is based on social, cultural and personal experiences, which collectively form individuals' self-concepts (Morgan, 2016). Rogers (1990) describes self-concepts as an individual awareness of oneself based on the acceptance or rejection of others. The figured worlds of the mathematics classroom present very specific and culturally powerful indicators of what is valued in these environments (Holland *et al.*, 2001). The emotion of helplessness is often associated with the mathematics classroom supporting the notion of a difficult subject (Lee and Johnston-Wilder, 2017). One factor contributing to this status may be the school itself and the contrast children experience between the school and home environments (Nunes, 1993). The learning environment of the school is generally in contrast to the learning environment of the home. An epistemology that privileges the teacher as the holder of knowledge can also encourage a giving into or handing over to authority by the pupil, which encourages helplessness (Lee & Johnson-Wilder, 2017). The school environment links to power and control, creating the rules to which pupils are encouraged to conform (Foucault, 1980). It also creates an environment where performativity and conformation are valued over understanding (Op 't Eynde, 2004). Dewey (1997) describes the complex family structure as one where children generally participate in a family social unit. Learning opportunities are placed within this social unit within various contexts, each with typically shared meaning, thus creating conflicting school and family environments.

It is not just the environment of school and home which creates contrast but also the mathematics activities themselves. These activities can be intertwined with the culture in which they take place. Lave (1988) demonstrated in her studies how the context in which mathematics was conducted influenced the attitudes and performance of the participants, for example, shopping, playing games, building and construction. Each activity has a cultural environment in which mathematics is played out. Saxe (1991) associated culturally constructed activities with the context in which it is socially connected. For example, the activity of column addition is associated with the school mathematics classroom. Just as the cultural activities of these environments can inform the figured worlds of those participants, so can the cultural artefacts determine how mathematics is taught and engaged in.

Nunes, Schliemann, Carraher, & Schliemann (1993) took the notion of the environment further by exploring the difference between the mathematics seen in the classroom and street mathematics. I will not distinguish between these different environments for this study, but the characteristics might be evident in the pupils' figured worlds and the teacher's pedagogical approaches. The figured worlds the children enacted contrasted between meaningful endeavours in the streets compared with the specific purpose of the mathematics in the school. Rowland *et al* (2009:108) stresses that a more meaningful way of ensuring progression and connectedness would be to build on pupils' knowledge already established outside the classroom rather than setting it aside. Rather than focusing on the different environments perhaps, it would be more beneficial to look at how teachers use pupils' prior knowledge to build new understanding.

As mentioned earlier, helplessness is a prominent emotion often associated with the mathematics classroom (Lee & Johnson-Wilder, 2017). Seligman (1975) coined the phrase 'learnt helplessness' to describe behaviour found in dogs. Boaler and Greeno (2000:171) describe the traditional pedagogical approach to mathematics as one in which students have to surrender their agency to follow the procedures, thus increasing helplessness. Behaviourist research with dogs, cited by Seligman (2006) and Williams (2003), occurred in the mid-1960s and inadvertently discovered an unexpected reaction. When the researcher put the dogs through negative experiences they could not control or avoid, in this case, electric shocks, they stopped taking action. Experiments were then carried out with humans using loud noises instead of electric shocks with similar results.

In comparison, the pupils in the classroom, when asked to do tasks that do not build on prior learning but add more confusion to an already complex and insecure understanding, could result in pupils feeling out of control of their learning. If this is repeated, similarly to the dogs in the experiment, pupils could stop trying and become helpless, relying on the teacher, thus creating teacher reliance. Williams research in 2003 looked at the characteristics of learning environments where pupils had the opportunities to explore novel mathematical ideas, behaving like thinking mathematicians remained in control. In this research, pupils appeared to demonstrate resilience in pursuing mathematical concepts avoiding helplessness.

Dewey (1997) describes an alternative to learnt helplessness through a constructivist model; when pupils are enabled to explore and understand mathematics and take possession of their learning, traits of helplessness appear to be reduced or avoided

(Williams, 2003. Seligman, 2006). Even when teaching aligns with the constructivist model, pupils can be at risk of helplessness if their understanding becomes weak or harbouring naïve conceptions the teacher cannot address. A possible consequence of the prevalence of helplessness and reliance on the teacher is that teachers now perpetuate the experience by replicating their own experiences and creating figured worlds in which helplessness is a part, continuing the cycle (Chinn, 2012).

With helplessness in a mathematics classroom comes a need for fitting into the figured worlds created. These figured worlds often result in over-reliance on the teacher and a need to be guided in performing. When pupils display confusion when challenged, teachers may simplify the question, often taking away the challenge altogether, smoothing the way, and removing with it any mathematical thinking (Wigley, 1992). Pupils are then unprepared to take on challenges and can lack resilience, relying solely on the teacher. (Lee and Johnston-Wilder, 2017; Mason, 2002). As a result, the subject's perception becomes one of being difficult.

Bandura, (1977) and later Dweck, (2013) have stressed there is a human desire to belong and fit into society and to know its socially constructed expectations. Once the social expectations have been established, understanding and reasoning is no longer a priority, and dependent behaviour and the need to belong can be more critical than understanding (Hogan, 2010). Pupils who continue to struggle can develop a cognitive belief that mathematics is not for them by focusing on avoidance and teacher reliance. Teachers will then respond by smoothing the way even more for pupils in preference to providing challenge. This intervention removes the one thing that could engage and motivate pupils, the excitement and reward of overcoming mathematical challenges. In my opinion, challenge is the purpose of the subject of mathematics. Smoothing the way will only perpetuate pupils' view of mathematics as confusing, and helplessness becomes an accepted feature of this environment and a key feature of their figured world.

Figure 1 shows the influences I have explored in this literature review on teachers’ figured worlds and that of their pupils. Due to the complexity of individual figured worlds and the transient nature, one aspect might have a greater influence at a particular moment in time but equally all aspects will be at work. The purpose of my study is to see what is influencing the figured worlds of the classroom at a moment in time. I will be listening to both the teacher and the pupils' accounts of that moment which might suggest the main influences at that point in time.

2.3 *The figured worlds of the mathematics classroom*

In this section, I want to examine if and how the social context of the mathematics classroom can influence identity. I feel the figured world of a pupil with a positive mathematical identity is essential for a pupil's mathematical success. Therefore I will consider the work of Boaler and Greeno (2000), and Wickstrom (2017), exploring the social environment of the mathematics classroom through the lens of the figured world, with a specific emphasis on identity influenced by the dialogue and cultural artefacts associated with the mathematics classroom. This study will collect and consider the perspectives of both the teacher and pupils, supported by a stimulated recall approach, thus building on these works.

Gee (2000) describes identity as the 'kinds of people' within specific contexts. Within the mathematics classroom, that could be; a confident mathematician, a risk-taker, a pupil who relies on the teacher or even rejects an association with that environment altogether. Identity is not fixed or entrenched but can change with context and time, mediated by a sense of self (Vygotsky, 1978. Bakhtin, 1981). According to Gee (2000), being recognised as a specific type of person is what is meant by identity. All pupils have multiple identities connected to their performance within a given social context and have a core identity (Gee, 2000). A pupil's identity is continually readjusting within their figured world. For this study, I will only be focusing on the identity portrayed within the figured worlds of the mathematics classroom.

Pupils' identities within their figured worlds are socially enacted through dialogue and cultural artefacts forming culturally constructed contexts (Holland *et al* 2001). According to Holland *et al* (2001), the social situation of the mathematics classroom can compel pupils. This compulsion can be socially through interactions with others, for example, through dialogue with and between the teacher and peer, and culturally through historical ways of doing things. As described by Bourdieu (1986), the rules and guidelines are in the unique possession of individuals, made up of their cultural experiences, developed over time. For example, teachers can use dialogue to give instructions. They can also use dialogue to negotiate understanding, creating a more interactive environment between peers and the teacher. Cultural artefacts can also signal differing roles and identity associations. For example, manipulative resources can signal remedial support suggesting pupils need help. Alternatively, these resources can be used as essential tools for all pupils to investigate, thus guiding pupils' behaviour. ‘Figured worlds represent the 'rules,' 'guidelines,' or social forces that influence (but do not completely dictate) the ways people speak, behave and 'practice' within social spaces' (Hatt, 2007:149–150). Figured worlds are continually changing.

Perceived ability and social status within the mathematics classroom are some of the characteristics that would inform the foci of the figured world (Holland *et al* 2001). According to Holland *et al.* (2001), three key elements of the figured world framework are identity, dialogue, and cultural artefacts. The way the social environment is constructed and accommodates these displayed qualities acts as evidence for forming self-concepts and deciding what is valued within the classroom (Boaler and Greeno, 2000; Urrieta, 2007). Each element provides opportunities for forming and influencing the specific figured worlds being acted out by each member of the mathematics classroom. How teachers and pupils behave generates implicitly or explicitly what is valued in classrooms. Considering both the teacher and the pupils' perspective could provide a greater understanding of these figured worlds.. I feel this would help to understand the complexities of the mathematics classroom.

2.3.1 The construction of identity

The social environment of a primary mathematics classroom can be considered unique. Evans (2000:4) describes the traditional environment as being led by negative emotions creating feelings of 'boredom, isolation and anxiety' through an abstract subject focused on and creating a figured world of isolation and speed. If this description were valid, it would have the capacity to influence the construction of a pupil's identity. Pupil identity is expressed through organised daily activities with others within this social practice. The role/s pupils take up within the mathematics classroom is continually self-constructed, with identities influenced by the cultural setting they are working within (Vygotsky, 1978. Bakhtin, 1981. Boaler & Greeno, 2000). Therefore, a theory of identity must consider the continual emergence of identity within the activities of the cultural setting, creating specific figured worlds. The building of that identity takes place over time within that setting. As stated in Holland *et al* (2001), status and value can be assigned to specific characters within a figured world, in this case, teachers and pupils. The pupil with specific cultural capital due to their upbringing can have a head start within an environment that values that capital (Bourdieu, 1973). Each character within the figured world assumes their role, changing, adjusting, and adapting that role over time according to the interactions taking place within that unique environment. This demonstrates how identity is ephemeral in nature. The world of the classroom is constructed socially and culturally, with guides offered on how to act and react. Each character will have their role and position depending on status, entitlement, cultural capital and sense of social standing, all relative to each other (Holland *et al.*, 2001).

Self-concept is a term that covers all aspects of self-judgement concerning self-image, self-esteem and ideal-self (Rogers, 1990). Boaler and Greeno (2000) considered the unique environment of the mathematics classroom in secondary schools and the specific impacts that the social structure within a classroom could have on a pupil's self-concept. The figured worlds of the classroom environment provide the framework for creating an image of self or identity, which works for the individual in the classroom's community of practice. By participating in communities of practice, identities can be formed and reformed. In short:

> *'Figured worlds provide the context of meaning and action in which social position and social relationships are named and conducted. They also provide the loci in which people fashion senses of self – that is, develop identities.'*
>
> (Holland *et al.*, 2001: 60)

Holland *et al* (2001) states identities are formed from within the context of a figured world. Within the lived world of the classroom, the place of knowledge is significant and can be used as a currency. Suppose a pupil pays equal attention to the level of knowledge the teacher values. In that case, the teacher might well extend an affordance through positive feedback, which encourages further knowledge. Pupils who do not pay attention or demonstrate the same value to the knowledge as the teacher might well not have the same response to an affordance offered by the teacher (Holland *et al*, 2001). This interaction is not restricted to cognitive knowledge but emotional experience too. If pupils are excluded from activities because of their perceived lack of interest or understanding, it can impact their identities (Brenneis, 1990). The structure of the classroom organisation in the form of ability groups emphasises the differing figured worlds being offered to pupils via their working groups and teacher expectations (Boaler, 2005. Boylan, 2021. Marks, 2016).

During mathematics lessons, the mathematical identities of both pupils and teachers are formed and reformed. Outside pressures such as groupings, teacher's theories of intelligence and their perceived relationship to mathematics can affect a pupils' mathematical identity (Dweck 2013). The teacher's identity will again be affected by outside influences, such as the curriculum, the pedagogies required in school, and internal influences, especially their relationship to mathematics. As illustrated in figure 1.

Vygotsky's (1978) social constructivist approach provides an opportunity to explore emotional responses within the figured world environment, thus enabling greater clarity of understanding by viewing the whole, and not just parts, of the experience (Varela, Thompson and Rosch, 1991. Op't Eynde, 2004). Activities can form emotional links within the figured world the pupils inhabit at that moment in time. Pupils subconsciously evaluate a situation concerning their personal goals when engaged in an activity (Hannula, 2006). Such evaluation can produce positive emotions linked to progress towards a pupil's goals or negative emotions when the way is blocked. The negative emotions associated with the mathematics classroom can develop into mathematics anxiety if pupils' opportunities for exercising mathematical thinking within their figured worlds are limited, and resilience is not formed (Lee, JohnsonWilder, 2017. Williams, 2003).

2.3.1.1. Personal and social identities within the figured world

According to Holland *et al* (2001), identity within the figured world consists of three aspects coming together that produce personal and social identities; Positionality, Self-dialogue, and World-making. Positionality is in relation to the personal activity within a particular social context, in this study the pupils' positioning is within the mathematics classroom. Self-dialogue is personalised understanding and part of a meaning-making process that starts with external dialogue, which is shared and practised. The external dialogue is then internalised and becomes part of the thought-making process and self-dialogue (Vygotsky, 1978). World-making is the creation of the individual figured worlds within the mathematics classroom. The coming together of these three aspects characterise the social enactment and cultural contexts of mathematics classrooms. In the following section, I will explore these three areas in more detail.

Boaler and Greeno (2000) assert that individuals do not have autonomy over positioning their own identity. They see identity as being negotiated through the social interactions taking place in the cultural space of the classroom. The specific social context of the mathematics classroom can see the teacher in a position of power, rank or influence that can foster specific characteristics within an individual. The teacher can do this by offering particular roles. For example, the labels of 'loud child', 'clever child' or 'bad child' would suggest to an individual their place within the group. An individual's positioning continually shifts, modifying or becoming entrenched based on history and circumstance (Urrieta, 2007). Urrieta (2007) suggests that all experiences are built on prior experiences. The idea of self-dialogue cannot exist in a vacuum because the individual exists within a context, and others influence that context. Vygotsky (1978) and Bakhtin (1981) view inner speech as dialogue based on experience. By working with a more experienced other, 'higher-order thought' can be achieved, supporting Vygotsky's theories of the zone of proximal development (ZPD) (Wertsch 1985: 201). (ZPD is discussed in greater depth in section 2.4).

In the classroom, teachers will draw on their ontologies and epistemologies, creating their own subconscious figured worlds of what they imagine a teacher to be, in effect what

Holland *et al* (2001) refer to as world-making. The teacher's history, social background and prior experiences form a 'tacit' knowledge. This tacit knowledge is built up over time, using a combination of intrinsic and explicit knowledge on the way. Eraut (2000) describes tacit knowledge as pre-conceptions and prejudice. The combination of this tacit and explicit knowledge is described as 'knowing in action' by Claxton (2000). The formation of a teacher's tacit knowledge could be restricted and lack self-dialogue and reflection due to the busy environment of a primary classroom. For my study, I introduce a challenge to that tacit knowledge through stimulated recall, dialogue and the introduction of the pupil's perspective.

The formation of tacit knowledge or knowing in action suggests that every action is built on prior knowledge and experiences, which does not explain how new ideas are created within a figured world. Bakhtin (1981) suggests pupils can do this through play. Claiming figured worlds are established from many figured worlds coming together and are never pure or singular and the place of play worlds allow for experimentation. Play or, in the case of mathematics, exploration and discovery of mathematical ideas and concepts provides opportunities for pupils to explore the cultural practices of the mathematics environment through problem-solving and trial and error. This approach can be done analytically, building up new layers of discovery or world-making. The use and development of the imagination, which can remove the boundaries of figured worlds, can also provide the opportunity to fantasise and create new figured worlds, exploring mathematical possibilities and conjecture. The figured world is a space created and inhabited by its participants, and those participants create their own imagined space socially, producing social and personal identities. For this study, I am building on Holland *et al's* (2001) work on identity formed within figured worlds, set within Vygotsky's social constructivist theories.

2.3.2 Dialogue

Dialogue and the use of language in the primary mathematics classroom contribute to cultural constructs and the social context of the figured worlds of both the teacher and the pupils. The following section will explore how language and dialogue influence these figured worlds and how questioning is embedded within classroom dialogue (Mason, 2021).

Learning a common language between people is part of building a community of practice (Lave and Wenger 2005) and is described as 'co-operative learning'. The language of the mathematics classroom supports the skills of comparison, imitation, representation, attention and generalisation (Newman and Holzman, 1993: 56). These features are all part of creating a particular figured world. Co-operative approaches are further explored in section 2.4. By mediating socially constructed norms within individual figured worlds, common meanings can be reached and agreed on (Gergen, 1995), although this may not be an easy pursuit. For example, in the mathematics classroom, the words 'take-away' are another way of saying 'subtraction', which removes objects from a collection that can also be described as 'the difference'. The language used in the mathematical classroom can lead to many complex or confusing situations, thus adding to the view of a 'difficult subject' and influencing self-concepts. The words and phrases used in mathematics can mean something very different in everyday situations (Gardener, 1993; Raiker, 2002). For example, a young child might well be waiting for the food to arrive because the teacher mentioned 'take-aways' or could be looking at the aesthetic differences of two sets of objects instead of the difference in quantities. The terminology the teacher and pupils use can subtly influence the relationship between the teacher and pupil by placing different emphasis on interpretation of events (Alexander, 2008; Barnes, 1992a). This mismatch intrigues me because of the possibility of adding more confusion to a pupil's figured world.

Dialogue is an integral part of the joint construction required to build a figured world. As mentioned in 'self-dialogue', words are not used in isolation but with an association of prior experiences and uses. Misunderstandings and confusion are more likely to be exposed if opportunities for open dialogic interactions are present, enabling pupils and teachers to uncover some of these misunderstandings (Mercer and Hodkin, 2008). Open dialogue offers teachers opportunities to glimpse pupils' figured worlds and an opening to address confusion. The present situation is compounded by what Alexander (2008) described as the expectation of the UK education system, in that the teacher imparts knowledge and prepares the child for adulthood, a political influence shown in figure 1. Once misunderstandings or confusion are illuminated, they can be addressed, and pupils can progress and develop their skills and knowledge, Demonstrating mathematical thinking. The figured world of the pupil can then become one of understanding and progress.

Understanding individual minds is problematic. Gergen (1995) suggests understanding can hinge on individual's subjectivity and be interpreted through actions and words. However, interpretation of forms of communication is heavily dependent on how different communities interact and what they value. To think of knowledge as just the content of the mind would risk ignoring the world and its socially constructed norms (Wertsch and Tulviste, 2005). Mercer and Wegerif (1999) point out that Vygotsky's research stops short of dealing with the classroom environment by focusing on the individual teacher and pupils within that environment. To view the teacher as the giver of knowledge would ignore the pupils' contribution. To consider the pupils' viewpoints would enhance the learning process (Sfard, 2008). Mercer and Wegerif (1999) suggest the role of talk in learning needs to be extended beyond Vygotsky's theories to include a broader range of dialogic discussion (Wertsch, 1985: 201). For my study, I am interested in both the teacher's and the pupils' views, including their classroom environment. Thus building on Vygotsky's research (1978).

Dialogic communication is the use of interaction by the pupil and teacher to stimulate and extend pupils' thinking, extending pupils opportunities to advance their learning and understanding. Dialogic communication is a two-way discussion that explores ideas, engaging and empowering the pupil. Barnes (1992b: 29) states that:

> *The more a learner controls his own language strategies, and the more he is enabled to think aloud, the more he can take responsibility for formulating explanatory hypotheses and evaluating them.*

Barnes (1992b) is suggesting talking and thinking are closely aligned. Wing (2016) describes dialogic interactions as 'patter' and stresses pupils ought to be encouraged to commentate on what they are doing, not to explain but to support the 'patter' which leads to understanding. This way, the teacher can also know what the pupil understands and help progress the patter. A dialogic approach would introduce understanding into the pupils' figured worlds. Barnes (1992b) has further shown that the construction of knowledge depends on the ability to make connections, explore, think aloud, make errors and develop mathematical identity. He considers that children can only do this if they engage in dialogue, enabling them to practise these skills. Alexander (2008) describes how small groups can enhance dialogue. Pupils are likely to feel comfortable in small groups exploring new ideas without the fear of ridicule. They can test out their hypotheses in preparation for the final audience. The use of dialogue can enable pupils to clarify and develop their understanding. Dialogue needs to go beyond practising the words and phrases to communicate understanding and be part of the learning experience (Alexander, 2008. Barnes, 1992a).

2.3.2.1 Questioning

A form of dialogue used extensively in schools is questioning. As Mason (2021) points out, questioning is a key feature of the types of dialogue used in the classroom. The use of questioning might suggest pupils' misunderstandings are uncovered in an environment with so many questions, thus offering them opportunities to share their answers. However, questions are used in various ways and for different purposes, each forming a different figured world for the pupils. Traditionally, and still today, summative assessment is carried out in the form of questioning to test pupils' knowledge. The Department for Education supports this questioning through statutory government testing. These types of questions are generally closed with specific answers to those questions. Closed questioning can create figured worlds of right and wrong, removing multiple possibilities (Holland *et al* 2001).

However, questioning by the teacher can create a different type of figured world. Open questions can invite pupils to share their thinking. Questioning can be used as a tool to scaffold pupils' understanding either by challenging them, for example; Do you think it would work if…? What would happen if you...? Alternatively, meta-questioning, for instance, if it is an odd number, can it be in the 2 times table? Or can this shape be a polygon, rectangle and square? (Wigley, 1992. Blair and Hindle, 2019). The challenging questions create a dialogue that can extend the pupil's learning opportunities introducing different possibilities.

In contrast, meta-questioning draws on the pupil's knowledge to justify and reason understanding. All forms of questioning will generate specific types of dialogue associated with different figured worlds (Holland *et al* 2001). Teachers and pupils can use a mix of varying question types at any one time, but when viewed as a cultural artefact, the predominance of one type of question over the others leads to a specific cultural environment.

2.3.3 Cultural Artefacts

Cultural artefacts can be both physical and psychological and allow pupils to organise their thoughts and emotions and direct their behaviour. There can be a range of cultural artefacts present in the mathematics classroom, for example, manipulative resources, worksheets, types of dialogue, questioning styles, and grouping pupils by their perceived ability. Wertsch (1998) advocates those cultural artefacts can be used as tools by which individuals can meditate and connect with each other and their environment. Pupils according to the specific figured world created see the cultural artefacts in a perceived way. Within the figured world model, Urrieta (2007) suggests that cultural artefacts bring past activities to the present and are assigned individual or group identities depending on the figured world of which they are part. Radford, Bardini, Sabena, Diallo and Simbagoye's (2005) research gives a theoretical account of the social and cultural natures of mathematical thinking, including cultural artefacts. They describe how pupils encounter the mathematics classroom's cultural artefacts and try to align their subject understanding with their cultural knowledge.

The worksheet is often used as a form of differentiation, and pupils are assigned specific worksheets based on some notion of mathematical ability, creating different possible scenarios and figured worlds. Boylan and Povey (2021) suggests that the grouping or segregation of pupils shapes the way mathematical capabilities are viewed by teachers and pupils. Fixed-ability thinking by many teachers in the United Kingdom has led to the cultural practice of pupils being placed in ability groups within the mathematics classroom. Each ability group would signal to pupils the figured world associated with the ability they are labelled. The culture of grouping has been suggested to negatively affect pupils by offering fixed abilities (Higgins, Katsipataki, Kokotsaki, Coleman, Major and Coe*,* 2013. Marks, 2016). To view pupils' abilities to be fixed in mathematics would indicate that the purpose of testing is to discover pupils' innate mathematical ability, void of cultural knowledge. If this were the case, pupils would not improve their scores through practise. This approach raises the question of what is being tested. Innate creativity and problem-solving skills are challenging to assess and test, leading the assessors to focus on just what can be tested, leaving out important information (Povey, 2017). An alternative to ability grouping is a whole class mixed ability approach. The features of such an approach have been discussed in a report by Boylan, *et al* (2019). This report evaluates the mathematics teacher exchange between China and England, which discusses features of the Mastery approach as mentioned in section 2.1. The report looks at the challenges of engaging the whole class by exploring high attainment instead of high ability, focusing on an acceptable pace ensuring a depth of knowledge instead of accelerated learning. Mixed ability groups and grouping by ability are based on cultural approaches and can create figured worlds of fixed or fluid abilities. I am interested in seeing if the assignment to a particular ability set could impact the pupils' figured world. Alternatively, how a mixed ability organisation could impact pupils figured worlds. I feel this area could significantly impact pupils' figured worlds, creating a specific view of mathematics.

Some of the more tangible cultural artefacts used in mathematics are manipulative resources, such as base ten blocks (also known as Dienes blocks) named after their creator Zoltan Pal Dienes. Pupils use the blocks to support the understanding of basic mathematical concepts. They can be considered a remedial intervention for pupils who need support understanding a specific mathematical concept. For example, they can use tokens assigned a particular value as place value counters when learning base ten. Alternatively, manipulative resources can be viewed as intellectual artefacts, used as an integral part of exploring theories, perhaps using counters to test an algorithm the pupils have developed. The way pupils are associated with a manipulative resource as a remedial or intellectual tool will influence how this cultural artefact fits in with the pupils' figured world. These different approaches to mathematics are discussed in detail in the 'Procedural ways of knowing' (section 2.6.4).

The teacher's epistemological position leads to the learning environment created through cultural artefacts in conjunction with pedagogical approaches. Therefore, the learning environment that the teacher has created and inhabits contributes to the figured world of the mathematical classroom (Boylan *et al*, 2019). My study is looking specifically at the pupil's figured world. I am interested in how the figured worlds created by the teacher can influence a pupil's view of mathematics.

2.4 Cooperative approaches to constructing concepts

The context of my study is the primary mathematics classroom and the figured worlds generated within it. A classroom is always a social environment, even if its teaching is traditional and allows minimal interaction. There will always be a degree of social interaction because teachers and pupils inhabit the same space simultaneously. My previous experience and the literature that I have reviewed in the last section lead me to believe that social interactions in oral and collaborative discourse allow pupils to create mathematical constructs more securely. Vygotsky (1896 – 1934) is the seminal author in this field, and therefore in this section, I will reflect on Vygotsky's social constructivist theories, which underpin my theoretical framework, which is based on Holland *et al's* (2001) figured worlds.

I will begin by exploring cooperative approaches to constructing concepts in the context of the learning environment of the primary mathematics classroom. A brief exploration of the social environment will follow, focusing on meaning-making through experience and reflection. I begin with Vygotsky's theory of Zone of Proximal Development (ZPD), which does not define what a pupil can or cannot do but instead focuses on the potential of a child to reach a particular developmental level with assistance from a more capable other and is described in this extract as:

> *…the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with a more capable peer.*
>
> (Vygotsky 1978: 86)

Vygotsky's (1978) ZPD describes the potential a pupil can have with the support of a teacher. Wertsch (1985) suggests that Vygotsky developed this theory in response to the need to find alternative ways of assessing children's academic development and not just testing ingrained learning patterns void of the potential. Vygotsky (1978:23) was interested in how a child could learn and develop through culturally mediated tools and language. The ZPD enables cultural and cognitive activities to occur together, creating a unique event negotiated between the teacher (or peer) and the pupil, creating specific figured worlds.

Forms of conceptual development identified by Vygotsky (1981, 1978) have been described in ways translated as 'scientific' and 'everyday' (Daniels, 1996: 11). Vygotsky linked ‘scientific concepts’ to schools, the classroom and learning through structures, placing concepts in a hierarchy, for example, prioritising the learning of 2-dimentional shapes before that of 3-dimentional shapes. Assimilation of such concepts is achieved through systematic teaching within schools. The alternative to ‘scientific concepts’ is what Vygotsky describes as ‘everyday concepts’. Pupils bring with them to the classroom their concepts of the world, relevant to their specific context, which can be unorganised and complex. The pupil’s figured world of the home can be in contrast to their figured world of the classroom as discussed in section 2.2.2. In this instance, the teacher supports pupils through activities to develop and organise these concepts. Where the scientific and everyday concepts come together, Vygotsky describes these as mature concepts (Newman and Holzman, 1993), perhaps were the pupil’s home and school figured worlds become more aligned. Sfard’s (1998) acquisition and participation approaches can be loosely compared with Vygotsky’s scientific and everyday conceptual development forms, which I discuss in the next section (2.5).

Vygotsky (1978) viewed constructing concepts through the lens of socially interacting with the environment. He considered that learning carried out actively and reactively could maximise learning opportunities. A learner follows their natural curiosity to investigate, collaborate and reflect on the experience to create their meaning of the world. In this theory, the role of the teacher is key, not as the leader or dictator of learning but as a facilitator and partner within the learning process, helping to create learning opportunities that challenge pupils’ thinking (Rogers, Kirschenbaum and Henderson, 1990). By mediating socially constructed norms, common meanings can be reached and agreed on.

Socially constructed norms is a constructionist viewpoint but one worth considering (Gergen, 1995). As discussed in section 2.3., the language of the mathematics classroom can be problematic, requiring more time and effort in constructing meaning through dialogic interaction between learners and more capable others. Thus, Vygotsky places the power of the community above that of the individual, impacting on the figured worlds formed within the classroom (Holland *et al*, 2001).

Understanding individual minds could be considered problematic, hinged on the subjectivity of individuals and interpreted through actions and words (Gergen, 1995). However, the interpretation of forms of communication is heavily dependent on how different communities interact. The view in the classroom from a pupil’s perspective is very different from that of the teacher (Sfard, 2008). To view the teacher as the giver of knowledge would ignore the pupil’s contribution to the learning process, echoing Vygotsky’s zone of proximal development (ZPD). Vygotsky’s view of collaborative learning is based on negotiation. The change of viewpoint by the participants can enhance or even change the original thought. The negotiation between the teacher and the pupil ought to create their communities of practice developing their figured worlds and expanding opportunities to develop conceptual understanding by reflecting on multiple viewpoints.

Holland *et al's* (2001) figured world embraces this collaborative approach by viewing the figured worlds of the pupils as a co-production of activities, moving towards a conceptual world beyond the pupil's immediate surroundings. The importance Vygotsky (1978:23) places on utilising dialogue and cultural artefacts to create a web of meaning supports the idea that new meanings can be attached or assigned to everyday objects. The role of the teacher is key in this process, not as a deliverer of knowledge as portrayed by Sfard's (1998) model of acquisition in section 2.5, or Belenky *et al's* (1986) received ecology of participation discussed in section 2.6. They create an environment where pupils' figured worlds can enact the role of involvement, creating an ecology of connected learning that resonates with what Sfard (1998) discusses in her participation metaphor. This discussion follows in the next section.

2.5 Metaphors for learning in contrasting figured worlds

In this section, I will explore two approaches to teaching mathematics: viewing knowledge as a fixed truth resulting in the learning of facts and negotiating understanding through enquiry, and investigation of multiple realities, thus creating different figured worlds as described by such theorists as Ernest, (1991), Sfard, (1998) and Perry, (1999). In an attempt to explore the variations in approaches, the polemic metaphors Sfard (1998) offers describes the teacher's approaches to learning and identify the way teachers can view knowledge.

Sfard's (1998) acquisition metaphor describes a traditional view of knowledge as a property, which can be physically 'collected' like personal possession, suggesting that the teachers 'giving' knowledge to pupils is a commodity transfer. A characteristic of this metaphor, which also chimes with Perry's (1999) dualism, is a linear approach to learning by following a set pathway to the end goal. The language of an acquisition-based classroom includes terminology that supports the assumption that there are clear 'right' and 'wrong' outcomes in mathematics. Pupils can miss critical understanding due to the desire to 'get to' the correct answer and focus on results. Conceptual naivety then becomes something to be corrected as opposed to explored. Boaler and Greeno's (2000) research found that secondary mathematics classrooms utilising this traditional pedagogical approach had unusually narrow and ritualistic social environments. A figured world of seeing learning as linear and punctuated with right and wrongs could introduce misconceptions where gaps start to appear and introduce confusion. Contrastingly, Wickstrom (2017) found when students were presented with opportunities to explore real life models; they were more inclined to view mathematics as a tool to be used to test out ideas. Pupils built on what they already knew, minimising the possibilities for gaps to appear.

Conceptual naivety can account for a pupils' lack of progress or ability to expand their knowledge in line with the classroom expectation (Swan, 2006). Gardener (1993) suggests conceptual naivety is a sign of incomplete understanding. Some pupils are more alert to knowing what step to take next instead of why they take a particular action, thus separating the procedure from the conceptual problem. Gergen (1995: 30) suggests knowledge is not a permanent state but 'to be knowledgeable is to occupy a given position at a given time within an ongoing relationship.' Gergen (1995) is implying that at any moment in time, knowledge is based on the experiences so far and will change with different experiences, creating figured worlds of possibilities by exploring multiple realities. So knowledge is not about facts but viability (von Glasersfeld, 1995), suggesting a more dialogic approach could help move from conceptual naivety through discussion and negotiation.

Sfard (1998) argues that participation, a more recent approach observed in classrooms, should be positioned at one end of the continuum and acquisition at the other. This metaphor objectifies knowing as an action or participation in activities, linking the participation metaphor to Dewey's (1933, 1964, and 1997) notion of a progressive child centred approach. In this approach, individuals within the classroom work in a democratic style, encouraging communication, reasoning and conjecture within a flexible curriculum. Reflective and reflexive practice are themes within Sfard's (1998) participation metaphor, accepting continual change and acknowledging that learning is about the experience rather than acquiring facts and concepts. The pupil's figured world would reflect active meaning-makers and they would view themselves as mathematicians within a discipline, growing within the subject, focusing on interpersonal relations. Pupils seek to understand other pupils' interpretations of the world (Sfard, 1998). Conceptual naivety is considered part of the learning process, viewing concepts as continual development and change, creating connections (Rowland, Turner, Thwaites and Huckstep 2009, Haylock, 1982). For example, exploring the properties of shapes develops with experience by observing similarities and differences and applying different criteria to different examples. This approach could be viewed as a much messier way of 'coming to know'.

The different epistemologies of teachers can be nurtured in the figured worlds of the mathematics classroom today. They can explore the established axioms of accepted truths of the subject based on purely cognitive processes or imbued within social factors (Nunes and Bryant, 1996). The Social Constructivists, Dewey (1997), Bruner (1972) and Vygoysky

(1978) challenged the figured worlds of a received or didactic classroom environment by introducing ‘meaning-making’. This introduction creates a different figured world of participation and active mathematical thinking behaviours (Ernest, 1991; Boaler and Greeno, 2000; Chinn, 2006). Meaning-making introduces a contrasting axiom to the view of a subject of right and wrong answers. It allows pupils to explore concepts by sharing their ethical and personal beliefs, basing their mathematics on logic and critical evaluation. So rather than acquiring
‘fluency’ ‘through practise and increasingly complex problems’ as advocated by the Department for Education (2013), pupils could learn concepts through understanding and critical evaluation within a context. This approach can increase ‘fluency’ through conceptual understanding.

In contrast to the Hirchian (1987:141-142) approach of a linear model that puts acquiring knowledge at the heart of the curriculum, Tall (2013) describes mathematical concepts as crystalline, in so far as a crystal is constructed through a maze of complex connections and bonds. The mathematics curriculum is the interpretation and understanding of how the concepts link together, building in micro and macro directions within a context that constitutes learning (Russell, 1938). For example, the statement ‘two and four’ can be interpreted at one level as six, organising the statement as an algorithm; a + b = c, thus following a linear process. Alternatively, it could be the relationship between ‘two and four’ introducing concepts of numeration and cardinality, or operational processes (Thompson, 2008). Many relationships and concepts can be explored beyond the simple representation of ‘two and four’. Ernest (1991) suggests that viewing mathematics, as a complex network of concepts would render the absolutist view of mathematics a myth because there are always new ways of exploring concepts in new directions. Pupils can view mathematics concepts in different ways through a crystalline lens (Tall, 2013).

Bruner (1972) also challenges the linear model, suggesting a spiral curriculum that revisits and builds on prior knowledge would ensure good coverage and depth of understanding. Using the example of ‘two and four’ above, the concepts concerning these two digits can be revisited repeatedly, building on and securing previous learning to develop new and stronger links. The concepts of conservation of number, classification, cardinality, ratio, and so on can all be explored and developed by using the digits ‘two and four’ as a starting point. This point is explored further under section 2.6.4; Procedural ways of knowing.

The use of axioms to describe the mathematics classroom is not new and can help to bring some order to an otherwise complex environment (Sfard, 1998). Sfard’s (1998) two metaphors may be helpful when observing the classroom and the figured world of the pupils. However, they risk introducing objectification of the phenomena, which are essentially ephemeral entities and concepts. Objectifying introduces the common language and everyday activities, risking introducing more complexity from the interpretations of the language used. The two metaphors (acquisition and participation) also present polemic interpretations of ways of coming to know in the classroom; however, Sfard (1998) emphasises that mathematics classrooms would have elements of both to a greater or less extent, offering a continuum when representing ways of knowing. I feel Sfard's (1998) metaphors for learning appear to be a blunt instrument for analysis by presenting an acquisition or participation model. However, I find the features of the two metaphors very helpful when tidying up the array of approaches evident in a classroom environment. They help quantify what is observed. Sfard's (1998) metaphors could help analyse how the teacher views knowledge and how this influences pupils' figured worlds.

2.6 Defining ways of coming to know.

In this section, I will explain how I can view the epistemological perspectives of the primary school teacher through the lens of Belenky *et al's* (1986) ways of knowing and ecologies of participation. By establishing how pupils can experience different pedagogical approaches within mathematics lessons and sharing this with the teacher, my study can provide greater insight into the approaches taken in the episodes observed. Attention will be on how teachers enact their practice, thus creating specific figured worlds in which pupils develop their identities with mathematics. For example, does the teacher model processes to be followed or present challenges to be explored? I will then explore other ways of knowing and coming to know that further illuminate this aspect of the mathematics classroom. Belenky *et al* (1986) offer a more nuanced look at the epistemological complexities present in the mathematics classroom by looking at contrasting ways of coming to know. The five ways of knowing as part of the ecologies of participation are: Withdrawal, Received, Subjective, Procedural and Connected ways of knowing, which I will briefly outline:

2.6.1 Withdrawal or isolation from the experience of coming to know.

Withdrawal describes the characteristics of not engaging, assuming learning is an external process governed and experienced by others, referred to by Povey and Burton (1999: 233) as 'mind blank' with no sense of knowing. Withdrawal, also referred to as silence, describes how some pupils present within the classroom, not a way of coming to know. It encapsulates total isolation from the experience of knowing. The figured worlds of these pupils would be unknown to the observer and could not be represented in the cases observed in this research. However, it does offer a category for the behaviours observed in some classrooms.

2.6.2 Received and connected ways of knowing

Received *and* Connected ways of knowing align, in part, with Sfard's (1998) notion of acquisition and participation, respectively. Received knowing relates to the traditional style of classroom teaching where the pupils receive their learning from a more knowledgeable other. The pupils enacting this particular figured world may view the teacher as the owner and giver of knowledge, placing them in a submissive position. Received knowing can remove the possibilities of exploration and discovery and becomes a journey of following the rules or processes to reach the required answer.

The epistemology associated with the received approach suggests that learning mathematics is a linear process, touched on in section 2.5. The received approach starts at the simplistic and moves in levels to the more complex, following Skemp's (1993) instrumental approach and Thorndike's (1998) behaviourist theories. Russell (1938) elaborates on the idea of a linear progression and describes the study of mathematics as having a telescopic effect going in two directions instead of one. One direction builds on the more complex models through the telescope as seen in school classrooms, moving from integers to fractions. However, in the other direction, there is also the analysis of and connectivity between concepts, which is often considered the more simplistic. By looking through the metaphorical microscope, it can be of equal value to the understanding of concepts but less often explored in the primary mathematics classroom (Clarke, 2009. Rowland, Turner, Twaites and Huckstep, 2009). To look through the microscope at the fundamentals of how numbers work, looking for patterns and connections could also provide challenge and learning opportunities, underpinning the more complex mathematical concepts.

The individual who is comfortable examining, questioning and developing systems constructs knowledge (Holland *et al*, 2001). Connected knowing, associated with an epistemology based on meaning-making, utilises a mix of reasoned intuition and expertise. Rational and emotional thoughts are woven together through integration and assimilation (Belenky *et al*, 1986). However, it does involve living with conflict because of the continual shifting of reality. This shifting reality is an example of constructivism, with knowledge constantly changing. The figured world of connected knowing opens up the possibilities for broader and more personal knowing through considering multiple realities.

Boaler and Greeno (2000), and later Urrieta (2007), found the figured world of the connected model, which features dialogic interaction, can be very different to that of the received model and will result in the teacher valuing other characteristics, impacting the self-concepts of the pupils in the classroom. The classroom, where knowledge is obtained from the teacher, puts the power and knowledge in the hands of the teacher and requires pupils to be compliant, have resilience, and be patient and obedient (Lee and Johnston-Wilder, 2017). Learning mathematics in this received way limits opportunities for pupils to have responsibility for their learning, denying them the opportunity to develop as thinking mathematicians within the mathematics classroom (Swan, 2006). An alternative to this figured world is where pupils can co-construct knowledge through social practice and sense-making through connected learning. In classrooms where discussion and meaning-making are valued, students can test out their ideas and theories, developing their identities concerning their peers. For a more meaningful dialogue, specifically for the mathematics classroom, Swan (2006) suggests creating a more organic communication through the mathematical challenge. Rogers, Kirschenbaum and Henderson (1990) advocated a pupil's self-concept will be impacted by the teacher engaging in discussion and putting a value on what is discussed. English (2016) describes how the dialogue used concerning mathematical challenges will impact pupil's mathematical thinking and their figured worlds.

Ball (1990) could see a conflict with an ideology that favoured children constructing their knowledge for meaning and making connections by building on pre-existing knowledge. He felt that the concept of motivation would be seen as a critical aspect of learning, or pupils would not make such constructions. Self-motivation is not necessary when learning is seen as received. In the received model, the pupil is the recipient of knowledge and has a figured world that supports a submissive role, with the pupil following the teacher's lead, responding to the teacher's instructions. Pupils react to their inner dialogue and their interactions with peers. In the connected model, the pupil's figured world reflects that of an active participant; thus, motivation is key to progress.

Csikszentmihalyi (1997:27) stated, 'innate intelligence cannot develop into mature intelligence… unless pupils [they] can control their attention.' Attention or concentration is difficult without motivation. If a child is interested and motivated, engagement can become effortless. Csikszentmihalyi described this as 'flow', wholly immersed in the experience. Sfard's (1998) acquisition and participation metaphors both require motivation, but the acquisition model relies on the teacher to ensure 'flow' for the pupils. The participation model depends on the pupil's motivation to create 'flow'. The place of motivation and engagement appears to me to be an important indicator of what might be happening concerning pupils' figured worlds.

2.6.3 Subjective ways of knowing.

Subjective knowing offers an alternative to the teacher's word when the pupil's figured world is based on intuition (Belenky *et al.*, 1986). This view challenges mathematics as an objectified discipline (Ernest, 1991). Subjective knowing requires the meaning-making person to exercise an initiative process that resides within the person in conjunction with outside facts, relying on what feels right and a gut reaction (Belenky, 1986). The internal authority fixes the idea of external authority, and meaning is dismissed, which leaves discussion and reasoning redundant and pointless. The emphasis moves from the 'right way' to 'my way' or 'your way' (ibid). However, this model of coming to know leaves the pupil's figured world exposed to a fear of being wrong and lost without understanding why. Any process or pragmatism is replaced with intuition (Povey, 1997).

2.6.4 Procedural ways of knowing

There are distinctive features to procedural knowing which can influence a pupil's figured world in different ways. In this section, I want to explore those features. To achieve this, I must distinguish between two divergent intrinsic objectives. One objective is to justify established knowledge, following a process (associated more with Belenky *et al's* (1986) received way of knowing). The alternative objective for procedural knowing is to inform and develop understanding to make connections and develop a conceptual understanding (Gray, 2008. Tall, 2013). This second view of procedural knowing is more akin to Belenky *et al's* (1986) connected ways of knowing. Gray (2008:88-90) described this procedural and conceptual understanding merger as a procept. This view is explored further in the following section.

Belenky *et al's* (1986) procedural ways of knowing resonate, to a point, with the translated description of Vygotsky's (1981, 1978) 'scientific' and 'everyday' classifications (Daniels, 2005: 107) as discussed in section 2.4. Vygotsky describes the merging of these concepts in the classroom as mature concepts (Daniels, 2005:11), suggesting a merging of understanding and justification. Rowland, Turner, Thwaites and Huckstep (2009) suggest learning procedures can prepare pupils to understand a concept. For example, learning the names of the numerals is a process as a precursor to gaining a conceptual understanding of the principles for counting (Gelman & Gallistel, 1978), thus using numbers as nouns and adjectives. However, this could run the risk of merging or blurring the epistemological stance of the teacher. Teachers can establish clarity by distinguishing between learning a process, which suggests justifying established knowledge, and following a process to organise and develop understanding, described earlier by Gray (2008) as a procept.

The discipline of mathematics has developed compressed and blended knowledge over many centuries to what we have in our classrooms today (Tall, 2013), lending itself to a procedural way of knowing. The expectation now is that children pick up these well-formed concepts and use them procedurally. The distinction between a process and a concept has been lost over time. Example A and B in figure 2, illustrate the respective differences:

Example A: 30 ÷ 5 process

Example B: $\frac{30}{5}$ concept

Figure 2: Examples of a calculation representing a process and a concept.

Example A, in figure 2, represents the process of dividing 30 by 5 thus producing the answer of 6. This process involves one straightforward process or algorithm of dividing 30 by 5 and resulting in one solution, taking a linear view. *Example B* represents the relationship between 30 and 5, opening up many more possibilities and a more comprehensive understanding of the relationship between the two numerals. For example, 1/6th of 30 is 5,
30 divided by 5 is 6, 30 shared 5 ways is the same as 60 divided by 10 or 6 divided by 1 etc. This representation illustrates the relationship between the numbers and mathematical concepts, in this case, fractions, division, and ratio (Rowlands, Turner, Thwaites & Huckstep, 2009).

The cross over from skills to applying these two approaches can create problems if skills are learnt without understanding. For example, learning to count is considered a traditional rote activity; however, a child also needs to have 'number sense' and be able to manipulate numbers to understand and become numerate (Anghileri, 2000). An understanding of ratio could open up more opportunities for learning than just the process of division. The merging of the two, or what we see through merging examples A and B, is what Gray (2008) describes as a procept. The link between a concept and a process becomes less clear-cut as pupils understand concepts through learning a procedure (Rowland, Turner, Thwaites & Huckstep, 2009). The figured world of conceptual or procedural understanding would be very different to a figured world that merges the two.

2.7 Synthesizing a theoretical framework from the literature review

A figured world model, drawing from Vygotsky's (1965) social constructivist theory, has informed the theoretical grounding for this research. It recognizes the classroom environment as a socially created and culturally imbued space. Teachers and pupils interact to make sense of mathematical ideas instead of constructivism, which focuses purely on learning as a cognitive process (Reid, 1996).

To scrutinize the complexities of the different teaching approaches of the mathematics classroom, I have adopted the works of Belenky *et al* (1986) and applied theories of ways of coming to know the mathematics classroom. I have also drawn from Sfard's (1998) metaphorical interpretations of contrasting ways of 'seeing' learning, offering further insight into the range of teaching approaches and how this can impact pupils' self-image and identity within the mathematics classroom. For example, from my own experience as a pupil, when the teacher took a didactic approach, I withdrew from the process. When a teacher introduced an enquiry approach, I started to engage with the learning process and felt like a mathematician. The different theoretical notions have been critiqued in chapter 2 and collated to develop a series of related conceptual framings. This has supported me with considering the learning in five different mathematics classrooms from various perspectives. Using a figured world framing to analyse learning interaction in the classroom, perhaps a better understanding of why and how pupils perform and are motivated in the mathematics classroom can bring multiple realities for both the teacher and pupils to the fore (RQ2).

2.8 In conclusion

I wish to explore how pupils experience mathematics learning within the different figured worlds they occupy, an area not well researched. My exploration of the literature shows complex interaction within the learning environment, as illustrated in figure 1. Therefore, my thesis will attempt to fill a gap in the literature. I will build on the work of Boaler and Greeno (2000) and Wickstrom (2017) by looking at primary classrooms, introducing the pupils' views, and using stimulated recall. Like Boaler and Greeno (2000) and Wickstrom (2017) my concerns are pupils' mathematical identities and how other figured worlds impact them. In particular, the teachers' pedagogical approaches and what aspects of mathematics they value. I adopted a social constructivist approach to a complex subject allowing for a rich, personalised exploration of specific multiple viewpoints. I consider pupils' independence and opportunities to enact mathematical thinking behaviours important for pupils, as this can encourage pupils to create a particular figured world.

In the next chapter, I will present my methodology, where I set out how I went about exploring this multi-faceted environment in a meaningful way, going beyond the initial rhetoric and producing a blended analytical framework on the themes of dialogue, cultural artefacts and identity. All three themes are present in Sfard (1998), Belenky *et al* (1986) and Holland's *et al's* (2001) theories. However, Sfard's (1998) metaphors for learning has a greater emphasis on the purpose of dialogue, Belenky *et al* (1986) scrutinise ways of coming to know and the impact on individuals' identities, and Holland *et al*. (2001) considers how cultural artefacts are utilised to form specific figured worlds. I hoped to achieve a richer dialogue to provide data for deeper analyses by bringing these three theories and critical features together.

Chapter 3

Methodology

3.0 Introduction

To take a fresh look at the interactions between the teacher and pupils in a mathematics classroom, I felt their figured worlds was an important area to study, as set out in section 2.3 of the literature review. Through wanting to extend my understanding of the challenges associated with mathematics learning for pupils in the classroom, I was open to new ideas and approaches. The divergent mix of historical, social and political hegemonies impacting teachers' epistemological views, my own included, adds to the field's complexity. Reflecting on the different approaches associated with teaching mathematics, pupils may harbour underlying barriers or naive conceptual understanding, which may not be evident due to the pedagogical approaches taken (Boaler 2016, Dweck 2013). Using a blended analytical framework, drawing together several individual frameworks, I searched for a more nuanced lens to view the primary mathematics classroom.

This chapter will bring together the philosophical discussion and theoretical frameworks that underpin the methodology used in this research. A qualitative research paradigm and interpretivist approach were taken to illuminate the nature of learning in the mathematics classroom. To allow a fine-grained investigation of the figured world themes of identity, dialogue, and cultural artefacts, aspects of which emerged from my literature critique in chapter 2, I have drawn from the theories of Sfard (1998), Belenky *et al.* (1986) and Holland *et al.* (2001). Section 3.8 provides detail of the blended analytical framework created. This blended analytical framework directed the focus of my research to contribute something unique to the field of mathematics education.

The research context acknowledges multiple realities (Schutz, 1962) by recognising the individual figured worlds of the participants in this study. Initially, I planned to present the information sequentially. However, due to the reflective nature of this research and the use of Mason's (2002) discipline of noticing approach, a far more cyclical method seemed apt. Mason (2002) focuses first on the teacher recounting a lesson void of assumptions and reasoning, then revisiting the same lesson but moving from recounting to reflecting and finally to the reflexive discussion. Revisiting and reviewing aspects of the research in this way revealed more insight. Table 3 in section 3.5.3 outlines the sequence of events taken. Adopting the blended framework allowed me to piece together the varying features of the differing figured world of five separate classrooms. By taking a range of perspectives, I built up layers for analysis (Denzin and Lincoln, 2000). The use of individual cases facilitated this.

While engaging teachers in reflection on a critical incident through stimulated recall, my focus has been on what teacher's appeared to privilege in pupils' ways of coming to know. Drawing on writers such as Selmo and Orsenigo (2014), Mason (2002), Grundy (1987), and Boud, Kaogh and Walker (1985) took a reflective and reflexive approach taken throughout this research project. Using reflective and reflexive practice to scrutinise the figured worlds within the mathematics classroom provided a range of information regarding what was happening (RQ1). To aid the collective reflection, I adopted visual methods using video for stimulated recall, drawing on the work of Banks (2001) and Rose (2016). The later sections of this chapter will describe and outline the methods used for the data collection, including sections on; participants, ethical considerations and data analysis.

3.0.1 The researcher

I came to this study with a background in primary teaching, which has precipitated my interest in the complex nature of the figured worlds of the mathematics classroom. It has been crucial to acknowledge my potential bias to ensure my research has validity and position myself within said research (Cohen, Manion, Morrison, 2018). Gouldner (1962) states that research is not a value-free enterprise. Ripley (2004) suggests that the researcher's background experience and personality can impact the teachers’ and pupils' actions during the research process. Estola (2003) and Goodley (2011) go further, recognising that the researchers own personal ontological approach is linked to their identity, impacting the interactions between the teacher and researcher. The teacher’s identity and mine was bound up with the narratives created. Therefore, a transparent approach was needed. As a researcher, I needed to be sympathetic to the teacher's ontological bias whilst being aware of my own.

My personal experiences as a teacher within the classroom suggested that the teacher and pupils could provide greater insight into the nature of the range of influences within their figured worlds. Observing lessons from video recordings and discussing their views of events made it clear that the teacher and pupils had different perspectives of the lesson. For example, the teacher could use recording strategies as visual support to develop the conceptual understanding of, say, division. On the other hand, the pupils might be focusing on the patterns within the notations representing grouping or sharing, both valid activities but lacking connection. Consequently, it appeared to me that the large data sets from the

Department for Education (2021b), routinely generated as part of accountability measures in England, hide important aspects of classroom life (Wragg, 1994).

The use of quantitative research methods can run the risk of losing valuable insight into the nuanced focus of individual pupils (Cohen, Manson and Morrison, 2018). According to Hogan (2010) and Jonas (2011), the mathematics classroom can risk having a narrow focus based on the collective. I wanted to focus on the individual for this study, suggesting that a qualitative, interpretivist approach was more appropriate. The methodology I developed included Mason's (2002) discipline of noticing to explore what individuals experienced in the classroom and how those actions and activities impacted its learners. It provided an opportunity to hear the individual stories of the teachers and pupils.

In 2009, I underwent training on the 'Numbers Count' intervention course from Edge Hill University as part of the Every Child Counts programme (2008). This experience provided me with a greater understanding of the importance of observation and non-intervention to establish the pupils' understanding before taking action. Thompson (2010) explains how conflict can occur in a busy classroom where the urge to be active by the teacher can override the invaluable act of listening to and watching the interaction between the pupils. However, Lewis (2008) points out it is from just one perspective in a room of many when watching and listening. There can be no definitive version of these observations because each noticing and retelling can be from a different perspective and a different time, including my view as the researcher (Van Manen, 2014). Therefore, my role as a Numbers Count Teacher was that of a curator, bringing together the different layers of interpretation to create something new based on individual figured worlds. This experience changed my ontology and, in turn, my epistemological stance. My focus shifted away from solely my perspective and interpretation of pupils' interactions to observing the socio-cultural environment in which these experiences had taken place. The socio-cultural environment provided different perspectives, valuing the multiple interpretations from various figured worlds. Taking the time to stop, look, and listen through digital video recording and discussion also allowed the participating teachers to consider their pupils' interpretations of the teaching they offered (Banks & Zeitlyn, 2015. Coles, 2013).

My mathematics classroom experiences have influenced the methodological approach for this research. These include my changing roles within that environment as a pupil, a teacher, teacher educator, parent, and researcher. The accumulation of all these factors has led me to want to work with teachers in the primary mathematics classroom. I have attempted to look explicitly at the figured worlds of the teachers and pupils in this research, mindful of separating them from my own. I remain acutely aware that these are through one lens and reflect the figured world I inhabit. My study fits with an interpretivist approach within a social constructivist framework (Wertsch and Tulviste, 2005).

3.1 Aims and objectives

This study aimed to produce a unique contribution to knowledge by viewing multiple perspectives of five individual episodes from five different primary school mathematics lessons. To achieve the aim both the teacher and pupils' viewpoints were collected, analysed, and discussed. The research questions below set out the overall intentions for this study. By taking a glimpse into the social world of the mathematics classroom to explore the figured worlds of the teacher and pupils, I aimed to build on the literature discussed in chapter 2. A more fine-grained approach was taken for my study, building on Mason's (2002) discipline of noticing.

3.1.1 Research questions

The following research questions have been created based on what I know from the literature to lead my exploration of the classroom using a figured world framework.

Overarching question:

How can reviewing the nature of learning in contrasting primary mathematics classrooms through the lens of a figured world illuminate significant and influential differences in pupils' experiences?

Research Question 1

How can viewing the mathematics classroom through a figured world lens provide access to pupils' experiences and bring a fresh approach to exploring the nature of learning?

Research Question 2

How can drawing together the multiple perspectives and experiences of the primary mathematics classroom through five illustrative cases bring to the fore the multiple realities at play?

Research Question 3

How can the pedagogical approaches of the teacher within the mathematics classroom transform the pupils' learning experiences?

3.2 Conceptual framework

The philosophical positioning behind this research is set out in the following section, illuminating the conceptual framework that formed the basis for this research. The research is characterised by my biography, ontology and epistemology and brings together the influences that have determined the underpinning paradigm. The use of collective reflection between the teacher and researcher, and pupils and researcher, aided by stimulated recall through reviewing videoed lesson material underpin the research methods employed.

3.2.1 Interpretivist research paradigm

The research aimed to seek insight into a complex situation through my research questions rather than to answer a specific closed question. Therefore an interpretivist paradigm was appropriate, taking the view that research focused on people is a perception of reality and consequently independent of a single reality (Pring, 2005). Cohen (2007) suggests an interpretivist stance involves an understanding of knowledge as "personal, subjective and unique" (Cohen and Morrison, 2007:7). The interpretation of an event is built up from layers of multiple views across time, including between those individuals creating an ever-moving interpretation: 'There is not and cannot be an agreement in perception, interpretation and language…interpretation changes with interaction' (Bassey, 1999:42). People see the world from their stance, but greater insight is generated by putting multiple views together. Merleau-Ponty (1981) infers that it is through the experiences that we 'know', not through our intellect, suggesting the world is how we live it, not what we think it is. Therefore, to understand both the teacher and the pupils; I needed to hear what those experiences were.

The initial motivation for this topic stemmed from a desire to understand why some children struggle with mathematics and reject it while others enjoy and succeed in the subject. This, as supported by Cohen (2007: 21) promotes interpretivist paradigm values, where research is often "characterised by a concern for the individual". This ideology could not have been reflected through what Lichtman (2006) describes as 'traditional research paradigms' of positivism, that often presume "there is an objective reality that researchers should try to uncover as they conduct their research" (Lichtman, 2006:4). Having said this, Cohen (2007:21) argu,es "just as positivistic theories can be criticised for their macrosociological persuasion, interpretive and qualitative theories can be criticised for their narrowly micro-sociological perspectives". Bearing this in mind, this approach had a variety of limitations as any findings were subjective, see section 3.6.5. However, according to Bassey (1999), the benefit of an interpretivist stance is not its ability to provide a wealth of generalisable information but to communicate a depth of information.

In exploring the social world of the mathematics classroom through a blended theoretical framework based on figures worlds and reflecting Vygotsky's social constructivism, the aim was to seek understanding, as opposed to testing out theories (Daniels, 2005). I required multiple viewpoints to explore the individual figured worlds of a primary mathematics classroom. The approach taken is designed to provide an opportunity to look at the unique figured world of each actor within each episode (Boaler and Greeno, 2000). Using discussion between the teacher and researcher and between the researcher and pupils allowed the generation of reflective discourse and collective reflection, creating a community of enquiry (Bignold and Su, 2013. Cobb, Boufi, McClain, & Whitenack, 1997). This was further enhanced by providing the teacher with the opportunity to assume the role of observer. Videoing the action created an opportunity to re-visit and review the episode multiple times (Reid, 1996). The multiple perspectives and a common goal of exploring the possibilities behind the actions in the episode constituted the seeking out of new knowledge. However, suppose a common focus or perspective is not established. In that case, the researcher could take one action within the sequence whilst the teacher or pupils' attention is captured by another altogether. Every attempt was made to follow the lead of the teacher and pupil as they viewed proceedings. Although attention can be directed on occasion to what the researcher notices. Every attempt to remain self-reflective and to self-critique is essential in keeping the research honest (Walker, 2017). If the teacher or pupils did not 'see' anything in the actions that the researcher points out, the issue was not forced.

Using social constructivist theories moves away from a simple cause and effect scenario to the cognition of the mind and body within the social context of the environment (Thompson, 2010; Nunes and Bryant, 1996), supporting Bruner's (1972) theory of seeing 'learning as doing' which underpins this framework. In this instance, I have looked at how the teacher and pupils act and react within the mathematics learning environment. This included an exploration of the beliefs and biases that impact their figured worlds. Social constructivism introduces flexibility and creativity within this enquiry and supports a theory of complex structures, creating and studying a network of continual change but allowing for interactions, creating and modifying properties (Reid, 1996; Wertsch and Tulviste, 2005). I have used this approach to find new ways of describing the world, replacing the old (Scott, 2005). All contributors used an extended version of Mason's (2002) discipline of noticing as a tool, which included multiple lenses and stimulated recall. This encouraged them to use reasoning to help them develop their thinking, from participating in an action to being aware of that action. It was what the teacher attended to that drove this research, and thus, it required the active involvement of the teacher.

3.2.2 Ontology

My ontological assumptions about the nature of reality reflect those of relativism, which assumes there is no external reality, simply interpretations of subjective experiences expressed in language. Such reality is temporary, creating unique individual realities (Denzin and Lincoln, 2000). The alternative axiom is realism, which assumes there is an external reality we can reach, interestingly an ontology reflected in a number of the cases observed in this research. The subject of mathematics often viewed as an objective discipline, suggesting realism, as discussed in section 2.2 (Morgan 2016).

3.2.3 A Social constructivist epistemology

My ontological assumptions guide my epistemological beliefs about how we gain knowledge. The view that knowledge is individually constructed through social and cultural practice again reflects a social constructivist model based on the theories of Vygotsky (Wertsch and Tulviste, 2005; Vygotsky, 1981; Lave and Wenger, 2005; Vygotsky, 1965; Vygotsky, 1978). Gergen (1995) builds on these views of knowledge through the development of the use of language and von Glasersfeld's (1995) evaluations of Vygotsky's Zone of Paraxial
Development (ZPD). Vygotsky's ZPD emphasises the importance of the relationship between the pupil and the more knowledgeable other in developing knowledge and understanding.

3.3 Research context and design

This study was designed for the researcher to work collaboratively through reflective discourse with the participants in a primary mathematics classroom. This was achieved by turning what was done in action during an episode into an object of reflection. The use of video has been used to aid recall, offering the teacher and pupils' time and space to view the lesson from an observer's perspective and assist the collaborative reflection. The following section details the reasoning behind the decisions made, starting with reflective discourse. Later I will consider the unique ethical considerations needed due to the research taking place in a school classroom and some of the participants being of school age, plus the use of video recording during a lesson.

3.3.1 Reflective discourse

The use of reflection and Mason's discipline of noticing (2002) is an element of the research design that can empower the people constructing the narrative (Bignold and Su, 2013). Habermas (1984) describe the activity of creating a joint account as a commutative action in pursuit of trying to persuade or influence the other when the talk is directed towards a mutual understanding. A relativist view has been taken in this research, using actions that acknowledge that knowledge 'truths' are not static but can be co-constructed between people and are therefore ever-changing (Byrne 2017).

To produce the rich data needed to understand the figured world in each case fully, I decided to use collective reflection, also described by Cobb *et al* (1997) as reflective discourse. Cobb *et al* (1997) introduced a collaborative, reflective process to the mathematics classroom by moving beyond an isolated individual reflection, a model associated with Piaget (1972). Disciplined noticing is about taking a new perspective on a subject by opening up the complexities of the moment and approaching the episode with an open mind. This process involves the participants being present in the moment, sensitive to detail and crucially prepared to challenge habitual behaviour (Mason, 2002). The educational and research environments share the same opportunities and need for disciplined noticing (Mason 2002) thus, extending the discussion of reflective and reflexive practice.

To experience an activity in the classroom or to observe data in isolation does not explicitly or implicit equal a learning process. Without disciplined noticing and reflexive practice, habitual mechanical behaviour can predominate. In this study, I have extended the action of disciplined noticing to include the pupils' perspective in the discussion. A common scenario is when a pupil is stuck and asks the teacher for help, and the teacher can assume a range of reasons for the disruption. Disciplined noticing, including the pupils' views, coupled with reflexive practice, can help look afresh at the pupil who is 'stuck', opening up new possibilities, thus enabling the pupil to move on in their learning and build on their knowledge.

A clear distinction needs to be made between describing an incident and making a justification or judgement of that incident to notice an incident or act objectively. The complexity of the classroom and the number of episodes available to notice can result in the teacher being very selective. It is easy to self-justify and jump to conclusions based on tacit knowledge when accounting for episodes. For example, when the teachers in this study described pupils actions as 'stuck', 'muddled', 'confused' or 'in a pickle', perhaps resonating habitual behaviour built up over time. Claxton (2000) describes judgements based on tacit knowledge as lacking creativity and rumination. Mason (2002) describes the spectrum of noticing within the classroom as passing through different levels of consciousness. First events go unnoticed, to noticing but not marking them, then marking an event to return to it later, and finally recording an event for analysis and further exploration. This process results in subsequent reflexive practice.

Using my extended version of Mason's (2002) discipline of noticing model, a structured approach to reviewing and analysing episodes of classroom practice was applied. The reflective discourse between myself (the researcher) and the teacher, plus the pupils' views, provided an opportunity for the teacher to reflect on their practice when observing an episode of their mathematics lesson. We then built on the reflective process by re-examining those first assumptions and separating the action and intentions of participants, introducing an emotional element and providing deeper insight, thus extending to a reflexive approach (Grundy, 1987).

In my role as researcher, I was present to mediate the process of collective reflection. According to Ellis (2008), collective reflection can provide insights into unique and specific details of daily life, in this case, the mathematics classroom. It was a process where the teacher and I were actively making accounts of what was happening and tidying up messy ambiguous interactions. It seemed to me that if the teacher and I worked together using stimulated recall to support reflective discourse, the data would potentially allow me to access the teacher's thinking, values, and beliefs as they acted within the case context. With this in mind, I decided to video the teaching acts to provide an opportunity to return to the lesson and offer another view of the events (Banks, Zeitlyn, 2015). I edited out video clips of episodes from the taught mathematics lessons where I could see that a perturbation had occurred. The clips were used for stimulated recall in the follow-up session to the lesson (Goodley, 2011). Stimulated recall sessions allowed reflection-in-action (Mason, 2002) and collective reflection (Ellis, 2008) between the teacher, augmented by the pupil's account of the event and myself. Section 3.5.1 goes into more detail about how the selection process worked. These additional perspectives allowed for building more layers of interpretation. The reflective discourse between the teacher and myself was designed to involve a fine-grained analysis not solely limited to the act of reflecting on the teacher's action(s) but also reflected on the learning taking place (Freire, 1972). This was achieved by discussing multiple views and providing insight into new possibilities for learning (Lewis, 2008). The methodological approach of placing the teacher as an observer provided an even greater opportunity for further insight for me as the researcher.

According to Heikkinen, Huttunen and Syrjälä (2007), transparency and framing are paramount to ensure that a shared narrative of a collective reflection is valid and credible. The contributors' role needs to be evident due to the possibilities of many interpretations of the same event. The relationship between the contributors and the environment, plus the researcher's background and specific interests, all impact the narrative produced. Clandinin and Connelly (2000) offer three dimensions to narratives: the first dimension is the opportunity to move in time, reviewing and reflecting on the past while discussing the present and conjecturing into the future. The use of video supports this dimension of the study. The second dimension looks outwardly at the environment, describing what can be seen and contributing to the narrative. The role of cultural artefacts and dialogue contribute to this dimension in this study. Finally, the emotional or inward-looking dimension can be the hardest to explore because of the tensions between the personal and emotional, reflecting on the experiences of experience. The process of telling the story of an episode can help support understanding but also creates a paradox between what is seen and what the participant thinks they know. I was particularly interested in how the contributors reacted when they engaged with the stimulated recall of the episode and the video footage of the pupil's reactions. There was no initial hypothesis because the narrative can continually change (Clandinin and Connelly, 2000). A potential disadvantage is raised by Gergen (1995), who suggests there may be a messy or chaotic feel to the data, including; no hierarchy of information or interpretation and a reliance on the power of language and how that can influence the interactions. I feel this 'messy data' reflects 'messy humans', and the research pivoted on the need for human sense-making.

There are two further drawbacks to constructing a narrative. The first and most challenging to overcome is the narrator using the narrative to signal to the audience what they want them to see, reflecting a need to create a desired perception, which may be what the participant knows to be their actual truth (Clandinin and Connelly, 2000). However, the data has provided insight into how both the teachers and pupils want to be perceived. The data also indicates what they see as essential or good practice (RQ2). Thus, this potential negative is turned into another layer of interpretation. The second barrier or consideration is poor or inaccurate memories. The use of the video helped to provide stimulated recall from the actual episode (Goodley *et al.*, 2004), so again this barrier did not undermine the trustworthiness of the research outcomes.

The approach taken for this study aimed to explore the figured worlds of the mathematics classroom (RQ1). To allow the multiple viewpoints to be heard, time had to be dedicated to an in-depth exploration of a classroom episode. Focusing on one sequence of interests that occurred during a mathematics lesson could explore the full story of that episode. A shorter episode can peel back the multiple layers in a way that focusing on a longer length of time cannot achieve (Clandinin and Connelly, 2000). Having a collective reflection on each of the chosen episodes provided a context and space in which the teacher had the opportunity to express their figured world (Polkinghorne, 1995). In doing so, this expression was what

Goodley (2011) describes as the 'private troubles' of the individual which could inform 'public issues' or provide new knowledge based on a new insight, and is thus valid as the research is doing what it intends to do (RQ2).

3.3.2 Stimulated recall through the use of video

According to Coles (2016) and Borko et al (2008), video can open up communication as a tool for stimulated recall. It can also provide an opportunity to look again in more depth at a given episode and perhaps see something new. Viewing the video multiple times can increase the chances to notice all the actions within the episode and give time and space for the contributor to observe, cogitate, discuss, and observe again, helping to provide a rich and authentic narrative. The video can also offer another perspective on the lesson and new possibilities for further discussion and insight.

The learning to notice framework used in this research builds on Mason’s (2002) discipline of noticing but also draws from the work of; van Es and Sherin (2002), Star and Strickland (2008), and Alsawaie and Alghazo (2010). Teachers can evaluate ‘why’ actions are being taken based on experience when working in a busy classroom. Often this results in the loss of noticing the detail of what ‘is’ happening in actual time. Mason (2002) distinguished between ‘accounts of’ and ‘accounts for’ what is happening. ‘Accounts of’ describe what you can see. ‘Accounts for’ evaluates what is happening, providing reasons for actions. I used the video for stimulated recall to provide an extra lens and an opportunity to revisit the episode for scrutiny. I asked the teacher to develop their role as the practitioner into an observer, standing outside of the events. Taking the time to observe the footage multiple times, I considered that this would increase opportunities to notice what was happening. The discussion was steered away from evaluative talk based on experience and established patterns of understanding by focusing on what was seen in the video. To notice was an attempt to revisit with ‘fresh eyes’. When viewing the video, it was an opportunity to observe and reason about what was happening, not judging or coming to any conclusions. It is impossible to know what a pupil may or may not be thinking. Coles (2016) advocates that the use of the video is to revisit an episode, allowing opportunities to observe again, no more, no less, providing time to learn to observe and not interpret. In the same way as Coles (2016) warns that you cannot observe ‘not paying attention’. A pupil may appear distracted or not ‘paying attention’, but that does not mean they are not.

The context and design of this study aimed to engage multiple viewpoints of an episode within the mathematics classroom, providing time and space for the participants to consider the event in greater depth, thus building on Mason’s discipline of noticing (2002). This approach could have raised tensions due to the context of the study being in five classrooms in five different schools, an environment controlled by rules and expectations of behaviour (Dockett, Perry, Kearney, 2013). More significant consideration needs to be given to the methods of data collection and the relationships within the classroom environment.

3.4 Participants

To help understand the figured world of mathematics classrooms, I required five teachers to participate in the research, allowing me to observe their teaching and video the selected episodes from a lesson within that teaching. By taking part in the collaborative reflection, I had hoped to capture the motivations, beliefs, and values that underpinned these teachers' actions in the classroom. However, to understand the episode, I needed the pupils' perspectives. By inviting the pupils to join me in a collaborative reflection, aided by the stimulated recall of the video, a better understanding of how the figured worlds of the teacher and pupils intertwine, collaborate or collide could be gained.

3.4.1 Recruitment of schools

As qualitative research methods have been used, employing a case-by-case approach it only required the recruitment of a small number of participants (Cohen, Manson and Morrison, 2018). Precise requirements were placed on the recruitment of schools. They had to be judged as 'Good' to 'Outstanding' by Ofsted, so there was no added pressure on the class teacher or risk of underlying problems that the research process may aggravate. This approach was in line with ethical approval. Convenience or opportunist sampling was used to recruit schools. If the contact did not want to participate in the study, I moved on to another contact. Once the headteacher or teacher showed interest, the school was invited to participate in the study (Bryman, 2012). Recruitment can be a complex process, as negotiating with the headteacher first could pressure the teacher to take part (Valentine 1999). The use of convenience sampling suggests that these five cases are not part of any specific group beyond this group; therefore, little if any generalisation can take place (Cohen, Manion and Morrison, 2018). The five schools involved in this project were selected to represent a range of size, location and socio-economic background to give a broader range of possible interpretations. The schools were selected from the south midlands region due to access requirements by the researcher.

Table 1 shows more anonymised details of the individual schools. The teachers who took part were all year two teachers and had been teaching for more than four years. They were key stage 1, year 2 classes with pupils aged 5 -7 years old. I felt this year group would have sufficient school experience but would still be at the start of their formal school education and just starting to take part in formal Key Stage Assessments (DfE, 2021a). I wanted the regular class teacher to proceed with the mathematics lesson as naturally as possible during the lesson to be studied (Cohen, Manion, Morrison, 2018). Furthermore, as a matter of course, the ethos of the mathematics classroom needed to support pupil interaction and have some elements of collaborative working; the interaction between students, therefore, provided richer material to be used as a stimulus for discussion.

Case name	**School, general catchment area and approximate number of pupils on roll**	**Teacher**	**Pupils filmed and Interviewed**
Case A Symmetry	Weston School Edge of a large market town 115	Ms Travis	**Asha, Tom**
Case B Word Problems	Langfield School Village school 130	Mr Smith	**Jaden, Otto and Andrew**
Case C Place Value	Sharp School Village school 120	Miss White	**Tim, Aaron,**
Case D Geometry	Northolt School Conurbation in the midlands 250	Mrs Armid	**Destiny and Maci**,
Case E Fractions	Felton School Small village school 30	Head teacher/Class teacher Ms Greenway	**Mai** *and Jill* (Only Mai was interviewed due to focus of episode)

Table 1: The case names and brief description of the participating schools, including the teachers and pupils in this research. All names are pseudonyms.

3.4.2 Five cases from five schools

A collection of five cases was studied to facilitate the greater depth and detailed analysis required of the events observed within the context of the mathematics lesson. A collection of illustrative cases were used, which is defined by Newby (2010), Stake (1995) and Denzin and Lincoln (2000) as a group of individual cases brought together, providing the opportunity for greater understanding. Each case focuses on what Gluckman (1961) described as a 'social situation', which I interpreted as a small collection of connected events and took place over a short time within a mathematics lesson. After an initial analysis of the individual cases, they were analysed as a 'collective' to understand commonalities and differences and to allow me to understand more about primary mathematics education. This process is how I could get to the crux of what I wanted to know. Nothing else would allow me the same level of access to these teachers' motivations, beliefs, and values.

The cases studied followed the three stages described by Newby (2010), which support Mason's (2002) discipline of noticing. First, I explored each case to identify what appeared to be happening in the classrooms by observing events as they happened, including viewing video footage for deeper analysis. The next stage was to describe the events as seen by each individual with no evaluation or comment on the actions observed, and this was to inform the reflective discourse with the teachers. The final stage was to explore possible explanations for the sequence of events that occurred. A collaborative reflection between the teacher and me throughout the three stages supported the reflexive process. I constructed a social situational case study to allow for meaning to be attached to the interactions between the participants. Approaching the cases in this heuristic style helped to reflect theoretical principles and to stimulate the imagination of both the teacher and myself to construct new insight into the figured worlds of the mathematics classroom (Clyde Mitchell, 2006).

Each case was unique and took place in different schools from different catchment areas. The lessons had different lesson objectives and pedagogical approaches, which would suggest limitations for the theoretical analysis. However, if typicality across cases were searched for, it would indicate a case had relevant characteristics with other cases. Van Velsen (1967) in Clyde Mitchell (2006) addresses this statement by arguing that the social processes involved are being reflected on, not the cultural structures, in this case, the figured worlds of the individuals within the classroom. Therefore, it is not the representation of the event that brings validity but the reflective process on the data (Clyde Mitchell, 2006).

Goode and Hatt (1952) suggest one drawback to the case study is defining boundaries, specifically in this case, the role of the participants, and how much the environment has to play? Stake (2005) suggests that boundaries between the roles can be defined by piecing together and looking for patterns linking concepts. The use of five individual episodes created five illustrative cases, providing a thick description. I needed to provide a thick description to address the complexity of the research questions while still offering flexibility in the research design (Geertz, 1973)

3.5 Methods of data collection

The data collected were the transcripts from; the lesson, the teacher's reflections and the pupil's discussion. The data collection process was broken down into distinct phases: The recruitment of schools prior to the day for data collection. The recording of the activities took place in school on the day of the lesson, and the follow-up actions involving data set organisation. In the following section, I outline the day's activities before placing them into the overall process of data collection.

3.5.1 The selection of the pupils and episode

In discussion with the teacher, I selected the initial groups of pupils to be videoed based on who had given assent and consent, plus which groups were of particular interest to the teacher and me. In keeping with my epistemological stance, I felt it was essential that the teacher expressed a desire to use this opportunity to find out more about their pupils' ways of working mathematically. It was imperative for the research that the teachers were interested in the selected pupils' understanding of mathematics. The final pupils were self-selecting from the initial videoed group, depending on the episode that stimulated my interest and the teacher.

The final selection of the episode used to support the stimulated recall depended on what happened during the mathematics lesson. The actual episode determined which pupils were selected. Ten pupils reviewed the video footage of themselves from the five schools. No other pupils were involved in this reviewing exercise. The pupils provided their narrative to the episode (not evaluation) prompted by the video and myself as the researcher. Not all the pupils would have the chance to review the video footage, which was made explicit. Pupils were consulted during the data collection process to ensure they were happy to continue. Chapter 4, the Findings chapter, provides more details of their responses.

3.5.2 Data collection during the day

The mathematics lesson and collective reflections were digitally recorded over the course of one day per case. The sequence of research activities during the day is summarised in Table 2. The use of stimulated recall through visual methods supported the data collection process. The following sections discuss each stage from table 2 individually.

	Participant	Activity	Time scale	Form of Data collected
Stage 1. The mathematics lesson	Pupil	Take part in group work during a mathematics lesson, which is being videoed.	30 minutes	Video recording
	Teacher	To teach the usual mathematics lesson.		
	Researcher	Observe the group session and ensure the videos were working correctly		
Stage 2 Selection of episode	Teacher / Researcher	Discuss any perturbations that occurred during the session. Select one perturbation to revisit and look at more closely. Identify the pupils to be interviewed.	30 minutes	Made field notes
	Researcher	Find the relevant section on the video for deeper analysis.	15 minutes	
Stage 3 Interviewing and videoing pupil recall	Researcher and selected Pupils (2)	Pupils construct a narrative using the video as a stimulus for the episode. Semi-structured interview questions were used to encourage dialogue but not influence the recall.	45 minutes	Video recording
Stage 4 Collaborative Reflection	Researcher and teacher	2.0 Recount the aims of the lesson and make a brief recount of what happened with no analysis (Teacher). 3.0 Construct a collaborative reflection based on the observation of the stimulus. 4.0 Share the narrative from the pupils with the teacher. d. Revisit the collective reflection in the light of the new information.	a. 5 minutes b. 10 minutes c. 10 minutes d. 20 minutes	Video recording

Table 2: The Stages of data collection undergone during the day in each school

Stage 1 Videoing the mathematics lesson

The group work section of the mathematics lesson was scheduled to run for 30 minutes and be digitally recorded by video. This approach ensured that recording the pupils would take place over a timespan that was no different from a typical mathematics lesson. Thus, it was as close to naturalistic episodes of learning as possible.

The length of the clip selected for the stimulated recall was intended to be five minutes, which Coles (2013) suggests is a reasonable length of time to capture an episode of interest but not too long to become unmanageable. However, I knew that each clip's final length would vary according to the individual episode. For example, episodes ranged from two connected shorter episodes of approximately two and a half minutes to a block of six minutes.

Using video in a primary classroom raises ethical issues, which I go into more depth in section 3.6.3. The justification for using video with the pupils in this study was to demonstrate that the pupil's perspective is relevant and vital and needs to be heard (Coles, 2016). To view pupils, using the video as a focal point, as co-researchers enabled a shared understanding to be negotiated (Rogoff 1990). Harcourt and Conroy (2011) emphasise the importance of establishing a trusting relationship to overcome the predisposition of pupils to respond to the conventions of the classroom.

Stage 2 Selection of episode

The selection of episodes used for further reflective analysis was by agreement between the teacher and myself, based on episodes that raised perturbation. Claxton (2000) describes the teacher's work as 'knowing-in-action'. The teacher draws on two banks of knowledge; the intuitive, which is based on tacit knowledge and the analytical, based on explicit knowledge. According to Claxton (2000), the craft of an effective teacher is to view and review their actions using a careful balance of the two, a process that takes practise. In this study, the teacher's and I selected an episode that interested them based on intuition and explicit knowledge. Due to the implicit and explicit relationship, 'moments' or 'episodes' are likely to occur during a lesson, which appears significant because the teacher's intuition raised questions. For example, a pupil's specific response could raise questions around the interpretation of a concept. Where the banks of knowledge do not align, perturbation or disturbance can ensue, resulting in episodes requiring further exploration.

I then isolated the relevant section from the video footage (approximately five minutes) for viewing, acting as a stimulus for both the teacher and pupils. The isolated clip allowed the teacher time to reflect on the lesson to 'notice' an episode that raised questions at the time. The video of the episode was also intended as stimulated recall for the pupils, so they could add their opinions to the reflective process, offering another point of view. The teacher and I discussed each episode from each case before selecting the perturbation. The section was edited from the video and reviewed following that discussion. As outlined above, this event is something the teacher 'notices' but under 'normal' circumstances does not always have the time or space to explore further within the lesson (Mason, 2002). The event, therefore, had significance to both the teacher and myself and deserved greater attention.

Stage 3 Interviewing and videoing the pupils' recall

The selected pupils viewed the excerpt separately from the teacher (a maximum of three pupils) and were asked to describe what was happening from their point of view. The pupils' descriptions added another layer of interpretations used in the teacher's collective reflection. The pupils thereby worked alongside me to provide further insight instead of being an object of enquiry (Christensen and James, 2017). Pupils are competent contributors and ought to be viewed as social agents in their own right. Valentine (2008:142) states:

> '…*we cannot assume that adult 'proxies' are able to give valid accounts of children's lives. Young people may have different values from adults or different perspectives on their experiences.*

Valentine (1998) states pupils have their own views and perspectives, which teachers are not necessarily privileged to assess. Dockett, Einarsdottir, and Perry (2009; 295) raise the question, 'How does our research recognize the importance of relationships in the research process?' Viewed by the pupil as a social agent creates the foundations of a democratic community and can inform adults (Harcourt, Conroy2011. Rinaldi, 2001). I decided to interview the pupils without their teacher. There is no getting away from the power differentials within a school environment, but I considered a quiet place for pupils to view and discuss the episode to go some way towards this. I videoed the discussion, but pupils had the time to reflect on their responses and to veto the video at any point. At the end of the recording, I reminded the pupils that I would be sharing it with their teacher, and if they were still okay with that, they were. Following the data collection, I transcribed the recordings for later analysis.

Stage 4 Collective reflection

The next stage in the research process was to frame the episode observed within the context of the lesson and the sequence of lessons already taught. The teacher and I discussed the lesson in general terms. I first wanted to establish why the teacher had selected this particular lesson on the day. I then asked them to describe the lesson without the help of the video. At this point, the emphasis was on the description. The teacher then viewed the excerpt and described what they saw. At this point, the focus was on noticing. Following the viewing of the episode, the teacher watched the video of the pupil's description of the excerpt. The teacher then reflected on the pupils' focus, comparing the similarities and differences to their descriptions. They were then asked if this helped them notice something missed earlier. The teacher could watch the video repeatedly if they requested, which they did. The final stage was collective reflection based on the lesson using the points noticed in the video of the episode. The video footage from these interactions was then transcribed, ready for analysis. This stage moved into the analysis, not making judgments but considering the implications for future teaching.

3.5.3 Data set organisation

The data collection processes are mapped out in table 3, showing the linear progression of events, starting with the preparation for working within the classroom and ensuring ethical guidelines had been adhered to (this is discussed in greater detail in the ethics section 3.6.) Thematic analysis was carried out on the narrative of each school, and then all five schools were combined, following the data collection (Braun and Clarke, 2006).

The preparation for the primary data collection activities on the day involved selecting participating schools, teachers and pupils, as discussed in section 3.5 above. I ensured the data collection met all ethical requirements due to pupils being underage to give consent themselves. I also sought pupils' assent and parents' consent. Finally, I familiarised the participants with the specific video equipment. The data collection was then transcribed, coding and analysed, using a combination of Nvivo and manual coding to sort data.

Timeline	Actions	Sequence of events for the methodology
Before the day of videoing the lesson	Contact Head teacher of the school.	Recruiting schools for research.
	Meet with the teacher and leave a camera.	Explain the research and if they agree to continue, leave the camera for familiarisation with the pupils.
	Collect consent and assent forms.	Consent and assent forms distributed, collected and checked, including information sheets.
	Familiarisation with the class.	I visited the classroom before the research took place so pupils would have met me before the day of videoing so I would not be a stranger.
On the day of videoing the lesson	Video mathematics lesson.	Video the group of pupils selected.
	Discussion with teacher.	What part of the lesson would be revisited as an episode for stimulated recall?
	Select five-minute section.	The researcher edits the video to isolate the specific episode agreed on.
	The researcher watches and records the section with the pupils.	The episode selected is shown to the pupils involved and their descriptions of the event are recorded.
	Collective reflection with the teacher.	At the end of the school day, the video of the excerpt and the video of the pupils was used as stimulated recall. From this, the collective reflection was generated.
After the day of videoing the lesson	Transcribe video	Familiarisation of the data.
	Code and analyse the narrative.	Fine-grained analysis of the data using Nvivo.
	Coding and individual thematic analysis of each schools data.	Initially using Nvivo for the fine-grained analysis of the data. Then manually coding and sorting data.
	Combine the thematic analysis across all the schools.	By combining all the data, a codebook was created and themes emerged, creating new knowledge.

Table 3: The sequence of activities undertaken; before, during and after data collection

3.5.4 Presentation of the data

Each case was framed individually using the following sequence as a structure to ensure transparency and rigour whilst also honouring the uniqueness of each case:

- *Context;* each case starts with a short description of the school, lesson and the emerging episode, followed by an illustrative account of the content of the narrative.

- *Teacher's initial response;* following the lesson and before viewing the video footage, I asked each teacher to give a brief overview of the lesson, providing insight for me as the researcher to understand what was intended from the lesson and set the context. This response is synthesised to establish how each teacher perceived the lesson.
- *Pupils' reflective consideration;* the pupils' interview was viewed immediately after viewing the episode, providing a different lens through which to reflect on it. (RQ1)
- *Teacher's and researcher's collective narrative*; the collective reflection created between the teacher and researcher in response to viewing each stage of the process informed the penultimate section.
- *Themes arising;* finally, to summarise salient features of each case and lead into the discussion section, deductive analysis introduced the use of the figured world framework. (RQ2)

3.6 Ethical considerations

The design of this research raised several ethical issues, which needed to be deliberated and addressed appropriately before, during and following data collection. From my experience of schools through being a teacher and teacher educator, I know there can be a wide range of challenges when working in busy schools. Recruiting and working with children and teachers in this challenging environment, using video within that environment, and my position as an insider researcher is discussed in the following sections. In terms of ethical considerations, the following bodies were considered and adhered to
BERA (British Educational Research Association), Oxford Brookes ethics guidelines and the National Children's Bureau guidelines (Shaw, Brady and Davey, 2011). Oxford Brookes university approval has been obtained to undertake the research.

3.6.1 The complexity of the environment

The complex relationship between the teacher and me required ethical considerations within a case study approach. I was very comfortable with my position having worked in a range of settings as both a teacher and teacher educator. I felt I was able to empathise with having visitors in my classroom. I was also very aware that each teacher would be regarding my presence very differently according to his or her own personal experiences. They did not know my personal experiences either. I was very mindful of the environment and worked towards minimising my impact on what was happening. When reflecting on a teaching episode, the delicate interactions and discussions could put the teacher in a vulnerable position, leaving them feeling exposed. The teacher may be worried about being judged or criticised. However, a good relationship between the participants would allow for richer data to be collected. Denzin and Lincoln (2000) stress that a respectful and supportive relationship needs to be established and maintained throughout, creating a careful balance of power and a safe environment. The power balance between the teacher and myself within the study and the more obvious adult-pupil power balance needed to be considered. Valentine (1999) heralds the importance of not treating pupils as vulnerable and making decisions for them. The teacher and I ensured the pupils had an opportunity to respond to the stimulated recall and their opinions were heard and respected.

Following the Oxford Brookes ethics guidelines, I initially sent an information sheet (See Appendix A) explaining the research to the headteachers and teachers. I then arranged a formal meeting with the headteachers and teachers. In this meeting, I gave a more detailed description and provided opportunities for the participants to discuss the research and pose any questions. The small size of the adult and child groups could potentially create data protection issues, such as the potential for participant identification and judgments to be made based on single episodes. The participants will be able to identify themselves due to the individual characteristics of each episode but the openness of the discussion and generalisation of the characteristics of the episodes should mitigate any possible judgements to arise. More generally, the use of pseudonyms for both participants and schools, to de-identify contributors and not presenting information identifying or locating schools mitigated these issues. All the data has been kept securely following the ethical guidelines and were promptly anonymised. Any lack of identity protection could impact the working relationships within the school and also with parents. For this reason, contributors were not given the option of being identified in any publication arising from the research.

Exploring the social world of the classroom presents ethical tensions between protecting the rights of the child (Convention on the rights of the child, 1989) and giving all participants an equal voice (Dockett, Perry, Kearney 2013). The social world of the classroom is one of conventions and rules, of which teachers are the gatekeepers. To have a researcher enter this environment could appear threatening to the teacher and become intrusive. As a former teacher, I know the social world of the classroom is a personal space and I liken it to inviting someone into your own home to look in detail at aspects of the world you have created. Therefore, I knew I had to share control of the process with the teacher, thereby allowing the teacher to explore what interested them, using me as a sounding board. Open questioning and discussion were used to support the reflective and reflexive process (See chapter 4). The pupil's role was to add their views to the narrative, providing another perspective on the episode studied.

The teacher's figured world has well-established norms and uses specific language, as mine does as a former teacher and now teacher educator. Links between behaviour and expectations are often made through experience and habit, looking for patterns and similarities between behaviours and children. However, working with individual teachers created individual challenges. Now as a researcher, I had to heighten my listening skills and develop my observations for when this was taking place (Banks, 2001; Rose, 2016).

3.6.2 Assent and consent forms

The information sheet was designed with illustrations to support pupils' interpretations of what being part of the research would involve (Dockett, Perry, Kearney, 2013). The pupils were made aware of the nature of the study throughout. The information sheet was shared by the teacher with them (Appendix B), the teacher also explained before the lesson what was happening, and I, as the researcher, explained again before viewing the video clip. Every effort was made to ensure pupils were informed and had opportunities to express their will.

Teachers and parents completed the consent forms, and pupils the assent form shown in Appendix C and D. Only pupils providing this information were videoed. This stage of the study again raises the issue of power differentials when researching in schools. The form could be construed as more schoolwork or parents wanting their pupils to be involved, offering consent without consulting the pupil (Docket, Einarsdoltir, Perry, 2009). The place of the pupil's assent form helped mitigate this by asking the pupils directly. However, assent needed to be an ongoing process and obtained throughout the study so that pupils felt they could withdraw at any time (Cocks 2007).

Many of the pupils were very keen to be part of the research so I explained they might not be selected for the research, and it would depend on what happened during a typical school day. For those pupils who were not selected, I spent time talking to them about their mathematics to help make them feel included. The pupils continued as usual, and the group working with the teacher on the research day were filmed. Pupils not wanting to be filmed or who had not returned their consent and assent forms were not included in the group. The way the classroom was organised and the positioning of the cameras ensured compliance with ethical considerations. Following the lesson, it was explained again to the pupils that they would be asked to recount the group's mathematics activity and that it would be recorded and shared with their teacher. They were asked at relevant points if they were happy to continue and if they felt comfortable with the process. If they wanted to withdraw at any point, they could re-join their class. They all appeared very happy to be part of the process.

3.6.3 The use of video

There are many considerations when using video as a visual method for stimulated recall. With video becoming an ever more popular medium in society with the explosion of social media and the use of video on phones, pupils are more familiar than ever with the use of this technology. In the classroom, teachers are also utilising video for data collection, pupils are becoming more familiar with its presence (Jewitt, 2012). However, there is a delicate balance between intrusiveness and benefits. To minimise the unfamiliar nature of having a video camera in the room, if they were not already in use in the classroom, the teacher was asked to familiarise the pupils with the camera to be used. This was done in the week leading up to the research day. The pupils saw the equipment in use and were able to handle it under supervision. This was hoped to allow them to become more comfortable with its presence. Any footage recorded during this stage was immediately deleted.

According to Atkinson and Claxton (2000), the use of images can provide greater understanding and support discussion by providing extra clues from body language and actions which can start to exhibit patterns of behaviour within a context in this case, the context of the mathematics classroom. However, it can also impact the environment where it is being used by its presence alone. Banks (2001) and Rose (2016) draw attention to the social context involved in creating and using visual images in research. To be videoed is not a clinical process but one that impacts the social context of what is being captured on camera. It could run the risk of becoming a performance of what is expected instead of a naturalist episode if the use of the camera is not familiar or if the camera dominates the environment because of its position in the classroom. The placement of the camera was integral to ensure it avoided dominating the lesson, so the familiarisation and positioning was crucial to minimise the impact on the pupils. I originally used an iPad placed on the tables but quickly found they were too large and intrusive because other pupils could see what was being filmed. The small flip cameras proved much less intrusive because of their size and could be positioned between pupils comfortable. Two and sometimes three cameras were placed within the group, providing different angles to view from.

The use of video formed part of the context in which the study was taking place, the reviewing of episodes was also within a social context, which signalled certain types of assumed behaviour. Each individual's own experience and knowledge of relationships within their own figured worlds of the classroom had a bearing on what each individual noticed and the way they recounted the episode (Rose, 2016; Banks, 2001; Berger, 2008).

During the lessons being observed and recorded, the video showed interactions taking place between pupils out of view of the teacher. The teacher could not always be privileged to view all interactions and happenings in the classroom or the full context of actions seen. The focus of this research was not just on the teacher's figured world of the classroom, but the figured worlds of all contributors to the classroom experience. The recording provided rich information for the teacher to observe and use for collective reflection later.

Banks (2001) and Rose (2016) point out areas for consideration when embarking on the use of visual methods. The questions of identity, power and privilege need to be deliberated. As a result of this research, the purpose of the video was shared with the teacher and pupils and the commentary and discussion were based on a co-constructed interpretation in an attempt to balance power and privilege. As a researcher and teacher educator, I wanted to find out more from the teachers view point and work towards addressing the imbalance of my privileged position. A collaborative approach was taken with the teacher and a shared objective. It was what the teacher chose to focus on and the language they chose to use to describe an episode that provided rich data reflecting their figured worlds, which informed this research (RQ1). The relationship ensured the teacher lead the discussion and using my skills as a teacher educator, I was able to act as a sounding board for their ideas.

Using the video camera and discussing the episode is a skill that needs to be practised. Fortunately, I had practised this during my 'Numbers Count' training and intervention work. The positioning of the cameras needed careful consideration to balance the quality of the recording and ensure only the pupils with ethics clearance were filmed, while also ensuring the presence of the camera was minimised, in the hope the pupils would forget it was there. To overcome this, one camera was positioned between each pair of pupils, focusing specifically on their actions, ensuring a good quality recording, capturing the dialogue between each pair and the camera was small enough to be discreet (Coles, 2013). The use of stimulated recall using visual methods fits with the methodology of this research, allowing freedom for the teacher to explore and articulate the figured worlds of their classroom and thus help to address research questions 1 and 3. The tensions between using video and ethical considerations remain in the balance but adhering to ethical considerations and ensuring pupils are empowered to make choices, the use of video remained a useful tool.

3.6.4 Insider researcher

The unique situation of researching within the classroom introduced another layer of complexity by introducing me as an observer. Just the presence of an observer changed what was observed, possibly subtly or noticeably, and this needed to be acknowledged. Therefore, the event is changed by interpreting the event due to differing perspectives of intention (Sirij-Blatchford, 1997). As a former teacher, intervention teacher and teacher educator, I have a wealth of classroom experience and I am aware that I need to be mindful of my own personal perspectives of what is happening within the videoed sessions. The observer is conditioned by their prior knowledge and will always be impacted by that. To be an insider researcher suggests a more nuanced prior knowledge, but not necessarily.

I see myself as a teacher with sixteen years of experience and a teacher educator of six years, thus feel that I can be considered an insider researcher. However, to the teacher, I am known as a representative of my Higher Education Institution, and thus they may see power differentials between us. In an attempt to break down these differentials, I became an active co-participant within the research, sharing responsibility with the teacher, thus creating an insight into the social world of each classroom (Sirij-Blatchford, 1997). A less mechanical approach was adopted through reflexivity and becoming part of the explored world. Therefore, my researcher identity consists of both insider and outsider views of the classroom.

Lesson observations can be stressful and make both teachers and pupils feel uncomfortable. They are a regular part of teacher training, professional development and assessment, in-school training, and external inspection. However, my role was to ensure both the teacher and pupils understood they were doing the observing and I was facilitating their opportunity to have time to observe and express their own figured worlds. For this research, the video footage was used as a stimulus for discussion and not as the primary focus, thus reducing the potential for causing stress. I also articulated that I would end the recording of the lesson at any point without any reason. Some pupils can become excitable or withdrawn in the presence of a new adult in the classroom, and I stressed that I know this and would make no judgements if this were to happen. The prevalence of learning support assistants, alternative teachers covering statutory non-contact time, and regular lesson observations in modern-day classrooms is likely to limit the impact of my presence significantly on teachers and their pupils. However, there will have been an impact. By taking all these actions in advance, I hope I have kept that impact to a minimum.

The teacher selected the activity for the pupils to do, in line with the mathematics planning for the class that week, limiting any impact on the pupils' education. Furthermore, I ensured the tasks the children undertook were similar to those undertaken daily in the classroom. The only two differences were video recording and my presence.

I ensured the pupils were at ease with the process by designing the study to form a relationship. Developing a relationship also created an environment where pupils might feel comfortable disclosing information. Working with pupils in this way raised questions around safeguarding issues. As a teacher, I have my 'Disclosure and Barring Service' (DBS) clearance but would refer back to the class teacher if I had any concerns or disclosures. The teacher had complete jurisdiction over their classroom.

3.6.5 Trustworthiness

To demonstrate rigour and transparency the five cases were inductively and deductively, coded. In addition, a subjective approach was taken, as described by Fereday (2006). The research has validity because it explores what it intended to explore. Initially, I took a deductive approach by viewing the data more generally and applying an appropriate template to the text organised by codes, in this case, based on the preliminary scanning of the data (Crabtree and Miller, 1999). I also took an inductive approach by noticing the important moments from the episodes before making interpretations, initially looking for patterns but then moving on to interpretations and generalisations from the data (Boyatzis, 1998). Concerning reliability, due to the small number of cases and the open-ended nature of the study, replicability has not been claimed. However, how the findings are transparently set out and explored ensures that I can establish trustworthiness, although I cannot wholly establish rigour.

3.7 Thematic analysis

The thematic analysis is primarily based on Holland *et al's*, (2001) figured world's framework drawing on the three themes of identity, dialogue and cultural artefacts, as illustrated in table .4. The analytical framework draws on Mason's (2002) discipline of noticing and builds on Holland *et al's* (2001) figured worlds, incorporating Sfard's (1998) metaphors for learning and Belenky *et al's* (1986) five ecologies of participation. Each position has been examined in sections 2.3, 2.5 and 2.6, drawing out the justification for using such a blended approach along with the strengths and weaknesses associated with each theory. The concluding section returns to the processes involved in the analysis including a reflexive approach.

3.7.1 Figured worlds framework

As a research model, figured worlds lends itself very well to social/cultural analysis, due to the nuanced complexities of the interactions and participations within an educational context. As a teacher educator visiting a wide range of schools it appeared to me the social cultural environment could impact pupils' engagement with mathematics. By using a figured world lens, I felt it would provide useful data to address my research questions. The Figured worlds framing is based on social contexts with the themes emerging based on identity, dialogue and cultural artefacts. Table 4 illustrates the characteristics displayed of the emerging codes, based on the three themes.

Figured world themes	Features present that informs the theme
Identity	Passive receivers of information Active constructors of knowledge Opportunities offered to exercise mathematical thinking Structured tasks modelled by the teacher and enacted by pupils, (procedural) Teachers responsibility to address naive concepts Pupil confusion Naïve concepts viewed as challenges Pupil engagement in activities
Dialogue (RQ1&3)	**Teacher/pupil balance** Pupil to pupil, teacher to pupil, pupil to teacher, silence **The role of dialogue** Instructional Challenge Justification Reasoning **The use of questions** Narrow and guiding Open and challenging Assessment focus
Cultural artefacts. (RQ1)	**Classroom organisation** The use of differentiated tasks by ability The cultural use of ability groups or mixed ability **The use of manipulative artefacts**: Remedial support Integrated tool to support investigations A view of mathematics as a difficult subject A view of mathematical challenge as positive or negative

Table 4: Emerging codes based on figured world themes of identity, dialogue and cultural artefacts

A mathematics classroom is a place where both teachers and pupils fashion a sense of themselves through their joint meanings and activities (Holland *et al.*, 2001). The culture of the school can equally influence the teacher’s figured world. It is specifically how the pupils act and interact in the world, intertwining their self-concepts and knowing, that fashions their identity (Rogers, 1990). The use of an inductive approach was introduced in the analysis of the data.

3.7.2 The analytical framework

The analytical framework used for this study drew on Mason’s (2002) model of noticing, enabling a finer-grained analysis to be possible. I have noticed that in busy classrooms the teacher has limited time to make decisions about what they see and hear pupils doing. With time, teachers are able to build up a bank of responses based on experiences. However, in my research I wanted to go beyond that tacit knowledge and provide time and space. This provided an analytical approach that extended beyond just a description and review of something notable about learning that teachers would claim, introducing a social-cultural view of the episodes. It was by moving beyond what Rorty (1980) described as the habitual response to particular actions that provided greater insight into the causes of the perturbations emerging in the episodes illustrated in chapter 4. To be able to move beyond a routine response is what Dewey (1933) regarded as the role of reflexive practice (Rolfe, 2014). Belenky *et al's* (1986) ecologies of participation provided a socio-cultural view of what was emerging as valued learning by the teacher.

To review the five unique mathematical episodes through the lens of the figured world, Belenky *et al's* (1986) ways of coming to know and Sfard's (1998) metaphors for learning have been integrated to create this unique analytical framework. Table 5 illustrates the key features of contrasting learning environments, and different forms of knowledge being mediated in response to different pedagogical approaches.

	Received model	Procedural model	Connected model
Origin of knowledge for pupil (Ontology)	Knowledge is an external entity to be obtained.	Knowledge is an external commodity to be obtained.	Multiple realities where knowledge which is social in origin is being generated and investigated by pupils through Social Constructionism.
Pedagogical approach of the teacher (Epistemology)	A Didactic approach enacted by the teacher requires pupils to listen and retain information that can be selected later when needed.	Strategies enacted by the teacher for pupils to actively apply, working towards external knowledge. (Scaffolding)	An enquiry approach to learning. Pupils are actively immersed in the learning process. Multiple realities (Sfard's (1998) participatory metaphor).
Identities afforded by the teacher to the pupils	The teacher dictates knowledge and the pupil is a passive receiver of information. Minimal opportunities for mathematical thinking or identity.	Pupils are given structured ways of engaging with tasks, following set procedures or routines, providing a constrained development of mathematical identity	Pupils are active constructors of knowledge. Through the immersion in the activity and taking their own decisions, they are afforded opportunities to develop a stronger mathematical identity.
Nature and direction of dialogue	Teachers as transmitters and pupils as relatively silent receivers. (Asymmetrical model) Recall focused questioning.	The teacher provides instructional direction and pupils follow procedural possibilities in the pursuit of an established solution. Teacher to pupil and limited pupil to pupil dialogue, pupils recalling the procedure.	All pupils and teachers are engaged in dialogue focused on challenge, criticality, conjecture and debate, collaboratively in an equal way (symmetrical model). Dialogic interactions.
Ways of using Cultural Artefacts	Resources used to transmit information. Teacher infers assumptions of fixed ability through the use of differentiated set tasks.	Teacher infers assumptions of fixed ability through providing manipulative resources to support the perceived lower ability groups, as scaffolding to the strategy.	All pupils have access to manipulative resources to test out a range of possible outcomes, inferring pupil's fluid ability.

Table 5: Holland et al's (2001) figured world themes, informed by Belenky et al's ways of knowing (1986) and Sfard's (1998) metaphors for learning, to create a unique analytical framework.

The different learning environments generated by the teacher of each episode had qualitatively different forms of knowledge being mediated, reflecting Sfard's (1998) metaphor of acquisition - knowledge to be obtained, and participation - knowledge to be jointly created. The privileged pedagogical approaches associated with each metaphor defines the knowledge produced. The identities developed within these figured worlds appear to be influenced by adapting to the constraints of the environment through the use of dialogue and cultural artefacts, enabling or constraining mathematical thinking.

3.7.3 A flexible approach to gathering views of the teachers and pupils

The aim of this research was to get closer to what is actually happening in primary mathematics classrooms. To do this the research needed to be both structured and flexible. Teachers and pupils can hold extensive knowledge acquired through personal experience, which they can share through narrative. However, that narrative can appear ambiguous and confused if just taken at a prosaic level (Yerushalmi, 2021). In this study, each school was considered separately and the video recordings underwent considered scrutiny through listening and watching the interviews, in an attempt to gather multiply perspectives this included facial expressions, gestures and mannerisms (Newby, 2010). First, the participants described what they saw with no analysis or conjecture, this is very hard to do but important to establish before the discussion and consideration takes place. The video recordings were then transcribed, read and reread numerous times. Initially, the whole transcript was read as a holistic social event. This overview was established before embarking on the fine-grained microanalysis that followed to establish codes and then themes. By using an inductive approach, a coding structure emerged from the transcriptions. For this research, the coding remained factual and descriptive, attempting to avoid interpretation. I could have made assumptions steering me in a different direction, influenced by my tacit knowledge. The other difficulty with this approach is the lack of boundaries and endless possibilities (Marshall, 2002). This process sounds linear, but it was far from that. At each stage, there were reflections, reviews and turning back to prior stages before moving on again demonstrating a reflexive process.

3.7.4 Synthesising how the approach enables a response to the research questions.

The research questions and overarching aim for this study involved the collection of multiple viewpoints of the figured worlds within the mathematics classroom thus creating complexity for both the collection of data and analysis (Holland *et al.*, 2001). These multiple viewpoints collected provided rich, if not 'messy' data, requiring the study to be flexible, reflective and creative, visiting and revisiting codes and themes. Looking at the cognation of mind and body within a social context provided a theoretical base for such complexities (Vygotsky, 1978).

Looking at individual cases allowed for contextual analysis of each episode viewed. However, this also raised ethical challenges by asking the participants to look more closely at their own actions and inactions. It is a big step from being the teacher and making the decisions to looking at your own practice and how it impacts the pupils. Due to the nature of the research taking place in school and using video, the ethical considerations were paramount. The use of stimulated recall enabled the teachers and pupils to take time to reflect and notice more than just relying on memory, so the added ethical considerations were deemed to be justified (Coles, 2016).

For clarity, I have presented the analytical framework in a linear form. However, the analysis was not a simple linear process a crystalline approach was taken, visiting and revisiting the data in different ways, building up layers of interpretation and creating rich analysis (Tall, 2013). An attempt was made to ensure clarity and transparency throughout this research by being open with all aspects; from my stance as a teacher educator and researcher, to the collection of data within schools through to the analysis and coding of data. The use of reflection and noticing was key to this process (Mason, 2002; Cohen, Manion and Morrison, 2018), supporting the interpretivist paradigm used to find insight as opposed to answering questions by looking at individual perspectives.

In teacher training, student teachers are asked to develop their reflective skills to encourage them to attend to what is happening and develop their practice beyond what they are doing to reflect on how their actions are impacting on pupils learning. Reflective practice can go further in research, actively using that knowledge to explore and inform future practice (Boud, Kaogh and Walker, 1985). Just reflective practice cannot achieve the creation of new knowledge, but also needs to be reflexive. It is through a conscious effort of piecing together the evidence collected through present experiences built on past learning, which will inform and create new beliefs and theories for the learner (Dewey, 1933).

3.8 In conclusion

This chapter has brought together the philosophical discussion and theoretical frameworks that underpin the methodology used in this study, including the analytical framework devised for the analysis of the data. The chapter also covered the processes involved in recruitment of participants and ethical considerations. Due to the study being set in the mathematics classroom, an area of complex and contrasting epistemologies concerning ways of coming to know, a blended framework was used to code the main features of the differing epistemologies present in the differing figured world of five distinct classroom. A version of Mason's (2002) discipline of noticing was adopted, alongside merging the models of Sfard (1998), and Belenky *et al* (1986) within Holland *et al's* (2001) figured worlds.

The next chapter will present the findings from the inductive and deductive analysis of the data, exploring the figured worlds and multiple viewpoints of a shared episode from a primary mathematics lesson.

Chapter 4

Findings

4.0 Introduction

This chapter aims to present just the findings of the multiple viewpoints of a shared episode within five separate mathematics classrooms. A broader discussion is offered in the following chapter of the analysis of those findings.

As set out in chapter 3, stimulated recall aided collective reflection between the teacher and the researcher. The teacher viewed the video recordings from two separate events to help memory; firstly, the recording of the mathematics lesson of a specific episode during the lesson and secondly, the recording of the pupils' narrative whilst viewing the same episode. The collective reflections from the five individual cases presented here draw directly on the conversations, illustrating interpreted accounts of the analysed data. To define and contextualise the points raised, I have included quotes to enable the reader to immerse themselves in the episodes and enhance the trustworthiness of my analysis. I have edited out the incidental sounds and pauses that could distract the reader from these quotations.

Initially, to strive for rigour and transparency, I took an inductive approach to gain a holistic view of each episode and ensure the context was clear and transparent before turning to the deductive approach.

It was an iterative process, applying the various levels and depths of analysis then coding using Nvivo software, followed by manually sifting codes (Cohen, Manion and Morrison, 2018). The initial themes emerging from the inductive approach, as discussed in section 3.7.1 of the Methodology chapter, were:

- Pedagogical approaches,
- Dialogue,
- Naive concepts,
- The use and place of manipulative resources,
- The teachers aims for the episode, for example, new knowledge or practicing skills.

Deductive coding was then applied through the adaptation of the figured world framework, as outlined in section 3.7.1. The deductive approach utilised the themes and codes of the figured world framework:

- Identity,
- Dialogue
- Cultural practices.

The narratives recorded here are snapshots in time, representing what the participants paid attention to at that moment. However, that provides an interesting focus that I examined from alternative perspectives, indicating what appeared to be pertinent to the teacher and pupils during the collective reflections.

The individual lessons taught in each case drew from different areas of the National Curriculum Programme of study for mathematics (NC, 2013). Three lessons were from the section on Number: Addition and Subtraction, Fractions, and Multiplication and Division. The other two lessons were from the section on Geometry and Properties of Shape: one looked at Symmetry, and the other looked at 3 Dimensional Shapes, names and properties.

Teachers generally follow a set pattern of delivery of the mathematics curriculum throughout the year. I was visiting schools for over two months, so I experienced a range of different subject areas as they progressed through the curriculum. Each case was framed individually using the structure outlined in section 3.5.4 to ensure transparency and rigour whilst also honouring the uniqueness of each case.

4.1 Case A - Weston School - symmetry lesson

Weston School has approximately 115 pupils on roll and is situated on the edge of a large market town in the middle of England. Weston School has experienced a period of poor Ofsted reports; deemed as requiring improvement in 2013, then rated as inadequate before going into special measures in 2015. The school underwent many changes over the next three years due to becoming part of an academy group and a change in the leadership team. In 2019 the school received an Ofsted rating of 'Outstanding.' Its approach to teaching mathematics has also changed, adopting the Mastery in Mathematics framework, which has its roots based in variation theory, discussed briefly in section 2.1.1. Weston School has above average challenges represented in its pupils, mainly due to disadvantaged households but achievement in the core subjects exceeded the national average in 2019.
The lesson observed for this study was on the theme of symmetry. The teacher, Ms Travis, started the main lesson by asking the whole class: '*What is symmetry? Talk to your partner. I want to see conversations with your partner. I want to see if you challenge or support your partner'.* From this initial whole-class introduction, a different shape was introduced to investigate individually, concluding with the final activity of the lesson identifying which shapes were symmetrical on the worksheet, therefore providing concluding evidence of

their decisions. One of the activities for this lesson was to discuss how a shape would need to change to become symmetrical. The shape in figure 3 is an example of the questions asked.

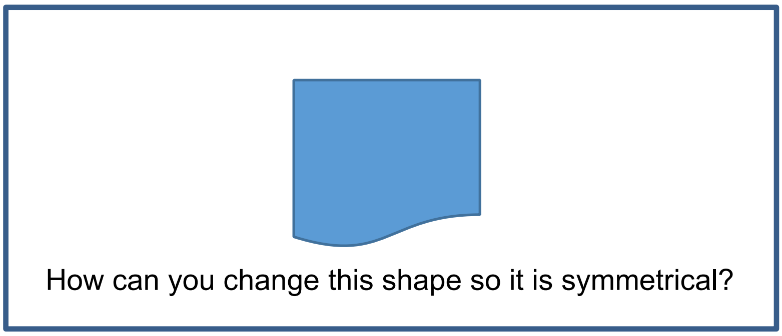


Figure 3: An example of the symmetry challenges set by Ms Travis at Weston school, case A.

During the lesson, Ms Travis focused on the way pupils verbalised their ideas and demonstrated their skills of conjecture and reasoning. Throughout the lesson, Ms Travis would stop the class and ask a question or make a statement to agree or challenge, thus sharing the pupils' reasoning and justification skills. The purpose of the activity appeared to be for pupils to co-construct dialogue. Ms Travis would then build on the pupil's response, making it a shared experience, creating opportunities for the pupils to contribute and provide support, as illustrated in this opening question to the class. The pupils had been given the question to consider before explaining. Ms Travis chose Tom to start the discussion:

Ms Travis:	*Tom what is symmetry?*
Tom:	*Symmetry means if there's a triangle shape on both sides it's the same.*
Ms Travis:	*So, if I have a shape, if on both sides it's the same. Can you clarify what you mean by on both sides? Because I'm unsure what Tom mean on both sides? I need to be a bit more mathematical about that.*
Asha:	*We have been folding it.*

Ms Travis: *So, you have been folding it in… (Class called out) half. That's a key word isn't it folding it in half. Casper do you want to add…*

(Transcript A)

Pupils were working with manipulative resources to test their theories, folding paper and using mirrors to support their reasoning when discussing the image in figure 4. The pupils' exploration of the image formed the excerpt for the episode recorded. There was particular attention to the chimney in this discussion. Ms Travis stated the house was not symmetrical and asked if pupils supported or challenged her statement. Then asked the pupils to explain their thinking, using the mirror to support their reasoning if they wanted to.

Figure 4: Image from the worksheet of the house used in case A for the symmetry activity by Ms Travis, Weston School

This except illustrates the types of conversations the pupils were having and the role of questioning between the pupils:

Tom: *It's not symmetrical. So this bit goes down. So this is the middle like that so not symmetrical because you have got this side going down and this side going up.*

Asha: *I support for that one. (Indicating she agrees and thought it was not symmetrical).*

Tom: *It's not the same is it? Look it's got this bit here.*

Asha: *What about a ruler? Try it with a mirror then?*

Tom: *(Tom uses a mirror) Nope.*

Asha: *(Using the mirror) this one here.*

Tom: *Yep.*

Ashe: *Support or Challenge?*

Tom: *Support because there is an edge there.*

(Transcript A)

At this point, Tom had indicated in his book that he thought the picture of the house was symmetrical, whilst at the same time agreeing with Asha that she was right and that the house is not symmetrical, creating a dichotomy. As a researcher, I was interested in how Tom and Asha used the resource of the mirror to support their reasoning. The follow-up conversation led to a discussion about the symmetry of the star they had worked on the day before. The two pupils disagreed on the symmetry of the star, this time folding the shape to test for symmetry. They both negotiated the answer, and I was interested in how they could follow each other's reasoning and remain in agreement.

4.1.1 Initial teacher response

Ms Travis acknowledged the lesson went as she had hoped. The pupils had built on their knowledge from the previous day. She explained how she was pleased with the conversations the pupils were having with each other, expressing, '*It was really lovely to see different children supporting each other.' 'They're not scared to be right or wrong with each other*.' Ms Travis' focus was on the pupils' dialogue and the way they communicated ideas with each other.

4.1.2 Pupil review

When talking to Tom and Asha about the house's symmetry, Tom said, *'the mirror proved it was symmetrical',* but the conversation between Tom and Asha went on, *'The house was symmetrical. If you put the mirror on the line, it will show the chimney here and here'* suggesting it was symmetrical but then goes on to say,' *I support because it is only one chimney'.* Tom and Asha watched the lesson clip, and I asked them to explain their reflective view. Tom starts by justifying his decision that the house is symmetrical because he thinks the mirror proves it but ends with supporting Asha because there is just one chimney. Tom does not appear to change his mind, but he does extend his explanation, which clarifies his interpretation. Tom acknowledges that Asha is approaching the task differently but accepts her explanation by following her line of reasoning.

Following on from the example in the actual lesson, Tom and Asha showed me a picture of the star they worked on the day before. They had cut the star out to enable folding but were then working with the image of the star on the worksheet, see illustration figure 5. Tom indicated the picture of the star as being symmetrical but meant the star that had been cut out as not being symmetrical:

> *Researcher: You were talking about the star from yesterday; we have got a star here. Straight away I can see you have got a cross here*

saying it is not symmetrical, so is that the same star as you had yesterday? Or different one?

Asha: *Same*

Tom: *Same*

Researcher: Is this helping you to see if it's symmetrical or not

Asha: *If we look on the next page. (Looking at the picture of the star in figure 5)*

Researcher: What do you think Tom, because you were thinking yours wasn't symmetrical

Tom: *It's because if we go back onto that page you see like that bits going upwards more than that one and that one's going there.* (Looking at the cut-out star in figure 4 and demonstrating to me how it could be folded.)

(Transcript A)

Tom and Asha were using the cut-out star to test if folding could help them prove or disprove if the shape was symmetrical or not. The accuracy of the cutting had introduced dissonance leading to the discussion.

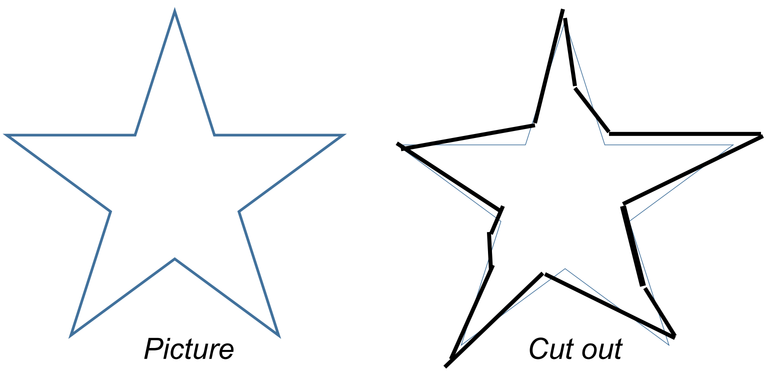


Figure 5: An illustration of cultural artefacts creating the discourse between Tom and Asha when discussing the symmetry of a star in case A. The picture of the star showing symmetry and a cut out star not showing symmetry.

The pupils were using reasoning and conjecture when discussing the properties of the shape, listening to each other's explanations and ideas.

It was the end of the day, so I thanked Tom and Asha for their time and for discussing their views of their work before they went home.

4.1.3 Teacher researcher narrative

Ms Travis observed from reviewing the footage of the episode that Tom appeared to be more actively involved than she had initially thought:

> *"…Tom didn't seem like he was interacting with Asha but he was listening to what she was saying? And then he chipped in with his thoughts. So even though he was busy, he was actively listening to what she was saying."*
>
> *(Transcript A)*

Ms Travis' focus throughout was Tom and Asha's communication skills which she found insightful and pleasing, stating; '*they're not scared to be right or wrong with each other. And that's really nice to actually see, I feel I've developed that ethos.'*

Opportunities to use enquiry-based learning has created a context in which pupils could actively co-construct knowledge, appearing empowered to discuss their own reasoning within different contexts. This is evident throughout the episode and the discussion with Tom and Asha. Their confidence and descriptions of their thought processes showed understanding of their ideas and thinking.

Ms Travis then goes on to express her pleasure in how these two pupils articulate themselves, explaining she had '*worked a lot on speaking in full sentences to try and articulate their [our] thinking, and these two particular pupils have limited communication skills which was not apparent from the video evidence*.' Observing, they seemed mature with the language they were using. Ms Travis was pleased and surprised at the understanding and content of Tom and Asha's conversations, which demonstrated active learning with good reasoning and understanding of symmetry.

After watching the clip of Tom and Asha, Ms Travis analysed the interaction, interpreting what she thought Tom was trying to say. She explained that Tom sometimes gets mixed up and confused with his mathematics, and she felt this was more to do with the way he tried to articulate his understanding. The symmetry challenge illustrated this point when he stated it was symmetrical and then went on to say the mirror proved it was not symmetrical.

By connecting, the visual images from the video of the episode and listening to Tom discuss the episode, a clearer picture emerged of Tom's understanding of symmetry and how he expressed himself. Ms Travis went on to give her interpretation of Tom's understanding, suggesting his implication was: if you look in the mirror, the image will be symmetrical. He was recording this in his book, but he knew the image was not symmetrical when listening to Tom talk.

The symmetry of the star was another example of dissonance and discussion. Due to the cut lines not following the template, it did not match identically when Tom folded the shape. Ms Travis observed, '*so it didn't fold equally. He knew it wasn't symmetrical*.'
However, Asha said immediately that the star was symmetrical by just looking at it regardless of the accuracy of the cutting out. Ms Travis goes onto deliberate the different perspectives from Asha and Tom, expressing Asha accepted the interpretation of the shape of a star with equal sides and angles, but Tom looked in more detail at the dimensions of the cut-out star, which did not match and was not symmetrical, creating a discussion point.

Asha was looking at the star and saying '*yes, it is symmetrical'*, but Tom was looking at the exact dimensions of the star after the cutting out and saying '*no, it isn't symmetrical'*. They showed each other their stars and agreed they were both right!

4.1.4 Themes arising

The purpose of the lesson was to explore the properties of shapes by using symmetry. Ms Travis was keen for the pupils to use the language of conjecture and justification when reasoning. So an enquiry-based approach was being used to explore new concepts. The initial response from the teacher occurred when revisiting the context of the lesson, stating: *'Tom does get himself mixed up a bit sometimes,'* suggesting a degree of confusion around his subject knowledge.

The lesson's focus for Ms Travis was on an active co-construction of knowledge. She gave the pupils the freedom to explore and investigate the properties of shapes but with the added challenge of communicating and justifying their findings. Changing their minds was considered part of the learning process. However, during the reflective process, the teacher noted, *'So on paper, it might look like Tom [he] said it is symmetrical, but actually, by listening to that, he knows it's not symmetrical'*. The teacher identified that through participating in social practice, Tom and Asha were able to engage in dialogue, demonstrated a very good understanding of symmetry, and challenged each other's responses. The discussion provided a different perspective instead of only looking at their books or following intuition based on limited classroom observations. Ms Travis' initial impressions were that Tom was not listening or engaging in dialogue during the lesson.

Applying a deductive codebook, dialogue, and cultural practices would suggest a connected model of knowing.

4.2 Case B - Langfield School – word problem lesson

Langfield School was similar to Weston school, with approximately 130 pupils on roll but had a very different catchment area. The progress scores for the school have been consistent over the last decade, and the previous Ofsted inspection in reading, writing and mathematics were above the national average of 64% at 82%. This limited statistical information would suggest a school in which its pupils present minimal challenges and have the ability to achieve well in the core subjects of English and mathematics.

The lesson observed for this study took place at the start of the second term (January) of the academic year, focusing on the pupil's mathematical understanding of number to solve word problems, with a specific focus on mathematical language. Mr Taylor introduced the lesson to the whole class. He emphasised the importance of the language within each question because it could give clues to the mathematical operation needed to answer the questions. For example, if the word 'altogether' appears, it would suggest an addition sum, and 'sharing equally' might indicate a division operation.

The focus group used for this research were three boys, Andrew, Thomas and Jaden, with a specific focus on Andrew due to his active role during the perturbation observed. These three boys had come to my attention in discussion with Mr Taylor before the lesson. They needed to decide if they agreed or disagreed with the statement and then justify their decision and record their individual reasoning. Figure 6 illustrates their task.

Choose a number that could be correct then explain why you chose that number.			
19 X 5 =	84	95	93
Choose a number that could be correct then explain why you chose that number.			
32 X 2 =	64	47	49

Figure 6: The questions used during the first part of the word problem lesson, case B, Langford School.

Mr Taylor had set out a clear sequence of steps for the approach to the lesson, with the aim of practising skills already mastered but in a new context. First, they selected a slip of paper with a question on, as in the example illustrated in figures 6 and 7. The answers were recorded in their books with an explanation and justification below. The questions became progressively more cognitively challenging, as the illustration in figure 7, which shows multiple choice questions being replaced with a ‘why’ question and the language of division.

Why can I divide 34 into two groups equally?	Why can I divide 65 into groups of 5?

Figure 7: The questions during the second part of the word problem lesson at Langford School, case B, showing progression to division, also from multiple choice questions to 'why' questions.

It was this critical episode that was selected for the research to see how Andrew approached the question, and what he and Thomas did to try to resolve the problem. This included the dialogue between the two boys and Mr Taylor. Andrew and Thomas appeared to become confused when they started working on the questions in figure 6. They were distracted and started watching what Jaden was doing and giggling. Andrew then went to get Mr Taylor for help. Mr Taylor sat at their table, reading the question and then asking if 34 was in the two times table, at which point Andrew started to recite the two times tables and replied ‘*yes*’. Mr Taylor says ‘*yes because’*. At this stage, Andrew started writing and Mr Taylor talked to Jaden. Thomas asked Andrew what he had written and Andrew read it back to him.

Unfortunately, this was inaudible on the recording. Andrew asked Mr Taylor: ‘*now what do I do*?’ Mr Taylor read the next question, see figure 7. The discussion then continued around the number 65. Andrew was focused on the number 65 being an odd or even number, perhaps suggesting following the strategy of the preceding question as illustrated in this dialogue:

Andrew:	*The first number is ‘6’ so yes it is even.*
Mr Taylor:	*Does it change how you divide it into 5? What does the five times tables end in?*
Andrew:	*Five or zero.*
Mr Taylor:	*So can you divide 65 by 5?*
Andrew:	*May-be*
Mr Taylor:	*You think?*

Andrew: *Yes?*

(Transcript B)

Andrew then recorded his answer in his book and the lesson ended. It was this episode suggesting insecure subject knowledge of odd and even numbers and reasoning around the five times tables, which was used for the stimulated recall.

4.2.1 Teacher's initial response

At the beginning of the discussion with Mr Taylor, he conveyed his intention for the lesson. He wanted to develop their reasoning skills, focusing on rules, patterns and explaining their understanding. Mr Taylor felt they had started to do that but wanted more depth to their explanations. The teacher's initial intentions for this activity would suggest using conceptual understanding of a process to identify possible rules to justify their answers. However, applying a deductive approach would mean that a procedural model of learning emerges from the data, with step-by-step instructions and the style of questioning being used.

4.2.2 Pupils' review

I asked the pupils the purpose of the lesson, why they were being asked to look for patterns and rules, and justify their reasoning. Andrew associated this with doing the government key stage assessments, often referred to as SATs, stating, '*Because sometimes, when you work it out, you get an extra mark in SATs.'*

I was interested in guiding the discussion towards the mathematical content and I wanted to find out more about the motivation for his actions. During the mathematics lesson, Andrew appeared to be an active learner, confidently engaged in his work and occasionally talking to Thomas. When he started working on the second set of questions on division and multiplication, he became less focused and distracted. At this point, he goes and gets Mr Taylor to help him. I wanted to know why he had decided to do that at that point in time:

Researcher: *So what made you go and get him [Mr Taylor], which question?*

Andrew: *…me and Thomas got stuck on, so we didn't really understand what question meant… It says; why can I divide thirty-four into two groups equally? That was the first one. And then the second one was, why can I divide sixty five into groups of five?*

Researcher: *Okay. So what was it, which bit of that was confusing?*

Andrew: *It was this bit. (Pointing at the question).*

Researcher: *The whole bit? It is about dividing thirty four into groups.*

Andrew: *Yes.*

Researcher: *So what happened when you, normally get stuck like that, what would you usually do?*

Andrew: *We normally go and get Mr Taylor.*

(Transcript B)

When asked, Andrew explained a little more about how he approached the second problem, *'Why can I divide 65 into groups of 5?*' his verbal response supported the visual clues in the video footage of the episode where Andrew appeared unsure about what to do and explained that he struggled with this question so '*he had a bit of a guess'* and then Thomas told him before he '*realised what to do.'*

I wanted to find out more about what was causing the uncertainty within this particular question and what they could tell me about the number 65, so I questioned further:

Researcher: *What can you tell me about the number sixty five?*

Thomas: *It's an even number because it's got a five in it.*

Researcher: *OK, is five an even number?*

Andrew: *No, odd.*

(Transcript B)

This would suggest some confusion. I concluded the interview by asking the pupils how they felt at the end of the lesson. Andrew replied 'tired' I also asked if they enjoyed mathematics lessons, to which Andrew and Thomas replied 'yes' and Jaden agreed, saying 'yes, I think we have learnt a lot about maths in year two.' I then asked if they had any questions, which they did not. I thanked them for their time and ended the session.

4.2.3 Teacher researcher narratives

In discussion with Mr Taylor before viewing the boys' video excerpt, Mr Taylor expanded on the subject of context, which he speculated was the reason behind the confusion from the boys during this activity. The mathematical focus for this question was division, but when the questions were framed as word problems, they became confused, and their understanding of place value came into question. Andrew had said 6 was an even number in the number 65. Mr Taylor had gone on to explain the point of using word problems was to see just how secure the pupils were with their understanding of multiplication and division. The lesson had revealed underlying insecurity with place value and the features of odd and even numbers. However, Mr Taylor also raised the topic of context. One of the purposes of the lesson was to provide the pupils with the opportunity to use their mathematical understanding of the 2 and 5 times tables in different contexts. The other purpose for the lesson that Mr Taylor verbalised was the preparation for the government key stage assessments:

> *'And that's the unfortunate thing about being in year 2 isn't it, it's that you know that the SATs are awful in the fact it takes it all away from them. They need to have these coping methods to work these problems out don't they…It was lovely that Andrew said that. He's obviously listened really well.'*
>
> *(Transcript B)*

4.2.4 Themes arising

As indicated by Mr Taylor at the start of the discussion, the initial purpose of the lesson was to prepare pupils for the forthcoming key stage tests in May. As part of that preparation, pupils were being asked to apply the skills and knowledge they already had to a new context. Mr Taylor was framing this in the context of using mathematical language as a link or thread to support or scaffold their activity. During the introduction to the lesson, Mr Taylor emphasised the use of procedures and following strategies. He also stated pupils need 'coping methods' for working out problems. However, taking a deductive approach, the activity hinted at a connected model of knowing due to the justification and reasoning needed to explain their workings. Mr Taylor emphasised this again in his assessment of the lesson and suggested they need to extend their explanations:

> *'I think they could have explained it a little bit deeper. I will hopefully address that in the marking afterwards as well and make them go back and reason a bit further.'*

(Transcript B)

The most frequent and observed ecologies of teaching during this episode would suggest an active learner following a procedural model. However, looking through the pupils' lens at this excerpt, when Andrew described the way he overcame the struggle he was having with one question as having a 'guess' and then 'checking' with Thomas, would suggest the desire to reach the correct answer was greater than following the procedure. Again taking a deductive approach might suggest that he relies on others and accommodates new ideas instead of challenging them, thus fitting with a received or didactic way of knowing.
This case would suggest a combined picture of a procedural and connected way of knowing. Mr Taylor set out clear procedures for the class to follow with elements of a connected approach. He expressed his intention was for the pupils to actively construct their reasoning using their understanding of the strategy in a new context. Andrew, however, suggests more of an imitation approach through his observed behaviour of guessing and checking the answer with Thomas.

Following scrutiny by the teacher through observing the video of the pupils' responses, it transpired Andrew was unsure of the mathematical language being used and had insecure knowledge of place value.

4.3 Case C - Sharp School – place value lesson

Sharp is a small village school with approximately 120 pupils on roll. It has recently joined a multi-academy trust. Progress scores for the school for the last Ofsted inspection in reading, writing and mathematics were comfortably above the national average of 64% at 80%.

Miss White, the class teacher, had started the week by introducing formal written methods for column addition with her class of year 2 pupils. She had decided to repeat the lesson with a group of four pupils, three boys and a girl, because she wanted to 'take them through it again using the practical equipment.' I decided to observe Tim in this group with his partner Aaron.

The lesson started with Miss White explaining the aims for the lesson to the small group. She explained they were being introduced to 'a different way of adding numbers.' The teacher then recorded the formal written method for addition on the board, as illustrated in figure 8, and explained they would work out the calculation together in steps using this new method before they tried it independently.

$$
\begin{array}{r}
35 \\
+ \quad \underline{21} \\
\underline{\quad\quad}
\end{array}
$$

Figure 8: The calculation modelled by Miss White to the group in the place value lesson, Sharp School.

The pupils were using base 10 manipulative resources, in this case, Dienes blocks, as a visual representation for the structure of the numbers. Pupils can manipulate the Dienes to deconstruct the numbers into tens and ones to represent exchanging ten unit blocks for a single ten stick. (The terminology of 'ones' and 'units' are interchangeable; however, the currently accepted term by the DfE is 'ones'.) Initially, the pupils were asked to make the two-digit number 35 with the Dienes, represented by 3 tens sticks and 5 blocks representing ones, illustrated in figure 9.

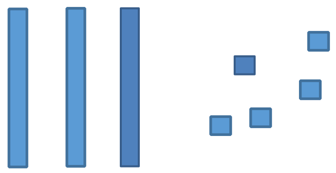
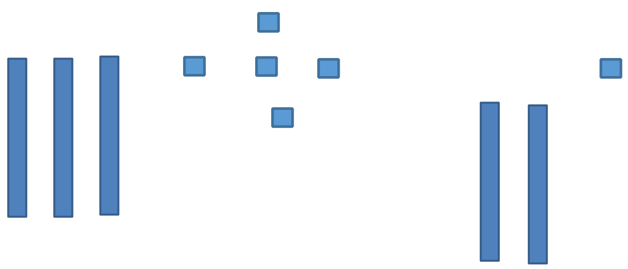


Figure 9: An example of the base 10 manipulative resources used to represent 3 tens and 5 ones during the episode at Sharp School, case C.

Next, the group were asked to make the two-digit number 21; two tens sticks and a single one block. Finally, they were asked to put them together to find the answer to the addition question recorded as a column addition. Miss White then modelled how it could be recorded in a formal written method. As illustrated in figure 10.



$$
\begin{array}{r}
35 \\
+ \quad \underline{21} \\
\underline{56}
\end{array}
$$

Figure 10: The representation of the column addition modelled, illustrating 3 tens and 5 ones plus 2 tens and 1 one, using Dienes during the episode at Sharp School, case C.

The next stage was to add two two-digit numbers but this time crossing the tens boundary, which means the ones column will be greater than 10, illustrated in figure 11. The pupils then needed to be able to exchange ten of their ones for a ten stick.

$$\begin{array}{r} 28 \\ +\underline{15} \\ \underline{\quad} \end{array}$$

Figure 11: The calculation used during the place value episode, case C, at Sharp School demonstrating a formal written notation for a calculation crossing the tens boundary.

It was the dialogue and actions at this stage that formed the episode for scrutiny.
Tim appeared to become very confused at this point.

4.3.1 Teacher initial response

Miss White recapped why she was teaching this particular session and how she changed it from the day before, acknowledging the pupils got a little lost towards the end:

> "*So yesterday we did a lesson with column addition without going through ten and these children didn't really get it, so I thought I would take them through it again. Back a step, get the practical equipment out. I think when I was bringing it into writing they were a bit lost." (Transcript C)*

Miss White indicated that she was interested in looking at the session again to pinpoint exactly where the pupils started to get puzzled.

4.3.2 Pupils review

My first question to the pupils was establishing what they understood they were learning, and Tim explained they *were 'learning how to make the number with units and tens.'* I went on to find out more about their understanding of tens and ones by asking *'why they had [have you] exchanged ones for tens?* Tim then became confused. He was unsure if he had exchanged 100 or 20 ones for a ten. I then returned to the original question of '*what have you learnt',* and Tim replied*, 'Well, I counted up the units and tried my best.'*

During the discussion with the boys, they focused on the answer, as shown here when I asked Tim and Aaron what they would like to do for the next lesson:

Tim: *I don't want to get my numbers wrong.*

Researcher: *When you say wrong what do you mean by wrong?*

Tim: *(making a cross sign with his finger) Just to like if I get it wrong – I'll do it again.*

(Transcript C)

I moved on to ask them what would help them practise, and Tim referred to getting the right answer; '*Knowing what we are doing and we know if we get the right answers.'*

Tim and Aaron indicated that they sometimes feel confused in maths lessons, so I tried to extend the discussion to find out more. They confirmed it was only in mathematics lessons, and they did not like it because it was hard. As the discussion went on, the boys became more distracted and fidgety, so I thanked them for their time and letting me watch their lesson before sending them back to the classroom.

4.3.3 Teacher researcher narratives

During the observation of the pupil's interview, we 'noticed' a pattern from the sequence of video clips; Tim appeared to get confused when he was working with the teen numbers, specifically fifteen and thirteen. Initially, in this episode, the individual pupils in the group successfully made the first number, 28, with the Deines blocks. Miss White followed this up by asking them all to make 15. Tim starts to look confused, looks at what Aaron was doing, and then selects his own Deines to represent 15. When asked how many ones and tens he had, he replied 'fifty ten'. Once the pupils had their Deines blocks representing the two numbers, Miss White asked them to exchange 10 of their ones for a ten stick. This stage appears to cause more confusion.

Tim: *(Looking on a bit puzzles but smiling)*

Miss White: *(To Tim) Can I have your ten?*

Tim: *(hands over a handful of ones, all thirteen of them plus all the tens)*

Miss White: *I only want ten of your units, count me out ten of your unit's*

Tim: *(counts out ten ones and gives them to the teacher in exchange for a ten) I have got a sticky (in a different voice, holding up the ten stick)*

(Transcript C)

Miss White states from this observation that she does not think *'he necessarily knows what he's got it for.'* Or that *'he has made that connection.'*

The group moved onto the next calculation 13 plus 30, this time avoiding the 10s boundary. They successfully represented each number with the Dienes blocks.

Miss White: *See if you can add them together*

Tim: *Thirteen one two three, four, five, six, seven, eight (Laugh) nine, ten, eleven, twelve (laughing) thirty twelve I've got thirty twelve*

Miss White: *Thirty twelve?*

(Transcript C)

Either Tim had jumped from twelve to thirty instead of thirteen or he had gone straight from twelve to adding thirty. It is not clear from the excerpt, and we cannot know what Tim was thinking or trying to communicate.

Miss White explains that she has separated the practical hands-on experience with the resources from the formal written strategies but felt that for a pupil like Tim, this introduces more confusion. Miss White discussed the context for learning and suggested Tim found the change of context very difficult.

4.3.4 Themes arising

Initially, the deductive coding for this episode would suggest a procedural model with Miss White talking the pupils through each stage of the process and then asking them to complete each task one step at a time. There was an element of active engagement when pupils were completing the tasks themselves at each step before checking with the group. However, Tim was finding it difficult to follow the steps. He was trying to imitate each step modelled to him by the teacher, fitting a received model. This resulted in confusion; by guessing, relying on his gut feeling, looking at the other pupils' work and answering in a questioning way, all in the pursuit of the right answer, it suggests Tim was lost and was trying to follow Miss White's procedural model.

Initially, the teacher noted when Tim was asked to record his findings, *'I think when I was bringing it into writing they [the pupils] were a bit lost.' T*he transition from the practical task to the more formal recording of the introduced confusion. The teacher and I observed Tim engaging in the social practice of constructing representations of numbers using manipulative resources. Tim appeared unsure, but the teacher also noticed Tim had withdrawn from the dialogue with the group and was reluctant to record his findings. From viewing the episode, the teacher noted *'They don't actually realise the maths we were doing was adding.'* and *'I don't think he's really got it at all. He obviously doesn't. I just can't understand why he didn't think; I have got three tens, and now I'll just count on from the thirty up to forty-three with the other ones.'* The observation by the teacher raised a question around Tim's subject knowledge of place value and his understanding of teen numbers.

4.4 Case D - Northolt School – geometry lesson

Northolt School is larger than the other four schools, with approximately 250 pupils on roll. The school has above average challenges with regard to the socio-economic background of its pupils but performs just above the average in the core subjects. The school is situated in the conurbation of the Midlands. It is an area of traditionally high unemployment due to the closure of the mines within

The year 2 mathematics lesson observed for this research was a continuation lesson from the day before. I decided to focus on the group of six pupils Mrs Armid, the class teacher, would be working with on that day. The pupils looked at the properties and names of two-dimensional (2D) and three-dimensional (3D) shapes. Mrs Armid had planned to work with this group because she felt they needed more practise naming 3-dimensional shapes by identifying their properties. I selected Destiny, a girl in the group, as the primary focus. Maci, her friend, also took part in the review process.

The teacher asked the group to play a form of bingo. Instead of having a number caller, they took turns picking up a card with the name of a 3-dimensional shape on and if they had an illustration of that shape on their bingo card, they covered it over. For example, if the pupil picked up the word ‘sphere’ and they had an illustration of a sphere; they kept the card and covered the picture.

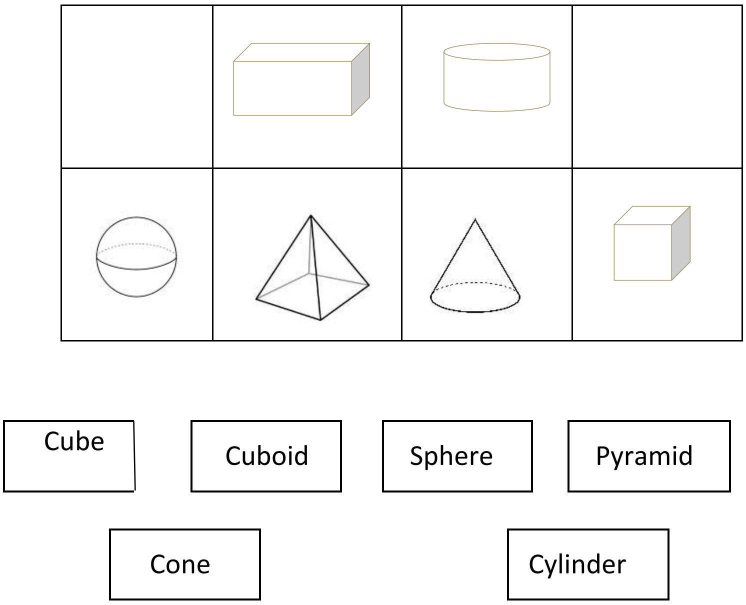


Figure 12: An example of the bingo and name cards used during the geometry episode at Northolt School, case D. The game is to identify and practise the properties of three-dimensional shapes by matching the word to the picture.

Before Mrs Armid joined the table, the pupils had set up the game and were taking turns selecting a word card and matching it with the illustration of the 3-dimensional shape on their card, if they had one. Destiny appeared to be following the game's progress, taking her turn, matching her card and observing pupils taking their turn. The pupils did not engage in discussion beyond whose turn it was next. Destiny appeared to follow what was happening around her and engaged in the activity, taking her turn and watching the other children take theirs.

Mrs Armid had ensured the whole class were on task and working in their groups (approximately 5 minutes) before joining the group. She asked the pupils to verbalise the names of their shapes when it was their turn and checked their knowledge of the names and properties of the shapes. A child on another table distracted Destiny, and Mrs Armid drew her back by asking her questions about her shapes. Destiny used the word 'square', a two-dimensional name, for the illustration of a cube, a three-dimensional shape. At this point, Mrs Armid decided to introduce the resources, in this case, three-dimensional shapes stored in a nearby cupboard. Mrs Armid demonstrated how to hold a 3-dimensional shape to see how it differed from a two-dimensional shape. The following interaction occurred between Mrs Armid and Destiny:

Mrs Armid: *Destiny maybe you could describe this shape? (Holding a tennis ball.)*

Destiny: *It's a tennis ball*

Tom: *It's just a tennis ball. (Playing with it.)*

Destiny: *It's a spear*

Mrs Armid: *Does it have any vertices?*

Destiny: *(Shakes her head.) No*

Mrs Armid: *(Holds up a square based pyramid.) Does this have any vertices?*

Destiny: *Yes*

Mrs Armid: *How many?*

Destiny: *(Points her finger towards the shape but doesn't touch it mimicking counting to five.) Five*

Mrs Armid: *Five vertices you can count them. (Handing the shape to Destiny and picking up the tennis ball.) This sphere is curved and has one edge, one side. (Puts down the tennis ball.) Can you describe this shape? It's got triangle faces and a square base. Can you describe the vertices, count the edges.*

(Transcript D)

Mrs Armid leaves the 3 dimensional shape of a square based pyramid with Destiny. Destiny talks to herself, counting the edges and faces: *There are five edges, four faces.* Mrs Armid returns to Destiny.

Mrs Armid: *Can you describe that now to me.*

Destiny: *Five vertices*

Mrs Armid: *Can you show me?*

Destiny counts as she points to each vertices.

Mrs Armid: *What else can you notice? How many edges?*

Destiny: *Five*

Mrs Armid: *How many edges look these are edges and so are these.*

Mrs Armid touches all the edges and hands the shape back to Destiny to count.

Destiny: *(touching each edge) Seven*

Mrs Armid: *Oh let's see (Destiny corrects herself and says very quietly eight) let's just wonder. Maci can you do all of us a favour. Children, Maci is just going to count the edges for me because we are getting a bit stuck here.*

(Transcript D)

This interaction formed the episode that was scrutinised for the purpose of this study.

4.4.1 Teacher initial response

Mrs Armid started by describing the activity to me and explaining why they were doing it; '*to secure the pupils' knowledge of 3-dimensional shapes'.* She felt it was important that the pupils had clear steps to follow and made sure she had modelled the process well. Mrs Armid also noted that she thought the pupils in the group already had a good idea of the properties of the shapes, but Destiny stood out from the group as needing more support. Destiny was conflating the names of the faces with the names of the 3-dimensional shapes.

4.4.2 Pupils review

During the lesson review, Destiny and Maci covered a wide range of topics, from films on YouTube to the colour of their eyes. However, they always came back to the mathematics lesson and 3-dimensional shapes. Destiny expressed her lack of experience around shapes, and the two girls discussed the pyramids. Maci had seen real pyramids from Egypt on a YouTube game, but Destiny said she had never seen a '*pyramint*', she also demonstrated insecurity with the names for some of the 3-dimensional shapes calling the pyramid '*pyramint*' and the sphere '*spear*.' Destiny went on to explain; '*I needed the shapes because I've not done that type of shapes.'* Destiny had an insecure understanding of the subject, as demonstrated in these three excerpts:

Researcher: *Can you tell me anything about 2D and 3D shapes?*

Destiny: *3D shapes are big and 2D shapes are flat.*

Researcher: *Did you need the shapes or did you already know?*

Destiny: *I needed the shapes because I am no good at telling the shapes.*

Researcher: *Is it the names of the shapes you find tricky? (Pause) Or is it anything else about the shapes you find tricky?*

Destiny: *It's, I don't know how to explain it.*

(Transcript D)

The two girls appeared to enjoy the time to discuss their mathematics, so I thanked them for their time and interesting ideas before they returned to the class.

4.4.3 Teacher researcher narratives

Mrs Armid immediately identified that Destiny was conflating two and 3-dimensional shapes, calling a pyramid, a triangle and a cube a square. When Mrs Armid recognised the misconception or naïve concept arising, she got the manipulative resources. She explained that a 3-dimensional shape could be held but not a 2-dimensional shape. Mrs Armid clarified that she felt Destiny was getting a bit confused, so she decided to get out the 3-dimensional shapes because the bingo game had 2-dimensional illustrations of the shapes. *Mrs Armid stated, "Destiny needed a bit more support and modelling and help with that."*

We discussed why the decision was made to use the 2-dimensional bingo game to teach the properties of 3-dimensional shapes. Mrs Armid explained she wanted to introduce a change of context so the pupils could transfer their knowledge of 3-dimensional shapes to 2-dimensional representations.

> *"I do feel that the children should be exposed to different ways of seeing things from different points of views as well. So I do feel, they've had this hands on experience of the different shapes… When they go further up the school, they're expected to know, and also, you look at exam, test papers, etc… I think to have both is important.* (Transcript D)

The discussion with Mrs Armid continued around teaching 2 and 3-dimensional shapes and the preparation for assessments further up the school. During the episode filmed, the pupils showed evidence of understanding 2-dimensional shapes in their dialogue. Still, Destiny was conflating the names of 3-dimensional shapes with the 2-dimensional ones, which could be either conceptual or terminological confusion.

4.4.4 Themes arising

Mrs Armid established that the learning experience was intended to practice the skills and knowledge already acquired and develop fluency. The class had been learning the names and the properties of 3-dimensional shapes, and Mrs Armid wanted this particular group to apply their skills to the bingo game providing an opportunity to assess their understanding. Mrs Armid explained that the bingo game made it more fun and engaging.

Destiny had conflated two and 3-dimensional shape names during the geometry episode. Initially, the teacher noted she had to; '*remind Destiny [her] again because she struggles to maintain retention and listening*'. Through deductive coding, Mrs Armid's focus on memory and retention would suggest a received model of knowing. The lesson began with a reminder of the names of the 3-dimensional shapes and their properties. When Mrs Armid joined the group playing the game, she asked questions to assess the pupil's knowledge, alluding to specific desirable answers. When a misconception or naïve concept emerges, Mrs Armid intervened and got the manipulative resources to demonstrate the 3Dness of the shapes to Destiny. Destiny took the role of a passive receiver, listening and watching Mrs Armid's explanation. Later through dialogue with Destiny, it appeared she had previously had limited opportunities to interact with 3-dimensional shapes. *'I never watched pyramids on YouTube, like Maci*,' suggesting a figured world with little experience of exploring 3-dimensional shapes, unlike her friend.

The game of bingo introduced an activity in which pupils could make choices based on their interpretation of the game's rules, but the teacher controlled the mathematical learning experience.

4.5 Case E - Felton School - fractions

Felton school is the smallest of the cases, with approximately 30 pupils on roll. The headteacher Mrs Greenway was also the year 2 teacher for mathematics. The school is situated in a small village in the centre of the country. Fractions were an area Mrs Greenway had identified as an area of weakness from the recent government key stage assessments.

Therefore, she had decided the theme of the mathematics for that week would be fractions. Before the lesson, we discussed specific children the teacher would be interested in observing closer. Two girls, in particular, Mai and Jill, stood out for me. Mrs Greenway felt Mai could do the work, but she regularly asked for help declaring she was '*stuck*'.

The main lesson started with all the pupils on the carpet to introduce that day's lesson. The pupils had been learning about fractions that week, and this was the second lesson. Mrs Greenway asked the pupils to record how they could find half of twelve on their whiteboards. To find half of twelve, Mrs Greenway demonstrated the equal sharing structure by drawing two circles and showing making a mark in alternate circles until she had made twelve marks. She then counted how many were in one circle for the answer. She then modelled each stage using jottings, see figure 13.

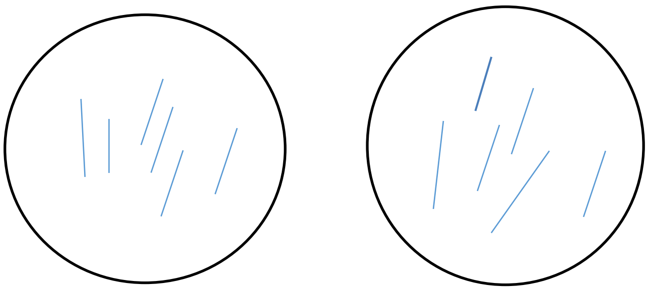


Figure 13: The teacher's jottings to model how to find 1/2 of 12 used at the start of the fractions lesson and referred to during the episode, case E.

Mrs Greenway worked through another example before sending the pupils off to their tables to start their challenges. She informed the class they could work in pairs if they wanted to. In practice, they all chose to work on their own independently. Mrs Greenway suggested this could be because they had just finished doing their key stage assessments and were now in the habit of working independently. The pupils appeared to be on mixed ability tables. The pupils were able to select from groups of differentiated questions. The groups of questions put on the board were categorised and labelled ‘Apprentice’, ‘Qualified’, ‘Master’ and ‘Master plus’, as illustrated in figure 14. The pupils selected the questions they felt were for them. They then recorded their work in their books, following the same procedure as the modelled part of the lesson.

Apprentice	Qualified	Master	Master Plus
½ of 8	¼ of 12	½ of 18	2/4 of 16
½ of 12	¼ of 16	¼ of 24	3/4 of 12
½ of 4	¼ of 20	1/3 of 12	2/3 of 12
½ of 10	¼ of 8	1/3 of 9	¾ of 16
½ of 16	¼ of 40	1/2 of 22	5/6 of 30

Figure 14: The challenge board presented to the pupils at Felton school, case E, which was used during the fractions episode. Pupils selected the column of questions they felt able to start with.

Mai started on the 'Apprentice' group of questions, recording the calculation and the answer with no working out for the first four questions;

½ of 8 = 4 ½ of 12 = 6 ½ of 4 = 2 ½ of 10 = 5

The fifth question; ½ of 16 she included jottings of two circles with eight dashes in each, following the model in figure 13 but using 16 instead of 12. Mai then moved onto the set of questions entitled 'Qualified' and had recorded just the calculation in her book; ¼ of 12 = 3. The next two questions she included more jottings with each question, illustrated in figure 15, she followed the same procedure as demonstrated at the start of the lesson.

¼ of 16 = 4

¼ of 20 = 5

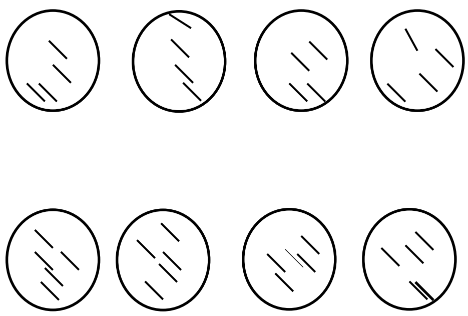


Figure 15: Mai's notations for two of the questions as recorded prior to the episode on fractions and then referred to during the episode. Felton School, case E.

Mai moved to the 'Mastery 'section at this point. It was the first time Mai had put her hand up to ask for help from the teacher. Mrs Greenway instructed her to: *'Look back at how you did the last one'.* She then wrote out the calculation again for Mai in her book. I was interested in why Mai had decided to ask for help on this particular question without trying to use the jottings she had done before.

4.5.1 Teacher initial response

Mrs Greenway acknowledged her satisfaction at the work the class had achieved during the lesson stating; '*Yes I think they managed that very well and were able to record in their books. Everyone was working quietly and there weren't any distractions.*' Mrs Greenway then focused on Mai and Jill:

> *"And they [Mai and Jill] had it. I was looking in the book because originally, previously Mai had been in a real pickle, but Mai had it on the whiteboards this morning and I thought 'oh they have remembered, let's let them go and do it in their books and see.' Her [Mai] and Jill had it so I thought there we are then."*
>
> (Transcript E)

Mrs Greenway expressed her opinion that Mai and Jill were in a position to be able to complete the task based on the evidence from the introduction to the lesson.

4.5.2 Pupils review

The discussion with Mai was limited. Mai had been working independently during the lesson, and Jill had not interacted with anyone. Mai's interaction with the teacher was one-to-one, so I felt it inappropriate to discuss this with another pupil. I set up the video review and discussion with Mai in a quiet area but in open view of the main hall. The intention was to ensure Mai did not feel isolated or under pressure. I also asked if she wanted to continue or preferred to return to class. Mai expressed she enjoyed talking and continued. Mai avoided answering any questions focused on mathematics but discussed her friendships. She also spoke about some of the things she had been thinking about during the lesson. As illustrated at one point during the episode when Mai started to fiddle with her hair and looked across the classroom, so I asked her what she was thinking:

Mai:	*About Rapunzel, I was looking at my book last night.*
Researcher:	*You weren't thinking about your maths?*
Mai:	*(Shakes head) Rapunzel has long hair.*

(Transcript E)

I asked Mai what would have helped her with her maths that day. She replied she would like to have had her '*Pizza thing'* to help her. There is a manipulative resource in the form of a pizza game used in school. The resource aims to provide a visual representation of ratio structures for division, thus looking at the size of the pizza sections representing different fractions of a whole pizza. I discussed this further with Mrs Greenway in section 4.5.3. I did not want to keep Mai from her class for very long on her own, so I thanked her and sent her back.

4.5.3 Teacher researcher

Mrs Greenway explained that Mai often worked well but would unexpectedly stop and ask for help, saying she was stuck, as illustrated in the excerpt. During the excerpt, Mrs Greenway observed that Mai had already completed a similar calculation to the one she was stuck on, so she explained to Mai that it was the same as what she had been doing and showed her similar ones from her book. At this point, Mai was able to continue with no further explanations. Mrs Greenway explains to me that the pupils can be stuck when moving from calculating halves to calculating quarters because they continue to record two groups instead of four, which could be a possible reason for Mai's confusion. However, Mai had already completed some questions with quarters correctly, and this was now halving again. Mai successfully worked out half of 16 using jottings, but when she had to halve 18, she immediately recorded 14 but appeared confused and asked for help without trying to use the jottings. Mrs Greenway observed: '*So she knows this doesn't she ½ of 12 is 6. But ½ of 16 she has had to work out. So when she is over here, and it is half of 18, she's forgotten that.'*

In the episode observed, Mrs Greenway reminded Mai to use visual representation or jottings to represent halving 18. She then wrote out the question again in Mai's book for her, and Mai drew the circles and recorded 9 as the answer. Further discussion with Mrs Greenwood and observation of the episode when Mai recorded half of 18 as 14 suggested that Mai might have forgotten to halve the 10, and knowing that half of 8 is 4, she just recorded 14. Still, she knew something was wrong because she asked for help. When Mrs Greenway had watched the discussion with Mai, she stated the pizza game would not have been appropriate for this lesson. Mai was practising an equal-sharing structure for division, sharing out ones equally into determined groups, as opposed to ratio structures for division, so the pizza would have been of limited if any value in this situation, as Mrs Greenway pointed out.

4.5.4 Themes arising

The purpose of this lesson was to revisit fractions and develop fluency. Mrs Greenway modelled how to work out the fractions through an equal-sharing structure, and then the class worked through examples, applying that model. Using deductive coding, Mai appears to be actively following a procedural model, following the steps modelled by the teacher. Mai asked for help when she moved from 'qualified' to 'mastery' level questions from the differentiated tasks. She was aware something was wrong, and her strategy was to ask for help, not to go back and look at what process she had followed before, suggesting a reliance on the teacher. Mai also displayed an interesting mix of working from memory and following the strategy, presenting a blend of the received and procedural model. Mai could not apply the strategy the teacher had modelled for her when faced with the next set of differentiated questions. Mrs Greenway construed that Mai's insecure knowledge of the strategy and memory had created the perturbation, stating she had been able to complete other questions using memory, or strategy, or a combination of the two. However, through further discussion with the teacher and observing the episode more closely, we observed when Mai moved
onto the next group of differentiated questions, the classification of the tasks might introduce a barrier or point of deliberation. The change of questions could signal to Mai the questions were becoming more complex and perhaps they were not for her, or maybe she felt she had just done enough!

4.6 In Conclusion

To review a selected episode, I intended to find out more about these five primary mathematics classrooms' figured worlds through multiple lenses. I inductively and deductively explored the data from the five cases to ensure transparency and rigour. Each case, in turn, provided a unique insight into the figured worlds of the individuals within the episodes observed within the classroom setting. Even though each case was unique, they also had similarities.

A similarity between the episodes selected from the five cases was that the episodes showed the figured worlds of a classroom when pupils got, in the words of the teachers, 'stuck', 'muddled', 'confused' or 'in a pickle'. All the teachers discussed the context they had placed the mathematics in for the pupils to apply their skills. They felt the transfer of skills from one context to another introduced perturbation. In four of the schools, the pupils outwardly expressed their confusion, audibly, through the action of asking for help or from their physical expressions. In the fifth school, the symmetry lesson, the disparity between the written and spoken word, as opposed to a possible lack of understanding of the concepts, raised my interest.

Using the figured world framework enabled the teasing out the ecologies of teaching established by the teacher and the positional identities of the pupils. The inductive accounts provided nuanced detail for each case, placing it within a context and offering clues to possible figured world indicators. The deductive approach helped categorise the events and put them within the figured world framework.

The use of inductive analysis helped to look at the finer detail of each case within its broader context, whilst the deductive approach allowed a more precise classification of codes and themes. Two of the cases fitted distinctly into two themes; received knowing in the geometry lesson; here, the teacher delivered the lesson, and the pupil relied on the teacher when naïve concepts appeared. The symmetry lesson clearly illustrated a connections model where pupils engaged in active co-constructed dialogue and had elements of control of their learning. The other three cases were not so clearly defined, and the link between the ecologies of teaching and the positional identity of the pupils was not as visibly distinguished.

The methodological approach to use multiple views of an episode and to exploit both inductive and deductive methods for the analysis has provided an opportunity for greater insight into the figured worlds of the mathematics classroom. Each case is unique, and every figured world is pertinent to the individual. Still, the interconnectedness within the mathematics classroom between the ecologies shows some connections, which I will detail in chapter 5.

Chapter 5

Discussion

'…the *practices* of learning mathematics define the knowledge that is produced. '

(Boaler and Greeno, 2000: 172)

5.0 Introduction

The previous chapter presented five separate mathematics episodes from multiple viewpoints, employing a case by case approach. This chapter will compare the different features within those cases, addressing the research questions and viewing the mathematics classroom through a figured world lens. As part of the methodological approach, a perturbation was selected from each lesson providing an episode for greater analysis. The episode was only a tiny section of the overall lesson, which could be considered restrictive. However, the intention was to look at the fine detail within the episode instead of a broader view. The episode selection was made following the lesson. I chose the moment for review and consulted with the teacher to ensure they also found this a moment of interest. A different person at a different time could select another episode on the day of data collection, so generalisability could not be claimed. However, patterns could be seen across the different episodes concerning the individual figured worlds and the themes of dialogue, cultural artefact and identity.

The methodological approach based on Mason's (2002) discipline of noticing provided a fresh and interesting insight into what these teachers' paid attention to during the reflective process. I considered each teacher's initial response to the lesson. The teacher was then put in the role of observer, reviewing the episode from a new perspective and in much greater depth, studying the video of the perturbation selected and the pupils' responses to that video. Both the teacher and pupils were able to take time and space to reflect on and recount their thoughts about their episode. The key emerging themes of identity, dialogue and ways of using cultural artefacts emerged from viewing the episodes through a figured world frame. The merging of Belenky *et al's* (1986) categories for ways of coming to know and Sfard's (1998) metaphors for learning offered the theoretical approach to analyse the events across the different episodes.

An outcome of studying these figured worlds was the extent to which the pupils had developed a stronger or weaker identity towards the subject of mathematics. A stronger mathematical identity would suggest pupils developing the belief that they are mathematicians through developing mathematical habits by engaging in mathematical activity (Aguirre, Mayfield-Ingram, and Martin, 2013. Boaler, 2002). A weaker mathematical identity would be associated with just doing an activity void of the associated mathematical habits and any belief of being a mathematician. Based on the level of mathematical thinking afforded pupils through dialogue and cultural artefacts, the suggestion emerges that differently figured worlds can generate distinct learning experiences and ways of working with mathematics.

Pedagogical approaches which, embrace and merge the individual pupil's development within the social environment of the classroom, would suggest that the teacher is paying attention to aspects of learning beyond curriculum considerations. Without this approach, it might lead to an imbalance of influences in figure 1 section 2.2.2. Finally, I recommend the implications for practice in teaching mathematics.

5.1 How the teachers from each episode construe the perturbations.

The findings from my research resonate with Boaler and Greeno's work in secondary mathematics classrooms in 2000 and Wickstrom's work on mathematical modelling in 2017. These studies found that pupils in differently figured mathematics worlds came to know mathematics depending on how the teacher enacted their pedagogy. The teacher's epistemology determined how pupils engaged with the mathematics based on their learning experiences. The studies adopted the same figured world framework and featured Belenky *et al's* (1986) ways of coming to know. However, Boaler and Greeno's (2000) methodological approach differed from mine in that they interviewed the teachers about identities regarding knowing and learning mathematics. Wickstrom (2017) introduced modelling tasks to challenge the teacher and pupils' perceptions of mathematics and positionality within the activities. In my study, I have engaged the teacher dialogically to support the reflective process to help them interpret the pedagogy and learning, and the implications of what they observe. To enable this, I facilitated stimulated and enhanced recall through video, allowing the teacher to stand outside of the event to review their practice. The teacher's perspective shifted from participant within the teaching episode to reflective onlooker (Van Manen, 2014).

The methodological process provided a good quality learning experience for both the teacher and myself, an opportunity that would develop both the teacher's practice and mine as a teacher educator. The practise could expand our understanding of what is happening in mathematics classrooms. To help recognise how pupils respond to events that happen within a mathematics lesson it is vital to be able to experience being put in the position of learner. It is often difficult to transition between being an educator and a learner when you are a teacher. However, it is essential to be a lifelong learner to understand the position the teacher can put their pupils in when learning mathematics. The episode in conjunction with the stimulated recall was observed. The teacher and pupils had time and space to recollect their learning experiences, being reflective and reflexive on the learning experiences within that episode.

5.1.1 Initial views of the five cases

The teachers from each case acknowledged that all the pupils were active participants, engaging and applying themselves to the learning task consistently. The episodes selected roused an initial response and judgement from the teacher and me, identified as a perturbation. The two clear categories, which appeared to be responsible for the perturbation across the five cases, according to the teachers, were; weak subject knowledge due to poor memory and problems when applying mathematical knowledge and skills to a new context. For example, in case C, the place value lesson, Miss White stated, *'I think when I was bringing it into writing they were a bit lost.'* Suggesting the transition from the practical task to the more formal recording of the mathematics introduced confusion. In case E, the fractions lesson, Mrs Greenway construed that it was Mai's insecure knowledge of the strategy and poor memory that had created the perturbation. Focusing on memory and context would suggest a Piagetian (1929) constructivist view of the five cases. The teachers construed their understanding by reflecting on the pupils interacting in their environment and its consequences.

By extending to teachers the opportunity to view the episodes as observers and to take part in reflective narrative, new possibilities for reflective and reflexive practice were possible. The teachers' emerging themes focused on the pupils' insecure subject knowledge due to inexperience and the use of cultural artefacts. Differentiated activities and insecure knowledge suggest a more social-cultural influence, introducing barriers or constraints within the learning environment (Boaler, 2016). For example, in case C, when observing the video of the pupil's responses, Mrs White stated: *'I don't think he's really got it at all… They don't actually realise that we were doing adding'.* The teacher saw that they were following their instructions but with little to no understanding of why. In case E, when Mai moved onto the next set of differentiated tasks, it appeared to signal that the questions were becoming more complex and perhaps inferred they were not for her.

By providing a different view of the learning environment, teachers developed from being reflective and accounting for their teaching to becoming more reflexive and developing greater insight into pupils' learning experiences within their figured worlds resulting from the teacher's privileged pedagogical approaches. The teachers were able to take the role of learner and investigate their own understanding of learning in relation to their pupils' perspectives. As a teacher educator, I was interested in the types of mathematical thinking and interactions the pupils were having with their peers. I could see the impact the process was having on the teachers as they observed and discussed the responses and comments of the pupils.

5.1.2 Synthesis of features from each case

Each of the five cases exhibited features that characterised ways of coming to know from across the received, procedural and connected models. Table 6 categorises the specific features concerning each episode with regard to the themes arising and particular ways of coming to know; it is colour coded using the following key:

Table 6 Key to colour code features of:
Received Model
Procedural Model
Connected Model
Ontology of fixed knowledge, transcending both received and procedural models. (Sfard's(1998) acquisition metaphor)

Concepts of ways of coming to know	**Case A Symmetry Case**	**Case B Word Problem Case**	**Case C Place Value Case**	**Case D Geometry Case**	**Case E Fractions Case**
Origin of knowledge for pupil (Ontology)	Knowledge created by pupils. Use of open questions e.g. How could you make this shape symmetrical?	Teacher transmitted knowledge. Modelled how to answer a question from a Sat's paper.	Teacher transmitted knowledge. Taught steps to complete a column addition calculation.	Teacher transmitted knowledge. The teacher recapped the properties of the 3D shapes before the pupils played the game.	Teacher transmitted knowledge. Modelled how the answer the question.
The pedagogical approach of the teacher (Epistemology)	Enquiry approach. Investigating the properties of shapes through their symmetry.	The teacher scaffolds the learning through strategies for pupils to enact. Drawing on prior knowledge to answer new questions e.g. Is it an even number?	The teacher scaffolds the learning through strategies for pupils to enact. Each step modelled. The pupils reproduce the step.	Pupils receive information to apply to a set task. The pupils practised their knowledge of 3D shapes through the bingo game.	Pupils receive information to apply to a set task. The teacher and pupils did a question together and then the pupils worked through the questions.
Identities afforded by the teacher to the pupils	Pupils are constructing their knowledge and developing their mathematical thinking skills through discussion and conjecture.	Pupils are afforded limited opportunities mathematical thinking through structured forms of engagement. The pupils referred back to the teacher when challenged.	Pupils are afforded limited opportunities for mathematician thinking through structured forms of engagement. The pupils waited for the teacher for the next step.	Pupils as passive receivers of information. When the pupil's knowledge was unsure, the teacher modelled with the resources.	Pupils were passive receivers of information. Pupils reproduced the strategy modelled to them. When challenged they referred back to the teacher.
Nature and direction of dialogue	All pupils engaged in communication focused on challenge, criticality, conjecture and debate. Dialogic interactions between pupils.	Teacher to pupil and limited pupil to pupil dialogue. Justification of procedure. The teacher used questioning to scaffold the direction of thought. Can the answer be in the five times table and even?	Teacher as transmitter and pupils as the receiver (Asymmetrical interactions). The teacher said 'Show me your tens, now show me your ones, now put them together.'	Teacher as transmitter and pupils as the receiver (Asymmetrical model) Recall focused questioning. The teacher told the pupils the properties of the shapes.	Teacher as transmitter and pupils as the receiver (Asymmetrical model). Recall focused questioning. When pupil asked for help, the teacher referred back to the example shown.
Ways of using Cultural Artefacts	Pupils access manipulative resources to test out a range of possible outcomes. Mixed ability pairs.	The use of power point and white boards to transmit information. Textbooks for pupils to record set tasks, where tasks are bound to ability groups.	The teacher provides manipulative resources as scaffolding to the strategy for this ability group.	The teacher provides manipulative resources as a visual aid for the pupils who could not recall the answers in this ability group	The use of power point and whiteboards to transmit information. Textbooks for pupils to record set tasks. Ability groupings.

Table 6: Synthesis of the features of each case concerning figured worlds and ways of coming to know, based on table 5.

The next section discusses the analytical framework used to draw on these themes.

5.2 Identity within the figured worlds of the mathematics classroom

According to Holland *et al* (2001), pupils' perceptions and experiences of the mathematics classroom and developing their identity, concerning learning mathematics is less about engaging in self-making and more about the social environment they occupy. It is appropriate to generalise by extrapolating from individual figured worlds that change from pupil to pupil and moment to moment. However, looking at small moments in time offers a glimpse of a new perspective of the mathematics classroom. The environment afforded to pupils by the teacher at that moment allowed pupils to engage in mathematical thinking to a greater or lesser extent concerning the mathematics they were learning (Priestley, Biesta and Robinson, 2015).

During the episodes observed, all the teachers included an element in their approach that encouraged an identity of independence to a greater or lesser extent, from setting enquiry based activities to playing a game or providing manipulative resources for pupils to create their images to support understanding. Teachers reported incorporating opportunities for greater independence through the planned activities, but this seemed to introduce tension within some of the cases. For example, in the game of Bingo in case D, Mrs Armid provided an opportunity for pupils to work independently of her, making their own decisions. However, when Mrs Armid intervened to assess progress and understanding, asymmetric dialogue was used, introducing a didactic approach characteristic of a received model. A model that does not support mathematical behaviour. Using the categories of received, procedural and connected ways of knowing can categorise mathematical identities afforded pupils by the teacher. The following sections explore each type.

5.2.1 Development of identity within a received model

A fundamental characteristic of a received model is the teacher adopting a didactic approach. This approach was evident in cases D and E. When Mrs Armid, in case D, intervened to assess the pupils’ understanding of 3-dimensional shapes through questioning, Destiny could not describe the shape concerning its properties using the appropriate mathematical terminology. Destiny explained during the stimulated recall session that she had not had very much experience of 3-dimensional shapes. Mrs Armid stated:

> "*I've gone back there to show her that that's a cuboid. So I need to go back and get the actual shape for her so that she can see it, she can feel it and learn a kinaesthetic way of learning as well."* (Transcript D)

Destiny could be construed as a receiver of knowledge, limiting the development of her identity as an independent thinking mathematician. Mrs Armid remained in control of the artefacts and the ecology of the environment.

In case E, the fractions episode, Mrs Greenway also enacted a received model, beginning the lesson by demonstrating how to answer the example question. The pupils then practised similar calculations in their books, imitating the strategy shown. Pupils were afforded a degree of independence by choosing from differentiated questions labelled by the teacher as; Apprentice, Qualified, Mastery, Master plus. Differentiated tasks introduced the cultural artefact of a fixed ability environment where pupils identify with a specific group (Boylan and Povey, 2021). Mrs Greenway noted: *'I would say go onto mastery level and she (Mai) would say 'no I want to stop now.*' Suggesting Mai exercised her limited agency to control her environment by choosing to stop, thus withdrawing from the experience.

5.2.2 Development of identity within a procedural model

In the procedural cases, the teachers' privileged pedagogical approach was to scaffold the learning through strategies that the pupils could then enact, similar to the received model, but which introduced an opportunity to develop and express their understanding of the strategy. Cases B and C illustrated this approach. The environment-specifics for these groups of pupils provided step by step instructions. Unlike the received model, it introduced smoothing the way through either structured questions or manipulative resources to support the pupils' understanding of the strategy.

In case B, the intention of Mr Taylor was for pupils to justify their answers during the lesson, with a focus on articulating their understanding of multiplication, suggesting an activity to encourage pupils to formulate their understanding, as illustrated here by Mr Taylor;

> *'…so the idea was to just get that multiplicative reasoning, …and then start thinking what are the rules, what are the patterns and understanding and getting them explaining that.'*
>
> *(Transcript D)*

However, in this case, it focused on maintaining a pupil's identity associated with reliance on a process owned by the teacher. Instead of exploring understanding through what Wing (2016) described as 'patter', thus justifying a procedure.

Mrs White, in case C, introduced a different way of getting pupils to engage in understanding the strategy. The teacher instructed the pupils to represent specific numbers with manipulative resources. The purpose of this was to introduce visual representations of the concept of place value as a pedagogical approach to support understanding. Mrs White, the teacher, gave her reasoning for this particular approach by saying:

> "*So yesterday we did a lesson with column addition without going through ten, and these children didn't really get it, so I thought I would take them through it again. Back a step, get the practical equipment out."* (*Transcript D*)

The teachers in these cases had offered carefully structured support for the pupils and focused on understanding the strategy, in doing so, had inadvertently limited pupils' opportunities to think mathematically.

5.2.3 Development of Identity within a connected model

Case A illustrated the connections model through pupils' identity. Ms Travis, the teacher, appeared to afford pupils a mathematical identity of 'thinking agent', through which they constructed their knowledge using enquiry methods and were, encouraged to use reasoning and conjecture. Tom and Asha had developed a stronger identity associated with an independent thinking mathematician. Pupils achieved this through the teacher's social practices of negotiation and interpretation through the engagement in certain types of activity. A stronger mathematical identity would be characterised through pupils taking the role of a mathematician independent of the teacher. Both the activity and pedagogical approach of the teacher helped to establish an ecological environment in which Tom and Asha could engage in mathematical thought, a feature of the connected model.

The data interpretation would suggest that the opportunities the teachers' afforded pupils influenced their identity formation within their figured world of the mathematics classroom. Table 7 summarises how the teachers' pedagogical approaches from all five cases have contributed towards this:

	The teacher's pedagogical approach	How teacher's pedagogy positioned the pupils to learn mathematics	Identity within the figured worlds
Case A Symmetry	Enquiry approach reflecting the connected model. Teacher as a facilitator.	Tom and Asha investigated the properties of shapes, using socio-cultural interactions. Investigating multiple realities.	The pupils took on the role of resolving a mathematical challenge and in so doing seemed to develop a stronger mathematical identity
Case B Word Problem	Procedural model, providing step by step instructions, scaffolding towards an external form of knowledge.	Andrew followed the step by step instructions and referred back to the teacher for help and confirmation of the external reality.	Andrew was given specific procedures, rather than opportunities to devise his own way of working out an answer, thus limiting opportunities for mathematical thinking and developing a weaker mathematical identity.
Case C Place Value	Procedural model, providing step by step instructions, scaffolding towards an external knowledge.	Tim tried to follow the step by step instructions and referred back to the teacher for help and confirmation of the external reality.	Tim was unable to make sense of the procedures he was given and so could not develop even limited mathematical thinking. He appeared to surrender his efforts and reject a mathematical identity.
Case D Geometry	The teacher transmitted knowledge to the pupils as in a received model during the introduction of the game and the assessment of pupil's knowledge during the game to address naïve concepts.	Destiny played the game with the group by taking her turn but did not communicate verbally with the other pupils. She responded to the teacher's direct questions.	Destiny appears to have an identity of limited self-development as a passive receiver of information, with a weaker mathematical identity.
Case E Fractions Case	The teacher transmitted knowledge to the pupils as in a received model.	Mai completed the task set confirming an external reality.	Mai appeared to handover responsibility and consequently did not develop a confident mathematical identity.

Table 7: Overview of the enacted roles suggesting the pupil's identity concerning the mathematics classroom.

The pedagogical approaches the teachers have taken within the connected and procedural cases supports Vygotsky's (1978) model, describing the teacher's role as a more knowledgeable other. As such, the teacher could maximise pupils' learning potential by working within their zones of proximal development (Vygotsky, 1978). However, Priestley, Biesta, and Robinson (2015) predicted, to form a stronger or weaker mathematical identity the cultural environment is important. It is the adaptations of the learning environment through the teacher's preferred pedagogical approach, which extends opportunities for pupils to think mathematically. For example, specific types of dialogue are used as cultural artefacts to define perceived ability, or develop, or limit opportunities. As discussed in the next section.

5.3 Dialogue within the figured worlds of the mathematics classroom

In this section, I will be exploring three themes of dialogue; the role of dialogue as used to transmit information or present problems to solve, the asymmetric and symmetric interactions of dialogue between the teacher and pupil, and questioning used to assess recall, justify answers or introduce enquiry (Barnes, 1992a; Alexander, 2008).

5.3.1 The Role of dialogue

Dialogue played a distinctive role in different learning environments depending on the teacher's privileged pedagogical approaches. Three distinct forms of talk can be associated with the three models;

> *Received model* – The transmission of the information from the teacher to the pupil and questioning used to test the pupil's acquisition of knowledge.
>
> *Procedural model* – Dialogue used to scaffold procedures and smooth the way to the established solution.
>
> *Connected model* – Dialogic communication between pupils to share multiple realities offering up their propositions for consideration by others. Questioning is used to extend possibilities.

Within each of the five cases, the social environment in which the pupils enacted the lessons created a divergent figured world for pupils based on dialogue, including forms of questioning, affording an identity of greater or less dependency as encouraged by the teacher. The following section discusses the characteristics of each model.

The role of dialogue in the received model was to transmit information, with either the teacher giving information or the pupil responding with answers. In case E, the fractions episode, Mrs Greenway stressed she had told Mai what to do at the start of the lesson, instructing her how to use the strategy to find the fractions of numbers. When Mai could not continue and asked the teacher for help, Mrs Greenway reminded Mai of the strategy, reflecting the learning environment associated with a received model. In case D, the geometry episode also used dialogue to transmit information, this time in the form of assessment. Mrs Armid explained she was asking Destiny questions to ensure she understood the properties of shapes and, in doing so, addressed naïve concepts where they arose. Suggesting the role of the dialogue in this episode was to assess pupils' knowledge and correct deviations from the established truth. Both Mai and Destiny's opportunities to think mathematically appeared limited.

The role of the dialogue in the procedural model was to guide the pupils, step by step, through each stage of the learning process, with the intention of pupils being able to justify their work. The teachers used the pedagogical approaches associated with scaffolding and the justification of the process. In Case B, the word problem lesson, Mr Taylor observed and commented on how the pupils followed the instructions he had given them to select answers from multiple-choice statements and justified their choice through reasoning. He noted he would like to have had longer and more detailed reasoning from the pupils. However, Mr Taylor had explained that introducing this particular task was to prepare the pupils for the forthcoming government statuary tests. It suggests the purpose of the justification and reasoning was for gaining marks in the test, as explained by Andrew; *'Because sometimes, in SATs when you work it out, you get an extra mark.'*

Case C, the place value episode, also used dialogue associated with a procedural model, but this time, manipulative resources were provided to help scaffold for understanding. The role of the dialogue was to guide the pupils, step by step, through each stage of the learning process but support their understanding through the use of base ten resources, in this case, Diene's blocks. When pupils completed each step successfully, they all moved on to the next step. The role of dialogue, in this case, was to give specific instruction at each stage ensuring pupils could follow. Miss White was surprised to see how confused Tim appeared and how difficult he found it to follow each instruction. The procedural model introduces dialogue for justification, suggesting opportunities to engage in mathematical thinking behaviours. However, justification in these cases related to the strategies rather than the mathematical conceptual understanding, thus limiting mathematical thinking.

Case A illustrates the role of dialogue in a connected model, the symmetry episode. The role of the dialogue was to privilege challenge, enquiry, conjecture, criticality and debate. The teacher started the main lesson by asking the whole class: 'What is symmetry? Talk to your partner. I want to see conversations with your partner. I want to see if you challenge or support your partner' (Transcript A). The pupils engaged in the social practice of asking questions followed by discussion and dialogue throughout the lesson. The teacher modelled using dialogue to privilege challenging ideas and encourage individual responses affording pupils the opportunities for mathematical thinking and developing a mathematical identity. A reminder by the teacher, Ms Travis, of the value to their learning of acting like thinking mathematicians through discussion would have afforded engagement that is more mathematical. Ms Travis expressed surprise and satisfaction at the confidence and sophistication of the language Tom and Asha used to discuss the challenge, reinforcing her intentions to privilege the use of dialogic communication between the pupils.

Boaler (2016:188) describes the difference in the dialogue between the received and connections model: the received model creates a subject of rules and tests, where pupils surrender their thoughts and ideas in favour of obedience and compliance. On the other hand, a connections model is a social environment in which pupils contribute to the judgements and validation of what is being learnt. However, my study of these five cases has suggested a more complex picture if the purpose of the dialogue is justification and reasoning. The procedural models focused on the justification and reasoning of the strategy, limiting opportunities for mathematical thinking. These models contrast with the connections model, which focused on the justification and reasoning of the pupils' understanding, affording more significant opportunities for thinking mathematically. The teacher's ways of knowing being privileged are difficult to ascertain without considering this point (RQ3).

5.3.2 Teacher pupil balance

Pupils were invited to work in pairs or groups collaboratively in line with my original request for the research, so groups could be observed working together. On the day of the study, two interaction styles emerged, suggesting differing social practices and participation within these figured worlds. In the received and procedural models, the dialogue was asymmetrical. The teacher transmitted information or instructions to be received or followed by the pupils in their groups. The connections model was more symmetrical in that there was a reciprocal discussion between peers and between teachers and pupils, with a prevalence of dialogic interaction. The lesson started with a whole class challenge in case A, setting a specific learning environment different from the other four in this study. Pupils communicated dialogically, being afforded time and space to ponder and debate with peers, exploring their understanding, developing their identity as thinking agents. The teacher, Ms Travis, encouraged the pupils to support and challenge each other, sharing mathematical dialogue, as illustrated here;

> *"Tom has told me that this can't be symmetrical because of one part of it, one corner, has broken off. Talk to your partner, why does it now mean that it can't be symmetrical?"*
>
> (Transcript A)

The balance of talk echoed, in this case, is not measured in the number of words spoken by each participant but reflects a dialogic balance of interaction (Barnes, 1992a). The teacher asks the pupils to solve the problem using the available equipment and discuss the challenge with a peer. A figured world with dialogic communication would suggest greater opportunities for pupils to engage in mathematical thinking.

In contrast, in cases C, D and E, the dialogue was asymmetrical. The teachers transmitted information while modelling the strategy for the pupils or giving direct instructions. In case C, the place value episode, Miss White noted from the stimulated recall session how she used the dialogue to transmit each step of the process for the pupils to respond to before moving to the next step. What emerged from the opportunity was her surprise at how much work she was doing in contrast to the pupils, creating asymmetrical dialogue as illustrated in this quotation from Miss White:

> *"…**Can we make it in ten's and ones?** Which we could, and then: **Can we add the numbers together?** I think that bit all went relatively well and then it was that **next step of bridging through ten**…'*
>
> (Transcript C)

In case D, the geometry lesson, pupils were encouraged to think mathematically through a game, but when Ms Armid intervened during the episode to assess pupils' knowledge, she introduced asymmetric dialogue, for example; *'Can you describe this shape*? *How many vertices does it have?*' Mrs Armid discussed the importance of checking the pupils had acquired the relevant information. This approach suggests introducing knowledge as an external commodity to be obtained, privileging a received pedagogical approach. The same was true of case E when Mrs Greenway transmitted the information for pupils to follow the strategy, see section 4.5.

In the asymmetric dialogue of case B, the word problems episode used dialogue to justify established knowledge. When the pupils asked for help, Mr Taylor used dialogue as a scaffold to lead the pupil through a sense-making procedure. He used dialogue to justify the answer, smoothing the way (Wigley, 1992), an approach used by teachers associated with Vygotsky's (1978) ZPD, being the more knowledgeable other guiding the pupil as demonstrated here:

Mr Taylor:	*Does it change how you divide it into 5? What does the five times tables end in?*
Andrew:	*Five or zero.*
Mr Taylor:	*So can you divide 65 by 5?*

Andrew: *Maybe*

Mr Taylor: *You think?*

Andrew: *Yes?* (Transcript B)

The teacher asked a question, and the pupil responded. This approach introduced asymmetric dialogue, limiting the pupils' mathematical thinking opportunities. However, symmetrical dialogue appeared to provide greater opportunities for pupils to engage in mathematical thinking through dialogic interaction, extending opportunities to explore concepts through investigation and conjecture, challenging ideas. The balance of dialogue within the different learning environments influenced the identity formation within these pupils' figured worlds.

5.3.3 Questioning

The role and ways of questioning are embedded in any classroom culture and Mason (2021:131) suggests could impact pupils' ways of coming to know. Specific approaches to questioning formed an intrinsic part of the learning environment observed and contributed to the pupils' figured worlds. See section 2.3.2.1. The style of questions ranged from enquiry and conjecture to meta-questioning used to scaffold understanding and summative assessment style questioning.

Using the connections model, Case A featured a pedagogy formed around an enquiry approach based on questioning. Ms Travis, the teacher, created a specific social learning environment by starting the lesson with a question for the whole class; 'What is symmetry?' and encouraged the pupils to discuss the answer before responding. Then she encouraged the pupils to build on one another's responses, creating an ethos of shared responsibility, which she felt she had achieved from observing the stimulated recall. This excerpt illustrates the types of conversations the pupils were having and the role of questioning between the pupils:

Tom: *So this bit goes down. So this is the middle like that so not symmetrical because you have got this side going down and this side going up.*

Asha: *I support the line. (Indicating she disagreed and thought it was symmetrical)*

Tom: *It's not the same is it? Look it's got this bit here.*

Asha: *What about a ruler? Try it with a mirror then?*

Tom: *(Tom uses a mirror) Nope.*

Asha: *(Using the mirror) this one here.*

Tom: *Yep.*

Ashe: *Support or Challenge?*

Tom: *Challenge because there is an edge there.*

(Transcript A)

When Ms Travis, the teacher, observed this episode, she commented: "…. *I was really pleased with the conversations I was listening into. They were … testing out their ideas."* The teacher's reaction to the pupils' interaction illustrated her intentions for a shared experience, thus creating opportunities for the pupils to engage in mathematical thinking.

Tom and Asha appeared to embrace the opportunity to work independently, supporting and challenging each other. These pupils appeared to be comfortable with a shifting reality, utilising reflection, rationalisation and conjecture in a meaningful way, merging pragmatism and personal integrity. The dialogic nature of questioning emerging within this social environment would suggest the teacher was enacting a connections model as a facilitator for the pupil to pupil discussion, with pupils asking and responding to their questions, creating enquiry approaches (Catlin and Willy, 2018; Christenson and James, 2017).

A pedagogical approach that uses questions as scaffolding to smooth the way, referred to by Mason (2021) as meta-questioning, was observed in case B, the problem-solving episode. The teacher used questioning to model coming to know with the possible intention of the pupils internalising the process (Wood, Bruner and Ross, 1976), illustrated in section 5.3.2. In case B, the word problem lesson, Mr Taylor noted from the stimulated recall session how he supported the pupils when they became confused through structured questioning (Wigley, 1992; Blair and Hindle, 2019). The difficulty with using this approach of metaquestioning, according to Mason (2021:134), is that it makes the pupil dependent on the teacher if they are unable to internalise the process. Therefore, a process that initially appears to be building self-reliance can result in teacher reliance. The excerpt from case B demonstrated how Mr Taylor took the role of the more knowledgeable other, modelling reasoning to Andrew through specific questioning. This approach created a reliance on the teacher to 'know the way' and created a figured world where Andrew had limited

opportunities to engage in mathematical thinking. When Andrew replies to Mr Taylor in section 5.4.2, a questioning 'Yes?' appears to be looking for confirmation. Mr Taylor expressed a need for this group of boys to become more confident in their work, observing their tentative responses during the stimulated recall session.
Questioning used for assessment was a feature of case D, the geometry episode. Mrs Armid used questioning to assess Destiny's understanding of three-dimensional shapes, as illustrated in this section of dialogue:

Mrs Armid:	*Does it have any vertices?*
Destiny:	*(Shakes her head.) No*
Mrs Armid:	*(Holds up a square-based pyramid.) Does this have any vertices?*
Destiny:	*Yes*
Mrs Armid:	*How many?*
Destiny:	*(Points her finger towards the shape but doesn't touch it mimicking counting to five.) Five*
Mrs Armid:	*Five vertices you can count them. (Hands the shape to Destiny.)*

(Transcript D)

Mrs Armid used questioning in this episode to assess the information Destiny had remembered about the properties of shapes, suggesting a received model of learning and limiting Destiny's opportunities to engage in mathematical thinking. The three types of question can be seen to contribute to the different figured worlds being formed:

- The connections model uses questioning to enquire and explore ideas, providing opportunities for thinking mathematically.
- The procedural model guides and smooths the way when pupils become confused, offering limited opportunities to engage in mathematical thinking.
- The received model assesses the pupils' knowledge, removing any opportunities to exercise mathematical thinking behaviours.

5.4 Ways of using cultural artefacts

As discussed in section 2.3.3, Holland *et al* (2001:61) describes cultural artefacts as a "*means by which figured worlds are evoked, collectively developed, individually learned, and personally powerful."* Vygotsky's (1978) notion of social interaction, which defines development in terms of mediation, includes tools forming cultural artefacts associated with different figured worlds (Wertsch, 1985: 15). This suggests it is the way the teacher and pupils use the mathematics resources that form the cultural artefacts of the mathematics classroom. Table 7 indicates the different ways cultural artefacts were used in these five cases as part of the pedagogical approaches the teachers adopted during the episodes. By viewing these episodes through a figured world lens, how cultural artefacts were being mediated alluded to the teachers' views on ability as being either fixed or fluid.

The cultural artefacts discussed in this study are based on the manipulative resources found in the mathematics classroom. Mediating those resources creates a cultural practice of ability setting (Marks, 2016). Manipulative resources associated with mathematical learning were available in three of the five episodes. The nature of the tasks and the way the teacher mediated the manipulative resources appeared to contribute to the identity formation within the figured worlds of the pupils, either an identity as a 'receiver of information' or as a 'creator of knowledge.' (Swann, 1985: 49).

5.4.1 The role of manipulative resources, as cultural artefacts and the links to ability thinking

The use of manipulative resources in the classroom can support conceptual understanding for pupils (Laski, Jordan, Daoust, and Murray, 2015) and can symbolise specific ecologies of learning within a classroom. A participation model might see the cultural use of manipulative resources as a way of 'scaffolding' learning by providing a visual aid, suggesting supporting understanding. Alternatively, in a connections model, manipulative resources can be used as a cultural tool to explore ideas and concepts. Forms of communication can also be embedded as cultural artefacts and do not need to be physical items. For example, Mr Taylor's response to Andrew's answer/question '*Yes?*' by saying '*You think?*' creates a specific environment looking for confirmation. Ms Travis asks pupils to challenge each other, using dialogue as a different cultural artefact.

During the episode in case A, the symmetry lesson, the way the teacher engaged the pupils in using the resources, consciously or unconsciously, shaped the learning opportunities to reinforce the notion of fluid ability. The teacher introduced the manipulative resources of mirrors and paper for folding to the whole class. The manipulative resources contributed to a mediation process that supports exploring ideas, in this case, to test out pupils' predictions and theories. The social identity associated with the resources was a mathematician’s tool kit used in the enquiry process. All the pupils had opportunities to test out and challenge their ideas.

In contrast, the teachers in the other two cases set tasks that reinforced the notion of fixed ability. The teacher explicitly used manipulative resources to scaffold the learning. If followed, Mason (2021:134) suggests the use of scaffolding and modelling can lead to conceptual understanding of the stages of ‘*direct questioning, indirect prompts and spontaneous use by the learner*’. If they are not, the pupils can become teacher-reliant. In case C, the place value lesson, Miss White explained they were using the resources because the group had not understood column addition the day before. Using the base 10 resources provided the pupils with manipulative and visual aids for conceptual understanding. In case D, the geometry lesson, Mrs Armid, introduced the resources, in this case, three-dimensional shapes, specifically to help Destiny identify the features of the shapes. Mrs Armid expounded during the stimulated recall session that she viewed Destiny as a kinaesthetic learner and, as such, needed the physical resource to support her understanding. The manipulative resources in both cases were used as a remedial tool and did not allow pupils to exercise ‘spontaneous use’, limiting opportunities to engage in mathematical thinking.

If the teacher, shaping the learning opportunities to reinforce the notion of fixed ability, mediates the manipulative resources, the pupils’ identity will be associated with ability. However, pupils will have an environment that encourages them to think mathematically if the teacher privileges fluid ability grouping and all pupils consider manipulative resources as part of the mathematicians’ toolbox.
In this case, cultural artefacts, manipulative mathematics resources and their use helped explain further complexity within the mathematics environment. Teachers can use manipulative resources to create an identity of dependency, reflecting a received model, or used in another way, they opened up possibilities for enquiry, reflecting a connected model.

5.4.2 Tasks set by the teacher

The nature of the tasks set by the teachers shaped the learning opportunities within the pupils' figured worlds, creating a specific environment in which pupils' identities were being formed. A common practice dominant in British mathematics classrooms today is to create an environment where the tasks are set within ability groups (Marks, 2016). The use of ability groups symbolises 'fixed ability thinking' (Boylan and Povey, 2021:55). Three cases in this study featured discrete environments of ability grouping, where pupils belonged to a specific set determined by the pupil's perceived ability. In contrast, the other two cases appeared to have a more fluid approach to classroom organisation.

Tasks were differentiated in similar ways in cases B and E, where the pupils were to complete their set of questions before moving on to the next set. The teacher had designed the sets of questions in levels of difficulty to reflect pupils' ability groups. In case B, the word problem episode, the pupils were allocated the tasks according to their set. Pupils in case E, the fractions episode, were able to select the task according to the level of difficulty they felt they associated with, which possibly reinforced the notion of fixed ability. When the pupils finished their questions and moved on to the next set, they became confused and asked the teacher for help. The lists signalled to the pupils the ability settings, suggesting to Mai that these questions might be beyond her ability, reinforcing a 'fixed ability thinking' (Boylan and Povey, 2021). Mrs Greenway observed during the stimulated recall session that the change of questions might have appeared to Mai that the task was getting more challenging and perhaps not for her. As Boaler (2016) discussed, the power of the fixed mindset might suggest why, when the pupils in these two cases reached the end of their differentiated tasks, they stopped and asked for help. Before watching the stimulated recall, Mrs Greenway had not considered how the change of questions could signal to Mai that they were not for her.

Case C, the place value group, was also the low ability group and worked with the teacher throughout the whole lesson. The teacher explained she wanted to go over the work again with this group because they had been confused. The teacher wanted to scaffold each step with the manipulative resources. In all three cases, the pupils depended on the teacher and referred back to the teacher when they became confused. The privileged pedagogical approach introduced activities to simplify and remove challenge for the lower ability groups, thus creating an environment where the teacher smoothed the way for pupils (Wigley, 1992). However, Miss White noted how confused the pupils looked and how much they relied on her.

In contrast, Case A, the symmetry case, introduced an inclusive pedagogical approach involving a whole-class enquiry, starting with a whole-class challenge, whereby pupils worked in mixed ability pairs favouring a more fluid approach. The teacher, Ms Travis, explained how she ensured all the pupils worked with different partners and in different groups regularly, showing an inclusive pedagogical approach. Ms Travis could see from the stimulated recall that the pupils were interacting and taking on the role of mathematicians.

There was evidence supporting a teacher's social environment of fixed ability in four of the five episodes, the fifth case planned for mixed ability groupings. Fixed ability can be associated with received and procedural learning models relying on cognitive processes of memory and retention of information, thus objectifying the subject. Fluid ability is more aligned with a connected model, and pupils develop enquiry, reasoning, and conjecture skills.

5.5 Synthesis of responses to the research questions

Boaler and Greeno (2000) stated that the mathematics classroom has traditionally held a narrow approach to mathematical identity formation due to the ritualistic nature of how the subject is perceived and taught, favouring a received model. However, by observing these episodes through a figured world lens, the interrelated teaching and learning processes are not clear-cut and provide a fresh interpretation of what unfolds (RQ1). The challenge was to use noticing so teachers could see beyond the immediate context and disrupt their current views. The introduction of the pupils' perspective and the opportunity to use video recording to review an episode of the lesson in detail helped achieve this.

All episodes introduced reasoning, choice and independent activities, through (for example) the use of a game and opportunities to explore concepts, features not associated with the narrow traditionalist approach observed by Boaler and Greeno (2000). However, applying the analytical framework as illustrated in table 5, identity formation appeared intrinsically linked to the nature of engagement with learning mathematics. The pupils and teachers' learning interactions confirmed these links (RQ3). There is a clear distinction within these episodes between a received and connected model of coming to know. The introduction of a procedural model introduced complexity. Focusing on the teacher's opportunities to encourage pupils’ mathematical thinking through the different aspects affecting their figured worlds builds a more detailed picture of the privileged social-cultural environment. Figure 16 offers a renewed version of figure 1, from chapter 2, which showed the influences that contribute to the figured worlds of the teacher and pupils in a mathematics class. In light of this study, I have identified if teachers afford to pupils opportunities to behave mathematically, it can create a figured world where pupils can take on the identity of a thinking mathematician. Figure 16 illustrates the key influences based on a figured world lens, which engage pupils in thinking mathematically. All the influences from figure 1 are still present but polarised into the teachers' objective or subjective views of mathematics and pupils' identity.

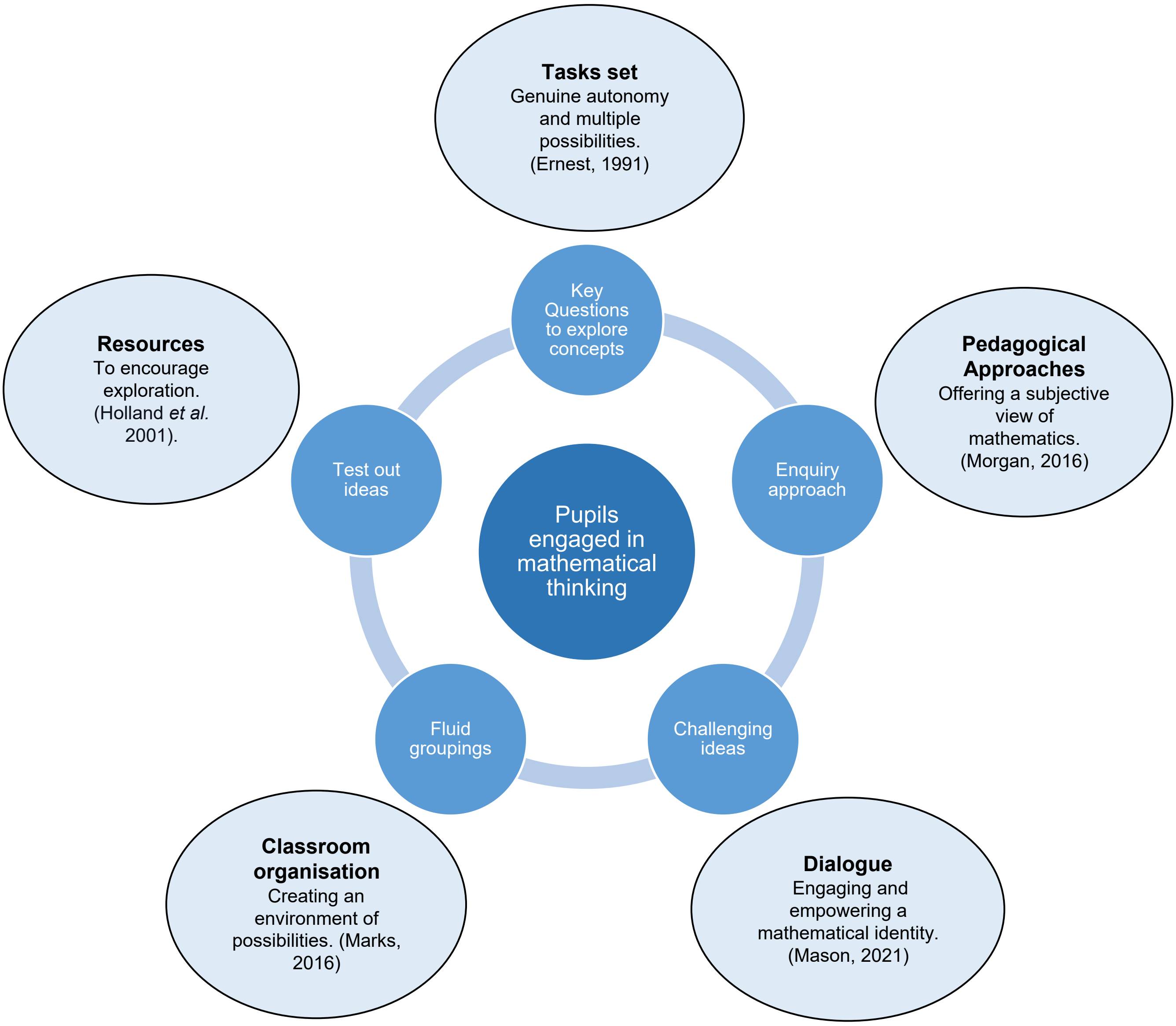


Figure: 16. The influences that contribute to pupils engaging in mathematical thinking I in light of this research.

The teachers in these cases had introduced and organised a pedagogical approach to engage pupils within activities that initially would suggest facilitating mathematical thinking. However, when viewed within the context with other aspects of the figured worlds, mathematical thinking was limited. When teachers enacted their practice, consciously or unconsciously, they were creating different kinds of figured worlds based on an objective or subjective view of the subject. This study would suggest that introducing aspects of a connected or procedural model through specific activities will not necessarily impact pupils' identities if the teacher continues to hold all the answers and sees knowledge as fixed, thus, viewing mathematics as an objective subject. As illustrated on figure 16, all five influences need to be supporting pupils mathematical thinking to develop a strong mathematical identity.

5.6 In Conclusion

To enable a fresh approach to observing learning in the mathematics classroom, I introduced the pupil's perspective and stimulated recall. This approach helped to disrupt existing assumptions the teacher might have. In conjunction with the extended version of Mason's (2002) discipline of noticing, it enabled me as the researcher and the teachers to gain a new informed view from which we could develop reflexive practice (RQ1).

My analytical framework provided;

- insight into the figured worlds of the primary mathematics classroom,
- a focus on the key areas of dialogue,
- the use of cultural artefacts and the impact on identity.

Looking back at figure 1 in section 2.2.2, the teachers' pedagogical approach, influenced by social, theoretical and political contexts, appears to impact pupils' mathematical identity. The data analysis suggests that teachers have to allow pupils to exercise mathematical thinking to develop an identity of a thinking mathematician. From my study, the mathematical thinking afforded pupils has a more significant impact than social and cultural influences set out in figure 1. In the next chapter, I will present my key findings and contribution to knowledge based on what teachers appear to privilege regarding their ways of coming to know and the influence this might have had on theirs and their pupils' relationship with the subject of mathematics.

Chapter 6

Conclusion

6.0 Introduction

This study aimed to explore the nature of the different figured worlds of the primary mathematics classroom. The purpose was to understand why some pupils reject the subject, and others embrace it. I wanted to know how the figured worlds of the mathematics classroom that teachers generate could have such a dramatic and varied effect on the relationship between the pupil and the subject of mathematics. From the five cases studied, I observed how the teachers' pedagogical approaches influenced the nature of the environment, reflecting aspects of figure 1 section 2.2.2. Then consequently, how the differently figured mathematical worlds impact the nature of learning in which the children engaged. There were clear distinctions between the cases, aligning them with the specific ways of coming to know. Each case also featured elements associated with the different approaches, creating a complex picture of mathematics teaching concerning the teaching and children's learning. Figure 16, in the light of this study suggests an emphasis on 'genuine' opportunities to think mathematically being afforded pupils, based on the teacher's view of mathematics as an objective or subjective subject.

6.1 Key findings

Section 2.2.2 argued that many view mathematics as a difficult subject compared with other curriculum areas. Mathematics is a subject that evokes learned helplessness, described by teachers as pupils being 'stuck', 'muddled', 'confused' or 'in a pickle'. I considered if teachers through a figured world lens viewed the mathematics environment holistically, perhaps a greater understanding of the complexities presented could be recognised and addressed. This view could lead to an environment associated with less confusion for both teachers and pupils. From my analysis, the themes of dialogue and cultural artefacts seem to be important in contributing to pupils' ecologies of learning, creating a unique environment where the levels of mathematical thinking offered to pupils can impact their identity as they move towards becoming thinking mathematicians. From this study, the following three key findings have emerged;

- This fresh approach to observing learning has suggested the teacher's organisation of the learning environment needs to remove the boundaries often created for pupils.

The boundaries of the specific use of dialogue and cultural artefacts. For example, setting by ability reduces manipulative resources to a remedial intervention.

- Viewing the mathematics classroom through a figured world lens and bringing to the fore the teacher and pupils' perspectives through stimulated recall allowed for a more nuanced analysis to take place. This approach disrupts the teachers established views.
- For pupils to develop an identity of a thinking mathematician, the teacher needs to provide opportunities for pupils to exercise mathematical thinking.

These key points help to illuminate possibilities for pupils to develop an identity of a thinking mathematician instead of creating dependency on the teacher. This study has identified how teachers have generated a particular kind of culture in the mathematics classroom by privileging specific ways of coming to know, creating the contrasting social environment in which pupils learn. The implications of this research suggest the pedagogical approaches teachers take needs to go beyond the curricular imperative, creating a figured world of multiple realities and possibilities to be explored by the pupils. An unexpected finding from my study was how the teachers viewed the range of perturbations. Either the teacher was responsible for resolving the perturbation, or the pupil viewed it as an opportunity to take on a challenge. The patterns of interaction between the pupils and teacher and peers and their tasks suggested the teacher's cultural influences created contrasting classroom cultures.

In cases B and E, when the pupils experienced perturbation, they appeared teacher-reliant and helpless, turning to the teacher to lead them through the task. In cases C and D, the teacher intervened with specific instructions when they observed naïve concepts emerging or if the pupils did not appear to follow the procedure set out. In case A, the symmetry episode, the pupils experienced a perturbation concerning the symmetry of a shape. To support them through the disruption, the teacher, in this case, had ensured that pupils had ways of justifying and trying out their ideas, using folding and a mirror to support their reasoning. This approach seemingly created identities of independence and self-help. The perturbation, in this case, was an integral part of the learning process. The perturbation in this instance created a rich learning opportunity and an opportunity for rich mathematical thinking.

6.2 Research questions

The overarching intention of this study was to review the nature of learning in contrasting primary mathematics classrooms through the lens of a figured world. The interplay between dialogue and cultural artefacts in identity development could help make sense of the differences in practice. It could help illuminate significant and influential differences in pupils' experiences.

This study aimed to:

- gain a greater understanding of why some pupils thrive in the mathematics classroom while others do not.
- explore the nature and impact of the different figured worlds of the mathematics classroom.
- apply and extend Mason's (2002) discipline of noticing approach to the primary mathematics classroom.
- interpret learning episodes by bringing together the multiple viewpoints of the participants in a primary mathematics classroom.
- use stimulated recall through digital video technology, enabling reflective exploration between the researcher, the teacher and the pupils.

These aims were achieved by the research addressing the three research questions. The following section summarises the significant findings.

6.2.1 Research Question 1

How can viewing the mathematics classroom through a figured world lens illuminate pupils' experiences and bring a fresh approach to exploring the nature of learning?

Exploring the figured worlds of the mathematics classroom provided an opportunity to consider the environment created by the teacher and enacted by the pupils. The picture was a complex one, but applying Mason's discipline of noticing (2002) and observing these five episodes through a figured world lens provided a fresh interpretation and opportunities for greater reflexivity. The study also provided time and space to observe the interactions within a primary mathematics classroom environment, thus taking reflection beyond just the habitual initial response. The themes of dialogue, cultural artefacts, and identity associated with the theoretical framework of a figured world characterise the teacher’s adopted ways of coming to know. The teacher's use of cultural artefacts to promote independent activity, leading to a specific mathematical identity, could be observed through didactic or dialogic approaches. The enactment of these themes afforded pupils differing levels of mathematical thinking behaviours, leading to particular identity formations concerning the mathematics classroom. This pedagogical enactment contributes to the formation of specific environments. The similarities and differences between the cases are demonstrable from the observer's viewpoint.

6.2.2 Research Question 2

How can creating and using a unique analytical framework drawing together Belenky et al's (1986) theories of coming to know and Sfard's (1998) metaphors for learning be applied through a figured world framing, bringing to the fore the multiple realities from both the teacher and pupils perspectives?

Applying a figured world framing and using both Belenky *et al* (1986) and Sfard's (1998) theories provided greater opportunities for nuanced analysis. The findings of this study showed that cases could not be assigned solely to one specific model of coming to know because they had elements of different models to a greater or less extent. Sfard's (1998) metaphors offered a way of viewing pupils' experience of acquiring knowledge through the characteristics of either of acquisition or participation. Belenky *et al's* (1986) ways of coming to know focused on the teacher's pedagogic apprache based on their epistemic stance. By exploring the different episodes through different lenses, finer distinctions were made. My study built a more detailed picture of the privileged social-cultural environment within the distinctly differing figured worlds. The analysis of the learning interactions from this study suggested an intrinsic link between the pupil's identity formation and the social-cultural environments of the individual figured worlds.

6.2.3 Research Question 3

How does the way of coming to know privileged by the teacher influence the pupil's relationship with the subject of mathematics?

When the teachers enacted their practice, consciously or unconsciously, they were generating different kinds of figured worlds. Pupils could develop a stronger or weaker identity concerning their engagement with mathematics within these figured worlds. This study would suggest that teachers only generating activities associated with a connections model, devoid of opportunities for pupils to demonstrate mathematical thinking, create a model associated with procedures. For example, engaging pupils in justification and reasoning whilst appearing to hold all the answers would not necessarily impact pupils' identity in the same way. In the procedural episodes, the teachers focused on the justification of the established strategy being enacted instead of reasoning involving multiply possibilities, thus giving a restricted choice. The teachers had introduced more opportunities for pupils to exercise mathematical thinking, but had still created a figured world where the pupils were reliant on the teacher. In these cases, teachers appeared to value the knowledge of the process and not allowed for ‘genuine’ mathematical thinking.

6.3 Contribution to knowledge

My study has provided two key areas to contribute to knowledge. The first being the methodological approach based on using a figured world lens and including the pupil's perspective. The second was a new understanding of what is happening within the primary mathematics classroom.

6.3.1 The research design

The research design was based on sharing different perspectives of the same episode of a mathematics lesson. The episode was videoed and used to stimulate recall for both the teacher and pupils. My findings showed that when these teachers viewed an episode of their mathematics lesson through different lens they were able to develop their reflective and reflexive practice. The teachers had the opportunity to observe and comment on the mathematics environment of the classroom. During the co-constructed narrative, the teachers could reflect on their own figured worlds as a teacher in a primary mathematics classroom. They had the opportunity to observe and reflect on the pupils' figured worlds as they presented them in the recorded episode and the follow-up interviews. The use of stimulated recall through visual methods provided a fresh look at the original episode in the classroom. This study has identified the importance of viewing learning through a figured world framing and taking into account multiple viewpoints to help construe a better understanding of what is happening in the mathematics classroom.

This methodological approach illuminated the teacher's privileged ways of coming to know. Initially, I viewed ways of knowing on a sliding scale between the received and connected models, with stages demarcated clearly between the two axioms (Belenky *et al*, 1986; Sfard, 1998). In section 2.6, the literature chapter, I described Belenky *et al's* (1986) five different ecologies of participation, suggesting a more nuanced categorisation between the different ways of coming to know. However, my study suggests a more complex environment with evidence of more than one model present in each episode.

Schools can use this approach for Continual Professional Development (CPD) to support and build teachers' understanding of the mathematical figured worlds on the classroom. A critical colleague, the mathematics coordinator or perhaps the pupils can replace the role of the researcher. The process needs to present challenge and to avoid just confirming habitual tacit knowledge. Setting up networks within schools and between schools would introduce different figured worlds providing different lenses to achieve this.

6.3.2 New understanding

The new understanding that emerged from my findings suggests that pupils need to immerse themselves in opportunities to explore multiple realities, which helps develop their identity as independent thinking mathematicians. The offering of opportunities to engage in isolated mathematical activities did not appear to replace the power of genuine autonomy through dialogue and cultural artefacts. The removal of the cultural boundaries of fixed ability and fixed truths enabled significant opportunities for independent mathematical thinking. My study would suggest that for pupils to occupy a figured world that sees them as active thinking mathematicians, the opportunity to exercise mathematical thinking as illustrated in figure 16 needs to be in place.

The use of cultural artefacts was evident throughout all five cases, but In case A, the symmetry lesson, Ms Travis did not organise her classroom in fixed ability groupings or use differentiated tasks, thus removing the socio-constructed boundaries associated with the cultural artefacts, both of which appeared to restrict opportunities to think mathematically for pupils in the other four cases. A further observation from the discussions with the pupils during the follow-up interviews showed some pupils engaged in mathematical discussion while others chose not to. The pupils from case A, demonstrated confidence and engagement in the mathematical discussion. The other four cases touched on the mathematical content but with a clear focus on reaching the correct answers, with no mention of the mathematical content, or possibilities of differences in opinions. As the interviewer, I had to draw the pupils back to the mathematics discussion through my questioning as they became distracted, suggesting to me a lack of engagement in the subject.

Dialogue also played an important role during the perturbations that arose. In this study, all the perturbations involved pupils looking confused and challenged by the activity. During the episodes from cases B and C, the teachers appeared to be taking the responsibility for resolving the perturbation, helping to smooth the way for the pupils' learning, using meta-questions to guide pupils. In case A the opposite was happening, the pupils were expected to discuss and explore the perturbation as they arose, using it as a learning opportunity. My study suggests giving pupils' responsibility and a means to clarify their learning for themselves were less confused and more engaged with the mathematics. The pupils appeared to be able to connect their thoughts and understanding. My study would suggest the removal of fixed ability through setting and differentiated worksheets would allow greater opportunities for pupils to take on a stronger mathematical identity. The removal of these cultural artefacts could allow pupils to explore and make connections for themselves, thus have ownership of their understanding. The dialogue between pupils and teacher and other pupils needs to be part of their exploration and learning.

My study did not seek to explain but to report on the figured worlds of the primary mathematics classroom, it appeared to show that when the teacher afforded pupils the opportunity to behave like thinking mathematicians, they took on the identity of thinking mathematicians.

6.4 Methodological approach

My methodological approach involves both teachers and pupils from each case. Rather than the study being solely the adult's domain, I have actively included the pupils in the process to break down the barriers associated with an adult-driven environment and provide a pupil's perspective. Through stimulated recall, the teacher and pupils were able to take time and space to reflect on an episode of mathematics and recount their thoughts about that episode, thus creating valuable data for analysis. As discussed in the methodology chapter, Mason's (2002) discipline of noticing approach informed my study, which extended the opportunities to explore what individuals experience in the classroom. This approach took my study beyond the initial interpretations of the pupils' observed behaviour of being 'stuck', 'muddled', 'confused' or 'in a pickle' to observe and reflect on the social and cultural environment in which the behaviour was taking place.

I have brought together the following methods to create a social constructivist approach in an attempt to address the research questions:

- Stimulated recall using audio-visual material
- The use of collective reflection between the teacher and researcher, and pupils and researcher, thus including the pupil's perspective
- The application of a new analytical framework viewing the data through a figured world lens and bringing together Belenky *et al's* (1986) ways of knowing and Sfard's (1998) metaphors for learning

Teachers undertaking action research within their classrooms can apply this analytical methodology. A colleague could take the researcher's place, providing a knowledgeable other to collaborate. This type of classroom research could offer powerful insight into the figured worlds of the mathematics classroom. By looking at the effects of the different pedagogical approaches on pupils and exploring the discourse between the intended and observed outcomes could develop a practice to enact change through teachers' continual professional development (CPD). This approach allows the teacher to take the role of learner.

6.4.1 Methodological challenges and limitations to the study

The potential limitations that emerge from this study that could impact validity and generalisability were; recruitment of participants, attempts at creating an authentic environment to observe and selecting episodes to explore in-depth for this study.

Convenience sampling was applied to recruit schools, as discussed in section 3.4.1. Due to the challenges of finding schools to participate in research, I contacted schools until I found five willing to be part of the research. The final schools selected offered a range of socioeconomic catchment areas and varying school sizes. Selecting different schools by different criteria might well have resulted in alternative findings. The recruitment of pupils to participate in the episodes videoed also suggested a limited selection. Unfortunately, some pupils were excluded from the research because they did not give assent and consent. Teachers could show a particular interest in any specific group of pupils before the selection process. All the teachers focused on pupils perceived as having particular challenges to their learning. The selection of this group in itself is an interesting point but risks limiting the study to pupils whom the teacher perceives as finding mathematics challenging. I feel this limited the research but also offered exciting data for this group of pupils. Viewing the figured worlds of differently attaining pupils could produce different data and different insights.

The presence of the researcher and the use of the video equipment introduces an audience for the pupils and teacher, suggesting a performance affecting individual behaviour. The video equipment's existence and the episode's selection also introduced limitations to the study by creating a possible distraction. Time was given to participants to familiarise themselves with the video camera in the week running up to the data collection, limiting the impact. Finally, the episodes selected introduced possibilities for different outcomes. It was the researcher's perceptions of what was being enacted that determined which episode was selected. Due to this study’s size and limitations, generalisability cannot be claimed. However, due to the patterns found in dialogue, cultural artefacts and identity, the features common to each case can be extrapolated from or looked for in other contexts.

6.5 Recommendation for further research

The implications for future research would suggest a study of varying mathematical environments and a wider range of pedagogical approaches, which could be attempted through further research addressing the following questions:

- What kinds of knowledge are being privileged by the teacher when working with pupils in ability groups in a mathematics classroom?

- How can the figured worlds of pupils on pupil premium differ from pupils from more affluent backgrounds?

- Which aspects of a mastery approach do teachers' privilege in school?

Greater autonomy for pupils and teachers could develop research approaches further by giving pupils and teachers a more active role. The researcher could ask the pupils to select the episode for analysis and respond to the teacher’s view of the episode. The researcher then becomes more of a facilitator and observer.

6.6 Implications for practice and policy

To improve the engagement and quality of the mathematics in our classrooms today a change in practice and policy is needed to redefine the subject. This research proposes that by introducing ‘genuine’ mathematical thinking to the classroom, where pupils can explore and develop mathematical challenges, would improve the mathematical content and pupils’ understanding. ‘Genuine’ implying all areas of the classroom support mathematical thinking, not just an activity or using open questions. Currently in school, pupils are offered a subject of fixed strategies to be learnt and practised to achieve the required answer already established. Developing opportunities for mathematical thinking would challenge the view of mathematics being an objective subject. By viewing mathematics as a subject of multiple realities void of cultural boundaries such as fixed ability grouping, pupils would have opportunities to behave like mathematicians.

The introduction of greater opportunities to engage in mathematical thinking would need a change in practice to show teachers an alternative approach. The basic skills of being numerate and knowing our number system needs a purpose beyond just a bank of knowledge. Pupils should be encouraged to use these skills to investigate, explore and create hypothesis to challenge and refine. Policy and practice need to shift the focus from just knowledge-based tests to genuine mathematical thinking by introducing a level of creativity. Figure 16 outlines the five main areas offering genuine opportunities for mathematical thinking. Teachers need to use dialogue to engage and empower pupils, not to direct and instruct. Pedagogical approaches need to offer a subjective view of mathematics, not based on knowledge acquisition and learning facts. The activities teachers set need to provide genuine autonomy with multiple possibilities, not just aiming for the correct answer void of understanding or ownership. The classroom resources need to be available to all void of stigma and pupils need to be able to be free of the categorisation by ability based on outdated approaches.

The introduction of the pupils' perspective alongside structured and supported reflective discussion has shown to be an effective way to enable teachers to take a fresh look at their practice, introducing a greater understanding of ways of coming to know. A new Policy that draws on the perspective of both the teacher, and pupils could challenge the focus of current professional development that appears to be based on tacit knowledge and data driven approaches.

6.7 A brief autobiographical reflection

My initial approach when coming to this study was to reflect on why some pupils find mathematics challenging. I wanted to look more closely at the actions and reactions of the pupils within the mathematics classroom, examining and trying to explain why some pupils reject the subject while others excel. I considered both the teacher and pupils' views and applied a figured world lens. In doing this, I found that the teacher's pedagogical approach to mathematics created particular environments shaped by the mathematical thinking the teacher afforded their pupils. The mathematical thinking generated was affected by the ways teachers engaged in dialogue with pupils through a didactic tone or a dialogic intention. The use and possession associated with the cultural artefacts of the mathematics classroom also restricted or supported mathematical thinking behaviours. The learning environment created by the teacher limited mathematical thinking opportunities by requiring pupils to justify facts. Alternatively, the teacher allowed pupils to exercise mathematical thought by promoting the exploration of multiple realities.

The methodological approach of detailed examination of the episodes with the children's responses put both the teacher and myself as teacher-researcher, in the position of student, thus, helping our professional learning around the teaching of mathematics. This study has highlighted the need for deeper scrutiny of classroom events from multiple viewpoints. I have developed a greater understanding of the mathematics classroom and I can see how reflective and reflexive practice is key to expanding my knowledge and understanding. Taking the time to allow thought and ideas to formulate and then to rigorously challenge those ideas formed a robust approach to learning and has enabled me to see beyond the habituated routines that are widely accepted and valued within the mathematics classroom. A fresh view is needed to avoid looking at what we have looked at in the past. It is exciting when a new perspective is taken and offers a unique insight.

Although I was only working with five cases and looking at one episode in each, the findings showed emerging patterns between the cases. I recorded, transcribed and detailed unfolding events at the micro-level within the classroom, which evidenced significant differences in mathematical figured worlds. By applying a figured world framework, I examined other characteristics that differentiated the teaching and learning I witnessed in the different episodes. It was clear that pupils' and teachers' identities were evolving and adapting instead of just 'being' (Urrieta 2007: 119). This identity change is evident in my identity, moving from an education practitioner to an education researcher. From this specific study, I have noticed that regardless of the individual teaching strategies applied to develop independent learning, the environment in which they are being conducted determines the degree of opportunities for genuine mathematical thinking. When teachers view knowledge as an acquired commodity, the relationship between the teacher and the pupil becomes that of provider and receiver. When teachers view knowledge as exploring multiple realities, the teacher's role can develop more as facilitators. This will allow pupils to explore multiple realities opening up opportunities for the teacher to be surprised by the pupils’ achievements.

The purpose of this research was to generate a reflective space for teachers, pupils and myself as a teacher-researcher to examine interactions during teaching episodes in primary mathematics classrooms to understand why some pupils appear to reject the subject and others embrace it. As a former teacher and now developing teacher-researcher, I wanted to consider how a new theoretical framework might illuminate new insights into teaching and learning mathematics issues. I wanted to know how the transient nature of the classroom could have such a permanent effect on the relationship between the pupil and the subject of mathematics. From the five cases, I observed how the teacher’s practice and pedagogical approaches influenced the nature of differently figured mathematical worlds and consequently impacted learning. The pupil's conative focus or disposition towards mathematics learning of avoidance and relying on the teacher or luck to get through is based on the cognitive beliefs that mathematics is a complex subject and not for them. This view can result from being offered limited or no possibilities to exercise mathematical thinking. Whereas Boaler and Greeno (2000:172) stated, '…the practices of learning mathematics define the produced knowledge. Perhaps it is the opportunities afforded the learner to think mathematically that defines the mathematician.

References

Aguirre, J. MayfieldIngram, K and Martin, D. (2013). *The Impact of Identity in K–8 Mathematics: Rethinking Equity-Based Practices.* Reston, VA: National Council of Teachers of Mathematics.

Alsawaie, O. and Alghazo, I. (2010) The effects of video-based approach on perspective teachers ability to analyze mathematical teaching. *Journal of Mathematics teaching*. vol 13, n,3, pp. 223-241.

Alexander, R. (2008) *Towards dialogic teaching.* Rethinking classroom talk 4th ed. New York. Dialogos.

Anghileri, J. (2000) *Teaching Number Sense*. 2nd ed. Continuum.

Artique, M. Blomhøj,M. (2013) Conceptualizing inquiry-based education. In *Mathematics. ZDM* vol 45, pages797–810 (2013)

Askew, M. (2015) *Transforming Primary Mathamatics. Understanding Classroom tasks, tools and talk*. Routledge.

Atkinson, T. and Claxton, G. (2000) *The intuitive practitioner: on the value of not always knowing what one is doing.* Buckingham [England]. Open University Press

Bakhtin, M. (1981). *The dialogic imagination: four essays by M.M. Bakhtin*. (M. Holquist & C. Emerson, Eds.). Austin: University of Texas.

Ball, S., J. (1990) *Politics and Policymaking in Education.* London: Routledge.

Bandura, A. (1977) *Social learning theory.* Prentice-Hall series in social learning theory Englewood Cliffs, N.J: Prentice Hall

Banks, M. (2001) *Visual methods in social research.* London; SAGE

Banks, M. and Zeitlyn, D. (2015) *Visual Methods in Social Research*, 2nd ed. London; Sage

Barnes, D. (1992a) '*Exploring Talk in School',* in Mercer, N. and Hodgkinson, S. (eds.) *Exploring Talk in Shool*: Sage.

Barnes, D. (1992b) *From Communication to Curriculum.* 2nd ed: Boynton/Cook publications Inc.

Bassey, M. (1999) *Case study research in educational settings.* Doing qualitative research in educational settings Buckingham [England]: Open University Press

Belenky, M. F., Clinchy, B., Goldberger, N. R. and Tarule, J. M. (1986) *Women's ways of knowing : the development of self, voice, and mind.* New York: Basic Books

Bell, A. (1994) Teaching for the Test. In: Selinger, M. (Ed), *Teaching Mathematics*. London: Routledge.

Berger, J. (2008) *Ways of seeing.* Penguin modern classics. London ;: British Broadcasting Corporation and Penguin Books, p. 165 pages

Bignold, W. and Su, F. (2013) The role of the narrator in narrative inquiry in education: construction and co-construction in two case studies, *International Journal of Research & Method in Education,* 36(4), pp. 400-414.

Bloom, B. S. (1971) Mastery Learning. Mastery learning: Theory and practice*,* in Block, J.H. (ed.) *Mastery Learning, theory and practice*. New York: Holt, Rinehart and Winston, pp. 47 - 63.

Boaler, J. (2000) Multiple perspectives on mathematics teaching and learning. *International perspectives on mathematics education, 1530-3993* Westport, CT: Ablex Pub

Boaler, J. (2002) “The Development of Disciplinary Relationships: Knowledge, Practice, and Identity in Mathematics Classrooms.” *For the Learning of Mathematics* 22 (1): 42–47.

Boaler, J. (2005) The 'psychological prisons' from which they never escape: The role of ability grouping in reproduceding social class inequalities. *Forum,* 47(2 and 3), 125-134

Boaler, J. (2016) *Mathematical Mindsets*. Jossey-Bass.

Boaler, J. and Greeno, J. (2000) Identity, Agency and Knowing in Mathematics Worlds*,* in Boaler, J. (ed.) *Multiple Perspectives on Mathematics Teaching and Learning:* Ablex Publishing, pp. 171 - 198.

Bonotto, C.(2013) Problem Posting in Mathematics Teaching and Learning: Establishing a Framework for Research. *Educational Studies in Mathematics.* Vol. 83, No. 1, PME Special Issue: pp. 37-55: Springer.

Boud, D., Kaogh, R. and Walker, D. (1985) *Reflection: Turning Experience into Learning.* London: Kogan Page.

Bourdieu, P. (1973) *Cultural reproduction and social reproduction.* London: Tavistock.

Bourdieu,P. (1986) *The forms of Capital*. In: Richardson, J. Handbook of theory and Research for the Sociology of Education. Westport, CT: Greenwood pp 241-258

Boyatzis, R. (1998) *Transforming qualitative information: Thematic analysis and code development. Thousand Oaks*, CA: Sage.

Boylan, M. and Povey, H. (2021) Ability thinking*, in*: Ineson, G. and Povey, H. (eds.) *Debates in Mathematics Education.* 2 ed: Routledge, pp. 55-65.

Boylan, M. Wolstenholme, C. Demack, S. Maxwell, B. Jay, T. Adams, G. and Reaney,S (2019). Evaluation of the mathematics teacher exchange: *China-England Final Report*. London: DoE. Available at:https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_ data/file/773320/MTE_main_report.pdf

Braun, V. and Clarke, V. (2006) Using thematic analysis in psychology. *Qualitative Research in Psychology*, 3, pp. 77 - 101.

Brenneis, D. (1990) Shared and Solitary Sentiments: The Discourse of Friendship, Play, and Anger in Bhatgaon*.* In Lutz, C. and Abu-Lughod, L. (eds.) *Language and the Politics of Emotion.* Cambridge University Press, pp. 113 - 125.

Bruner, J. S. (1972) *The relevance of education*. Allen & Unwin, p. 175 pages.

Bryman, A. (2012) *Social research methods* (4th ed): oxford University Press.

Byrne, G. (2017) Narrative inquiry and the problem of representation: 'giving voice', making meaning, *International journal of research & method in education,* 40(1), pp. 36-52.

Carraher, T., Carraher,D., & Schliemann, A (1985). Mathematics in the streets and in school. *British Journal of Developmental Psychology*, 3(1), 21-29. 1985.

Chinn, S. (2006) *The Trouble with Maths. A practical guide to helping learners with numeracy difficulties:* Routledge Falmer.

Chinn, S. (2012) Beliefs, Anxiety, and Avoiding Failure in Mathematics. *Child Development Research.* pp. 8.

Christensen, P. M. and James, A. (2017) *Research with children: perspectives and practices*. Third edition. London: Routledge, Taylor & Francis Group

Clandinin, D. J. and Connelly, F. M. (2000) Narrative inquiry : experience and story in qualitative research. *The Jossey-Bass education series* 1st ed. San Francisco: Jossey-Bass Publishers

Clarke, S. (2009) Thinking Psychosocially About Difference: Ethnicity, Community and Emotion in Day, in Sclater, S., Jones, D.W., Price, H. and Yates, C. (eds.) *Emotion: New Psychosocial Perspectives*. Basingstoke: Palgrave MacMillan, pp. 1 - 16.

Claxton, G. (2000) The anatomy of intuition, in Atkinson, T. and Claxton, G. (eds.) *The Intuitive Practitioner*: Open University Press, pp. 32 - 52.

Clyde Mitchell, J. (2006) Case and Situation Analysis, in Evens, T.M.S. and Handelman, D. (eds.) *The Manchester School. Practice and Ethnographic Praxis in Anthropology*. America: Berhahn Books, pp. 27 - 32.

Cobb, P. Yackel, E. (1995) Constructivist, Emergent, and Sociocultural Perspectives in the Context of Developmental Research. *Paper Presented at the Seventeenth Annual Meeting for the Psychology of Mathematics Education* (North American Chapter) October 21-24, 1995

Cobb, P. Boufi, A. McClain, K. & Whitenack, J. (1997) Reflective Discourse and Collective Reflection. *Journal for Research in Mathematics Education* Vol. 28, No. 3 , pp. 258-277: National Council of Teachers of Mathematics

Cocks, A. (2007) The ethical maze: finding an inclusive path towards gaining children's agreement to research participation. *Childhood* 13(2): 247-66

Cohen, L., Manion, L. and Morrison, K. (2018) *Research methods in education*. 8th ed. London: Routledge

Cohen, L. M., L. and Morrison, K. (2007) *Research methods in education*. 6 ed. Abingdon: Routledge.

Coles, A. (2013) Using video for professional development: The role of the discussion facilitator, *Journal of Mathematics Teacher Education*16-3, pp. 165-184.

Coles, A. (2016) *Facilitating the discussion of video with teachers of mathematics: The Paradox of Judgment.* University of Bristol UK.

Crabtree, B. and Miller, W. (1999) A template approach to text analysis: Developing and using codebooks, in Miller, C.a. (ed.) *Doing qualitative research*. Newbury Park CA: Sage, pp. 163 - 177.

Crosby,N. (1996) Creating an authentic voice of the people. *Annual Meeting of the mid-west Political Science Association,* Jefferson centre Minneapolis. https://nedcrosby.org/wpcontent/uploads/2018/05/Creating-Authentic-Voice-of-the-People.pdf

Csikszentmihalyi, M. (1997) *Finding flow: the psychology of engagement with everyday life. MasterMinds.* New York: Basic Books

Daniels, H. (1996) *An introduction to Vygotsky*. 1st ed. edn. London: Routled

Daniels, H. (2005) *An introduction to Vygotsky.* 2nd ed. edn. London: Routledge

Davis, J. (2003) *The theory of the individual in economics:* London: Routledge

Department for Education (1988) *Mathematics in the National Curriculum*.London: HMSO http://www.school- maths.info/downloads/Maths_National_Curriculum_1989.pdf

Department for Education and Employment (1999) *National Numeracy Strategy (NNS)*. Office of standards in education.

Department for Education (2013) *Primary National Curriculum in England: Primary Framework.* London: HMSO

Department for Education (2021a) *The Standards and Testing Agency*. London: HMSO. https://www.gov.uk/government/organisations/standards-and-testing-agency/about

Department for Education (2021b) *National Pupil Data Base* https://find-npddata.education.gov.uk/categories

Dewey, J. (1933) *How we think: a restatement of the relation of reflective thinking to the educative process.* (revised edn). Boston: D C Heath.

Dockett, S. Einarsdottir, J. Perry, B. (2009) Research with children: Ethical tensions. J*ournal of Early Childhood Research.* Vol 7(3) 283-298

Dockett, S. Perry, B. Kearney, E. (2013) Promoting children's informed assesnt in research participation*. International Journal of Qualitative Studies in Education,* 26:7 802-828

Dweck, C. (2013) *Self-Theories. Their Role in Motivation, Personality, and Development.*: Psychology Press, Taylor & Francis Group.

Ellis, C. (2008) *The Sage Encyclopedia of Qualitative Research methods.* Sage Publications.

English, A. R. (2016) Dialogic Teaching and Moral Learning: Self-critique, Narrativity, Community and 'Blind Spots'*, Journal of Philosophy of Education*, 50(2), pp. 160-176.

Eraut, M. (2000) Non-formal learning and tacit knowledge in professional work, *British Journal of Educational Psychology*, 70(1), pp. 113-136.

Ernest, P. (1991) *The Philosophy of Mathematics Education.* The Falmer Press.

Estola, E. (2003) Hope as Work - Student Teachers Constructing Their Narrative Identities, *Scandinavian Journal of Educational Research* Vol 47 (2)(181 - 200).

Evans, J. (2000) *Mathematical thinking and emotions.* Routledge: London.

Every Child Counts (2008) Edghill University. *Department for Education.* https://everychildcounts.edgehill.ac.uk/

Fereday, J. and Muir-Cochrane, E. (2006) Demonstrating Rigor Using Thematic Analysis: A Hybrid Approach of Inductive and Deductive Coding and Theme Developmen. *International journal of qualitative methods.*

Foucault, M. (1980) *Power/Knowledge, Selected interviews & other writings 1972 - 1977*. Pantheon Books.

Foy, P., Garden, R., Gonzalez, E., Hastedt, D., Joncas, M., Kulik, E., Malak, B., Mullis, I., O'Connor, K., Smith, T. A. and Yamamoto, K. (2000) TIMSS 1999 Technical Report, Massachusetts: 2000 *International Association for the evaluation of Educational Achievement.*

Freire, P. (1972) *Cultural action for freedom.* Penguin education Harmondsworth: Penguin

Gardener, H. (1993) *The unschooled mind: How children think and how schools should teach.* Fontana Press UK.

Gee, J. P. (2000–2001). Identity as an analytic lens for research in education. *Review of Research in Education*, 25, 99–125.

Geertz, C. (1973) *Thick Description: Towards an Interpretive Theory of Culture, The Introduction of Cultures: Selected Essays*. New York: Basic Books.

Gelman, R. Gallistel,C. R. (1978) *The Childs Understanding of number*. Cambridge, Mass: Harverd University Press.

Gergen, K. (1995) Social Construction and the Educational Process, in Staffe, L. and Gale, J. (eds.) *Constructivism in Education.* 1st ed: Lawrence Erlbaum Associates, pp. 17 - 40.

Gluckman, M. (1961) Ethnographic data in British social anthropology. *The Sociological Review.*

Goode, W. J and Hatt, P. K (1952) *Methods in Social Research.* New York: McGraw-Hill.

Goodley, D. (2011) Narrative Inquiry, in Bannister, P. (ed.) *Qualitative Methods in Psychology: A Research Guide.* 2nd ed: M Graw-Hill Education.

Goodley, D. Lawthom, R. Clough, P. and Moore, M. (2004) *Researching life stories: Methods, theory and analysis in a biographical age*. London: Routledge Falmer.

Gouldner, A. W. (1962) Anti-minotaur: the myth of value-free sociology*. Social Problems.*Vol 9 Issue 3 pp199-213.

Gove, M. (2013) *Curriculum, Exam and Accountablility Reform.* Oral statement. Department for Education. https://www.gov.uk/government/speeches/curriculum-exam-and-accountabilityreform.

Gray, E. (2008) Compressing the counting process: strength from the flexible interpretation of symbol. In Thompson, I. (2008) *Teaching and Learning Early Number*. Open University Press.

Grundy, S. (1987) C*urriculum: Product or Praxis.* Lewes: Falmer Press.

Guskey, T.R. (1997) *Implimenting Mastery Learning* (2nd ed) . Belmont, CA: Wandsworth.

Habermas, J. R. (1984) The theory of communicative action. Vol. 1: *Reason and the rationalization of society*. London: Heinemann

Haladyna, T.M. (2006). Perils of Standardised Achievement Testing. *Educational Horizons,* 85(1), 30-43.

Hannula, M. Evans, J. Philippou, G. and Zan, R. (2004) RF01: Affect in mathematics - Exploring theoretical frameworks. Proceedings of the 28th Conference of the I*nternational Group for the Psychology of Mathematics Education*. Vol 1 pp107-136.

Hannula, M. (2006) Motivationin Mathematics. *Educational Studies in Mathematics:* Kluwer Academic Publisher.

Harcourt, D. Conroy, H. (2011) Informed consent Process and Procdures in seeking research partnerships with Young children. In: Harcourt, D, Perry, B. Waller, T. (Ed) *Reaseraching Young Childrens perspectives. Debating the Ethics and dilemmars of educational research with children.Ch 3.* Routledge.

Harris, A. and Jones, M. (2018) The unintended outcomes of PISA, in Rycroft-Smith, L. and Dutant, J.L. (eds.) *Flipping the system UK. A teachers Manifesto*: Routledge.

Hatt, B. (2007). Street smarts vs. book smarts: The figured world of smartness in the lives of marginalized, urban youth. *The Urban Review,* 39(2), 145–166.

Haylock, D. (1982) Understanding Mathematics: making connections. *Mathematics teaching,* 92:5458.

Heikkinen, H. L. T. Huttunen, R. and Syrjälä, L. (2007) Action research as narrative: five principles for validation, *Educational Action Research,* 15(1), pp. 5-19.

Hernandez-Martinez, P. and Williams, J. (2013) Against the odds: Resilience in mathematics students in transition. *British Educational Research Journal,* 39(1), pp. 45-59.

Higgins, S. Katsipataki, M. Kokotsaki, D Coleman, R. Major, L.M. and Coe, R (2013) *The Sutton Trust -Education Endowment Foundation Teaching and Learning Tool kit.* Sutton Trust

Hirsch, E. D.(1987) *Cultural Literacy. What every American needs to know.* Vintage Books.

Hogan, P. (2010) *The Significance of Learning: Imaginations Heartworks.* Routledge.

Holland, D., Lachicotte, W., Skinner, D. and Cain, C. (2001) *Identity and agency in cultural worlds. Harvard University Press.*

Jaworski, B. (2012) How we teach; Inquiry in Teaching and Learning. *Turkish Journal of Computer and Mathematics Education.* Vol 1 No. 2 105-121

Jewitt, C. (2012) An introduction to using video in research. London instutute of Education.: *National Centre for Research methods* working paper 03/12.

Jonas, M. (2011) Dewey's Conception of Interest and its Significance for Teacher Education. *Educational Philosophy and Theory:* Blackwells Publishing.

Kullberg, A., Runesson Kempe, U. and Marton, F. (2017) What is made possible to learn when using the variation theory of learning in teaching mathematics?, *ZDM Mathematics,* 49(4), pp. 559 - 569.

Laski, E., V, Jordan, J., R, Daoust, C. and Murray, A. (2015) What Makes Mathematics Manipulatives Effective? Lessons From *Cognitive Science and Montessori Education.* Sage Open, pp. 1-8.

Leach,J. Moon, B. (2008) *The Power of Pedagogy.* Sage Publications Ltd.

Lee, C. and Johnston-Wilder, S. (2017) The Construct of Mathematical Resilience, in Xolocotzin Eligio, U. (ed.) *Understanding Emotions in Mathematical Thinking and Learning*: Academic Press, pp. 269-291.

Lewis, J. (2008) *Through the looking Glass: A study of teaching, Journal for research in Mathematics Education.* Monograph, http://www.jstor.org/stable/0037739 National Council of Teachers of Mathematics., Vol.14 A study of teaching: Multiple lenses, Multiple Views, pp. 1 -12.

Lichtman, M. (2006) *Qualitative Research in Education: A users guide.* Sage publications.

Lord, M. (2020) Going deeper: Exploring ways to achieve ‘mastery at greater depth’ in the primary mathematics classroom. Proceedings of the *British Society for Research into Learning Mathematics* 40(3) November 2020. Cambridge University.

Mansell, W. (2007) *Education by numbers: The tyranny of testing*. London: Politico.

Marks, R. (2016) *Ability -Grouping in Primary Schools. Case studies and critical debates. Critical Guides for Teacher Educators.* Critical Publishing Ltd.

Marshall, H. (2002) What do we do when we code data*. Qualitative Research Journal*.

Marton, F., & Booth, S. (1997). *Learning and awareness*. Mahwah, NJ: Lawrence Erlbaum Associates.

Marton, F. (2015) *Necessary Conditions of Learning.* Taylor Frances.

Mason, J. (2002) *Researching your own practice – The discipline of Noticing.* Routledge.

Mason, J., Burton, L. and Stacey, K. (2010) *Thinking Mathematically.* 2nd ed. Pearson.

Mason, J. (2021) Effective questioning and responding in the mathematics classroom, in Ineson, G. and Povey, H. (eds.) *Debates in Mathematics Education.* 2 ed: Roultledge, pp. 131-142.

Mercer, N. and Hodkin, S. (2008). *Exploring Talk inSchool.* Sage Publication.

Mercer, N. and Wegerif, R. (1999) Is 'exploratory talk' productive talk?, in Littleton, K. and Light, P. (eds.) *Learning with computers. Analysing productive interaction.*: Routledge, pp. 79 - 102.

Merleau-Ponty, M. (1981) *Phenomenology of perception.* Routledge & Kegan Paul ; Humanities Press, London, New Jersey.

Morgan, C. (2016) Studying the role of human agency in school mathematics. *Research in Mathematical Education.* Vol:18 iss.2. *A discursive approach to the investigation of school mathematics.*

Moyles,J. Hargreaves,L. Merry,,R. Paterson,F. Esarte-Sarries,V. (2003) *Interactive Teaching in the Primary School: digging deeper into meaning*. McGrew-Hill Education (UK).

Newby, P. (2010) *Research Methods for Education.* Pearson Education Ltd.

Newman, F. and Holzman, L. (1993) *Lev Vygotsky: Revolutionary scientist*. New York: Routledge.

Nunes, T. and Bryant, P. (1996) *Children doing mathematics. Understanding children's worlds.* Oxford: Blackwells.

Nunes, T., Schliemann, A. Carraher, D. & Schliemann, A. (1993) *Street mathematics and school mathematics. Learning in doing.* Cambridge [England]: Cambridge University Press.

Nussbaum, M. (2010) *Not for Profit, why democracy needs the humanities.* Princeton University Press.

Ofsted, (2008) *Understanding the Score*. London, Ofsted.

Ofsted, (2011) *Good practice in primary mathematics: Evidence from 20 successful schools. Manchester:* Ofsted

Ofsted, (2012) *Mathematics: Made to Measure* https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_d a ta/file/417446/Mathematics_made_to_measure.pdf. Ofsted

Perry, W. G., Jr. (1999) Forms of intellectual and ethical development in the college years : a scheme. *Jossey-Bass higher and adult education series* 1st ed. edn. San Francisco, Calif.: Jossey-Bass Publishers

Piaget, J. (1929) The child's conception of the world. I*nternational library of psychology, philosophy and scientific method.* London: Routledge and Kegan Paul

Piaget, J. (1972) *Principles of genetic epistimology*. London: Routledge and Kegan Paul

PISA, (2014) *'PISA in Focus 34: Who are the strong performers and successful reformers in education?*, (Accessed 02 January 2020).

Polkinghorne, D. E. (1995) Narrative configuration in qualitative analysis*, International Journal of Qualitative Studies in Education,* 8(1), pp. 5-23.

Povey, H. (1997) Beginning mathematics teachers' ways of knowing: the link with working for emancipatory change*, Curriculum Studies,* 5:3, pp. 329-343.

Povey, H. and Burton, L. (1999) Learners as authors in the mathematics classroom, in Burton, L. (ed.) *Learning Mathematics: from Hierarchies to Networks.* London: Routledge, pp. 232-245.

Priestley, M., Biesta, G. and Robinson, S. (2015) Teacher agency: what is it and why does it matter?, in Kneyber, R. and Evers, J. (eds.) *Flip the System: Changing Education from the Bottom Up.* London: Routledge.

Pring, R. (2005) *The Philosophy of Education.* Bloomsbury Publishing.

Radford, Bardini, Sabena, Diallo and Simbagoye's (2005) On embodyment, artifacts, and signs: A semiotic cultural perspective on mathematical thinking: In Chick, H. L. & Vincent, J. L. (Eds.). *Proceedings of the 29 th Conference of the International Group for the Psychology of Mathematics Education,* Vol. 4, pp. 113-120. Melbourne: PME.

Raiker, A. (2002) *Spoken Language and Mathematics.* Taylor and Francis.

Reid, D. (1996) Enactivism as a methodology*. The Twentieth Annual Conference of the International Group for the Psychology of Mathematics Education,* Valencia, Spain, 203-210.

Robeyns, I. (2006) Three models of education Rights, capabilities and human capital*,* Theory and *Research in Education,* 4 (1), pp. 69-84.

Rogers, C. R., Kirschenbaum, H. and Henderson, V. L. (1990) *The Carl Rogers reader.* London: Constable.

Rogoff, B. (1990) A*pprentership in Thinking: Cognitive Development in Social Context*. NY Oxford University Press

Rolfe, G. (2014) Rethinking reflective education: What would Dewey have done? *Nurse education today*, 34(8), pp. 1179-83.

Rorty, R. (1980) *Philosophy and the mirror of nature.* Oxford: Blackwell.

Rose, G. (2016) *Visual methodologies : an introduction to researching with visual materials*. 4th edition. edn. London: SAGE Publications Ltd.

Rousseau, J,J. (1956) *Emile for today.* Boyd,W. (ed.). London: Heinemann.

Rowland,T. Turner, F. Thwaites,A. Huckstep,P. (2009) *Developing Primary Mathematics Teaching.* Sage Publications.

Rowland,T. (1999) *The Pragmatics of Mathematics Education : Vagueness and Mathematical Discourse.* Taylor and Frances.

Russell, B. (1938) *Introduction to mathematical Philosophy.* The Macmillian Co.

Russell, B. (1946) *History of Western Philosophy.* Routledge.

Salmon, P. (1995) *Psychology in the classroom.* Cassell Education.

Schutz A. (1962) On Multiple Realities. In: Natanson M. (eds) Collected Papers I. *Phaenomenologica* (Collection Publiée Sous le Patronage des Centres d'Archives-Husserl), vol 11. Springer, Dordrecht. https://doi.org/10.1007/978-94-010-2851-6_9.

Scott, D. (2005) Critical Realism and Empirical Research Methods in Education, *Journal of Philosophy of Education,* 39(4), pp. 633-646.

Seligman, M. (1975) *Helplessness On Depression, Development, and Death.* W H Freeman and Company San Francisco.

Seligman, M. (2006) *Learned Optimism. How to Change your Mind and your Life.* Vintage Books.

Selmo, L. and Orsenigo, J. (2014) Learning and sharing through reflective practice in teacher education in Italy*. Procdia Social and Behavioural Science,* 16, pp. 1925 - 1929.

Sfard, A. (1998) On Two Metaphors for Learning and the Dangers of Choosing Just One*. Educational Researcher,* 27(2), pp. 4-13.

Sfard, A. (2008) *Thinking as communication.* Cambridge University Press.

Shaw, C., Brady, L. and Davey, C. (2011) National Children's Bureau Guidelines for research with Children and Young People: *National Children's Bureau Research Centre*. Charity Registration Number258825.

Simon, B. (1981) Why no pedagogy in England*?* in B. Simon & W. Taylor (Eds) *Education in the Eighties*. London: Batsford.

Skemp, R. (1993) *The Psychology of Learning Mathematics.* Penguin Books.

Stake, R. (2005) Qualitative case studies. in Denzin, N., K and Lincoln, Y., S (eds.) *The Sage handbook of qualitative research*: Sage publications Ltd, pp. 443 - 466.

Stake R. E. (1995) The art of case study research. Thousand Oaks, CA: Sage.

Star, J. and Strickland, J. (2008) Learning to observe: using video to improve preservice mathematics teachers' ability to notice*. Journal of Mathematics Teacher Education,* 11(2), pp. 107 - 125.

Swan, M. (2006) Collaborative learning in mathematics : a challenge to our beliefs and practices. London: *National Research and Development Centre for Adult Literacy and Numeracy.*

Swann, M. (1985) Swann Report: *Education for All. Report of the committee of enquiry into education from minority groups*. Her Mageisty's Stationary office: London.

Tall, D. O. (2013) *How humans learn to think mathematically : exploring the three worlds of mathematics. Learning in doing: social, cognitive and computational perspectives Cambridge*: Cambridge University Press.

Thompson, I. (2008) *Teaching and learning early number.* McGraw Hill Open University.

Thompson, I. (2010) *Issues in teaching numeracy in primary schools Berkshire, England*: McGrawHill, Open University Press. Available at: http://public.eblib.com/choice/publicfullrecord.aspx?p=771428. Available at: http://site.ebrary.com/id/10413337.

Thorndike, E., L (1998) Animal intelligence: An experimental study of the associate processes in animals. *American Psychologist,* 52(10), pp. 1125-1127.

UNICEF (1989) *The United Nations Convention on the Rights of the Child*. London; UNICEF Treaty no. 27531. United Nations Treaty Series, 1577, pp. 3-178. Available at: https://treaties.un.org/doc/Treaties/1990/09/19900902%2003-14%20AM/Ch_IV_11p.pdf (Accessed 3 July 2020).

Urrieta, L. (2007) Figured Worlds and Education: An Introduction to the Special Issue. *The Urban Review : Issues and Ideas in Public Education,* 39(2), pp. 107-116.

Valentine, G. (1999) Being seen and heard? The Ethical complexities of workin gwith children and young people at home and at school. *Ethics, Place and Environment*, 2:2, 141-155

van Es, E. and Sherin, M. (2002) Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education,* 10, pp. 571 - 596.

Van Manen, M. (2014) *Phenomenology of Practice*. Left Coast Press.

Varela, E., Thompson, E. and Rosch, E. 1991. *The Embodied Mind: Cognitive Science and human experience.* MIT Press.

von Glasersfeld, E. (1995) A constructivist Approach to teaching. in Staffe, L. and Gale, J. (eds.) *Constructivism in Education.* 1st ed: Lawrence Erlbaum Associates (LEA), pp. 3 - 16.

Vygotsky, L. S. (1965) *Thought and language*. The M.I.T. paperback series Cambridge, Mass.: M.I.T. Press.

Vygotsky, L., S (1978) *Mind in Society: The development of higher psychological processes.* Cambridge MA: Harvard University Press.

Vygotsky, L., S (1981) The development of higher forms of attention. in Wertsch, J. (ed.) *The concept of activity in Soviet Psychology*: Armonk NY: M.E. Sharpe, pp. 189-240.

Walker, R. (2017) Naturalistic Research, In: Coe, R. Waring, M. Hedges, Arthur H & J (2nd) *Research Methods and Methodologies in Education.* Sage: London.

Watson, A. & Mason, J. (2006) Seeing an exercise as a single mathematical object: Using variation to structure sense-making. *Mathematical thinking and learning*. 8(2),91-111.

Wertsch, J. (1985) *Vygotsky and the Social Formation of Mind.* Harvard University Press.

Wertsch, J. (1998) *Mind and Action*. Oxford University Press.

Wertsch, J. and Tulviste, P. (2005) L.S. Vygotsky and contemporary developmental psychology. *I*n H Daniels (Ed2)*, An Introduction to Vygotsky p. 59-80. Routledge.*

Wickstrom,M,H. (2017) 'Mathematical Modelling: Challenging the Figured Worlds of Elementary Mathematics', in Proceddings of the 39th annual meeting of the North American Chapter of the International group for the Psychology of Mathematics Education. Galinda, E., & Newton,J., (Eds.)

Wigley, A. (1992) Models of Teaching Mathematics. *Mathematics Teacher.* 141:1.

Williams, G. (2003) Associations between student pursuit of novel mathematical ideas and resilience, in Mathematics Education Research: Innovation. *26th Annual Conference of the Mathematics Education Research Group of Australia*, Deakin University, Geelong: Deakin University, Geelong, Vic, 752-759.

Wing, T. (2016) *Learning Patter. Mathematics Teaching:* www.atm.org.uk.

Wood, D., Bruner, J. and Ross, G. (1976) The role of tutoring in Problem Solving. *Journal of Child Psychology and Psychiatry,* 17, pp. 89-100.

Wood, E. (2014) Free choice and free play in early childhood education: troubleing the discourse. *International Journal of Early Years Education,* 22:1, 4 – 18.

Wragg, E., C (1994) *An Introduction to Classroom Observations*. Routledge.

Yerushalmi,H. (2021) Authentic Voices in Supervision. *British Journal of Psychotherapy* 37, 1 (2021) 116–129.

Glossary of Terms

Conative – The attempted action associated with a cognitive belief.

Conceptual naivety – a lack of understanding or a misconception

Cultural artefact - Something that is in the “consciousness” of a particular culture for a particular reason.

Human capital model – In relation to education, human labour considered as a commodity

Perturbation – Something that raises questions because it deviates from the norm.

Procept – In mathematical terms the merging of concepts and procedures.

Self-concept – a collection of beliefs about one’s self

World making – how an individual interprets their world

Appendices

Appendix A. Parent Information Sheet

Study title: **Reflective practice in the mathematics classroom**

My name is Ms Jane Fletcher and I am a doctoral student within the School of Educational at Oxford Brookes University. I am asking if you would be prepared to participate in a research study to explore reflective practice in the year two mathematics classroom. Before you decide to take part please read the following information carefully; it is important that you understand why this research is relevant and what it will involve.
What is the purpose of the study?
This research is to look at the complex nature of the mathematics classroom and to take time to look at one mathematics activity through the eyes of the teacher and pupils separately. The two worlds of the teacher and pupil will be brought together, providing different views for the same activity, allowing for insight into what is being learnt. The aim of this research is to create a space, using video and audio equipment to support the recall of the session and compare the same activity from your own personal perspectives. It is hoped that this will create dialogue which will support the understanding of mathematical concepts. The research will be part of the normal school day and will focus on the group work section of the mathematics lesson.

Why have I been invited to participate?
I am approaching a range of schools in the Oxfordshire region and your Head-teacher has shown an initial interest in this project.

Do I have to take part?
Taking part in this project is voluntary and if you decide to decline the invitation you can do so without providing a reason. If you are willing to take part in the study please complete the attached consent form. Consent to use the data from the study will be implied by the return of the completed consent form. If you wish to withdraw from the research, at any stage, you may do so without explanation and any data previously collected and/or subsequently analysed will be immediately destroyed.

What will happen to me if I take part?
The research will take place over the duration of one school day and will include an interview of approximately one hour to one and a half hours maximum. If you agree to take part in the project, the recording will take place during the normal course of teaching, which would be a pre-agreed timetabled mathematics group-work session and should last for no longer than thirty minutes and will be videoed. The video recording will focus on the activity the pupils will be doing.
An information sheet similar to this and a consent/assent form will be sent to the parents of the pupils informing them of the action they should take if they want their child to participate in the project and be video-recorded. The group of students selected will be the group you would normally be teaching on the day the research takes place. If there is a pupil without permission to take part in the group, then the research will not take place and we will reschedule for another day. This is to keep the process as naturalistic as possible. The two pupils selected from the video footage will depend on which part of the lesson we choose to focus on. Children will be asked for their assent because they are under eighteen years old. They will be able to withdraw at any time if they are not happy to participate, even if their parents have given their consent.
The next stage will be for you, in consultation with the researcher, to select two pupils from the group and a five minute section of the recording to use to support recall. An audio recording will be

taken of your reflections on the section; this process will take place at your school at a time which is convenient to you and should last no longer than 45 minutes.
The two pupils will also be asked to commentate on the activity using the same video clip that you have selected and to recount their experience. This will be audio recorded. The final phase of the process will be to play you the audio recording of the pupils' feedback, creating an opportunity for deeper analysis. The pupils will not be part of the final discussion and this part will no longer than 45 minutes.

What are the possible benefits of taking part?
This in depth reflective process could help to clarify some of the complexities of the mathematics classroom. Plus it could help develop your skills in reflective practice.

Will what I say in this study be kept confidential?
All data will be coded and identities of contestants will remain confidential. The participating school, teacher and class pupils will remain anonymous. All information collected will be kept strictly confidential throughout the study but because the number of staff participants and the size of the school it may be impossible to guarantee. All raw data, will be scrutinised and analysed and returned to storage and subsequently archived in a safe locked place at all times between processing. Data will be destroyed ten years after the conclusion of the study.

What should I do if I want to take part?
If you wish to take part please retain this information sheet and sign the consent form provided, with the appropriate boxes initialled or ticked. Please post it in the self-addressed envelope and a copy will be returned to you prior to observation or return electronically.
What will happen to the results of the research study?
The project will conclude within a two year time period with the completion of a doctoral thesis in 2019; a summary of the findings can be made available for you to read upon request.

Who has reviewed the study?
This project has been approved by the University Research Ethics Committee, Oxford Brookes University.

Contact for Further Information
The researchers contact details and those of the supervisors respectively are:

Ms Jane Fletcher	Dr Richard Newton	Dr Catharine Gilson
Oxford Brookes University	Oxford Brookes University	Oxford Brookes University
School of Education	School of Education	School of Education
Harcourt Hill Campus	Harcourt Hill Campus	Harcourt Hill Campus
Oxford OX2 9AT	Oxford OX2 9AT	Oxford OX2 9AT
Tel: 01865488603	TEL: 01865 488499	Tel: 01865 488167
Email:	Email:	Email:
j.fletcher@brookes.ac.uk	rnewton@brookes.ac.uk	cgilson@brookes.ac.uk

If you have any concerns about the way in which this study is being conducted please contact my named supervisors above or the Chair of the University Research Ethics Committee at Oxford Brookes (email: ethics@brookes.ac.uk).

Thank you for taking the time to consider your involvement with my project

Parent Information Sheet

Study title: **Reflective practice in the mathematics classroom**

Dear

My name is Ms Jane Fletcher and I am a doctoral student within the School of Educational at Oxford Brookes University.
__________________ has been invited to take part in a research study to find out more about ways of using video equipment to support the learning of mathematics. Please read the following information carefully; it is important that you understand why this research is relevant and what it will involve.

What is the purpose of the study?
The aim of this research is to create an opportunity for teacher and pupils to reflect on a lesson with the help of video and audio equipment. This will provide a chance to revisit the lesson and talk through their understanding of mathematical concepts. The research will be part of the normal school day and will focus on the group work section of a mathematics lesson.

Why has my child been invited to participate in the observed mathematics lesson?
Your child has been invited to take part because the school's head-teacher and your child's class teacher have shown an interest in better understanding how video can be used in the classroom to improve teaching and learning in mathematics. Therefore all the children in your child's class, and you as a parent, are being approached to ask if they would like to participate in this research.

Do they have to take part?
Taking part in this research is entirely voluntary. Not all of the classroom participants will be recorded on camera and any parent who does not wish their child to be filmed can be assured they will still be able to continue with their lesson as normal and will be out of the field of view of the camera. An assent form, because they are under the age of eighteen, will be provided for your child to sign, stating that they understand that the researcher will film the lesson. They will also receive an age appropriate, child-friendly information sheet, there will be electronic and paper copies available. Choosing to either take part or not take part in the study will have no impact on their marks, assessments or future studies.

What will happen to my child if they take part?
The observation will take place during the normal course of teaching and will be a timetabled group mathematics session, pre- arranged in consultation with the teacher, in June and should last for no longer than thirty minutes. The observation will focus on the activity taking place in the lesson. Audio visual equipment will be used to capture one of the mathematics sessions but only with the signed consent/assent of participating teacher and the pupils involved in the mathematics session. Following the videoing of the session a maximum of just two children will be invited to view a five minute clip and describe the mathematics taking place. This will be recorded on audio equipment. Not all the pupils who give assent will necessarily take part due to the small scale of this research. The group of students selected will be the group the teacher would normally be teaching on the day the research takes place. This is to keep the process as naturalistic as possible. The two pupils selected from the video footage will depend on what happens during the session. All data will be coded and identities of participants will remain confidential.

What are the possible benefits of taking part?
The outcomes of this project will aim to help inform teacher, schools, policy makers and others to develop skills in using video and audio equipment to enhance and develop practice in the mathematics classroom and help to clarify some of the complexities of the mathematics classroom.

Will what my child says in this study be kept confidential?
The participating school, teacher and class pupils' will remain anonymous. All information collected will be kept strictly confidential throughout the study. Each school and child will be given a pseudonym to retain confidentiality and safeguard their identity. All identifying and coding information

will be securely stored separately to the raw data. All raw data, paper or electronic, will be scrutinised and analysed but returned to storage and subsequently archived in a safe locked place at all times between processing. The video footage and all raw data will be destroyed ten years after the conclusion of the project.

What should I do if I want my child to take part?
If you agree to your child taking part in this research, could you ask your child directly whether they would like to be involved in the research and support your child to complete the assent form and help them understand that the researcher will film the lesson? They will receive an age-appropriate, child friendly information sheet. Choosing to either take part or not take part in the study it will have no impact on their marks, assessments or future studies.

What will happen to the results of the research study?
The project will conclude within a two year time period with the completion of a doctoral thesis in 2019; a summary of the findings can be made available for you to read upon request.

Who has reviewed the study?
This project has been approved by the University Research Ethics Committee, Oxford Brookes University.

Contact for Further Information
The researchers contact details and those of the supervisors are:

Ms Jane Fletcher	Dr Richard Newton	Dr Catharine Gilson
Oxford Brookes University	Oxford Brookes University	Oxford Brookes University
School of Education	School of Education	School of Education
Harcourt Hill Campus	Harcourt Hill Campus	Harcourt Hill Campus
Oxford OX2 9AT	Oxford OX2 9AT	Oxford OX2 9AT
Tel: 01865488603	TEL: 01865 488499	Tel: 01865 488167
Email:	Email:	Email:
j.fletcher@brookes.ac.uk	rnewton@brookes.ac.uk	cgilson@brookes.ac.uk

If you have any concerns about the way in which this study is being conducted please contact the supervisors, contact details are above or the Chair of the University Research Ethics Committee at Oxford Brookes (email: ethics@brookes.ac.uk).

Thank you for taking the time to consider your involvement with my project.

Yours sincerely

Jane Fletcher

Doctoral Student

Programme Coordinator BA(Hons) Primary Education

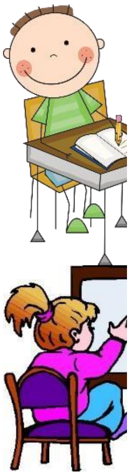
Appendix B. Pupils information Sheet

What's happening in your mathematics classroom?

My name is Jane Fletcher and I would like to invite you to take part in a research study using video cameras in your mathematics lesson.

You might have a chance to be recorded and then watch part of your mathematics lesson and talk about it to see if that helps your learning.

You do not have to take part if you don't want to. If you choose to take part we will ask you to sign a form to say you are happy to be videoed. You can change your mind at any time.



Your teacher will tell you when the lesson will be recorded and if you are in the group. It will be your normal mathematics lesson. After the lesson you and a friend might be asked to watch a clip of the video where you are doing some mathematics and be asked to describe what is happening.

You might enjoy taking part, plus having a chance to see your lesson again could be interesting. The only people who will see the video will be your teacher, you, your friend and me.

If you would like to take part you just need to tell your parents and teacher.

The information collected will be written down and you, your teacher and parents can read it later if you like.

Thank you for your time.
Date: June 2018

Appendix C. Teacher and Parent consent forms

OXFORD BROOKES UNIVERSITY

Reflective practice in the mathematics classroom

CONSENT FORM FOR THE TEACHER TO PARTICIPATE IN THIS RESEARCH

Please tick the box if you agree with the sentence.

I confirm that I have read and understand the information sheet for the above study and have had the opportunity to ask questions. ☐

I understand that my participation is voluntary and that I am free to withdraw at any time, without giving reason. ☐

I agree to take part in the above study. ☐

I agree that my data gathered in this study may be stored (after it has been anonymised) in a specialist data centre and may be used for future research. ☐

I agree to the group mathematics session being video recorded. ☐

I agree to my analysis of the session being audio recorded. ☐

I agree to the use of anonymised quotes in publications. ☐

Name of Teacher	Date	Signature
Name of Researcher	Date	Signature

Thank you for taking the time to consider your involvement with my project.

OXFORD BROOKES UNIVERSITY

Reflective practice in the mathematics classroom

CONSENT FORM FOR THE PARENTS TO ALLOW THEIR CHILD TO PARTICIPATE IN THIS RESEARCH

Please tick box if you agree with the sentence.

I confirm that I have read and understand the information sheet for the above study and have had the opportunity to ask questions. ☐

I agree to the researcher using a video camera to record my child's group session in Mathematics at school. ☐

I understand that my child might be asked to describe part of the lesson and I agree to that being audio recorded. ☐

I agree to my child taking part in this study. ☐

Parent's Name	Date	Signature
Pupil's name	Date	Signature
Name of Researcher	Date	Signature

Thank you for taking the time to think about your involvement with my project.

Appendix D Pupils Assent Form

Reflective practice in the mathematics classroom

ASSENT FORM FOR PARTICIPANTS UNDER THE AGE OF 18 TO PARTICIPATE IN THIS RESEARCH

Please tick the box if you agree with the sentence.

I understand that I do not have to take part in the study. ☐

I agree to the researcher recording the group session using a video camera. ☐

I agree to take part in a discussion about my lesson if I am asked to and it will be audio recorded. ☐

I have had the opportunity to ask questions. ☐

I agree to take part in the above study.

Pupil's Name	Date	Signature
Name of Researcher	Date	Signature

Thank you for taking the time to think about your involvement with my project.

Transcripts

The following five transcripts are samples of the original transcripts from each episode from each school. This will provide the context in which excerpts have been taken.

Transcript A

Weston School, Symmetry, Case A

Researcher: So I've got sort of two little clips I showed them. They were very good, very eloquent, they were, you know, can we stop the camera now, we'll talk about it, type of thing. Let me get to the bit where, the more I watch it the more I see, so you may watch it and think, oh. So what we'll do, if we just watch the video, see what you think. If you want to stop, we'll stop and have a chat, and we can re-run. So there's a little five minute slot here and there's a little bit at the end I thought was quite good.

Watching Clip……………

Researcher: I'll stop it back there because they're getting into what they're doing there. And the other bit towards the end, it's when you gave them the little sheet I think. Did I go, actually, no, I think I gave it, I will show you the thirty because I can't remember, I put it down, so they'll be a reason why I put that.

Researcher: I think I stopped there because of the proof, because they were saying, the proof, I'm not quite sure what he's proving, which was quite interesting.

Ms Travis: Yes, that's interesting, isn't it? I think they had the shape and because she has said x, that it wasn't symmetrical, and when she was looking in the mirror she could see that it wasn't, they could see it was not the same on either side.

Researcher: So that was the proof?

Ms Travis: But that was the proof, yes.

Researcher: No, that's good.

Ms Travis: I think that is that, on that one.

Researcher: I do ask them at the end of the interview as well, which, so they're quite confident ……………………

Ms Travis: They seem so mature.

Researcher They're so confident, the way they talk.

Clip:

Because you were talking about the x in the mirror, does that help to do the x or not? So up here, put your x up here. You've got your x there and your x there, so if you put the mirror there or there, does that help you knowing if it's symmetrical or not?

Not really. We haven't really tried that yet. At home I always use the mirror to see which side needs to go off, my mirror at home.

It makes everything go backwards.

But the house was symmetrical because if you put the mirror on the line, it would show the chimney here and here.

Clip:

…………………

Researcher: So what do you think there that's, you're surprised with or?

Ms Travis: No, it's really interesting. I'm not surprised, what's really lovely is the conversation. That they're not scared to be right or wrong with each other. And that's really nice to actually see because you don't ever, I feel like I've developed that ethos but to actually see, and these two wouldn't normally, they're not friends, so they wouldn't normally, well they all get on in my class but, you know, they wouldn't naturally

Researcher: It's not their natural?

Ms Travis: So the fact, and you could see in the beginning bits where Tom was not really, he kind of didn't seem like he was interacting with Asha but he was listening to what she was saying, wasn't he? And then he chipped in with his thoughts. So even though he was busy, he was actively listening to what she was saying. And she was a very good partner actually, to keep drawing him back in, wasn't it?

Researcher: Yes.

Ms Travis: And that's really nice to see because I don't, without having that, I wouldn't have ever really seen it.

Researcher: No.

Ms Travis: And what's the other thing, I know I'm not really massively talking about the maths at the moment.

Researcher: No, no, that's fine.

Ms Travis: Their full sentences. We've worked a lot on speaking in full sentences to try and articulate our thinking. And I would say, these two are one of the ones that find it tricky. And you wouldn't really

Researcher: You wouldn't have got that from there.

Ms Travis: Got from that, they seem really mature and really, they're using the language.

Researcher: She's stumbling over the words at the end but it was just, it wasn't noticeable, it wasn't stopping her from speaking those full sentences at all.

Ms Travis: No, I think that's because she was reading.

Researcher: Oh yes, of course she was, yes.

Ms Travis: She was reading, so that's why she's stumbling over it, but yes. And so that's really positive for me, to see that all that is working. But yes, I think it did show, Tom does get himself mixed up a bit sometimes, and I think it's also his way of articulating what he's thinking. Because, you know, with the mirror, he was saying, it is symmetrical, but he didn't go on to say, it's symmetrical when I look in the mirror. But that's probably, I think when you probed him a bit more, that is actually, what he actually meant. So on paper, it might look like, he said it is symmetrical, but actually, by listening to that, he knows it's not symmetrical but when he looks in a mirror, that is the symmetrical image. So this kind of gives another layer to what they're actually recording.

Researcher: It's actually showing that his, he's thinking much deeper or thinking different things to what he's articulating. Very articulate but he's been very, almost literal about it, isn't he?

Ms Travis: Yes, he is being very literal about it.

Researcher: Yes. This is a reflection that is symmetrical. Because there wasn't any shock or anything, or change, when he got to the bit, oh yes, the chimney on it, the chimney's not on it.

Ms Travis: Yes. He was agreeing, wasn't he, with Asha, when she said, no, like it's not symmetrical because there's an extra bit, there's a chimney. And he was like, yes, I support that. So you can tell he actually, he knew it wasn't symmetrical but he was more talking about when you're looking, that it wasn't clear. And if he had wrote in his book

Researcher: He'd have got that wrong.

Ms Travis: It would have looked like he got it wrong but actually, he did actually understand it. And he was in a deeper level about, thinking about, whenever I look in a mirror, if I look into the mirror, that's always going to be the symmetrical, it's going to be symmetrical. So that was really interesting.

Researcher: So they understand why they're using the mirror, what the mirror does, which I thinks really interesting.

Ms Travis: That is good. They did use it really well. On the clips, on the table, you could see, I didn't know whether they would all use the mirror that well because it can be quite a tricky concept to know that when I look in it, that's what I should be seeing on the other side.

Researcher: Because you can look at it and think, oh yes, that's symmetrical, and you forget to look, oh no, it's not.

Ms Travis: Look on the other side, yes. So that was really, it's really good to see. And it just shows me that, especially at this age, later on he will become more articulate and be able to like have the stamina to write everything that he's thinking. But just to have those like conversations about what they're recording in the books if I'm unsure as to what they mean, that's really, just flag that up a bit more.

Researcher: Yes, it does show that deeper meaning. And the discussion around the star was quite interesting.

Ms Travis: That was, wasn't it? About how the cut, and he was being, it showed, because he had cut it, so it didn't fold equally. He knew it wasn't symmetrical.

Researcher: It's interesting because between the two of them, her style was less symmetrical than his. But she was saying it was symmetrical and he was saying it wasn't. But actually, if you want to be really pernickety, hers was far less symmetrical.

Ms Travis: Far less symmetrical because of her cutting.

Researcher: Yes, which was quite interesting. And yet, she just went, yes, this is it.

Ms Travis: When she kind of got, so he's very literal, isn't he? Whereas, she kind of gets the general gist from the results, and she's like, oh that's, like oh I can see my cutting, like she's kind of jumped and gone

Researcher: The next process.

Ms Travis: Yes, it is symmetrical, it's just how I've cut it. She's kind of on that step. Whereas, he is also thinking deeply because he's thinking, well

Researcher: It doesn't cover, it doesn't match.

Ms Travis: It doesn't, it doesn't match.

Researcher: It was like, I can see things behind it.

Ms Travis: Yes, exactly. So he is going deeper. So in both respects, by the cutting

Researcher: They've both got different lines of thought going.

Ms Travis: Yes, doesn't it?

Researcher: It just shows that maths isn't black and white.

Ms Travis: Yes, it definitely isn't.

Researcher: Completely, and they're both as equally correct. So there isn't

Ms Travis: Yes, and as long as you can, I always say that to them, as long as you can justify to me your answer, I don't mind.

Researcher: And the way they were supporting each other with that discussion, there was no

Ms Travis: Yes, they were.

Researcher: It was, I disagree with you but it was also, I do agree with you.

Ms Travis: Yes, and they were listening, listening to each other, weren't they?

Researcher: Yes.

Ms Travis: Yes, no, that's really lovely to see. And I'm just, it really shows that they're using the vocabulary. And things like top tip

Researcher: Yes.

Ms Travis: Yes, like I do say, don't give the answer to your partner, because they do so much partner talk, I don't often go, give a top tip. I just sometimes refer like, oh like today, oh Tom, did Tom give you a top tip? And it was, she's picked that really, that up, so that was really good.

Researcher: And between them as well, watching them work together, because occasionally she was working on her, but like you said, he was listening in. But they were both equally giving to the conversation, there wasn't one dominating at all.

Ms Travis: No, I'd agree.

Researcher: And she was definitely, is she quite friendly with Tom as well?

Ms Travis: Yes.

Researcher: Because she was sort of talking to him and coming back again.

Ms Travis: Yes.

Researcher: So it was really nice.

Ms Travis: Very collaborative, wasn't it? But it was so much so that it kept pushing their own, they were pushing their own learning on without me being there, weren't they? Because they were talking and discussing and supporting and challenging each other, they were going deeper without me having to actually

Researcher: She'd moved on to the curvy, the wibbly, wobbly one, for the folding, because they were all talking about the corner coming off, but she was actually, she'd gone to the other one.

Ms Travis: Having a go at that.

Researcher: And looking at it and showing you that, which was really nice, yes.

Ms Travis: Yes.

Researcher: OK, well I think that's, that's lovely.

Ms Travis: No, that's really lovely. And I like this, I think I might think about

Researcher: Get some batteries.

Ms Travis: Yes, but just having a, that's quite a nice, because if you have some children that you really want to focus in on, just here, because that, I know you wouldn't be able to listen to everyone, but just to get a snap, to get a snapshot of how they're feeling about it.

Researcher: Because even though that was quite a long session to film and it appeared, because when I was looking at it I was thinking, what have we got in here? And then you get a little bit like that and as you know the children, you can unpick that even more.

Ms Travis: Yes, definitely. I found it really, I just found it really, yes, interesting, their dynamic, because, as I say, they wouldn't naturally, I think it just really shows, yes, that they were confident in talking with what they were thinking about, weren't they? But not, they're not shying away from it.

Researcher: No, and they were engaged, they were engaged in what they'd been doing and they were very good at justifying what they were doing. It wasn't, oh I'm going to, he did go off and talk about the computer but that was at one point. And, like you say, if that's his normal default, that's, and this is all new to him, so it's something different for him to look at. So you expect that a little bit.

Ms Travis: Yes.

Researcher: And their conversations together, I thought was really revealing. Each bit of it seemed to build on what they'd been doing in the first bit, which shows that they were following that line of thought.

Ms Travis: Yes, they were, and they were following, yes.

Researcher: Yes, oh good.

Ms Travis: That was really interesting.

Researcher: Yes, so you found the video bit useful?

Ms Travis: I actually do think that's a really useful tool to see more in-depth, so if you had a child, for example, who you weren't sure why, if they were struggling and you couldn't work out why, that's quite an in-depth understanding of them. And like that's given me a little bit of insight into Tom actually, because it really shows that he doesn't get a lot on paper. And so sometimes it could look wrong, like, as I say, like if he does the house, but actually, by having that conversation or listening to what he was actually meaning about that, it really shows that he did understand. He was, just hadn't like deepened it on what he's actually recording in his book.

Researcher: Because watching him, when he said about the cross at the beginning and I thought, oh this is the mirror, is this something, because that was, to me, it was a trigger that perhaps he hasn't understood that. But actually, as it goes on, again, he knew what that was without having to explain it really. That was leading me down a rabbit hole, which I didn't need to go down.

Ms Travis: Yes. So I think the use of film is really, that's really quite a handy tool to, especially with the whole class approach, where talk is vital, it is vital. That's the whole point, it's them deepening their own understanding by following their own line of enquiry. Everyone's doing the same but once they've started on the question, they can push themselves to wherever they, wherever their thinking takes them. And just to have some snapshots of that is quite nice because you can't get round everyone. So you could be like, well this week, or today, I'm going to focus on this pairing and see what they get.

Researcher: And have a quick flick through and see what you've got.

Ms Travis: Yes, see what you've got, yes.

Researcher: Yes.

Ms Travis: Or even just using it as a, as you say, like her talking to the camera to explain. So if there's a child that really finds it hard to articulate what they're thinking, they could do it to the camera, so yes.

Researcher: Well it takes away the person, doesn't it? It makes it more private for them.

Ms Travis: Yes, definitely, like a little private showing, isn't it?

Researcher: You Tubing.

Ms Travis: That was cute.

Researcher: Or you could almost get them to, because you get them to explain to the rest of the class, but actually, if they did it on there and then they showed the rest of the class

Ms Travis: What they were doing, yes.

Researcher: So if you got somebody who did it really well, if there was a concept they weren't quite getting or you wanted to reinforce it, it would be quite interesting to see children teaching children.

Ms Travis: It would be, wouldn't it? Yes.

Researcher: And if you've actually got them to film it and then edited it, so you've got it really precise, that would be great.

Ms Travis: Yes.

Researcher: That would be really interesting to look at actually.

Ms Travis: Yes, it would be, wouldn't it?

Researcher: Yes, children teaching children.

Ms Travis: Because that's, essentially, what this whole, because they do so much discussion and they support and challenge each other, they are adjusting their thought processes all the time. And I think that's really

Researcher: That showed that, didn't it?

Ms Travis: They're not worried about making a mistake. They're listening and if what the person has said makes complete sense and they've got proof and it's justified well, then they are OK with accepting what that person says. And if they're not OK they'll go, well no, I disagree.

Researcher: They're not aiming for the end result, which I found very, it really stood out within these two. It wasn't the end result, it was the discussion all the way through. Because even the star, how many lines of symmetry with the star, and this was quite interesting because he kept rubbing it down, no, I did this to show that it doesn't go that way. But then he got the line down there, he goes, yes, yes, but it has got a line.

Ms Travis: Yes. So, initially

Researcher: I thought, oh he doesn't know this.

Ms Travis: He's kind of saying, it's not symmetrical when you look at it that way but actually, when, and it's that other, just that other probing question, but what did Asha do differently? And he was very matter of fact, well she folded it a different way, so it was symmetrical. And it was very matter of fact, wasn't it?

Researcher: Yes.

Ms Travis: As in, yes.

Researcher: And oh you've got a line down. Yes, yes, that's a line of symmetry. Oh right.

Ms Travis: Yes.

Researcher: So it is symmetrical? Yes. Oh OK, right OK. So yes, so it's quite interesting, the conversation, yes, which was nice. Yes, it's really good. Well thank you ever so much.

Ms Travis: No, thank you and if you ever need anymore, let me know.

Transcript B

Langford School – Word Problem, Case B

Researcher: From the actual lesson itself um they are working quite hard and doing different things but before we talk about that do you want to tell me a bit about the lesson? What you felt about how it went. What you felt by the end of it and what you will do next time that kind of thing?

Mr Taylor: Um so the idea was to just get that multiplicative reasoning, just to read up to 12 x 5 table and whatever then start thinking what are the rules, what are the patterns and understanding and getting them explaining that. They explained it well. I think they could have explained it a little bit deeper. I will hopefully address that in the address marking afterwards as well and make them go back and reason a bit further. Um we haven't done multiplication since after, before Christmas.

<u>Watching clip</u>

Researcher: So you went to talk to your Mr Taylor, is that right? Is that right, Andrew, did you go and get him?

Andrew: Yes.

Researcher: So what made you go and get him, which question?

Andrew: Because we were stuck on this one and me and Thomas didn't know how to do it.

Researcher: OK, can you talk us through what it was and what you got stuck on?

Andrew: So like what me and Thomas got stuck on, so we didn't like really understand what the question meant.

Researcher: Right. So what was the question?

Andrew: It says, why can I divide thirty four into two groups equally? That was the first one. And then the second one was, why can I divide sixty five into groups of five?

Researcher: OK. So what was it, which bit of that was confusing?

Andrew: So it was the first bit and we didn't like really know if it was like, well so we were like, got like quite stuck on like this bit here.

Researcher: On the number thirty four or was it on the divide bit?

Andrew: It was on this bit.

Researcher: The whole bit. It's about dividing thirty four into two groups, yes?

Andrew: Yes.

Researcher: So what happened when you, so when you normally get stuck like that, what would you usually do?

Andrew: We normally go and get Mr Taylor.

Researcher: Right, okay. Do you talk about it, do you sometimes talk to each other about it?

Thomas: Yes.

Andrew: That's only, that's quite rare.

Researcher: Had you tried that already?

Andrew: Yes.

Researcher: Do you ever use resources or equipment or anything, things to help you?

Andrew: Well we used to when we started in the first week, we did it and then we stopped. Then we started drawing.

Researcher: OK. And why do you think that happened then, why do you think you stopped using the resources and started drawing?

Andrew: Because our SATS, because they're coming up. It might, I think it's in this summer. Yes, so it will be quite soon because it's nearly summer.

Researcher: So why would you not use the resources then, why does the SATS stop you using the resources, do you know?

Andrew: Because sometimes, like when you like work it out in, in SATS, when you work it out, you get an extra mark.

Researcher: OK.

Andrew: And it will give you extra points.

Researcher: Ah I see. So that's why you're practicing working it out as well as doing the answer?

Andrew: Yes, so then we don't have to like get it, the tables all messy too. Because then, I remember getting all the tables messy with Dienes, so we're just drawing them in our books now.

Researcher: Oh OK. So did you enjoy using the Dienes?

Andrew: Like that.

Researcher: Oh yes, I saw, yes. Did you enjoy?

Andrew: Yes.

Researcher: How about you Jaden, what do you think to all this? Do you agree with what they're saying?

Jaden: Yes.

Researcher: So what do you agree to, do you like using the resources or do you like drawing it or do you like doing it in your head or all of those?

Jaden: I like all of them.

Andrew: Well I usually do it in my head when it's pluses.

Andrew: Yes, because they're easier to do in your head. You don't have to like use Dienes or anything.

Researcher: Oh so you know that?

Clip of episode:

Mr Taylor: Do the next one. So why can't I divide sixty five, why can I divide sixty five into groups of five?

Andrew: It has an even number, like on the start of it.

Mr Taylor: Does that change the way that you can divide it into five then?

Researcher: Ah now, I think, hang on. You said, it's got an even number at the start of it, didn't you? Where's the question again? Let's all have a little look at this.

Andrew: Oh yes, it was here.

Researcher: This one here. So you've got, why can I divide sixty five into groups of five? And you said, the first numbers even, didn't you? And you were looking for an even number. Can you explain that to me?

Andrew: So where I struggled with like, why can I divide sixty five into groups of five, yes, so I didn't really know what to do on like this bit. So I had a little bit of a guess and then I realised, when Thomas told me, and then I like, then I realised, yes, so when I realised I was like, oh yes, I have to do that.

Researcher: What was it that, you said you had a guess but what were you actually guessing?

Andrew: I was guessing like

Researcher: Because that was really interesting because you said, you were looking for even numbers, which was this one here, wasn't it? So perhaps you were thinking of that, I don't know. But then you said, the first number is even.

Andrew: So it was this one.

Researcher: Okay, and what's that number?

Andrew: Six.

Researcher: Six. And what does it actually represent? Is it six or is it six or something?

Andrew: Just sixty five.

Researcher: It's just sixty five, Okay. Does it matter, if you're looking at numbers in the two times table or even numbers, is that, does that tell you much, that six?

Andrew: Well I don't really know.

Researcher: Do you know Thomas? If we're looking at the number sixty five here, what can you tell me about the number sixty five?

Thomas: It's an even number because it's got a five in it.

Researcher: Okay, is five an even number?

Thomas: No, odd.

Researcher: Hang on, is it Jaden?

Jaden: Oh yes, odd.

Researcher: It's an odd number. So sixty five, is that an even number or an odd number?

Andrew: Odd.

Researcher: Odd, because?

Andrew: Because it's not in the two times tables.

Researcher: Okay, right, lovely. So are you clearer on that now, are you happy?

Andrew: Yes.

Researcher: Yes, because you were looking for an odd number not an even number. Right, now, I'm going to move us back to right at the beginning. This is where we started at the beginning, wasn't it? Let's see.

Teacher / Researcher

Researcher: I am going to stop it there because it just carries on a bit. So what do you think of that? Anything you notice?

Mr Taylor: That's really nice, that's really great I think they have obviously really listened and understand and why the purpose of the drawing I think they really get that, they say well it helps us work it out.

Researcher: Do you think it's a fair representation.

Mr Taylor: Yeh, no I think I do I do I think I think they, they do all understand why they are doing it. And that's the unfortunate thing about being in in year 2 isn't it it's that you know that the Sat's are awful in the fact it takes it all away from them. They need to have these coping methods to work these problems out don't they. Um and what I hope is that they get from these is that actually they are still real life resources but just been drawn down. I would like to think so. It was lovely that Andrew said that. He's obviously listened really well.

Researcher: Also they were using the resources up here as well. (Pointing to the poster on the wall) A lot they were talking about that.

Mr Taylor: Oh yeh, we have got it up here, they used that, so they are using other resources. Good.

Researcher: Yes they seemed very aware of what they are doing.

Mr Taylor: Yeh and it's nice that, we try to with our 5 b's try to make sure that they are independent enough to just go off and look for themselves.

Researcher: And leading on into that. I was just.. at how and what strategies they have got to support them as well as just coming to get you.

Mr Taylor: Yeh laughed.

Researcher: Obviously they do but not the main one

Mr Taylor: Yeh

Researcher: So the next bit was a little discussion about even numbers

Clip of episode:

Andrew: The first number is '6' so yes it is even.

Mr Taylor: Does it change how you divide it into 5? What does the five times tables end in?

Andrew: Five or zero.

Mr Taylor: So can you divide 65 by 5?

Andrew: May be

Mr Taylor: You think?

Andrew: Yes?

Teacher / Researcher

Researcher: This is where I was really interested because he just slipped in that 65 the first.

Mr Taylor: He said, he said 6 was even.

Researcher: I was just trying to unpick that with him.

Mr Taylor: Yeh.

Researcher: Let's see what else he says about that.

Mr Taylor: Yeh.

Researcher: What do you think about that one then? How does that fit in with what you would normally be doing? Were you surprised about the 65 or not.

Mr Taylor: Yeh a little bit, yeh a little bit I think he got himself a little bit confused with what divide means, I don't. Because you know his place value is pretty sound. But then, he obviously had a bit of a wobble there.

Researcher: Different situation?

Mr Taylor: New type of question.

Researcher: Or whether he needs to do more of it. To get used to it to see how, to apply what they know?

Mr Taylor: Yeh.

Researcher: They are applying their other knowledge into this. Making sure it's working that just shows.

Mr Taylor: He needs a little bit of work on it, absolutely. I found that bit interesting as well. He told me that as well, the 6 is even (in 65) is that the number we are looking at though? Oh what what that's 60 oh.oh, 65 oh ok.

Researcher: Hadn't picked up the whole thing, I just caught that snippet of the 6 and the first number is even. And then when I talked to him about place value you know, what is 65? He was well well it's 65.

Mr Taylor: Laughing. Yeh yeh yeh I found that interesting as well when I was talking to him.

Researcher: So Place value wise normally absolutely fine?

Mr Taylor: Yeh but I think it is the different context which is why I guess the whole point of doing these word problems and real life maths isn't it….making sure it is really secure not just in one way.

Transcript C

Sharp School, Place Value, Case C

Researcher: What was the lesson about and what did you think about it?

Teacher: So yesterday we did a lesson with column addition without going through ten and these children didn't really get it so I thought I would take them through it again. Back a step, get the practical equipment out. Can we make it in ten's and ones which, we could and then can we add the numbers together. I think that bit all went relatively well and then it was that next step of bridging through ten which, I think they sort of just starting to get the hang of with the practical stuff but I think when I was bringing it into writing it they were a bit lost.

Researcher: Okay, let's have a little look. We are going to do it in two sections the first bit were he gets a bit confused and then the second bit where he just goes for it. So we will start with the first section. Just say stop if you want to stop and talk a bit.

<u>**Clip from the episode**</u>

Tim: (Looking on a bit puzzled but smiling).

Miss White: (To Tim) Can I have your ten?

Tim: (Hands over a handful of ones, all thirteen of them plus all the tens).

Miss White: I only want ten of your units, count me out ten of your unit's.

Tim: (Counts out ten one's and gives them to the teacher in exchange for a ten). I have got a sticky (in a different voice, holding up the ten stick).

Miss White: See if you can add them together.

Tim: Thirteen one two three, four, five, six, seven, eight (Laugh) nine, ten, eleven, twelve (laughing) thirty twelve I've got thirty twelve.

Clip from the pupil interview

Researcher: How's that, what's that, can you do that? Exchange it for a 10.

Tim: Yes yeah yes

Researcher: So why is that then, why have you exchanged ones for tens?

Tim: Uh pause, because I have too much ones.

Researcher: Ah okay so how many ones can you have at one go?

Tim: Uh pause.

Researcher: What's the maximum?

Tim: 100.

Researcher: 100 before you can change it for tens?

Tim: 20?

Researcher: Okay shell we move on and see what's next? Here we go.

Watching the video

Researcher: Good counting skills

……………………

Teacher / Researcher

Teacher: They don't actually realise the maths we were doing was adding. I don't think he necessarily knows what he's got it for.

Researcher: No.

Teacher: I don't think he has made that connection at all.

Researcher: His face tells the story that he is confused. So what do you think might be confusing him on that then?

Teacher: I don't think he's really got it at all. He obviously doesn't, I just can't understand why he didn't think 'I have got three tens and now I'll just count on from the thirty up to forty three with the other ones. He knows they are completely different.

Researcher: Just watching that time because it didn't click before – is it the teen numbers. Is it the fact that he has a thirteen and thirty?

Teacher: Maybe.

Researcher: And perhaps he's not confident with his place value. I don't know it was just a thought.

…….

Researcher: That fits in with the 15 doesn't it, perhaps he just has a block on the 15.

Teacher: That's really strange.

Researcher: It is quite common for children to get confused
Teacher &

Researcher: With their teen numbers.

Researcher: And it's just the 15, the 19 might be absolutely fine.

Teacher: Yes.

Researcher: It might be just.

Teacher
& Researcher: That number.

Teacher: And you would just have to practice that number a few time and that would be it, sorted.

Researcher: It's very subtle isn't it?

Teacher: Yes.

Researcher: Easy to miss.

Teacher: Yes.

…………………

Teacher: Other than that there is quite a lot of freedom to do it how I'd want to do it. It's that element which is kind of on it. I think there is like a black cloud around maths. One thing that parents always say to me is 'I don't want to teach it in case I do it wrong.' Ultimately you are more likely to teach them something in English wrong because English is much more complicated. Maths is just, so long as you get to the answer the method is sort of.

Researcher: It's which way you're going to get there.

Teacher: Yah.

Researcher: I have got the last bit but I'm not sure you will want to see it now.

Teacher: Is it really awful 'do you like being at school and maths? No!'

Researcher: They didn't like doing the maths no, they felt the math's…

Teacher: Why because it was too hard?

Researcher: I asked them what would help them with their maths next time.

Teacher: What did they say, having all the equipment out?

Researcher: No they didn't, no.

Teacher: What did they say?

Researcher: They said getting to the answer.

Pupil / Researcher

Researcher: So what do you think you learnt from that math's lesson this morning?

Tim: Well I counted up the units and tried my best.

Researcher: And do you think you were okay making the numbers with the blocks?

Tim: Yes.

Researcher: Okay so you were good at doing that. Do you feel you were able to add the two numbers together using the blocks?

Pause (Someone walk past)

Tim: Uh (pause) kind of.

Researcher: Kind of. And what about you (Looking at other pupil)

Pause

Researcher: What do you think about swapping the ten units with the one ten? Is that easy to do or difficult or not sure?

Tim: (Quietly) Not sure.

Researcher: Not sure. For your next maths lesson what would you like to practise more or learn next?

Tim: Ur to not destroy your work to use it to make a different number.

Researcher: So you make sure you keep your two numbers safe. So you don't mix them up. That sounds like a really good idea.

Pupils talking over each other: In a safe place. I don't want to get my numbers wrong.

Researcher: When you say wrong what do you mean by wrong?

Tim: (Making a cross sign with his finger).

Tim: Just to like um if I get it wrong ur, I'll do it again.

Researcher: You like to get the right answer do you? Is that the most important thing to get the right answer or is it more important to understand what you are doing?

Tim: Using a different voice, Yes

Researcher: Which do you think or both?

Tim: Both (Different voice).

Researcher: Both.

Tim: Yeh, both.

Researcher: What about adding your column addition. (Pupil puts his head on the desk). Can you do that now do you think? Or do you think you need a bit more practise?

Tim: More practise.

Researcher: More practise.

Tim: (Giggling and using different voices.)

Researcher: What do you think would help you with that practise? Last question what do you think would help you with that practise?

Tim: So that we know when we are next doing it and know what we are doing and we know if we get the right answers.

Researcher: Do you always know what you are doing or do you sometimes feel a bit confused?

Aaron: Confused.

Tim: (Using different voice) Confused.

Researcher: Is it always maths that you feel confused or every subject?

Tim: Yes, yes.

Aaron: Just in maths.

Tim: Maths maths.

Researcher: Really, okay.

Transcript D

Northolt School, Geomatry, Case D

Mrs Armid: So it's your turn now to describe the shape. You are very good at naming the shape. Now you can move on to describing the shapes to a partner using a little voice.

Destiny: (*Showing Maci the word card*) It has six sides. *(Maci looks away and Destiny counts the side on the picture of a cube*).

Mrs Armid: May be you can describe that shape? (*Repeated round the table to different pupils. Destiny looking at her own shape. Destiny picks up a prisms and puts it down*).

Mrs Armid: (*Then looking at Tom*). Maybe you could describe this shape to Destiny (*Holding out a tennis ball*) and Destiny can say the shapes.

Destiny: It's a tennis ball.

Tom: It's just a tennis ball. (*Playing with it*).

Destiny: It's a spear.

Mrs Armid: Does it have any vertices?

Mrs Armid: (*Shakes her head*). No.

Mrs Armid: (*Holds up a square based pyramid*). Does this have any vertices?

Destiny: Yes.

Mrs Armid: How many?

Destiny (*Points her finger towards the shape but doesn't touch it mimicking counting to five).* Five.

Mrs Armid: Five vertices you can count them. (*Hands the shape to Destiny*).

Mrs Armid: (*Picks up the tennis ball*). This sphere is curved and has one edge, one side. (*Puts down the tennis ball and hands Destiny the square based pyramid again.)* Can you describe this shape? It's got triangle faces and a square base. Can you describe that the vertices, count the edges?

Destiny: (*Holding the shape*). There are five edges. There are four faces and a base here. (*She touches each edge and the base. Then holds it in one hand.*) It's like a cup!

Mrs Armid: Can you describe that now to me.

Destiny: It has five sides

Mrs Armid: Can you show me now where they are?

Destiny: (*Touches each point*). Five.

Mrs Armid: Five. What else do you notice? How many edges, how many edges does it have?

Tom: (*Points to the shape*). It has five.

Mrs Armid: Are these are the vertices (*Pointing at them*). How many edges are there? Don't forget these ones. (*Destiny try's to take the shape and the teacher moves it away)*. Destiny watches attentively. (*The teacher touches each edge one at a time*).

Destiny: Four.

Mrs Armid: What about these ones you have only done four. Watch carefully.

Destiny: (*Takes the shape and counts the edges*). Seven.

Mrs Armid: Ooh you think there's seven?

Destiny: (*Very quietly*) eight. Seven eight.

Mrs Armid: Just a minute. Maci can you do all of us a favour.

Destiny: (*Very quietly*) eight.

Mrs Armid: Children on this table. Maci is just going to count the edges for me because we are getting a bit stuck.

Destiny: (*Very quietly*) eight.

Mrs Armid: So we know these are all the edges. Can you count them for us? (*Handing the shape to Maci).*

Destiny: (*Quietly*) eight

Maci: (*Touching each edge with her finger*) one, two, three.

Destiny: (*Quietly mouthing eight*).

Maci: (*Touching each edge with her finger*) four, five, six, seven, eight.

Excerpt of the Interview with the pupils School D Northolt

Showed the clip to Maci and Destiny and asked them to describe what was happening. They were unable to tell me anything so we watched the clip a few times.

Watching the clip

Destiny: Um

Researcher: (*Stops video*). Do you think you know what was happening now? What was she saying Destiny?

Destiny: Flat.

Researcher: Flat.

Researcher: Why was she saying flat? What was she talking to you about?

Destiny: Um 2D shapes and 3D shapes.

Researcher: Oh right I see. Can you tell me anything about 2D and 3D shapes?

Destiny: Oh I already know this.

Destiny: 3D shapes are big and 2 D shapes are flat.

Researcher: Okay, is that what your teacher was telling you there when she was going like that? (*Smacking hands together*). Okay watch the next bit and tell me what you think of the next bit.
(*They watch a little more of the clip*).

Researcher: Where has she gone? (*The teacher walks out of camera shot on the video*).

Destiny: To get the shapes.

Researcher: Why was she getting the shapes?

Destiny: To show me.

Researcher: What do you think she was trying to show you do you think?

Destiny: Um the 3D shapes and um the 3D shapes.

Researcher: Did you need the shapes or did you already know?

Destiny: I needed the shapes because I am no good at telling the shapes.

Researcher: Are you not, why is that then? What is it that you find difficult?

Destiny: Because (*pause*) um (*pause*) because (*Very long pause*) because I am not good at the shapes names.

Researcher: Is it the names of the shapes you find tricky? (Pause) Or is it anything else about the shapes you find tricky?

Destiny: Um It's its, I don't know how to explain it.

Researcher: Shall we look at a bit more of the film and that might help you.
(*Addressing Maci*) What about you what do you think?

Maci: (*Appears to be thinking).* So when we were thinking about the pyramid.

Researcher: Yep.

Destiny: I thought of it.

Researcher: Hang on, what were you about to say?

Destiny: UM that a triangle is a 2D shape and a pyramid is a 3D shape and so I get confused.

Researcher: That is very tricky isn't it, yes? I can understand that. Can you understand that as well (*Talking to Maci, Maci nods her head*).

Researcher: Do you get stuck with that sometimes? (*Maci vigorously shakes her head*). You have worked that out now have you? I still get stuck with that sometimes.

Destiny: And a spear and a circle.

Researcher: A sphere and a circle gets confusing as well doesn't it.

Maci: Because I have seen a pyramid two times. In !!!!! video and the equent!

Destiny: And the equent!

Maci: How did you know that?

Researcher: So why do you think you get confused between triangles and pyramids, circles and spheres?

Destiny: Because because cus….I never watched pyramids on YouTube. I never saw !!!! on YouTube except what Maci said.

Researcher: What like a real life pyramid?

Maci: and D: Yeah

Maci: Yah that uh yeah

Destiny: The game master

Maci: Yes 'Haircorders!' (Pointing her finger very animated) . The Aquent and the Chadroyal. There's a lie detector guy. called Daniel.

Researcher: So that all helps with your shapes does it?

Maci: Yeah.

Researcher: (*To Destiny*) what would help you with your shapes?

Destiny: Um.

Researcher: To learn the difference between the 3D and the 2D shapes. What do you think would help you with that? Any ideas?

Destiny: Knowing my shapes.

Researcher: Knowing the names of the shapes. Shall we watch a little bit more and see what else there is?

Maci & Destiny: Yeah

Researcher: We'll stop again and see what you think of the next bit.

Teacher view of Video

Interview between the researcher and the Teacher at School D

Researcher: So do you want to describe what went on there?

Mrs Armid: So it's a game they're all playing, a shape bingo game. And the idea of the game is for them to look at the 3D shapes and to match the words, so correctly identify the 3D shape. So the children are taking it in turns to pick up a card, check it against the one that they have on their mat, on their shape mat, and then match it to their shape and eventually, try and win the game. But the objective of the game is for the children to identify the shape, name the shape, which, obviously, they were doing through a game, yes.

Researcher: So do you want to describe what was happening with the shapes with this?

Mrs Armid: Yes, with this child, the other children seemed to be quite knowledgeable about the shapes and were able to name the shapes and read the labels for the shapes, the names of the shapes. And Destiny was, the misconception I found there, was that she was naming the shapes as 2D shapes. So she saw the faces and recognised the face shape, so she was saying, I can see the prism, the triangle.

Researcher: She said the triangle there, didn't she?

Mrs Armid: Yes. So that was the, there was a pyramid that she could see but she could see the face of the triangle on there, so she was calling it a triangle. And then again, I think she did the same with the cube or the cuboid.

Researcher: It wasn't the sphere, was it?

Mrs Armid: Not the sphere, no.

Researcher: That was alright.

Mrs Armid: She just was using the 2D face name for the shape. So she was getting a little bit confused. She was supposed to be using the 3D shape names but she was using the 2D shape names. So what I decided to do then, because, obviously, their pictures are flat, and it's easier to show them that, look, we're looking at solid shapes, something that you can hold, and that's why I got out the 3D shapes for her again, to look at, which we have had out previous lessons. But I think Destiny needed a bit more support and modelling and help with that. That's why I kind of decided to help her a little bit with that activity. So, I think by the end of it she was able to match them and name them. So I think, yes.

Researcher: Yes. Shall we go back to the, just the beginning bit again? If anything comes up now that you want to stop.

Clip: *3D shape is a solid shape. So what shape is this? It's the first shape we looked at on the white board. A cube, so that's the cube. What is that? A cube as well.*

Mrs Armid: So if we just stop there, here, where she's identified that it's a cube, I think previous to that she said, it's a square. So that's the misconception, I didn't want her to use the wrong name for the shape because that, obviously, the language vocabulary, it will not be correct. So she needed to have said cube but she said square, which is a 2D shape name.

Researcher: Yes. So you pointed over to what you were doing previously, haven't you? Towards the whiteboard, trying to remind her.

Mrs Armid: Yes.

Researcher: And sort of bring that in.

Mrs Armid: Yes.

Researcher: So you were scaffolding for her, weren't you?

Mrs Armid: Yes. And that's why it was really important to write the words on the board as well, because then she was able to go back, look and read that actually, those are the shape names that we are learning today as well. So I was trying to get her to use those as well.

Researcher: To link it through. And then that helps again with the words here, doesn't it?

Mrs Armid: Yes.

Researcher: Yes, so that's nice. And do you think she's getting it?

Mrs Armid: I think, towards the end, the very end, she was able to, but then I think she still needs more. I think she still needs a little bit more support and maybe going back and look at it.

Clip: *And what's this one? So this is cuboid.*

Mrs Armid: I've gone back there to show her that that's a cuboid. So I need to go back and get the actual shape for her, so she can see it, she can feel it and learn a kinaesthetic way of learning as well.

Researcher: Because you brought this, she started off with that, didn't she? And you described it.

Mrs Armid: Yes.

Researcher: Then you went and got this. And then, so you were, yes.

Mrs Armid: Kind of

Researcher: Building, scaffolding.

Mrs Armid: As much as I can, yes. As much as I can to support.

Clip: *We have a cuboid. Can we say, a cuboid?*

Researcher: Was it just you getting them to say it as well?

Mrs Armid: And she knew she didn't have it, so the word down.

Researcher: Oh the word back down.

Mrs Armid: So that's where I'm actually saying to them, hold the 3D shape. Whereas, if you just said a square, that will be just a flat

Researcher: You can't pick it up.

Mrs Armid: Yes, that's a flat shape. A square is a flat shape. You know, we've done that, we've already done that part and it was me having to remind her again because she does have, she struggles to maintain retention and listening. I'm sure, even in the video, you can see her turning around and under the table.

Researcher: She's looking for a sticker, she's lost her sticker.

Mrs Armid: Oh that's really important.

Researcher: Very important to get the sticker.

Mrs Armid: Very important. But generally, as well, you can note with her, on carpet time especially, she does struggle with a bit of retention and listening. So I think she needs a bit more reinforcing.

Researcher: But she is actually listening to you really well, isn't she?

Mrs Armid: She is, yes.

Researcher: She's very attentive.

Mrs Armid: She did there, she knows I'm helping and she probably feels that she's getting that attention and support to probably feel better to play the game with them.

Transcript E

Felton School, Fractions, Case E

Mrs Greenway:	When I say go back here where she says 'I'm stuck'.
Researcher:	Yep
Mrs Greenway:	So I say why are you stuck on that? This is what we were doing (pointing at book).
Researcher:	Yah
Mrs Greenway:	When you go back into her book and look, she hasn't actually done the groups has she.
Researcher:	Let's have a look, well spotted.
Mrs Greenway:	Yah, I realised that at the time she just knew these didn't she. (Pointing to the book) This one she has done, half of sixteen.
Researcher:	But she has used that there.
Mrs Greenway:	Yes.
Researcher:	That's interesting.
Mrs Greenway:	This is what I am saying to her. Why are you stuck with this because this is what you were doing? (Pointing at book with previous work in) This is what, this is.
Researcher:	And actually she had moved on from this. She has done it once as an apprentice (System used in class).
Mrs Greenway:	And she has done a quarter. This is the mistakes they were making before. Even though it was a quarter they would still do two groups not realising it should be four groups.
Researcher:	Ya.

Mrs Greenway: So she could do this.

Researcher: She has done three.

Mrs Greenway: Yes.

Researcher: But then how long ago was that?

Mrs Greenway: Yesterday look (pointing at book).

Researcher: So that's yesterday, she was doing that yesterday?

Mrs Greenway: No this is today, this is what she had done this morning and then when she turns over half of 18 is so…

Researcher: So she has gone from, so she whizzed through the apprentice.

Mrs Greenway: And she has not done any working out has she?

Researcher: So she knows what ½ is she…

Mrs Greenway: She knows her ½'s. She has got to ½ of 16 and she knows she has to have her two groups. Which we have modelled.

Researcher: Would she know what ½ of 16 is?

Mrs Greenway: Well she talked to you didn't she about ½.

Researcher: She could do, she knew half of 8, she knew the 8 was a 4 but she didn't know what to do with the 10 did she?

Mrs Greenway: Yah, no that's right, that's what she talks about. So that's why, that's why this is classic Mai, when she says 'I'm stuck' and it's this bit. Why are you stuck? Because we were doing it. We have been doing halving.

Researcher: Well she has got, but she was halving she was happy with 10 and that's where the trouble came.

Mrs Greenway: Yah.

Researcher: But actually she has done one of the halves and then she has gone to ¼.

Mrs Greenway: So she.

Researcher: She needs more.

Mrs Greenway: Yah, yah maybe she is ready to see she can do it. She just knows half of 16 is 8 although she has worked that one out.

Researcher: Yah.

Mrs Greenway: So she knows this doesn't she ½ of 12 is 6. But ½ of 16 she has had to work out. So when she is over here and it is half of 18 she's forgotten that.

Researcher: But she did do it didn't she.

Mrs Greenway: Yes she did. She asked me before she got to that bit.

Researcher: That's right.

Mrs Greenway: That's where she said 'I'm stuck'.

Researcher: She got 14 when she was talking to me. Half of 18 is 14.

Mrs Greenway: So 4 and 8 yah.

Researcher: She tries to do this and forgets to do this (Pointing at the 10 and the 8) and she knows it's wrong but she doesn't know how to do the next step.

Mrs Greenway: So, which I said what have we been doing back here? You need to get your two groups and then of course she does do it and I tell her she has done it wrong. (Laughing) but she did do it right.

Researcher:	That is fine but the fact that she listened to you saying it's wrong, She didn't have the confidence to say 'no that's right' because if she had had the confidence she would say.
Mrs Greenway:	Yah yah it's true.
Researcher:	She is hanging onto you isn't she? The other thing, you say she has done ½ ½ ½ (Pointing to calculations in book) then ¼ ¼ ¼ . Those are all right, these are all good. Did you model that at the beginning?
Mrs Greenway:	We did one on the board, one ½ and one ¼.
Researcher:	Yah.
Mrs Greenway:	And they had it. I was looking in the book because originally, previously Mai had been in a real pickle but she [Mai] had it on the whiteboards this morning and I thought 'oh they have remembered, let's let them go and do it in their books and see. Mai and Jill had it so I thought there we are then.
Researcher:	Look she has gone to ¼ so she is happy with the ¼'s?
Mrs Greenway:	Yah
Researcher:	And she has drawn it very nicely.
Mrs Greenway:	And she had checked and put her 10 by each one.
Researcher:	Then she has gone to the Master level (Next level up of difficulty on the questions on the board). So on the master is she trying to do it in her head and actually she hadn't used. You suggested.
Mrs Greenway:	She just had the question written and this is where she said 'I'm stuck'.
Researcher:	Yes and I can see why she is stuck.
Mrs Greenway:	Yah

Researcher: Because she is doing ½ of 8 and getting 14 and she has forgotten the 10.

Mrs Greenway: Yap, so I reminded her to do two groups and then she gets it right and.

Researcher: Yes and I think this is all okay, this continues…

Mrs Greenway: And she knew they had to be equal so she so she knew this couldn't be right.

Researcher: So at that point what does she need to do for her to progress on with her having to ask you?

Mrs Greenway: She needs to know it is not right she just tries again doesn't she? Without me coming to her and saying test it again.

Researcher: Yah, yah it is almost just asking and that's the same here, wasn't it. This was a real confusion but you can see where the misconception is. That's quite interesting.

Mrs Greenway: It would be interesting to see if she could do this now. If she's gone away, come back, would she know this was three groups? I think she probably would.

Researcher: Interesting.

Mrs Greenway: It's knowing what to do when you think it is wrong.

Researcher: Yah

Mrs Greenway: Yah

Researcher: It's the confidence. Is it an independence thing?

Mrs Greenway: I thinks it's just a maturity thing isn't it. Because actually she has come on such a long way. This is her second book of the year. You would see from the beginning but she has come on such a long way, in the volume of work that we do and get done as well. It used to be we would just have a couple in that session and I would have been doing loads more work with…whereas now at this point in the year, I can put it on the board and the routines embedded.

……………..

Mrs Greenway writes in book.

Mrs Greenway: I really need to stop doing this bit. I need to say.

Researcher: You are feeding her do you think?

Mrs Greenway: Yes because there was an awful lot of scribbling out and really aware of someone else looking at it. It's telling the story isn't it. It's saying well we marked these and then there's help and then we did it again.

Researcher: So perhaps the help bit is 'So what are you going to do?

Mrs Greenway: Yes so what are you going to do? She then says I am going to write it. I'm going to do it again

Mrs Greenway: So it is throwing it back to her, the mirror.

Watching video.

Researcher: She's not doing anything

Mrs Greenway: No because she was….

Researcher: She was stuck on that and was waiting for the next one.

Mrs Greenway: She's waiting she's waiting for me to say.

Watching the video as two girls attempt a problem

Researcher: I suppose at that point you could have got them together.

Mrs Greenway: Yeah.

Researcher: I don't know.

Mrs Greenway: They are really not doing anything here.

Mai keeps looking round.

Listening to the video

Mrs Greenway:	5 minutes back on carpet. Come back and we will finish on the carpet. Please leave your maths books open where they are.
Mrs Greenway:	(Impersonating the child) NOT DOING THAT ANY MORE and slamming the book down.
Researcher:	She did quite a lot.
Mrs Greenway:	She did, they did do quite a lot and I think we fall into the …If there weren't that many in the group they wouldn't be getting this much done would they in maths – it would be…
Researcher:	I don't know it all depends.
Mrs Greenway:	Yah.
Researcher:	You know them better than me it's…They are producing a lot aren't they. What she's doing is good isn't it. She's actually producing there. She sat down, she did the first lot.
Mrs Greenway:	Yah.
Researcher:	Happily confident and she didn't ask for help until – (pointing to the book).
Mrs Greenway:	No she did all that. It was here wasn't it with half of 18.
Researcher:	So what, when she was on a role she was all right.
Mrs Greenway:	Yah.
Researcher:	Then her confidence is knocked.

Mrs Greenway: She has forgotten that we can do the grouping. And yet she has done it. And that's it that is Mai all over. This is exactly what happens.

Researcher: Do you, I don't know. This is just a thought. She has done all that do you think she is tired?

Mrs Greenway: I wonder. We have started to do this, we have started to do the active classroom ethos. And I am in it sometimes and I have sat back and they are all working and I see A do this, I will say 'Mai and Jill go out the door, run up to the tepee and then come back.' Just that fresh air run and back in.

Researcher: That sounds really useful…

Mrs Greenway: May be.

Researcher: I suppose if we look from the very beginning, but we are not going to do that. If we look from the very beginning through, we might see a bit more of a pattern. I don't know.

Mrs Greenway: You have to watch them every day don't you. So what point after 30 minutes Mai has had enough?

Researcher: But it's obvious she had got onto a harder lot hadn't she? She had gone onto the mastery one.

Mrs Greenway: I wonder if that's a block.

Researcher: Yah.

Mrs Greenway: You see, I do worry about that as well. Ivy, Ivy for ages would do all of 'Apprentice, all of …. Get them all right. I would say go onto mastery and she would say no I want to stop now and we would only have 5 or 10 minutes from the end, so I would say okay you stop now. Then they go through to the other bit (of the classroom) and have a bit independent Lego, independent book time. Um it's almost like a psychological barrier.

Researcher: You are asking them to do more and more!

Mrs Greenway: It's going to be harder.

……………………….

Researcher: Yes at the end I asked what would help you. 'I really like doing my times tables wasn't it. So obviously that is something she is quite proud of.

Mrs Greenway: Yah.

Researcher: She said she would like to have her pizza thing but I don't think that would help her.

Mrs Greenway: I don't think that would have helped her.

Researcher: Because it's fractions?

Mrs Greenway: Because it's making that link between sharing shapes and cutting things in half, to sharing out numbers.

Researcher: I suppose when I have done all the, they have had the, I suppose she could have tested it, if she had had the resources she could have just tested it.

Mrs Greenway: But again you see it's that nonsense of Sat's isn't it. Because year 2 Sat's they can't have any resources out, can they and so we would have done loads of sharing out cubes 1 for me 1 for you and then we say , now we are going to do it without but you can, you will always have a pencil and paper this is what this is about. We have done loads of on the carpet, in pairs, number sentences on the white board, putting the cubes actually in the groups.

Researcher: That's interesting the thing you were saying about the test and that was very much the (thought pattern!!) and Sat's.

Mrs Greenway: Yes we have just done that.

Researcher: It's really interesting how that's impacted on….

Mrs Greenway: Perhaps it's really now we need to model and teach them that if you are stuck it's okay to ask somebody on your table. If you don't want to put your hand up and ask me, lets, let's build on that, who else can you ask on your table?