73rd International Astronautical Congress (IAC), Paris, France, 18-22 September 2022.

IAC-22-C1.7.9

Deterministic and Stochastic Exploration of Long Asteroid Fly-by Sequences Exploiting Tree-graph and Optimal Substructure Properties

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Abstract

In the past, space trajectory design was limited to the optimal design of transfers to single destinations. However, a somewhat more daring approach is today making the space community to consider missions that visit, with one single spacecraft, a multitude of celestial objects; such as asteroid tour mission proposals CASTAway or MANTIS, which both proposed to visit 10 or more asteroids in a quick succession of asteroid fly-bys. The design of these so-called asteroid tours is complicated by the fact that the sequence of asteroids is not known a priori, but is the objective of the optimisation itself. This leads to a complex mixed-integer non-linear programming (MINLP) problem, on which the decision variables assume both continuous and discrete values. Beyond the obvious complexity of such problem formulation, preliminary mission design requires not only to locate the global optimum solution but, also, to map the ensemble of solutions that leads to feasible transfers. This paper analyses the complexity of such search space, which can be efficiently modelled as a tree-graph of interconnected Lambert arc solutions between two consecutive asteroids. This allows to exploit the optimal substructure of the problem and enables complete tree traverse explorations for limited asteroid catalogues. Nevertheless, the search space quickly grows in complexity for larger catalogues, featuring a labyrinthine multi-modal structure and extreme non-linearities. This underlying complexity ultimately renders common stochastic heuristics, such as Ant Colony Optimization, rather inefficient. Mostly, due to the fact that the metaheuristic processes are not able to gather any real understanding, or knowledge, such that it can efficiently guide the search. Instead, an astrodynamics-lead heuristic based on the distance between spacecraft and asteroid at the asteroid's MOID-point crossing epoch, enables an efficient pruning of the asteroid catalogue. Then, deterministic processes based on dynamic programming and beam search can be efficiently applied, providing solutions to both the global optimum and the constraint satisfaction problems.
Keywords: Mixed-integer Trajectory Design, Asteroid Tours, Optimization, Constraint Satisfaction.

1. Introduction

Given the complexity and risks of space operations, designing and/or planning spacecraft trajectories have traditionally focused on single destination transfers, such as Earth to Moon or Earth to Mars transfers. New technologies, as well as much more daring approaches to space operations, are however giving rise to the challenge of missions that target several or, potentially, many different destinations; be it celestial objects, such as different moons or asteroids, or simply different orbital configurations. NASA's Dawn mission, which visited two different minor planets of the main asteroid belt [1], is one such example of multi-target mission. Nevertheless, many other much more challenging examples are being planned and/or studied, such as ESA's JUICE mission [2] with more than twenty close passages to Jovian moons; commercial concepts for On-orbit Servicing [3], such as multiple Active Debris Removal, or asteroid tour missions such as CASTAway [4] or MANTIS [5], which plan to visit tens of asteroids within one single mission.

Given the intricacies of space flight, designing the spacecraft trajectory that visits multiple orbital waypoint (planets or specific orbits) is a notoriously challenging problem. The problem itself is defined as a Mixed-Integer Non-Linear Programming (MINLP) problem, where a set of integer parameters encode the sequence of orbital waypoints to visit, while continuous variables define all the necessary operational aspects to course the path visiting the given set of waypoints (e.g., departure times, time of flights, thrusting arcs, fly-by configurations, etc).

In particular, this paper focuses on the mission design of a tour within the main asteroid belt. There is clear scientific interest in exploring this region of the Solar System, to better understand asteroids composition and evolution from early stages of the Solar System. The aim is to pass-by as many asteroids as possible to obtain key information about the composition and geomorphology of the objects by close encounter analysis [4]. A mission that could fly-by for example 10 asteroids will double the number of objects visited to date, possibly within a short mission lifespan and relatively meagre budget [6].

Section 2 describes the asteroid tour problem and the presents the design of these trajectories as a general backbox. Section 3 describes the pipeline of processes that lead to the full design of an asteroid tour trajectory, such as that described in Section 2. Section 4 describes the optimal substructure property of the problem and discusses the possibility to completely explore the full tree graph of the search space. Section 5 briefly describes Ant Colony Optimization as a paradigm of incomplete tree search exploration using stochastic branching. Section 6 describes its deterministic counterpart, i.e., the beam search. Section 7 presents how dynamic programming principle allow still to guarantee the localization of the global optimum, despite the high dimensionality of the search space.

2. The Asteroid Tour Problem

The term tour, in the context of a space mission, simply indicates a mission that aims to visit not one but several celestial objects (or orbital *waypoints*). Asteroid tours would thus refer to missions that aims to visit a long sequence of asteroids with one single spacecraft. There have already been multiple propositions of asteroid tour mission problems, such as those in 6 out of 11 editions of GTOC, or in more mission design-oriented studies, such as in [7, 8]. Any of these examples will imply many constraints and boundaries that are specific to the problem, however all will require dealing with comparable complexities in a similar dynamical framework.

Figure 1 shows an example of an asteroid tour mission. This begins with a direct escape from Earth, defined through an Earth escape $\boldsymbol{v}_\infty$ vector, which is then followed by 11 fly-bys. One of these fly-bys is in fact a gravitational assist with Mars, which increases the orbital energy of the spacecraft and allows it to obtain a better reach within the main asteroid belt (MAB). The remaining flybys, or encounters, are all asteroids; all of which provide negligible gravitational perturbations.

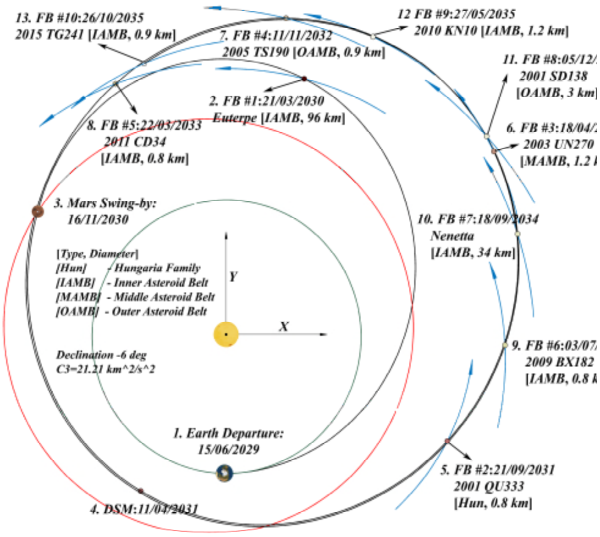

Figure 1. CASTAway asteroid tour trajectory as presented in Sanchez et al. [7].

The dynamic framework used to compute the asteroid tour in Figure 1 is the so-called MGA-DSM model [9], which considers a coast-arc followed by a Lambert arc for each individual leg of the transfer. The initial coast-arc is defined either by the escape $\boldsymbol{v}_\infty$ vector for the Earth, or by 2 fly-by parameters for any other planetary body. The deep space manoeuvre is then computed as a result of a discontinuity in the velocity at the end of the coast-arc and the velocity necessary to reach the following asteroid or planet at a given time. Table 1 summarises all the design variables necessary to define the asteroid tour as proposed here and the associated values for the specific example in Figure 1.

Table 1. Input variables for to describe an asteroid tour in MGA+DSM model.

Integer variables	Description
$x_i, i = 1, \ldots, n_{\text{int}}$	Asteroids and planet integer identifier in the sequence.
x=[39, 4, 77165, 77205, 63041, 87449, 83853, 357, 26508, 50905, 87189]	Earth (assumed departure from Earth)- 27 Euterpe – Mars – 2001 QU333 – 2003 UN270 – 2005 TS190 – 2011CD34 – 2009 BX182 - 289 Nenetta – 2001 SD138 – 2010 KN10 – 215 TG241
Continuous variables	**Description**
$y_1 = t_0$	Earth Launch date – 15/06/2019
$y_{2,3,4} = \mathbf{v}_\infty$	Earth Escape Relative Velocity $\mathbf{v}_\infty = [4.621 \text{ km/s} \quad 0.234 \text{ rad} \quad -0.174 \text{ rad}]$ as modulus, in-plane and out-of-plane angles with respect to the Earth velocity at epoch.
$y_5 = \alpha_0$	coast-arc/ToF fraction for the first leg coast-arc. $\alpha_0 = 0.09$

$y_{6,\ldots,16} = ToF_i, i = 1,\ldots,n_{\text{int}}$	Time of flight to reach each of the objects encountered along the trajectory. $ToF = [279.2 \;\; 239.6 \;\; 309.3\ldots$ $209.7 \;\; 206.8 \;\; 130.9 \;\; 468.5\ldots$ $77.3 \;\; 77.9 \;\; 123.4 \;\; 177.4]$ days
$y_{17,18,19} = \begin{cases} g_k = \gamma \\ g_{k+1} = r_p \\ g_{k+2} = \alpha \end{cases}$ $k = 1,\ldots,n_{GA}$	Fly-by and coast-arc/ToF fraction, where: n_{fb} is the number of objects in the sequence; r_p and γ are periapsis and inclination of the fly-by plane, respectively; α is the time at which a DSM is performed after the fly-by with the planet. r_p and γ are only relevant for bodies of non-negligible mass (i.e. planets), while example in Figure 1 does not optimize coast-arc for the asteroid-asteroid legs. Hence: $y = [0.0717 \text{ rad} \;\; 1.0736 \cdot \text{R}_M \;\; 0.4635]$ completes the design variables set (R_M is mars radius).

The design variables in Table 1 represent thus the input variables of a function $f(x, y)$ that defines a full asteroid tour. The design problem would typically be defined as a mixed-integer non-linear programming (MINLP) problem, where function *f* may output a single or multiple objective quantitative criteria (i.e., fitness criteria) to be optimized. These may typically include trajectory design characteristics such as the total Δv of the mission or the total time of flight.

Indeed, given their inherent complexity of multiple target MINLP trajectory design, these have been a recurrent proposition in many editions of the Global Trajectory Optimization Competition (GTOC), where the winner of the competition is the team who submits the solution with the *best* fitness criteria. Many techniques have been used to solve these types of problems. These techniques are generally categorised as either deterministic or stochastics. Stablished deterministic approaches such as Branch & Bound [10] or Dynamic Programming [11] take advantage of discretization and the consequent tree-graph data structure of the problem. However, as the dimensionality of the parameter space increases, the computational costs are yet difficult to handle. Thus, over time, stochastic approaches such as Genetic Algorithms [12], Particle Swarm Optimization [13] and Ant Colony Optimization [14] have gained terrain, and are now often the technique of choice to solve the most arduous problems [15].

However, designing a real mission is a substantially different challenge than winning an optimization competition. For a start, it is often the case that the goodness of a mission design is not easily quantifiable. See for example the case on which the goodness of a given asteroid tour, such as that in Figure 1, is based on the opinion of a set of scientific experts. All of which may have a distinctly different research interest and expertise and so may value asteroid sequences in different ways. For example, one may prefer a sequence that visits several fast rotation asteroids, while another one a sequence that visits a rare spectral type, etc. Secondly, early stages of mission design require a true exploration of the search space, rather than only the localization of some isolated notable solutions. Hence, if one seeks true insight into the topography of the feasible search space a consistent exploration of the search space must instead be ensured.

Hence, one may argue that a Constraint Satisfaction Problem (CSP) formulation such as:

Given a set of variables: (x,y) where $x \in \mathbb{Z}^{n_{\text{int}}}$ and $y \in \mathbb{R}^{n_{cont}}$, $n_{\text{int}}, n_{cont} \in \mathbb{N}$

And their domains:

$$x_{lb} \le x \le x_{ub} \qquad (1)$$

$$y_{lb} \le y \le y_{ub}$$

And a set of constraints: e.g., $f(x, y) \le 0$

Find all, or as many as possible, sets of (x,y) that satisfy all defined constraints

is indeed as relevant problem to be solved as a global optimization formulation of it.

3. Multi-fidelity paradigm

Solving Eq. (1) with no a-priori knowledge of the problem would only be feasible for problem formulations with very small search domains for both the integer and the continuous-varying variables (*x*,*y*). Hence, it is evident that solving the mixed-integer formulation of the asteroid tour exploration requires a process of refinement to manage this complexity efficiently. This section describes very briefly, the multi-fidelity pipeline proposed to efficiently solve the asteroid tour MINLP both in its global optimization and its CSP formulation.

3.1 Preliminary Steps

As of September 2022, slightly over 1 million catalogued asteroids are known within the MAB. Exploring all the, for example, ten-asteroid-long sequences within the known population would require computing more than 10^{53} trajectories. Computing such number of trajectories would take many orders of magnitude more than the age of the universe; hence a compelling reason to prune out the overall catalogue, and target only a sensible set of interesting MAB objects.

Here, a pruned database of ~102,000 main belt asteroids is used to search for main-belt asteroid sequences. This database provides a prefiltered population of MAB objects. All asteroids larger than 10 km in diameter are retained in this database, while smaller objects are pruned out maintaining a representative diversity of asteroids in size and orbital distribution. The orbital elements and other physical information of the asteroids in the database was downloaded from Jet Propulsion Laboratory Small-Body Database. It should be noted that an asteroid set of ~102,000 objects is larger than any GTOC-related asteroid set, and that the addition of one or multiple planetary gravity assists, as in Figure 1, adds extra complexity to the mixed-integer global optimization.

Ultimately, different planetary sequences will allow the spacecraft to enter the MAB with different paths and, thus, spend more or less time within the bounds of the MAB. See for example Figure 2, which showcases a simple Earth direct escape with no planetary gravity assists, as well as an Earth-Mars and Earth-Venus-Earth sequences. In the remainder of the paper only an Earth-Mars sequence will be considered, since as shown in Figure 3 and analysed by Gallego [16], it enables a decent reach within the MAB at low Earth escape velocities, specially if short missions are considered (~5 years).

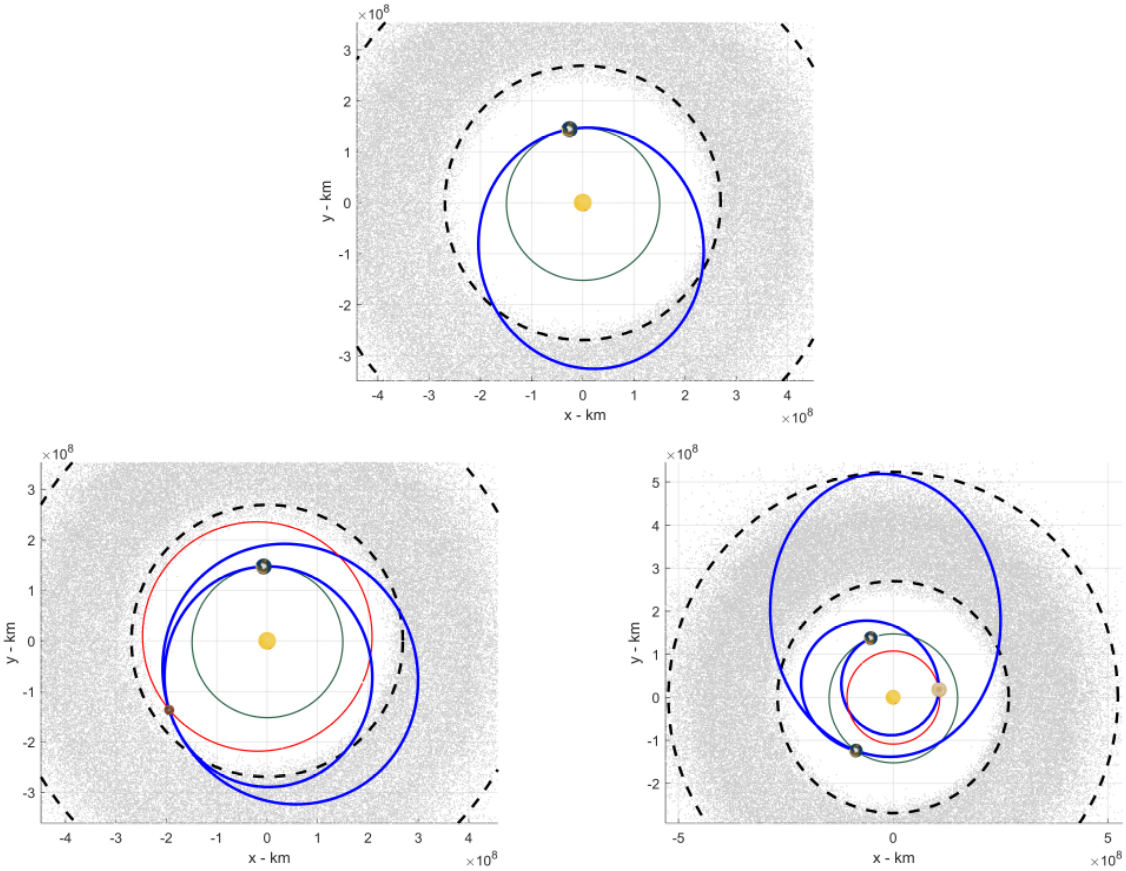

Figure 2. Three trajectory options (i.e., planetary sequences) to reach the MAB: with Earth alone (top), Earth-Mars (bottom left) and Earth-Venus-Earth (bottom right).

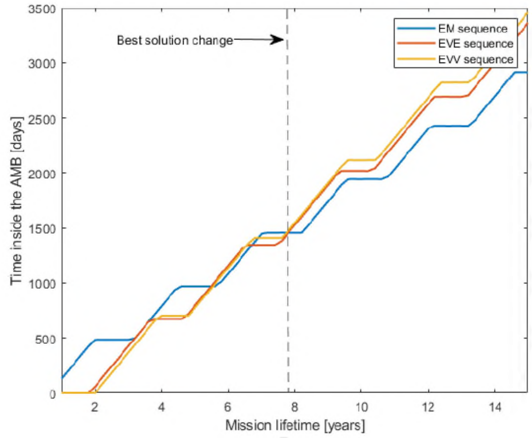

Figure 3. Maximum time spend within the MAB for a spacecraft departing the Earth with heliocentric escape velocity of 5 km/s. reproduced from Gallego [16].

The following subsections will describe the process to explore all asteroid tours possible for one single Earth departure in an Earth-Mars planetary sequence. The global and CSP problems are thus solved only for the reference trajectory described in Table 2.

Table 2. Details for the Earth-Mars reference trajectory.

Event	Value
Earth departure (y_1)	24th of December 2030
Earth Escape Velocity v_∞	4.23 km/s
Mars Fly-by Date	3rd March 2033

3.2 Asteroid Tour Transcribed Problem

Recall that at this stage two different sub-problems need to be solved to design an asteroids tour. Firstly, the right sequence of asteroids (i.e., completing the integer vector variable x), among the ~102,000 targets, need to be appropriately chosen, i.e., solving a discrete combinatorial problem. However, the quality of a given asteroid tour can only be assessed after identifying the actual dates for each asteroid encounter (i.e., continuous-varying vector variable y). It should be noted that the Δv cost of an asteroid tour will be highly sensitive to the actual dates of the asteroid encounters. A priori, the possible dates for each asteroid fly-by (t_{fb}) could be any date that satisfies $y_1<t_{fb}<y_1+TOF_{max}$ (i.e., any date within the allowed by the maximum mission timespan TOF_{max}). The fact that both of these two sub-problems are tightly associated presents the crux of the asteroid tour problem and differentiates it from other classic combinatorial problems, such as the Travelling Salesman Problem.

Given that, in this pipeline, the asteroid tour sequence is refined over one specific baseline trajectory (i.e., Table 2), one may consider that only asteroids that are close to the spacecraft at any one point during this trajectory can be feasibly encountered (as with small Δv manoeuvres). An analytical algorithm identifying the minimum orbital intersection distance (MOID), combining both Bonanno [17] and Milisavljevic [18] algorithms, is implemented to identify asteroids within a MOID range, as well as their true anomaly at the epoch of the MOID point crossing. The distance between the spacecraft and the asteroid at the epoch at which the asteroid crosses its MOID point is used as a quick estimate of the asteroid minimum distance (note multiple asteroid epochs may exist within the mission timespan TOF). One can thus prune out all asteroids that do not satisfy a given distance threshold d_{thr}. Table 3 summarizes the number of asteroids within a set of spacecraft distance, following this proceeding.

Table 3. Largeness of the Asteroid Set Identified

Distance threshold d_{thr}	Number of Asteroids
0.03 au	49
0.04 au	98
0.05 au	158
0.10 au	562
0.15 au	1026

Note that each asteroid in the set also has an epoch associated to it, which identifies the epoch at which the asteroid crosses its MOID point. Hence, effectively the problem is now transcribed into a purely combinatorial problem that requires to select only a set of asteroids, whose encounter times are now predetermined to happen at the time the asteroid crosses its MOID point. The continuous optimization with the actual dates for each asteroid encounter may then be refined in the final part of the pipeline.

3.3 Asteroid Tour Refinement

Figure 4 synthetises the results of the process of refinement from the transcribed problem (P1) to the full mixed-integer optimization (P2). The continuous-variable optimization is implemented using the standard genetic algorithm solver available within MATLAB global optimization toolbox.

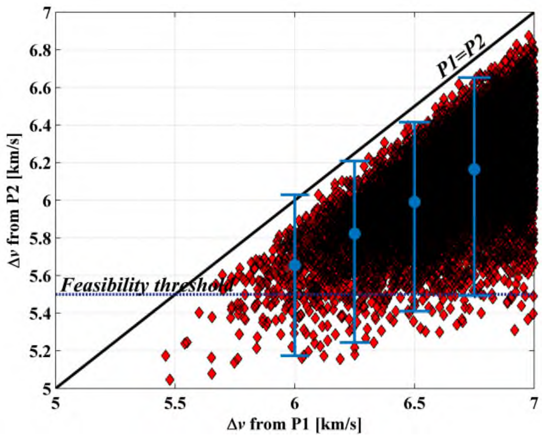

Figure 4. Summary of continuous-variables optimized solutions (P2) and their combinatorial transcription (P1) estimates.

It should be noted that the transcribed problem P1 provides a real solution of the full MINLP and thus the refined solution must be equal or better. Indeed, Figure

4 shows that the refinement of P1 solutions may achieve some improvement of Δv, but the transcribe problem still provides a rather accurate estimate of the goodness of the trajectory, without the need to deal with the continuous variable optimization. Tackling the combined mixed-integer problem, without the process described in this pipeline (Subsection 3.2 and 3.3) would be a rather daunting and inefficient endeavour. Figure 5 in fact showcases this point by plotting the best asteroid tour found for the reference trajectory in Table 2, after 10^6 sequence evaluations with 12 asteroids. (x,y) variables are both randomized for each evaluations within the possible bounds for each design variable.

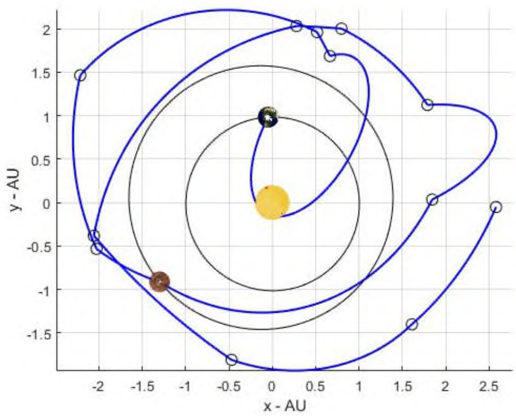

Figure 5. Best asteroid tour found within 10^6 f(x,y) evaluations with randomization of both x,y variables within their possible bounds. Final Δv of the tour is 300 km/s.

Instead, the so-called transcription process into a purely combinatorial problem enormously simplifies the problem and renders the possibility of a much simpler search for feasible and optimal solutions. Note that the approximation of the fly-by time, as the asteroids' epoch at its MOID point, is a very accurate approximation of the real fly-by time. Figure 6 indeed shows how the optimal Δv for each leg of a sample sequence is indeed within only a few days of the asteroids' crossing of the MOID point.

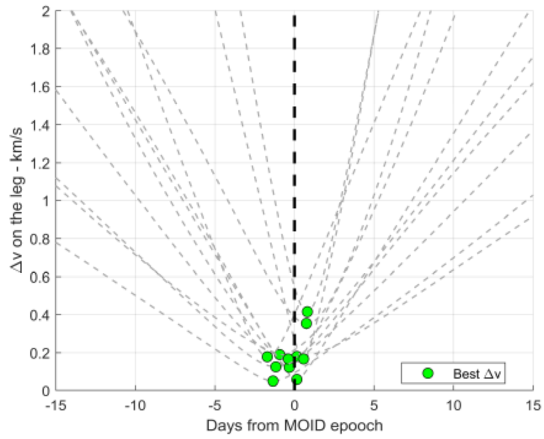

Figure 6. Δv variation on the different legs of a asteroid tour (reference trajectory as in Table 2) with respect to the MOID epochs for each of the visited asteroids.

4. Graph structure of the search space

In the Asteroid Tour Transcribed problem, P1 thereafter, the cost of the path connecting two asteroids is associated with the Lambert arc connecting both asteroids at their respective MOID point epochs. Figure 7 illustrates the spacecraft trajectory between two asteroids of indices j and k. Since asteroid j and k have each an associated MOID point epochs $t^{(j)}_{MOID}$, the time of flight between the two is uniquely defined (i.e., $\Delta t_{tof}=t^{(k)}_{MOID}- t^{(j)}_{MOID}$) and, consequently, also the zero-revolution Lambert arc between these two points. The spacecraft cost of connecting asteroid j and k at their respective MOID point epochs is given by the impulsive manoeuvre $\Delta v_{(j,k)}=|\mathbf{v}_{j,departure}-\mathbf{v}_{j,arrival}|$, where $\mathbf{v}_{j,arrival}$ is the spacecraft velocity at arrival at asteroid j and $\mathbf{v}_{j,departure}$ is the departure velocity defined by the Lambert arc between asteroids j and k. Consequently, the cost of a given leg is not unique, but depends upon the asteroid prior to asteroid j, which will define the $\mathbf{v}_{j,arrival}$. Thus, to uniquely define the cost of a given leg between A_j and A_k, one needs to consider also the previously visited one, say A_i, so for the triplet (A_i,A_j,A_k), one has a unique cost. One should note that this optimal substructure property in the form of a triplet of individual nodes is common to all problems where fly-bys are to be considered, thus also to multiple gravity assist trajectories with planets/moons/etc.

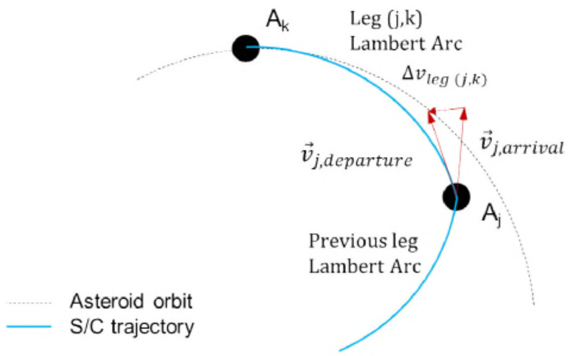

Figure 7. Sketch of spacecraft trajectory and Δv between asteroid A_j and A_k. The spacecraft previously visited asteroid A_i and then followed to reach A_k (blue path).

Because of this substructure of unique triplets, the search space is a graph that can be modelled as a multi-dimensional space of connected nodes, each made by a couple of asteroids. When linking two consecutive nodes, the first asteroid in one node is equal to the second asteroid encoded in previous node. The connection is then made by $\Delta v_{(j,k)}$. In this space, the cost of the paths between the nodes is unique, which is the main advantage of modelling the search space in this way. Being the $\Delta v_{(j,k)}$ a tri-asteroids dependent cost, unique for each of the legs of the search space, a tri-structured score matrix can be created. Figure 8 shows an example of score matrix for the smallest set of 49 asteroids (see Table 3). Each row, in *y* axis, represents a couple of asteroids, i.e., a trajectory between two objects, and each column, in *x* axis, is encoded with asteroids in the catalogue that completes the triplet. The third dimension is completed with $\Delta v_{(j,k)}$.

From \ To	A_1	A_2	A_3	A_4	A_5	⋯	A_{47}	A_{48}	A_{49}
(A_E, A_1)	x	$\Delta v^E_{1,2}$	$\Delta v^E_{1,3}$	$\Delta v^E_{1,4}$	$\Delta v^E_{1,5}$	⋯	$\Delta v^E_{1,47}$	$\Delta v^E_{1,48}$	$\Delta v^E_{1,49}$
(A_E, A_2)	x	x	$\Delta v^E_{2,3}$	$\Delta v^E_{2,4}$	$\Delta v^E_{2,5}$	⋯	$\Delta v^E_{2,47}$	$\Delta v^E_{2,48}$	$\Delta v^E_{2,49}$
⋮	⋮	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮
(A_E, A_{47})	x	x	x	x	x	⋯	x	$\Delta v^E_{47,48}$	$\Delta v^E_{47,49}$
(A_E, A_{48})	x	x	x	x	x	⋯	x	x	$\Delta v^E_{47,49}$
(A_E, A_{49})	x	x	x	x	x	⋯	x	x	x
(A_1, A_2)	x	x	$\Delta v^1_{2,3}$	$\Delta v^1_{2,4}$	$\Delta v^1_{2,5}$	⋯	$\Delta v^1_{2,47}$	$\Delta v^1_{2,48}$	$\Delta v^1_{2,49}$
(A_1, A_3)	x	x	x	$\Delta v^1_{3,4}$	$\Delta v^1_{3,5}$	⋯	$\Delta v^1_{3,47}$	$\Delta v^1_{3,48}$	$\Delta v^1_{3,49}$
⋮	⋮	⋮	⋮	⋮	⋮	⋱	⋮	⋮	⋮
(A_{47}, A_{48})	x	x	x	x	x	⋯	x	x	$\Delta v^{47}_{48,49}$
(A_{48}, A_{49})	x	x	x	x	x	⋯	x	x	x

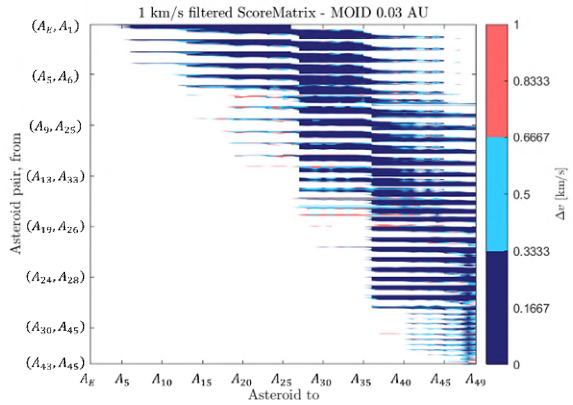

Figure 8. Score matrix structure (top) and values (bottom) for the catalogue of 49 asteroids (d_{thr}=0.03 au). Asteroids are ordered with respect to their MOID epochs.

To usefully exploit the substructure of unique triplets, one can either precompute all the possible triplets in the score matrix or store the cost for each triplet at the first instance that is computed during the search process.

Having define the search space as a graph of connected nodes, each of which defined as pairs of asteroids, it may then be tempting to consider complete tree travers algorithms to solve both the global optimisation and the CSP problems. Depth First (DF) or Breadth First (BF) strategies, combined with Branch & Bound and efficient pruning, may allow to systematically cross the tree graph representing the entire search space of the problem. By means of the binomial coefficient, one can quickly see how the total number of possible paths in a tree quickly grows to unfeasible values. Table 4, for example, shows all the possible sequences of 12-asteroid-long paths that exist for each set identified for different distance threshold d_{hr} (see Table 3).

Table 4. Number of combinations of 12 asteroids sequences.

Number of Asteroids	**Number of Sequences with 12 asteroids**
49	9×10^{10}
98	8×10^{14}
158	3×10^{17}
562	2×10^{24}
1026	2×10^{27}

Considering pruning criteria derived from realistic mission design scenarios, one can still complete tree travers explorations for the smallest catalogues in Table 4. Nevertheless, given the exponential growth of the number of combinations, the task quickly becomes an impossible endeavour for larger sets. Beyond complete searches such as DF/BF, incomplete tree-traverses can be divided in searches that perform the branching either deterministically or stochastically. A widely use strategy for the latter is Ant Colony Optimization [19], which is discussed in the following section.

5. Ant Colony Optimization

Among tree-search strategies that employ stochastic branching, Ant Colony Optimization (ACO) is perhaps the most popular for solving complex combinatorial problems. This is a metaheuristic algorithm whose search strategy mimics the behaviour of ant colonies in searching for food [19]. In an ACO algorithm, artificial ants construct candidate solutions by traversing a discrete graph. The branching procedure is stochastic and guided by a probability function, which defines the likelihood of the artificial ant at node (*i*,*j*) to move to node *k*:

$$P_{(i,j),k} = \frac{\tau^{\alpha}_{(i,j),k} \cdot \eta^{\beta}_{(i,j),k}}{\sum\limits_{\left(A_i^{m-2}, A_j^{m-1}, A_l^{m}\right)} \tau^{\alpha}_{(i,j),l} \cdot \eta^{\beta}_{(i,j),l}}$$

$$P_{(i,j),k} \forall \left(A_i , A_j , A_l \right) \text{ where } l \in N, \text{being } N$$

$$\text{the sets of available nodes from } \left(A_i , A_j \right)$$

The probability $P_{(i,j),k}$ is thus driven by the *pheromone* $\tau_{(i,j),k}$ and the *heuristic* $\eta_{(i,j),k}$ matrices, as well as parameters α and β. The heuristic $\tau_{(i,j),k}$ matrix is here defined as a constant matrix, whose entries are the inverse of the Score Matrix value for each asteroid triplet. The pheromone $\eta_{(i,j),k}$ matrix represents instead the knowledge gained by the ensemble of ants as they explore the search space. The synthesis of this knowledge is here implemented as:

$$\tau_{(i,j),k} \leftarrow \left(1-\rho\right) \cdot \tau_{(i,j),k} + \sum_{a=1}^{n_{ants}} \Delta\tau^{a}_{(i,j),k}$$

where the *pheromone* $\tau_{(i,j)k}$ is updated at each iteration of n_{ants} (i.e., an entire colony) with each ant individual knowledge $\tau^{a}_{(i,j),k}$ being the invers of the Δv cost of the asteroid tour, only if the triplet (*i*,*j*,*k*) is among those explored by the ant and zero otherwise. Taking the ant analogy further, the parameter ρ represents the evaporation of the pheromone trail. The search is completed iteratively by different colonies with n_{ants} each. The overall implementation used here is thus relatively simple.

As it is reported in Carrillo [20] and Gonzalez-Pardo et al. [21], the performance of the ACO is highly sensible to the α, β and ρ parameters, which define the weights for the use of the heuristic and pheromone information. The best performance of ACO was achieved by heavily weighting the heuristic information (i.e., α=1, β=5 and ρ=0.25). This set up allowed ACO to find the global optimum for the 98-asteroid set; albeit, not consistently, given the stochastic nature of ACO. A detailed description of the ACO implementation, as well as the tests performed, results, and specific discussion on the performances of ACO can be found in Carrillo [20].

If the global optimum for a given catalogue is known, the following test brings some light to the performance of ACO. Figure 8 shows the value of the probability function $P_{(i,j),GO}$ and $P_{(i,j),B}$, where the subindex *GO* and *B* refer to the path leading to the global optimum and the path leading to the best explored solution of a given iteration, respectively. The specific run of ACO shown in Figure 8 did encounter the global optimum, which can be observed as a perfect correspondence between $P_{(i,j),GO}$ and $P_{(i,j),B}$ at iteration 1083. It is also worthwhile to note the sharp increase of the probability $P_{(i,j),GO}$ for the first level, or path choice. The increase of $P_{(i,j),GO}$ is associated with the effect of the pheromone trail of predecessor ants, which are able to efficiently indicate the path of the global optimum, but only at the very first level. The improvement in $P_{(i,j),GO}$ increases the overall probability of a single ant to find the global optimum from the initial 1.4×10^{-7} to a final 7.2×10^{-7}. Hence, for the case of 65000 ants, exploring the search space associated with the 98-asteroid catalogue, the overall probability of successfully finding the global optimum is still only 5%.

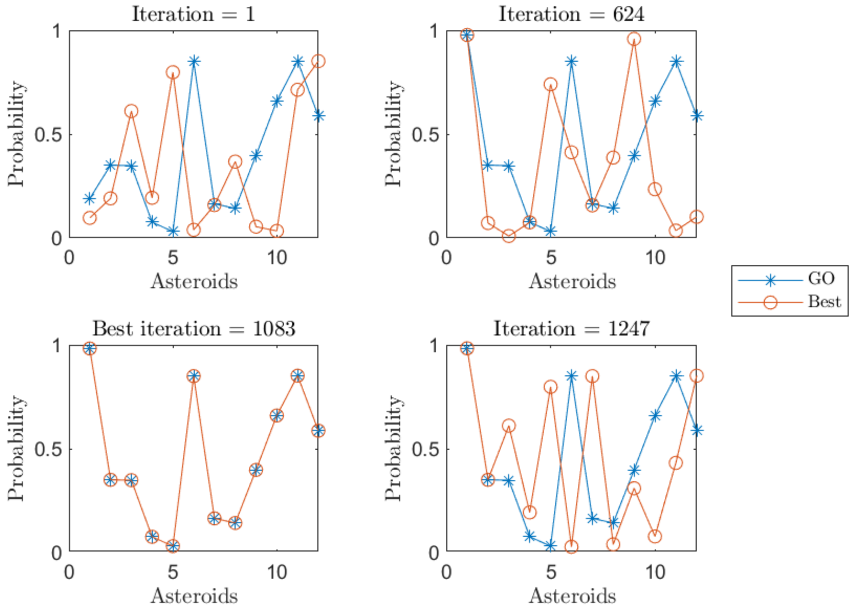

Figure 9. Probability plot for ACO Search with 98 asteroids catalogue.

6. Beam Search

Beam Search strategies are the counterpart of ACO algorithms, on which rather than branching stochastically, the branching is defined completely deterministically. In a Beam Search, the computational effort associated with tree exploration is bounded by capping the number of branches that can be expanded at any one level. More specifically, from all the branches generated at one level, only a limited set of them, referred as the beam, is selected to be expanded at successive levels. The beam selection is performed deterministically, meaning that nodes at each depth-level of the tree are sorted with respect to a heuristic criterion, for example the total Δv incurred so far, and only those with the most optimal heuristic are selected for further consideration. The size of the beam is called beam width BW. A search where BW=1 would thus correspond to a nearest-neighbour search, while a BW=∞ would be a complete tree travers.

Selecting the proper BW is thus a compromise between solution quality and number and computational effort. Performances of the BS algorithm on the problem at hand are evaluated over a grid of settings for the BW for the different asteroids' catalogues considered. The BW varies from 0 to an arbitrary high number, e.g., 200x10³, that allows to consistently find tens of thousands of solutions, thus providing a good approximated solutions to the CSP formulation.

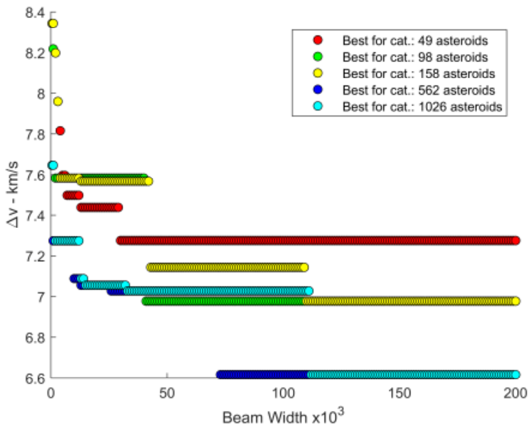

Figure 10. Best Δv solutions with respect to BW for different catalogues considered.

The analysis in Figure 10 show several plateaus, with transitions at a priori unknown BW. One can first observe first that transitions to the same Δv level occur at a larger BW for larger catalogues: see for example the 98 and 158 asteroids catalogues, which both reach 6.977 km/s level, but the 158 asteroid catalogue requires more than twice the BW to reach it than for the 98 catalogue. This feature suggests an underlying multi-modal structure of rapidly increasing complexity.

The last of these plateau transitions must thus coincide with the global optimum. However, one does not know a priori the BW necessary to reach this final transition. In fact, for the results shown in Figure 10, the catalogues with 49, 98 and 158 asteroids reach the global optimum in the search space, while the catalogues with 562 and 1026 do not.

7. Dynamic Programming

A possible workaround to guarantee the global optimality of a solution is enabled by the application of dynamic programming principles. Dynamic programming is applicable here since the asteroid tour combinatorial problem P1 is in fact a combination of independent sub-problems, i.e., the transfers between triplets of asteroids. Figure 11 represents the application of Bellman's principle of optimality [11] in the exploration of asteroid tour sequences. It is clear that during a tree exploration that seeks solely to find the global optimum, it is useless to keep in memory anything beyond the optimal path that reaches a given node of the tree.

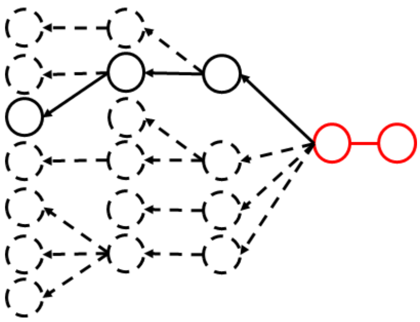

Figure 11. The optimal sequence containing a given pair of asteroids must contain the optimal subsequence arriving and departing from the pair.

Figure 12 shows the computational effort with respect to decision stages (i.e., tree-levels). The computational effort is represented by the number of solutions kept in memory when terminating the exploration of a given level. Figure 12 also summarizes the global optimum solutions as found by the dynamic programming application and the best solution found by the beam search (BW=200x10³). Hence the application of dynamic programming guarantees finding the global optimum solution for the P1 transcribed problem. Moreover, it does it with a substantially lower number of function evaluations for all cases. However, it does not store a very large number of solutions and its diversity needs still to be properly investigated.

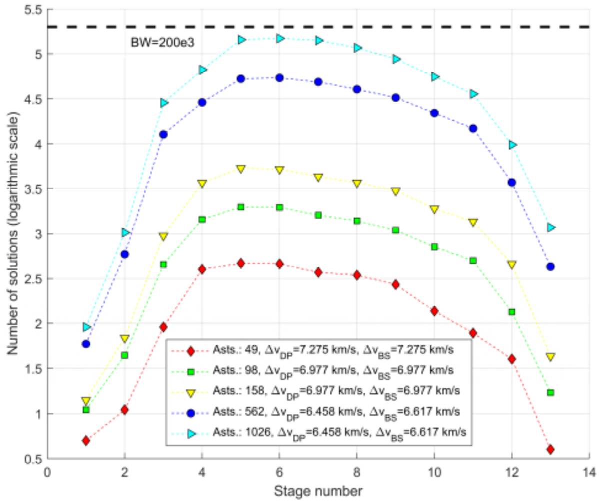

***Figure 12.** Number of solutions (in logarithmic scale) for each decision stage of dynamic programming (DP) for different asteroids datasets. In the legend, global optimum Δv as from DP and best Δv as from the Beam Search (BW=200*10^3) are also reported. Horizontal line highlights the number of solutions of the Beam Search.*

8. Conclusions

This paper has presented an efficient pipeline process that enables the design complex asteroid tour sequences within a medium fidelity dynamical framework that considers patched conics dynamics, planetary gravity assists and deep space manoeuvres (i.e., MGA+DSM). Moreover, a multifidelity approach is presented on which the planetary sequence to reach the MAB is first explored, followed by the integration of near-by asteroids along the path. The use of the asteroid's epoch of the MOID point crossing provides a extremely efficient means to uncouple the mixed-integer problem, into a separate combinatorial problem and a refinement of the continuous variables on a later stage. The process allows for the description of the combinatorial only problem using a tree graph structure and the implementation of dynamic programming principles following its optimal substructure property. Finally, a combination of dynamic programming and beam search is proposed to solve both the global optimization and the constraint satisfaction problem of asteroid tour trajectory design.

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