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THE RACE BETWEEN INTERNATIONAL TRADE AND AUTOMATION IN EXPLANING WAGE POLARIZATION

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Abstract

The rise of the skill premium since the 1980s gave rise to the development of the Direct Technical Change (DTC) literature. This literature links the increase in the relative supply of skilled workers with the technological-knowledge bias towards those workers, which induces a higher relative demand for this labor type. However, more recent and more detailed data point to a polarization of wages concerning the distribution of skills, requiring the literature to address modeling approaches focused on automating and/or relocating different types of tasks and considering more than two labor types. Based on a standardized developed country where robotics is more intense and the relocations of production is more effective, this paper reconciles the two approaches, DTC and wage polarization, when considering: (i) routine and non-routine tasks; and (ii) relocation of routine tasks towards developing countries, which generate international trade. Such reconciliation implies that (i) the skill premium is positively affected by an increase in unskilled-labor over the skilled labor, (ii) inter-country wage inequality depends positively on relocation and negatively on automation, (iii) for the expected values of the parameters, wage polarization increases with robotization and relocation, and decreases with the rise of skilled and unskilled domestic workers; (iv) economic growth is also boosted by robotization and relocations. In terms of competitiveness, the race between robotization and relocation is won by robotization if exceeds relocations.

Keywords: Directed technical change; International trade; Economic growth; Wage inequality and polarization.

JEL Classification: J23, J31, F16, F43, O30, O41.

Resumo

O aumento do prémio de competências (i.e., o salário relativo dos trabalhadores qualificados) desde os anos 80 deu origem ao desenvolvimento da literatura Directed Technical Change (DTC). Esta literatura liga o aumento da oferta relativa de trabalhadores qualificados com enviesamento do conhecimento tecnológico em relação a esses trabalhadores, o que induz uma maior procura relativa para este tipo de trabalho. Contudo, dados mais recentes e mais detalhados apontam para uma polarização dos salários no que diz respeito à distribuição de competências, exigindo que a literatura aborde abordagens de modelação centradas na automatização e/ou relocalização de diferentes tipos de tarefas e considerando mais de dois tipos de mão-de-obra. Com base num país desenvolvido padronizado onde a robótica é mais intensa e a relocalização da produção é mais eficaz, este documento concilia as duas abordagens, DTC e polarização salarial, ao considerar: (i) tarefas rotineiras e não rotineiras; e (ii) a relocalização de tarefas rotineiras para países em desenvolvimento, que geram comércio internacional. Tais a reconciliação implica que (i) o prémio de qualificação é positivamente afetado por um aumento do trabalho não qualificado em relação ao trabalho qualificado, (ii) a desigualdade salarial entre países depende positivamente da relocalização e negativamente na automatização, (iii) para os valores esperados dos parâmetros, polarização salarial aumenta com a robotização e deslocalização, e diminui com o aumento de trabalhadores domésticos qualificados e não qualificados; (iv) o crescimento económico também é impulsionado pela robotização e deslocalizações. Em termos de competitividade, a corrida entre a robotização e a deslocalização é ganha pela robotização se exceder deslocalizações.

Palavras-chave: Enviesamento do conhecimento tecnológico; Comércio internacional; Crescimento económico; Desigualdade salarial e polarização.

Classificação JEL: J23, J31, F16, F43, O30, O41

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1 Introduction

There is consensus in the literature that inequality has been increasing in most advanced economies since the 1980s (e.g., Alvaredo et al. 2018). Initially, the prevailing literature attributed this phenomenon to the bias of technological-knowledge progress in favor of skilled workers vis-à-vis unskilled workers (e.g., Bound and Johnson 1992; Katz and Murphy 1992; Juhn et al. 1993; Acemoglu 1998, 2002).[1] In the context of this Directed Technical Change (DTC) literature, unskilled and skilled workers are complemented by specific types of technologies. An increase in the supply of one type of labor causes an expansion of the market size of the technologies it complements (a market-size channel), which, given the associated profitability, creates additional incentives for R&D directed at those technologies. Consequently, changes in technological knowledge, as a result of R&D activity, are biased toward those technologies; i.e., toward a specific sector. In turn, the bias increases the demand for the type of complementary labor, which would supplement the increased supply. Thus, the proposed modeling was able to explain the increase in skill premium as a result of the observed increase in the relative supply of skilled labor in the same period (e.g., Akerman et al. 2015; McAdam and Willman 2018).

Alternatively, considering three types of workers, as suggested by the finer analysis of the existing data, several authors have found that medium-skilled workers are employed in routine tasks, while unskilled and skilled tasks are mainly employed in "purely manual" and "abstract/cognitive" non-routine tasks, respectively (e.g., Acemoglu and Autor 2011; Autor and Dorn 2013; Michaels et al. 2014; World Bank Group 2016; Wang et al. 2021); for example, according to the World Bank Group (2016), around 57% of current jobs in the OECD are at risk of being replaced by robots or by relocations of tasks towards developing countries,[2] mainly at the level of complementary routine tasks for medium-skilled workers (Blanas et al. 2019) since these tasks only require methodical repetitions (Autor et al. 2003). Both types of non-routine tasks – purely

[1]Less intensely, the literature has also analyzed the role played by institutional changes in the labor market, especially in terms of minimum wages and unionization (e.g., DiNardo et al. 1996; Neto et al. 2019), and by globalization (e.g., Feenstra and Hanson 1999; Deardorff 2005; Grossman and Rossi-Hansberg 2008, 2012; Rodriguez-Clare 2010; Criscuolo and Garicano 2010; Afonso 2012; Antras and Yeaple 2014; and Acemoglu et al. 2015).

[2]Relocations, materialized by worldwide firms, take the form of some of the tasks being transferred from the stylized developed country toward a developing country where wages are lower.

manual and abstract/cognitive – are difficult to reduce to a specific set of instructions (Chui et al. 2016; and Acemoglu and Restrepo 2018). Indeed, the purely manual tasks require human and physical elements (e.g., service occupations), and the abstract/cognitive tasks require complex cognitive processes (e.g., managers, technicians, accounting, consulting, planning, and even in various medical specialties, etc.).

In the context above, there are two main explanations for the mentioned "wage polarization" observed in developed countries:

- automation (or robotization), which, by leading to an increase in routine tasks performed by machines/robots, makes labor less productive in the routine sector and decreases the relative demand for medium-skilled workers – e.g., Autor et al. (2003), Acemoglu and Autor (2011), Autor and Dorn (2013), Hémous and Osborne (2014), and Lankisch et al. (2019).[3]

- offshoring / outsourcing / foreign direct invest (FDI) through which worldwide firms – i.e., firms operating in developed/innovator countries and, direct or indirectly, in developing/follower countries – transfer productions to countries with lower costs (hereinafter, relocations that promote international trade) that potentially benefit all workers in the world since create efficiency gains (e.g., Grossman and Rossi-Hansberg 2008; Rodriguez-Clare 2010), but also have strong distributional effects that can, for example, have negative implications on wages of medium-skilled workers in developed countries – e.g., Feenstra and Hanson (1996, 1999) Deardorff (2005), and Criscuolo and Garicano (2010).[4] That is, greater imports of cheap medium-skilled inputs produced by worldwide firms in developing countries may lead to a decline in the medium-skilled wage and a rise "wage polarization" in developed countries.

Therefore, this dissertation focuses on the explanation of "wage polarization" observed in many developed countries, based on the "race" between relocations (i.e., international trade) and the

[3]Other works studying the decline of wages in routine tasks include Lee and Shin (2017), Gregory et al. (2018), Jaimovich et al. (2020), and Atalay et al. (2020), among others.

[4]In addition, relocations could lower income in developed countries by penalizing the respective technological-knowledge advantage in a set of tasks (Samuelson 2004).

automation of tasks. Given the scope of the Master's in International Business (MEGI) and the importance of relocations,[5] it is justified to seek to understand the importance of each channel – relocations and automation – in explaining wage polarization. In accordance with the data and the assumptions of the theoretical model developed, hereafter, we find that there is a positive relationship between robotization and relocations and, respectively, the number of medium-skilled workers at home and abroad. That is, medium-skilled domestic workers impose the size of tasks that can be robotized, although, of course, the intensification of robotization makes this supply of workers less productive. Similarly, medium-skilled foreign workers are considered to impose the size of the tasks that can be relocated, but the intensification of relocations may make these workers less productive.

In the light of the above, we can state that the paradigm shift has led to a shift in the literature away from the DTC conceptual framework, since the latter, by considering only two types of skills (skilled and unskilled labor), is unable to provide an adequate justification for the phenomenon of wage bias leading to "wage polarization" (e.g., Acemoglu and Restrepo 2018). Here, we start by contributing to this literature by proposing a theoretical framework, based on Acemoglu (2002) and Afonso (2012), that reconciles the two approaches, DTC and "wage polarization". In the proposed extension we also consider the impact on economic growth induced by the relocation and/or automation of tasks.

Detailing a little further, we develop a dynamic general equilibrium endogenous growth model where the aggregate output (i.e., the numeraire good) is produced by a continuum of non-routine and routine tasks, as is previously addressed by Autor et al. (2003), and is used in consumption and investment. The non-routine sector is country-specific and is composed of tasks that require high-level abstract skills (non-routine abstract/cognitive tasks) and others that require physical dexterity and proficiency in human interactions (non-routine manual tasks). The routine sector, in turn, can be robotized or relocated abroad by global firms. Hence, non-routine tasks

[5] Since the 1980s and at least until the emergence of the Covid19 pandemic, there has been a growing trend to transfer production to less developed countries. This transfer of production was operated by firms that directly or indirectly operated in several countries. For example, the share of imported intermediate goods (mostly produced by global firms in developing countries) in the total use of intermediate goods used in the US increased from about 2% in 1974 to more than 27% in 2010 (e.g. Feenstra and Jensen 2012; Goel 2017).

are performed by domestic skilled and unskilled workers, and non-routine tasks are produced by medium-skilled workers in the developed country or by a mix of different types of workers in the developing countries (Blanas et al. 2019), where the representative worker in the developing country corresponds to the medium-skilled worker in the developed country. Therefore, in either sector, non-routine and routine, a continuum of competitive firms use specific labor, and production is complemented by specific quality-adjusted machines – vertical R&D. Each (quality-adjusted) machine consists of a continuum of monopolistic producers, each one using a specific design sold by the R&D sector.

Through the proposed model, we intend to analyze the relative impact of automation and relocation (hence, trade) of routine tasks on competitiveness, wages, and economic growth. Relocations immediately affect the country's competitiveness by decreasing the number of tasks produced in the developed country in contrast to automation. Both – relocations and automation – provoked the emergence of some effects on wages – *labor*, *market-size*, and *price effects*. The two last effects operate through the bias of technological-knowledge progress.[6] Relocation and automation of tasks performed by medium-skilled workers, immediately increase the relative labor supply – *the labor effect* – thus generating wage polarization. Furthermore, the technological-knowledge bias, generated by the dynamics of *market-size* and *price effect*s, also decisively affects wages, and the observed technological-knowledge progress affects economic growth. Economic growth frees up resources that become partially available for investment in R&D activities, thereby increasing the probability of successful research, which accelerates the technological knowledge. The effects on aggregate technological knowledge affect firms' productivity: when it increases, it generates higher demand and labor productivity. If technological-knowledge progress is biased towards the non-routine sector then it contributes to the emergence of wage polarization.

The proposed theoretical model developed is then confronted with real data from countries, considering The United States of America (The US) as representative of the developed country, and Argentina, Brazil and Turkey as representative of developing countries.[7] In general, the

[6] See Acemoglu (2002) on the *price effect* and *market-size effect* on the technological-knowledge bias.

[7] Since the main variables of interest for our analysis are linked to employment and earnings, we use data acquired in the International Labor Organization database (ILOSTAT) at `https://www.ilo.org` and evaluated based on the International Standard Classification of Education (ISCED) and the International Standard Classification of

theoretical results obtained confirm the data in the Figures 1-8 that: (i) the relative increase in the number of unskilled workers positively affects the skill premium, as proposed by the DTC literature; (ii) inter-country wage inequality depends positively on relocation and negatively on automation; (iii) wage polarization increases with robotization – in line with, e.g., Acemoglu and Restrepo (2017), Graetz and Michaels (2018), and Lankisch et al. (2019) – and relocation, and decreases with the rise of skilled and unskilled domestic workers; (iv) economic growth is also boosted by robotization – e.g., Zeira (1998), Acemoglu and Restrepo (2017), and Graetz and Michaels (2018) – and relocations – in line with the empirical studies performed by, e.g., Li and Liu (2005), Baharumshah and Almasaied (2009), Wang (2007), Kramer (2010), Cuadros and Alguacil (2014), and Su and Liu (2016). Furthermore, in terms of competitiveness, the race between robotization and relocation is won by robotization if the level of domestic medium-skilled workers exceeds the number existing abroad – that is, if the level of robotization exceeds the level of relocations – and if the absolute advantage of domestic medium-skilled workers outweighs the absolute advantage of the same type of workers abroad.

After this brief introduction, the next Section specifies the current status of the literature review performed so far, and in Section 3 the descriptive data is presented, highlighting the relationships between wages and growth with relocations and automation. In Section 4 the methodology is detailed; here, we start by modeling the preferences (demand side of the model) in Subsection 4.1, the productive side of the model (Subsections 4.2-4.5). In Section 5 is performed the model general equilibrium and depicted the main theoretical results. At the end, in Section 6, the main conclusions are presented.

Occupations (ISCO). Relocations are evaluated by the offshoring rate, which is measured as imported intermediate goods from a developing country by the developed country divided by imported and exported intermediate goods in the developed country (available from the World Bank database, World Integrated Trade Solution, at `https://wits.worldbank.org/`. Following Graetz and Michaels (2018), we measure the robotic density by the stock of operational robots, and we use data obtained from the International Federation of Robotics.. Finally, as far as the economic growth rate is concerned, the data was obtained from the World Bank database at `https://data.worldbank.org`.

2 Revising the related literature

This is how the productive world can be described nowadays:

> John, after sleeping on a cotton pillow (made in Egypt), started the day very early, woken up by the alarm clock (made in Japan) at 7 am. After a shower with soap (made in France) and while the coffee (imported from Colombia) was brewing in the machine (made in Chech Republic), he shaved with the electric shaver (made in China). He put on a shirt (made in Sri Lanka), designer jeans (made in Singapore) and a pocket watch (made in Switzerland). After preparing wheat toast (made in USA) in his toaster (made in Germany) and while drinking coffee in a cup (made in Spain), he picked up the calculating machine (made in Korea) to see how much he could spend that day and checked the Internet on his computer (made in Thailand) to see the weather forecast. After listening to the news on the radio (made in India), he still drank some orange juice (produced in Israel), got into his Saab car (made in Sweden) and continued on to his job in his home country, Italy, where he is an engineer in a highly robotized firm producing automobile components that were once sourced in Turkey.

Considering the current state of the productive world and the dissertation objectives, in this session we revisit the related literature and we follow the following structure. In subsection 2.1 we discuss the relationship between "relocations and wages", in subsection 2.2 we analyze the literature on "relocations and economic growth", in subsection 2.3 the literature on "automation, wages, and growth" is presented and, finally, in subsection 2.4 the closer literature is detailed.

2.1 Relocations and wages

Our dissertation is related to a growing literature on (inter-country) relocations materialized by worldwide firms from developed to developing countries. They are one means by which developing countries penetrate into the production of some goods, improving the respective competitiveness and contributing to the called *efficiency effect*. Indeed, through relocations, developing countries cover domestic shortfalls (poor and inadequate economic infrastructure), gain access to technology, managerial know-how and marketing networks (e.g., Smeets 2008). Moreover, technology transfer from worldwide firms reduces the X-inefficiency and improves productivity of the domestic firms (e.g., Gorg and Greenway 2004; Smeets 2008). They also improve allocative efficiency of resources (e.g., Caves 1974), leads to inefficient domestic firms leaving the market (e.g., Aitken and Harrison 1999), and the connections with the domestic firms enhance productivity of these ones (e.g., Javorcik 2004). They introduce competition and competitive pressure that enhances competitiveness (UNCTAD 1999).

In particular, evidence shows that global firms – the ones that, directly or indirectly, operate in several countries, involving many production and service tasks that were previously produced domestically now being sourced from abroad – relocate more to improve efficiency than to expand market size or access to natural resources in order to improve their competitiveness (e.g. Sashidharan and Ramanathan 2007). For this reason, the literature shows that global firms have become more productive and, in turn, more competitive (e.g. Bhattacharya et al. 2008; Gorg and Strobl 2000; Chuang and Lin 1999).

Many theoretical approaches have been proposed to analyze the impact of globalization, including the action of global firms, on the skill premium. Recent theoretical contributions that have studied the effect of relocations on wages include Antras et al. (2006), Burstein and Vogel (2012), Costinot et al. (2013), and Goel (2017), among others. Most recent studies analyze the effect without attending to the impact that relocation has on the direction of technological knowledge in favor of certain labor types (e.g. Glass and Saggi 2001; Naghavi and Ottaviano 2009; Dinopoulos and Segerstrom 2010; Rodriguez-Clare 2010; Branstetter and Saggi 2011). In turn, seminal DTC models address the direction of technological knowledge, but ignore relocations (e.g. Acemoglu 2002, and 2007; Acemoglu and Zilibotti 2001; Gancia and Zilibotti 2009), while some other DTC models only link international trade to the technological-knowledge bias that thereby affects the demand for skills (e.g. Acemoglu 2003; Thoenig and Verdier 2003; Epifani and Gancia 2009; Gancia et al. 2013).

Table 3, in Appendix A.1, synthesizes the studies that somehow study the impact of the developed-developing relationships, including global firms, on the skill premium. Summing up, globalization, including the action of global corporations, increases the skill premium by widening the wage gap between groups of workers; indeed, in the 27 theoretical studies reviewed, 12 argue that global firms increase wage inequality, 3 consider that the action of global firms causes a decrease in wage inequality, 9 assume that there is a bi-causality relationship between both variables, and 6 state that there is no significant effect between the variables.

2.2 Relocations and economic growth

Regarding the contribution of global firms to economic growth, endogenous growth models driven by technological progress, horizontal or vertical, tend to provide the basis for empirical applications, considering that the diffusion of technological knowledge is the central driver of positive externalities emerging from foreign presence in developing countries (e.g. OECD 2002).

Global firms, highly technologically advanced and world leaders in investments in R&D activities (e.g. Borensztein et al. 1998; OECD 2002; Wei and Liu 2006), are key to growth because they allow: (i) disseminate technological knowledge between countries affecting production on a global scale according to comparative advantages (e.g. Borensztein et al. 1998), (ii) train and assist in the production process at all levels improving production efficiency on a global scale (e.g. OECD 2002; Javorcik 2004), (iii) "produce" human capital by providing training to the local workforce of the on-the-job-training type (e.g. Borensztein et al. 1998), (iv) raise local competitiveness (e.g., Buckley et al. 2002), (v) restructuring, readapting and strategically upgrading production equipment and processes, as well as introducing new goods and processes (e.g., Un 2016).

Thus, the dominant empirical applications support that global firms have positive direct and indirect impact on productivity and economic growth (e.g., Li and Liu 2005 for 84 countries; Baharumshah and Almasaied 2009 for Malaysia; Wang 2007 for 40 countries; Kramer 2010 for 47 countries; Cuadros and Alguacil 2014 for 28 countries; Su and Liu 2016 for China); however, there are also minority studies that found a negative direct relationship and a positive indirect relationship (e.g., Borenztein et al. 1998 for 69 countries; and Glas et al. 2016 for Brazil, Russia, India and China), considering that developing countries enhance the presence of global firms when they have the ability to internalise/absorb foreign technological knowledge (e.g., Buckley et al. 2002; Hermes and Lensink 2003; Wei and Liu 2006; Kummer-Noormamode 2015). Moreover, there are also studies that, namely for the case of developing countries, emphasize the heterogeneous effect in general (e.g., Alguacil et al. 2011), others that correlate the effect with the supply of skilled labor (e.g., Borensztein et al. 1998; Xu 2000; Li and Liu 2005), and still others that do not find a relationship between global firms and economic growth (e.g. Carkovic and Levine, 2002 for 71 countries).

2.3 Automation, wages, and growth

There is a consensus that, in recent decades, machines, computers and robots have been transforming the labor market (Blanas et al. 2019), particularly in more developed countries where, effectively, the adoption of industrial robots has been increasing significantly (Graetz and Michaels 2018). According to the International Federation of Robots, the number of robots performing activities previously performed by humans has already reached values between 1.5 and 1.75 million and this amount is expected to increase to between 4 and 6 million by 2025 (Acemoglu and Restrepo 2017).

The substitution and/or complementarity between humans and machines has been a matter of concern (DeCanio 2016) in relation to future jobs and wages (Acemoglu and Restrepo 2017).[8] According to the World Bank, around 57% of current jobs in the OECD are at risk of being replaced by robots (World Bank Group 2016). This replacement is mainly connected with routine occupations of unskilled and especially medium-skilled workers (Blanas et al. 2019) as these tasks require basically methodical repetitions (Autor et al. 2003; Michaels et al. 2014; Acemoglu and Restrepo 2017, 2018a,b,c). However, as some studies illustrate, automation can also be complementary to skilled workers as they are specialized in complex tasks, although in relative terms it is less frequent (Acemoglu and Restrepo 2018b). Due to the replacement of unskilled and especially medium-skilled workers, the increased use of robots in production influences the gap

[8] In this process, considering that substitutability dominates, the macroeconomic implications of taxing robots have also already been raised to soften some consequences (Thuemmel 2018), but there are no studies on the specific implications. This important issue will not, however, be addressed in this dissertation as it is not the focus of the analysis.

between the earnings of skilled, medium-skilled and unskilled workers (Lankisch et al. 2019).[9]

Regarding the impact of automation on employment and wages, Autor et al. (2003) and Autor and Dorn (2013) suggest that, as expected, unskilled and especially medium-skilled workers are the most affected. As Hémous and Osborne (2014), for example, note the replacement of unskilled and medium-skilled workers by machines, increases the skill premium and decreases unskilled and medium-skilled wages, and is expected to penalize more the medium-skilled workers and hence the emergence of wage polarization. Acemoglu and Restrepo (2017) and Graetz and Michaels (2018) show that indeed the areas most exposed to robots experienced adverse effects (mainly unskilled and medium-skilled employment and wages). In fact, Acemoglu and Restrepo (2017) estimate that one more robot per thousand employees decreases employment and wages by 0.18 to 0.34 percentage points (pp) and 0.25 to 0.5%, respectively.

Automation also stimulates labor productivity, leading to an increase in economic growth (Graetz and Michaels 2018). Zeira (1998) revealed that technology adoption, as happens whenever automation is introduced or intensified, increases productivity gaps between countries, which leads to accentuated differences in real GDP. Acemoglu and Restrepo (2017) estimated that an increase of one robot per thousand employees induces a 0.13% increase in GDP. Graetz and Michaels (2018) found that the observed automation increases the annual real GDP growth and labor productivity by about 0.37 and 0.36 pp, respectively.

2.4 Closer literature

There is literature that comes closer to our proposal and that is given account of here.

As stated, we intend to explore the effects of relocation and automation of tasks on the wage inequality between skilled, medium-skilled and unskilled workers – considering the division between routine and non-routine tasks, previously addressed by Autor et al. (2003). Also, we plan to evaluate the behavior of the economic growth rate. In order to assess these impacts, we

[9] This could naturally be mitigated by combining the taxation of robots with equity-promoting redistributive policies. Guerreiro et al. (2017) denote that taxes on robots are optimal in the short run: the use of robots decreases the non-routine wage premium and taxes on robots permits the redistribution of earnings to routine workers. Recently, Acemoglu et al. (2020) showed that reducing or combining other taxes with automation taxes can increase employment; however, as already mentioned, this will not be addressed in this dissertation.

will develop an extended dynamic DTC growth model, based on Acemoglu (2002) and Afonso (2012).

Most existing related studies on relocations depreciate the technological-knowledge progress and bias. There are, however, notable exceptions. Glass and Saggi (2001), Rodriguez-Clare (2010), and Goel (2017) argue theoretically and Boler et al. (2015) reveal for Norway that technological-knowledge progress increases with relocations; in turn, Naghavi and Ottaviano (2008) observe that relocations reduce R&D activities. We contribute for this literature also by analyzing the implications of relocations on the competitiveness of the developed country and on the endogenous rate of the technological-knowledge progress and bias. This bias, affected by both the *market-size effect* and the *price effect*, influences the skill premium and wage polarization: increases the skill premium when technological knowledge is biased towards skilled workers and wage polarization when technological-knowledge is biased towards the non-routine sector.

Indeed, there is a spirited debate on the accurate contribution of the DTC explanation and the international-trade proposal to the observed skill premium in developed and developing countries. The broad consensus is that a faster increase in the demand for skills than in their supply has been at the origin of the observed skill premium. According to the DTC explanation, prominently explored by, e.g., Katz and Murphy (1992), Berman et al. (1998), Autor et al. (2003), Autor et al. (2008), and Acemoglu and Autor (2011), more DTC, at given factor supplies, tend to increase the skill premium. The trade explanation, adopted, for example, by Wood (1998), depends mainly on the application of the Stolper-Samuelson theorem: imports (exports) of goods produced by unskilled (skilled) labor reduce (increases) unskilled (skilled) wages. As relocations (hence trade) affect the DTC, our developed theoretical model is also a contribution for the DTC-Trade debate on the puzzle “rise in the skill premium and rise in the proportion of skilled labor”, allowing to connect the two explanations. Such as the DTC literature, it will be considered that the technological-knowledge bias has a decisive impact on the skill premium. However, while the DTC literature generates endogenous DTC from the supply side – the technological-knowledge bias responds to changes in labor supply (e.g., Acemoglu 2002) –, our model instead generates endogenous DTC from the demand side, and from considering three types of workers – unskilled, medium-skilled, and skilled workers.

Papers most closely related to ours are Chu et al. (2014), Acemoglu et al. (2015), and Goel (2017). The former paper studies the effect of changes in labor supply in China on the technological-knowledge bias in a model with relocations, and results are closed to those observed in models with DTC under international IPRs protection. The remaining two papers – Acemoglu et al. (2015) and Goel (2017) –, such as in this dissertation, analyze neither the competitiveness nor the imitation process by developing countries.

An exemplification of the effects of relocations can be done considering the iPod Apple product for which the vast majority of unskilled/medium-skilled assembly and production jobs are relocated to developing countries (Linden et al. 2011). Absent these relocations, it may not have been profitable for Apple to introduce the commercialized iPod due to higher US unskilled/medium-skilled labor costs, which would have likely diminished the skill engineering demand and design jobs in the US, and the *price effect* on the technological-knowledge bias. Moreover, the iPod would be different, being formulated to reduce the dependence on the expensive US unskilled/medium-skilled labor and, thus, owing to the *market-size effect*, R&D would be directed toward unskilled/medium-skilled-biased technological change.

We also extend the models based on tasks originally developed by Acemoglu and Autor (2011) and Acemoglu and Restrepo (2017), considering a continuous set of routine and non-routine tasks. Routine tasks are performed by medium-skilled labor and a continuous set of machines/robots whose quality can be improved with successful R&D activities. Our dissertation is also related to the empirical literature on the effects of automation and robotics on the labor market. Autor et al. (2003) documented the decline in employment in routine tasks due to the computerization of such tasks. Michaels et al. (2014) show that the replacement of routine tasks with robotic labor caused a decline in employment opportunities for unskilled and medium-skilled workers – e.g., also Acemoglu and Autor (2011), Gregory et al. (2018). Acemoglu and Restrepo (2017), documented that, from 1990 to 2007, areas of the US with industries most exposed to the use of industrial robots experienced a significant decline in employment and real wages. Using a panel of industries in 17 countries between 1993 and 2007, Graetz and Michaels (2018) show that robotic work brought about faster productivity and wage growth, but also negatively affected unskilled and medium-skilled employment.

3 Descriptive data

The context described above translates into a diverse pattern of empirical relationships between key macroeconomic variables, such as wages – skill premium, inter-country wage inequality, and wage polarization –, domestic labor levels and relocation levels. Here, we present descriptive data for an illustration for the US, as an innovator/advanced country, and a number of follower/developing countries – Argentina, Brazil and Turkey – for which it was possible to obtain data. The data pertain to the period 2005-2019. The offshoring rate (our relocation measure) is measured as imported intermediate goods from a developing country by the developed country divided by imported and exported intermediate goods in the developed country (available from the World Bank database, World Integrated Trade Solution, at `https://wits.worldbank.org/`), the skill premium is measured as the ratio of mean nominal monthly earnings in US dollars of employees with tertiary education to the earnings of employees with primary level of education (available from the ILO database at `https://www.ilo.org`), and the inter-country wage inequality is the ratio of mean nominal monthly earnings in US dollars of employees in a given country versus the earnings in the US (also available from the ILO database). According to Graetz and Michaels (2018), the robotic density will be measured by the stock of operational robots and the data will be retrieved from the International Federation of Robotics. Finally, as far as the economic growth rate is concerned, the data used was obtained from the World Bank database at `https://data.worldbank.org`.

Figure 1 depicts the relationship between the US skill premium and the US unskilled-labor over the US skilled labor; this data suggests a positive correlation.

Figure 2 illustrates the relationship between the inter-country wage inequality in favor of the US (against Argentina, Brazil and Turkey) and, respectively, the US relocations (with respect to a given follower/developing country) and the US Operation stock of robots; for all cases in our sample, the sign of the first relationships is positive and the sign of the second relationships is negative.

Figures 3, 4, 5, 6, and 7 show the relationship between the US wage polarization and, respectively, the US Operation stock of robots, the US middle-skilled labor, the US relocations (with respect

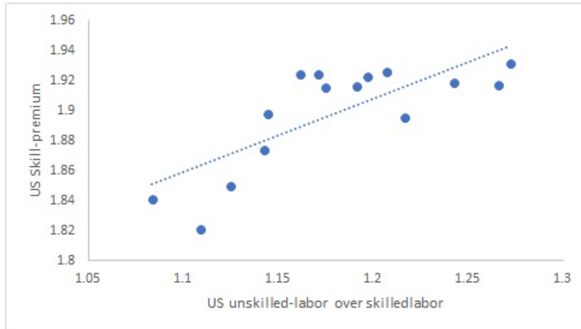

Figure 1: The US skill premium and the US unskilled-labor over the US skilled labor. Data period: 2005-2019. See data sources in the text.

to the considered developing country), the US unskilled labor and the US skilled labor. The relationship in Figures 3 and 4 are, as expected, both positive since there is a positive relationship between robotization and the employment of medium-skilled workers.[10] The relationships in Figure 5 are all positive.[11] In turn, the relationship in Figure 6 is negative. Finally, the relationship in Figure 7 is not, in general, a well-defined pattern regarding the sign of the correlations.

Finally, Figure 8 illustrates the positive relationships between the US economic growth and, respectively, robotization (or automation) and relocations.

Overall, these descriptive data suggest the existence of underlying compensation mechanisms in economies, the net effect of which seems to depend on the specific characteristics of the non-routine and routine sectors and, within the latter, the characteristics of automation and relocations.

[10]Indeed, the supply of medium-skilled workers imposes the size of the tasks that can be robotized, although, of course, the intensification of robotization makes this supply of workers less productive.

[11]A similar result emerges from the relationship between the US wage polarization and the labor levels in developing country since, as in the previous case, the supply of workers in developing countries available to perform relocation productions imposes the size of the tasks that can be relocated, although, of course, the intensification of relocations makes this supply of workers less productive. However, to avoid repeating similar relationships, a new Figure with the respective six relationships is not presented.

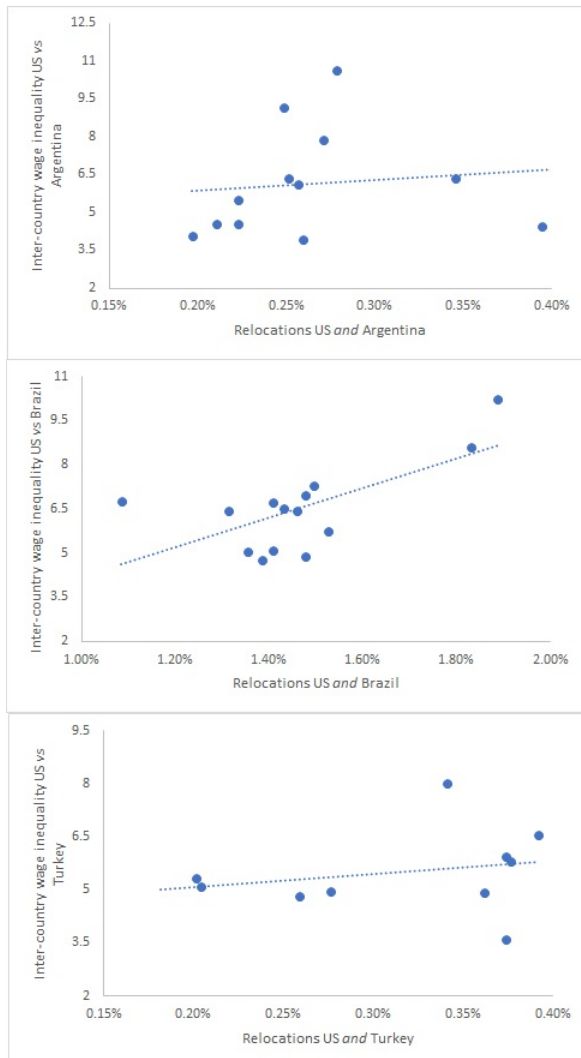

Figure 2: Inter-country wage inequality in favor of the US (against Argentina, Brazil and Turkey) and, respectively, the US relocations (with respect to a given developing country). Data period: 2005-2019. See data sources in the text.

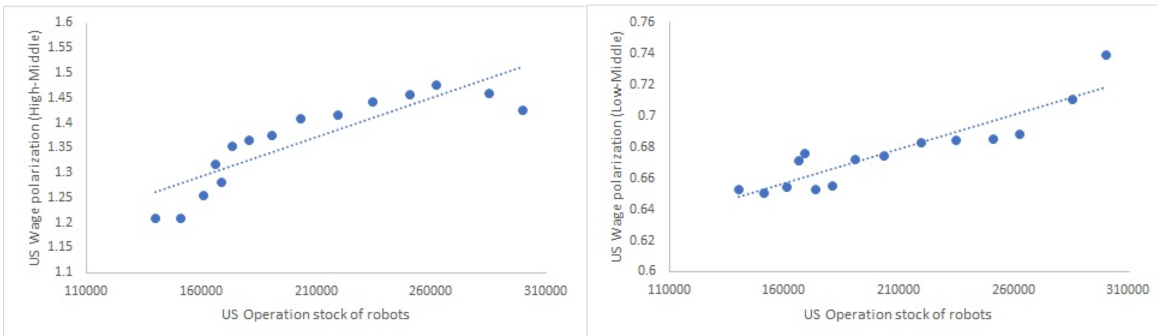

Figure 3: The US wage polarization and the US Operation stock of robots. Data period: 2005-2019. See data sources in the text.

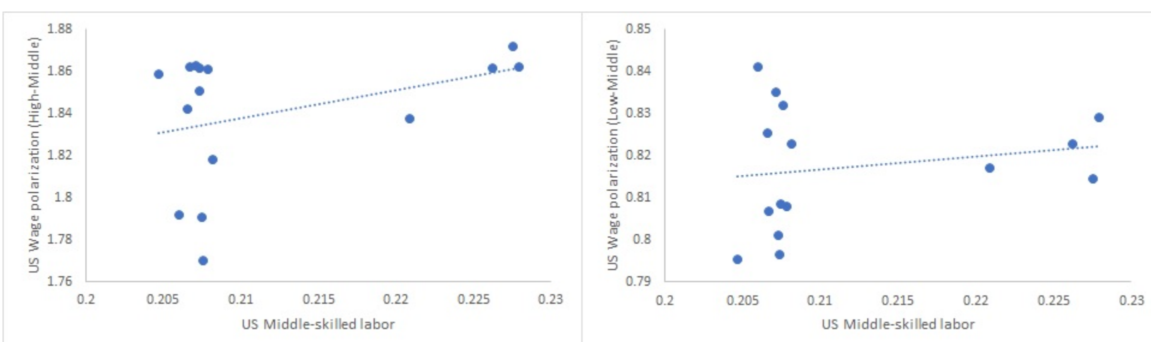

Figure 4: The US wage polarization and the US middle-skilled labor. Data period: 2005-2019. See data sources in the text

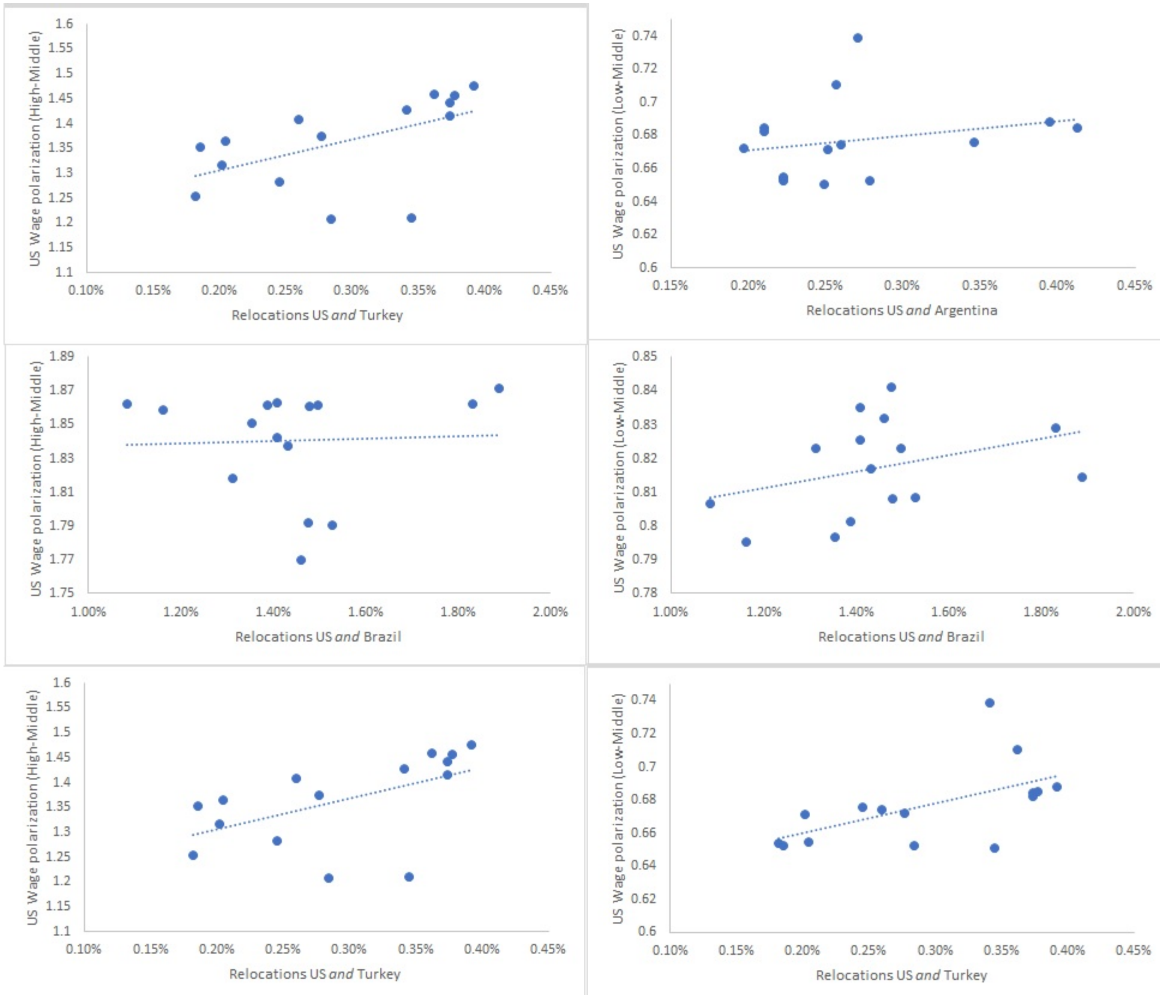

Figure 5: The US wage polarization and the US relocations (with respect to the developing country). Data period: 2005-2019. See data sources in the text.

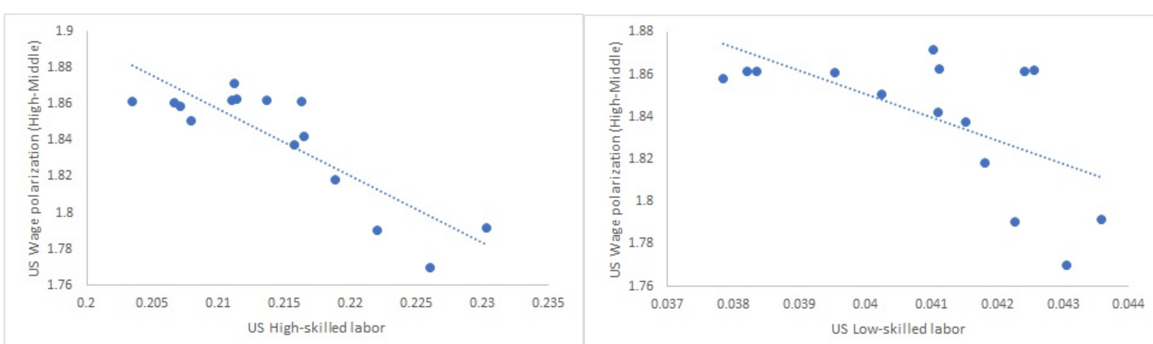

Figure 6: The US wage polarization and the US unskilled. Data period: 2005-2019. See data sources in the text.

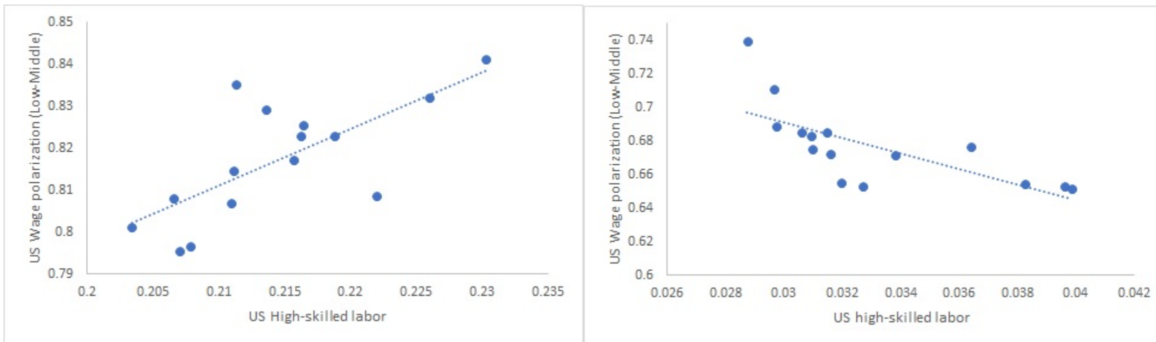

Figure 7: The US wage polarization and the US skilled. Data period: 2005-2019. See data sources in the text.

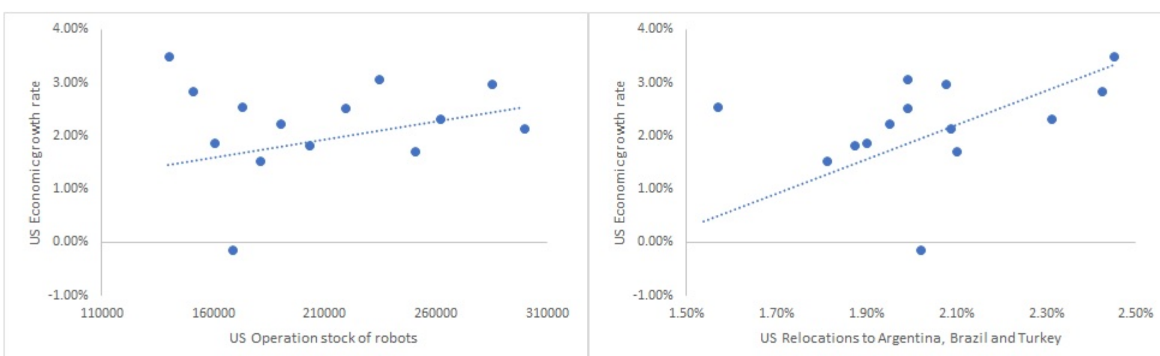

Figure 8: US Economic growth rate and, respectively, the US Operation stock of robots and the US Relocations to Argentina, Brazil and Turkey. Data period: 2005-2019. See data sources in the text.

4 The model – relocations vs automation

Our findings are thus in line with two perspectives – the perspective proposed by the DTC literature and the perspective proposed by the "wage polarization" literature, which, in turn, is mainly based on tasks automation. We connect the two perspectives by considering routine tasks that can be relocated or automated. In line with the DTC literature, we hope to obtain a positive relationship between the relative supply of skilled workers and the skill premium. Moreover, we also hope to have results consistent with empirical evidence for the existence of wage polarization.

In particular, we propose a dynamic general equilibrium endogenous growth model to better understand the mechanisms. It will be considered that infinite lifetime households supply labor inelastically, maximize the consumption utility of the aggregate final good (the numeraire), and invest in firms equity. In the productive side of the model, the aggregate final good will be a composite of the product of two sectors, the non-routine and the routine, each with many competitive firms producing a continuum of tasks. As already stated, the country-specific non-routine sector includes manual tasks that require physical dexterity and situational adaptability performed by unskilled workers, and also cognitive/abstract tasks that require mastery of high-level mental abilities performed by skilled workers. In the routine sector, some tasks will be performed by medium-skilled workers and they can be relocated or automated. The continuum of tasks in each sector also requires a continuum of specific non-durable quality-adjusted machines. As is standard in the DTC growth model, each machine consists of a continuum of industries and is characterized by monopolistic competition. Each monopolist uses a design, sold by the R&D sector and protected by a patent, and a numeraire to produce at a price that maximizes profits. In the R&D sector, in turn, each potential competitor devotes a numeraire to inventing successful vertical designs to be supplied to a new monopolist, so R&D by increasing the quality of machines improves technological knowledge.

In the context of the model, relocations will impose an improvement in the absolute advantage of labor in the non-routine sector over the routine sector (medium-skilled labor), which increases the technological-knowledge advantage of the non-routine sector over the routine sector; i.e., this leads to a technological-knowledge bias in favor of the non-routine sector. Although all types of workers benefit from relocations, the relative wages of workers in the non-routine sector (unskilled and skilled labor) relative to workers in the routine sector (medium-skilled labor) increase, thus generating a wage polarization – wages reflect skills but, due to relocations, medium-skilled workers lose out to skilled and unskilled workers. Moreover, depending on how automation impacts, the polarization may be concomitant with the increase in the skill premium.

In particular, this Section describes the economic set-up of the closed Economy in which infinitely-lived households inelastically supply labor, maximize the utility of consumption from the aggregate final good, and invest in a firm's equity. The inputs of the aggregate numeraire good, Y, are two final goods, Non-routine (Y_N produced in the N-sector) and Rotine (Y_R produced in the R-sector), each one composed by many competitive firms that produce a continuum of tasks; i.e., there are two sectors, $s = N$ and $s = R$. In $s = N$ there are a positive fixed level of skilled-labor type, L_N^+, and unskilled-labor type, L_N^-. In $s = R$, the production of the tasks can be carried out domestically by medium-skilled workers if automated, L_R^A, or by foreign workers if relocated, L_R^B. The continuum of tasks of each sector, $s = N, R$, uses, in addition to the specific labor, a continuum of specific non-durable quality-adjusted machines,[12] and is characterized by monopolistic competition: the monopolist in industry j uses a design, sold by the R&D sector and protected by a patent, and numeraire to produce at a price that maximizes profits. In the R&D sector, each potential entrant devotes numeraire to inventing successful vertical designs to be supplied a new monopolist machine firm/industry; i.e., R&D allows increasing (not the number, but) the quality of machines and, thus, the technological knowledge. Therefore, some endogenous technological knowledge complements skilled labor, unskilled labor, medium-skilled labor, or foreign labor.

4.1 Preferences

Infinitely-lived households obtain utility from the consumption, C, of the unique aggregate final good, whose price we normalize to 1, and collect income from investments in financial assets (equity) and from labor. They supply labor to the N-sector or to the R-sector. Preferences are

[12] In this model we assume that only vertical innovation takes place. As a result, the number of machines is exogenous, which does not affect the main results.

identical across workers L_N^i, L_N^h, L_R^i and L_R^h. Thus, there is a representative household with preferences at time $t=0$ given by $U_C = \int_0^\infty \left(\frac{C(t)^{1-\theta}-1}{1-\theta}\right) e^{-\rho t} dt$, where $\rho > 0$ is the subjective discount rate, ensuring that U_C is bounded away from infinity if C were constant over time, and $\theta > 0$ is the inverse of the inter-temporal elasticity of substitution, subject to the flow budget constraint

$$\dot{a}(t) = r(t) \cdot a(t) + \sum_{s=N,R} \left(w_s^h \cdot L_s^h + w_s^i \cdot L_s^i \right) - C(t), \tag{1}$$

where $a(t) = \sum_{s=N,R} \left[a_s^h(t) + a_s^i(t) \right]$ denotes household's real financial assets/wealth holdings (composed of equity of machine producers, considering the profits seized by the top-quality producers), r is the real interest rate, and w_s^h and w_s^i are the wage for labor type h and i employed in sector $s = \{N, R\}$. The initial level of wealth $a(0)$ is given and the non-Ponzi games condition $\lim_{t\to\infty} e^{-\int_0^t r(s)ds} a(t) \geq 0$ is imposed. The representative household chooses the path of aggregate consumption $[C(t)]_{t\geq 0}$ to maximize the discounted lifetime utility, resulting in the following optimal consumption path Euler equation,

$$\frac{\dot{C}(t)}{C(t)} = g = \frac{1}{\theta} \cdot [r(t) - \rho] \,. \tag{2}$$

Moreover, the transversality condition is also standard: $\underset{t\to\infty}{lim} e^{-\rho t} \cdot C(t)^{-\theta} \cdot a(t) = 0$.

4.2 Technology, output and prices

Aggregate economy. In the Economy, aggregate output Y is produced with a CES aggregate production function of Non-routine and Routine competitively produced final goods:

$$Y(t) \;\; = \left[\textstyle\sum_{s=N,R} \chi_s \cdot Y_s(t)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad , \;\; \varepsilon \in (0, +\infty) \,, \tag{3}$$

where: Y_N and Y_R are the total outputs of the N- and the R-sectors, respectively; χ_N and χ_R, with $\sum_{s=N,R} \chi_s = 1$, are the distribution parameters, measuring the relative importance of the sectors; $\varepsilon \geq 0$ is the elasticity of substitution between the two sectors, wherein $\varepsilon > 1$

$(\varepsilon < 1)$ means that they are gross substitutes (complements) in the production of Y.[13] The the assumption of competitive final-good firms, implies the following maximization problem: $\max_{Y_s} \Pi_Y = P_Y \cdot Y - \sum_{s=N,R} P_s \cdot Y_s$. From the first-order conditions emerge the inverse demand for Y_s, $s = \{N, R\}$:[14]

$$\frac{P_s}{P_Y} = \chi_s \left(\frac{Y}{Y_s}\right)^{\frac{1}{\varepsilon}} \Leftrightarrow Y_s = \left(\frac{P_s}{P_Y \cdot \chi_s}\right)^{-\varepsilon} Y. \tag{4}$$

Thus, the we obtain the following expression for relative demand for output from the N-sector:

$$\frac{Y_N}{Y_R} = \left(\frac{\chi_N}{\chi_R}\right)^{\epsilon} \left(\frac{P_N}{P_R}\right)^{-\epsilon}, \tag{5}$$

which depends positively on the relative share in production and negatively on the relative price of output from this sector. Replacing (4) in (3) we have that $P_Y = \left[\sum_{s=N,R} \chi_s^{\varepsilon} \cdot P_s^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$, where P_N and P_R are the prices of the outputs of, respectively, the N- and the R-sectors, and the right-hand side of the expression is the unit production cost. From (4) we also have that $P_s \cdot Y_s = P_Y \cdot Y^{\frac{1}{\varepsilon}} + \chi_s \cdot Y_s^{\frac{\varepsilon-1}{\varepsilon}}$ and summing across sectors results $P_Y \cdot Y = P_N \cdot Y_N + P_R \cdot Y_R$.

Sectors of the economy. The output Y_s of each sector $s = \{N, R\}$ is produced in perfect competition by the following production function with constant returns to scale$Y_s = \exp\left(\int_0^1 \ln Y_{v_s} dv_s\right)$, i.e., Y_s is a continuum of the output produced by tasks Y_{v_s} indexed respectively, by $v_N \in [0, 1]$ and $v_R \in [0, 1]$. We think of the tasks v_N as Non-routine tasks that are in the early stages of automation via artificial intelligence, big data, and a new phase of robotics. Tasks v_B as Routine tasks, many of which have been automated in the past 30 years through the use of information-processing technologies or industrial robots. The producer of Y_s maximizes profits given by $\Pi_s = P_s \cdot Y_s - \int_0^1 P_{v_s} \cdot Y_{v_s} dv_s$, subject to the restriction imposed by the functional form of the production function of Y. Assuming perfect competition, the maximization problem results in the following first order conditions: $\frac{\partial \Pi_s}{\partial Y_{v_s}} = 0 \Rightarrow Y_{v_s} = \frac{P_s \cdot Y_s}{P_{v_s}}$. Therefore, from here $P_{v_s} \cdot Y_{v_s} = P_s \cdot Y_s$ is a constant, which replaced in the profits function and in the production

[13] In particular, cases where $\varepsilon = 0$, $\varepsilon = 1$ and $\varepsilon = +\infty$ we have, respectively, a Leontief, a Cobb-Douglas, and a Linear production function.

[14] Throughout the dissertation we suppress the time argument t whenever this does not cause confusion.

function results, respectively, in $\Pi_s = P_s \cdot Y_s - \int_0^1 P_s \cdot Y_s dv_s = 0$ and also in

$$Y_s = \exp\left(\int_0^1 \ln \frac{P_s \cdot Y_s}{P_{v_s}} dv_s\right) \Leftrightarrow P_s = \exp\left(\int_0^1 \ln P_{v_s} dv_s\right). \tag{6}$$

Tasks in each sector. Task producers in sector $s = N$ must choose to produce them either with unskilled-labor type "h" or with skilled-labor type "i", and task producers in sector $s = R$ must choose to produce them with foreign labor in relocated tasks in developing countries "h" or with domestic medium-skilled labor employed in automated tasks "i", which implies choosing between the following Cobb-Douglas production functions:

$$Y_{v_s}^h(t) = \left[\int_0^J \left(q^{k(j,t)} \cdot x_{v_s}^h(k,j,t)\right)^{1-\alpha} dj\right] \left[(1 - v_s(t)) \cdot l_s^h \cdot L_{v_s}^h\right]^\alpha, \tag{7}$$

$$Y_{v_s}^i(t) = \left[\int_J^1 \left(q^{k(j,t)} \cdot x_{v_s}^i(k,j,t)\right)^{1-\alpha} dj\right] \left[v_s(t) \cdot l_s^i \cdot L_{v_s}^i\right]^\alpha. \tag{8}$$

Each uses two factors: labor of type L^i or L^h (the second term on the right-hand side) and machines (the first term on the right-hand side) with a share in income of α and $1-\alpha$, respectively. Each machine j used in v_s production is quality-adjusted: the constant quality upgrade is $q > 1$ and is constant, k is the top-quality rung at t and $x_{v_s}^h(k,j,t)$ and $x_{v_s}^i(k,j,t)$ represent the units of machines demanded for task v_s if it is produced to be used by L_s^h or by L_s^i, respectively. The labor term includes the quantities employed in the production of v_s, $L_{v_s}^i$ or $L_{v_s}^h$, and two types of corrective factors accounting for productivity differentials such that workers are assigned to tasks according to location and the most efficient firm in production; i.e, we take into account

- the absolute net advantage of labor. For $s = N$, we consider $l_N^i > l_N^h$ since L_N^i is more qualified than L_N^h, implying that L_N^i operates in increasingly abstract/cognitive Non-routine tasks, while L_N^h operates in "purely manual" Non-routine tasks. In $s = R$, the production can be performed by foreign workers if relocated, L_R^h,[15] or by domestic medium-skilled workers if automated, L_R^i, or and the quantities used should be corrected by the term l_R^h and l_R^i due to factors that are specific to automation and relocations; we consider

[15] L_R^h can be seen as a measure of the willingness of developing countries to host relocations; it increases with globalization and decreases with de-globalization.

that $l_R^i > l_R^h$ since, on the one hand, automation improves the labor productivity and, on the other hand, domestic medium-skilled labor has an absolute productivity advantage over foreign labor in developing countries.[16]

- The relative productivity advantage of labor. Following the point of view proposed by, e.g., Acemoglu and Zilibotti (2001) and Afonso (2012), through the terms terms $(1 - v_s)$ and v_s: for $s = N$, L^i is relatively more productive in tasks indexed by larger v_s, and vice-versa.

Hence, it is assumed that in each sector $s = N$ and $s = R$ there is substitutability between tasks that use labor type h and tasks that use labor type i. On the other hand, it is assumed that, regardless of the labor type used by sector s, there is complementarity between labor and a specific set of machines. However, as will become clear later, the level of machinery used by each type of labor in s, depends on the "effective" labor level. To determine the tasks that use labor type "h" and labor type "i" in each sector, firstly we need to solve the respective maximization problems:

$$\max_{x_{v_s}^h(k,j,t), L_{v_s}^h} \Pi_{v_s}^h(t) = P_{v_s}^h(t) \cdot Y_{v_s}^h(t) - \int_0^J p(k,j,t) \cdot x_{v_s}^h(k,j,t) \cdot dj - w_s^h(t) \cdot L_{v_s}^h, \tag{9}$$

$$\max_{x_{v_s}^i(k,j,t), L_{v_s}^i} \Pi_{v_s}^i(t) = P_{v_s}^i(t) \cdot Y_{v_s}^i(t) - \int_J^1 p(k,j,t) \cdot x_{v_s}^i(k,j,t) \cdot dj - w_{L_s^i}(t) \cdot L_{v_s}^i, \tag{10}$$

bearing in mind 7 and 8, where: $P_{v_s}^h(t)$ and $P_{v_s}^i(t)$ are the price of task v_s produced by labor type h and i, respectively, at time t; $p(k,j,t)$ denotes the price paid for the machine j with quality

[16] Indeed labor depends positively on the quality of country's institutions non-international trade related, namely tax laws and government services, that are better in developed countries, and the operationalization of production in developing countries requires higher labor requirements due to coordination, organizational, transportation, and communication costs (e.g., Grossman and Rossi-Hansberg 2008, 2012; Acemoglu et al. 2015) and, for reasons of simplicity, we reflect in these parameters another crucial feature the original firms need to support the cost of an initial outsource agreement, or to pay a one-time set-up cost to offshore production to a partner firm in a developing country or, in case of FDI filial firms, to pay a one-time set-up cost to control and manage domestic firms via cross-border acquisitions of existing firms or to establish a new firm in a developing country.

k, at time t; $w_s^h(t)$ and $w_s^i(t)$ are, as already stated, the price of each unit of labor type h and i, respectively, at time t – these prices are given for the perfectly competitive producers of the tasks. The first order conditions with respect to machines allow us to obtain the following:

$$x_{v_s}^h(k,j,t) \quad = \quad \left[\frac{P_{v_s}^h(t)\cdot(1-\alpha)}{p(k,j,t)}\right]^{\frac{1}{\alpha}} \quad \cdot \quad q^{k(j,t)\frac{1-\alpha}{\alpha}} \quad \cdot \quad (1-v_s(t)) \quad \cdot \quad l_s^h \quad \cdot \quad L_{v_s}^h, \quad (11)$$

$$x_{v_s}^i(k,j,t) = \left[\frac{P_{v_s}^i(t)\cdot(1-\alpha)}{p(k,j,t)}\right]^{\frac{1}{\alpha}} \cdot q^{k(j,t)\frac{1-\alpha}{\alpha}} \cdot v_s(t)\cdot l_s^i \cdot L_{v_s}^i. \qquad (12)$$

Replacing 11 and 12 in the corresponding production functions 7 and 8, we have that:

$$\begin{aligned} Y_{v_s}^h(t) &= \left[\frac{P_{v_s}^h(t)\cdot(1-\alpha)}{p(k,j,t)}\right]^{\frac{1-\alpha}{\alpha}} \cdot Q_s^h(t)\cdot(1-v_s(t))\cdot l_s^h\cdot L_{v_s}^h, & (13)\\ Y_{v_s}^i(t) &= \left[\frac{P_{v_s}^i(t)\cdot(1-\alpha)}{p(k,j,t)}\right]^{\frac{1-\alpha}{\alpha}} \cdot Q_s^i(t)\cdot v_s(t)\cdot l_s^i\cdot L_{v_s}^i & (14)\end{aligned}$$

where $Q_s^h \equiv \int_0^J q^{k(j,t)\frac{1-\alpha}{\alpha}}dj$ and $Q_s^i \equiv \int_J^1 q^{k(j,t)\frac{1-\alpha}{\alpha}}dj$ are measures of the quality level of machines used in sector s to be endogenously determined in Section 3, thereby originating the dynamic effects of the model.

Wages and threshold task in each sector. The first order conditions with respect to labor units allow us to obtain the following:

$$w_s^h(t) = \frac{\alpha\cdot P_{v_s}^h(t)\cdot Y_{v_s}^h(t)}{L_{v_s}^h} = \left[P_{v_s}^h(t)\right]^{\frac{1}{\alpha}} \cdot \left[\frac{1-\alpha}{p(k,j,t)}\right]^{\frac{1-\alpha}{\alpha}} \cdot Q_s^h(t)\cdot(1-v_s(t))\cdot l_s^h, \quad (15)$$

$$w_s^i(t) = \frac{\alpha\cdot P_{v_s}^i(t)\cdot Y_{v_s}^i(t)}{L_{v_s}^i} = \left[P_{v_s}^i(t)\right]^{\frac{1}{\alpha}} \cdot \left[\frac{1-\alpha}{p(k,j,t)}\right]^{\frac{1-\alpha}{\alpha}} \cdot Q_s^i(t)\cdot v_s(t)\cdot l_s^i. \qquad (16)$$

In equilibrium, there is a threshold task that ensures that each type of labor gets the same wage regardless of the task it is used for. To this end, we can define the following price indexes as constants:

$$\left[P_s^h(t)\right]^{\frac{1}{\alpha}} = \left[P_{v_s}^h(t)\right]^{\frac{1}{\alpha}} \cdot (1 - v_s(t)) \ and \ \left[P_s^i(t)\right]^{\frac{1}{\alpha}} = \left[P_{v_s}^i(t)\right]^{\frac{1}{\alpha}} \cdot v_s(t). \tag{17}$$

As shown in appendix A.2, in sector $s = \{N, R\}$ (i) tasks with a very low (high) v_s have a lower price if produced by L_s^h (L_s^i) rather than L_s^i (L_s^h), such that perfectly competitive producers use L_s^h (L_s^i) to avoid being driven out of the market, (ii) there is a threshold task v_s where prices are equal and is given by the following expression:

$$\overline{v}_s = \left[1 + \left(\frac{Q_s^i}{Q_s^h}\frac{l_s^i}{l_s^h}\frac{L_s^i}{L_s^h}\right)^{\frac{1}{2}}\right]^{-1} \tag{18}$$

Bearing in mind the labor levels and the net absolute productivity advantage of labor L_s^i over labor L_s^h, $l_s^i > l_s^h$, results in a given $\overline{v}_s$ and thus in a given number of tasks produced by each type of labor in sector s. Hence, an increase in $\overline{v}_s$ means a larger space for production with L_s^i, thus evaluating its "comparative advantage" in s. In particular, an increase in l_s^i increases the labor's L_s^i "comparative advantage".

Proposition 1. *The threshold tasks $\overline{v}_R$ and $\overline{v}_N$ are small, implying that the number of: (i) automated routine tasks produced with medium-skilled labor is large when both the relative medium-skilled-labor supply and the respective relative absolute advantage are high in face of the foreign-labor supply and the respective absolute advantage; (ii) non-routine tasks produced with skilled labor is large when both the relative skilled-labor supply and the respective relative absolute advantage are high in face of the unskilled-labor supply and the respective absolute advantage – see 18.*

In particular, whenever labor abroad becomes more productive, it boosts the number of tasks relocated and, therefore, promotes globalization. Automation, on the other hand, by improving the productivity of medium-skilled domestic labor, l_R^i, boosts the production of routine tasks at home, thus penalizing the competitiveness of the globalization.

4.3 Machines sector

In the machines sector, the production of the top quality k of each j needs an initial R&D cost to achieve the new prototype/design. This initial cost can only be recovered if, with the production of the new quality of the machine, profits are made over a certain period of time in the future. This is assured by a system of Intellectual Property Rights that protect the leader firm's monopoly, while at the same time, this technological knowledge is accessible, practically free of charge, from other firms – this is why it is said that the technological-knowledge progress is made on the shoulders of a giant. Hence, each firm that holds the patent for the top quality k of j at t supplies all respective tasks, v_s, in sector $s = \{N, R\}$. If we consider that each unit of machine j requires one unit of final output Y, since its price is 1 to 1 and the producer of j gets profits $\pi_s(k, j, t) = [p(k, j, t) - 1] \cdot x_s(k, j, t)$, where $x_s(k, j, t) = \int_0^{\overline{v}_s} x^h_{v_s}(k, j, t) \cdot dv_s + \int_{\overline{v}_s}^1 x^i_{v_s}(k, j, t) \cdot dv_s$ is the demand for machine j from all the producers of tasks v_s that use such input, regardless of the labor type used in tasks.

Assuming that the monopolist charges the same price, $p(k, j, t)$, for all these firms, we can find the optimal price by replacing $x_s(k, j, t)$ by the demand of the producer of a single task v_s , i.e., either by $x^h_{v_s}(k, j, t)$ or by $x^i_{v_s}(k, j, t)$ and then maximizing with respect to $p(k, j, t)$. This can be seen by $\pi_s(k, j, t) = \int_0^1 \pi_{v_s}(k, j, t) \cdot dv_s = \int_0^{\overline{v}_s} \pi^h_{v_s}(k, j, t) \cdot dv_s + \int_{\overline{v}_s}^1 \pi^i_{v_s}(k, j, t) \cdot dv_s$, where $\pi^h_{v_s}(k, j, t)$ and $\pi^i_{v_s}(k, j, t)$ denote the profits of the producer of j for selling this machine to the producer of task v_s. Therefore, we can find $p(k, j, t)$ by solving the following maximization problems $\max_{p(k,j,t)} [p(k, j, t) - 1] \cdot x^h_{v_s}(k, j, t)$ and $\max_{p(k,j,t)} [p(k, j, t) - 1] \cdot x^i_{v_s}(k, j, t)$, where $x^h_{v_s}(k, j, t)$ and $x^i_{v_s}(k, j, t)$ can be done by (11) or (12). From the first order condition $\frac{\partial \pi_s(k,j,t)}{\partial p(k,j,t)}$, we have that $p(k, j, t) \equiv p = \frac{1}{1-\alpha} = q$, assuming that the limit pricing strategy is binding.[17]

Taking also into account $p = q$, (17), (11), and (12), the demand for the machine j used in sector s together with L^h_s is,

[17]In this setup we assume that only the top quality rung of each machine input is used in the production. If we generalize and consider that the machine input j used by the producer of task v_s is $\widetilde{x}^{L^i_s}_{v_s}(k, j, t) = \sum_0^{k(j,t)} q^{k(j,t)} \cdot x^{L^i_s}_{v_s}(k, j, t)$, we have that a machine of quality $k + 1$ corresponds to q machines of quality k. This implies that the price of a machine of quality k, $p(k, j, t)$, can be at most $\frac{p(k+1,j,t)}{q}$. Therefore, if the producer of the machine with the highest quality adopts a limit pricing strategy and sets the price to $q - \epsilon$, where ϵ is an infinitesimal, than none of the inferior qualities would be able to survive since their profits would be negative. Since the monopoly optimal price is $p(k, j, t) \equiv p = \frac{1}{1-\alpha}$, assuming that the limit pricing strategy is binding, implies that $p = q$.

$$x_s^h(t) = \int_0^{\overline{v}_s} x_{v_s}^h(k,j,t) \cdot dv_s = \left[\frac{P_s^h(t)\cdot(1-\alpha)}{q}\right]^{\frac{1}{\alpha}} \cdot Q_s^h(t) \cdot l_s^h \cdot L_s^h, \tag{19}$$

and together with L_s^i is,

$$x_s^i(t) = \int_{\overline{v}_s}^1 x_{v_s}^i(k,j,t) \cdot dv_s = \left[\frac{P_s^i(t)\cdot(1-\alpha)}{q}\right]^{\frac{1}{\alpha}} \cdot Q_s^i(t) \cdot l_s^i \cdot L_s^i. \tag{20}$$

Therefore, total demand for machine j used in sector s is $X_s(j) = x_s^h(k,j,t) + x_s^i(k,j,t)$, and the profits for the machines used in sector $s = \{N, R\}$ by labor type h and i are $\pi_s^h(t) = (q-1) \cdot x_s^h(t)$ and $\pi_s^i(k,j,t) = (q-1) \cdot x_s^i(t)$, respectively.

4.4 Allocation of resources

Once determined the threshold task for each sector s and the price of the machines, we can now determine, for a given factor/input levels: (i) the price indices of each sector, P_N and P_R; (ii) the output performed by each type of labor in each sector, Y_s^h and Y_s^i; (iii) the absolute and relative output of the sector, Y_N, Y_R, and $\frac{Y_N}{Y_R}$; (iv) the relative price of sectors, $\frac{P_N}{P_R}$; (v) the relative value of the output, $\frac{P_N Y_N}{P_R Y_R}$; (vi) the wage differences between types of labor in the various contexts – intra-country wage inequality (skill premium), intra-country wage polarization and inter-country wage inequality. In this Subsection the analysis will be conducted without the adjustment of the technological-knowledge progress in each sector.

We can start by determining absolute values for price indexes. To this end, we use the definition of the price of output underlying the producer's output maximization problem in sector s, Y_s, which implies $P_s = \exp\left(\int_0^1 \ln P_{v_s} dv_s\right)$ – see (6). We also make use of the result that the value of each task, $P_{v_s} Y_{v_s}$, is a constant for all v_s, and we use (17) and (18) to have $P_s^i = \left(\frac{\overline{v}_s}{1-\overline{v}_s}\right)^{\alpha} P_s^h$. From this analysis, we obtain the following expressions – see Appendix A.3:

$$P_s^h = P_s \cdot \exp(-\alpha) \cdot \overline{v}_s^{-\alpha} and P_s^i = P_s \cdot \exp(-\alpha) \cdot (1-\overline{v}_s)^{-\alpha} \Rightarrow \frac{P_s^i}{P_s^h} = \left(\frac{Q_s^i}{Q_s^h}\frac{l_s^i}{l_s^h}\frac{L_s^i}{L_s^h}\right)^{-\frac{\alpha}{2}}, \tag{21}$$

where P_N and P_R are also determined in Appendix A.3. An increase in the labor level of sector

s has market-size on the demand for machines through the term $\overline{v}_s$. However, by affecting $\overline{v}_s$ the same effect has, in addition, a price effect since increases the supply of output of sector s that induces a decrease in the absolute price of this output and, therefore, a decrease in the price index of tasks in this sector. This decreases the output of each task, which decreases demand for machines in this sector – see (19) and (20).

Proposition 2. *The price index of the tasks produced by a certain type of labor, P_s^h or P_s^i, depends positively on the price of the output of the sector, P_s, and depends negatively on the number of tasks produced, evaluated by the respective threshold task – see* (21).

From Proposition 2, the relative price index of tasks produced with some type of labor in a sector is higher the lower the respective effective labor level in the sector. In case of the sector $s = R$, since $l_R^i > l_R^h$ due to factors specific to automation and relocations, assuming, on the one hand, that automation improves labor productivity and, on the other, that domestic labor with medium skills has an absolute productivity advantage over foreign labor in developing countries, *ceteris paribus*, given the values of l_R^i and l_R^h the price index associated with the production of tasks produced with automation is lower than the price index of relocated tasks.

From the profit maximization problem of the producer of Y and since in each sector some tasks are produced by labor L_s^h and other part are performed by labor L_s^i, the aggregate output is the following: $P_s Y_s = \int_0^1 P_{v_s} Y_{v_s} dv_s = \int_0^{\overline{v}_s} P_{v_s}^h Y_{v_s}^h dv_s + \int_{\overline{v}_s}^1 P_{v_s}^i Y_{v_s}^i dv_s = P_s Y_s^h + P_s Y_s^i$. On the basis of these definitions and taking into account (13), (14), (17), and (21), the outputs in sector s performed by labor type L_s^h, Y_s^h, and labor type L_s^i, $Y_s^{L_s^i}$, are:

$$Y_s^h = \exp(-1) \cdot \left[\frac{P_s \cdot (1-\alpha)}{q} \right]^{\frac{1-\alpha}{\alpha}} \cdot \frac{Q_s^h \cdot l_s^h \cdot L_s^h}{\overline{v}_s}, \tag{22}$$

,

$$Y_s^i = \exp(-1) \cdot \left[\frac{P_s \cdot (1-\alpha)}{q} \right]^{\frac{1-\alpha}{\alpha}} \cdot \frac{Q_s^i \cdot l_s^i \cdot L_s^i}{1 - \overline{v}_s}. \tag{23}$$

Therefore, from $P_s Y_s = P_s Y_s^h + P_s Y_s^i = P_s \left(Y_s^h + Y_s^i \right)$, the output of each sector is:

$$Y_s = \exp(-1) \cdot \left[\frac{P_s \cdot (1-\alpha)}{q}\right]^{\frac{1-\alpha}{\alpha}} \cdot M_s, \tag{24}$$

where, bearing in mind (18), $M_s = \frac{Q_s \cdot l_s^h \cdot L_s^h}{\overline{v}_s} + \frac{Q_s \cdot l_s^i \cdot L_s^i}{1-\overline{v}_s} = \left[\left(Q_s^h \cdot l_s^h \cdot L_s^h\right)^{\frac{1}{2}} + \left(Q_s^i \cdot l_s^i \cdot L_s^i\right)^{\frac{1}{2}}\right]^2$ evaluates the market size. Similarly, we can obtain an expression for machines produced for each sector s:

$$X_s(t) = \exp(-1) \cdot \left[\frac{P_s \cdot (1-\alpha)}{q}\right]^{\frac{1}{\alpha}} \cdot M_s, \tag{25}$$

where the aggregate resources devoted to machines production in sector s, X_s, is also expressible as a function of the currently given technological knowledge in sector s.

Proposition 3. A. *The output performed by each labor type in each sector, Y_s^h and Y_s^i, and the output of each sector, Y_s, are expressible as a function of the current technological knowledge, measure by the aggregate quality indexes, and thus the technological-knowledge progress is the driving force of the economic growth. Moreover, the technological-knowledge and labor levels determine the relative output level, $\frac{Y_s^i}{Y_s^h} = \left(\frac{Q_s^i \cdot l_s^i \cdot L_s^i}{Q_s^h \cdot l_s^h \cdot L_s^h}\right)^{\frac{1}{2}}$.* **B.** *Given the technological-knowledge levels, an increase in labor levels has the following impacts:(i) scale effects since through terms $l_s^h \cdot L_s^h$ or $l_s^i \cdot L_s^i$ in (22) and (23) increase the output produced and thereby the relative demand for machines; (ii) price effect since through the terms $(\overline{v}_s)^{-1}$ and $(1-\overline{v}_s)^{-1}$ in (22) and (23) increase the share of tasks produced by the respective labor type, which decreases the price index where such increase took place. This, in turn, decreases the output of each task, thereby decreasing demand for machines and profits in the sector.* **C.** *Inter-sector analysis shows that the relative level of production between sectors depends essentially on the technological-knowledge bias and on the sector's factorial allocations.* **D.** *An increase in l_s^h or l_s^i increases the sector output and thus the aggregate output, contributing to higher wages for both types of labor (or higher prices for all factors).*

Proof. **A.**, **B.**, and **D.** result directly from (22), (23) and (24), and from the calculated intra-sector output ratio

$$\frac{Y_s^i}{Y_s^h} = \left(\frac{Q_s^i \cdot l_s^i \cdot L_s^i}{Q_s^h \cdot l_s^h \cdot L_s^h}\right)^{\frac{1}{2}}. \tag{26}$$

C. Results directly from the computed inter-sector output ratio

$$\frac{Y_N}{Y_R} = \left(\frac{\chi_N}{\chi_R}\right)^{\frac{\epsilon}{\epsilon\cdot\alpha+1}} \left(\frac{M_N}{M_R}\right)^{\frac{\epsilon\cdot\alpha}{\epsilon\cdot\alpha+1}} . \tag{27}$$

□

In particular, from 26, *ceteris paribus*, the intra-sector output ratio in sector $s = R$ is more biased towards automated tasks the larger l_R^i is relative to l_R^h. Moreover, from (5), $\frac{P_N}{P_R} = \frac{\chi_N}{\chi_R}\left(\frac{Y_N}{Y_R}\right)^{-\frac{1}{\epsilon}}$ and then considering (24), the relative price of the output in the N-sector is – see Appendix A.3:

$$\frac{P_N}{P_R} = \left(\frac{\chi_N}{\chi_R}\right)^{\frac{\epsilon\cdot\alpha}{\epsilon\cdot\alpha+1}} \left(\frac{M_N}{M_R}\right)^{-\frac{\epsilon}{\epsilon\cdot\alpha+1}} , \tag{28}$$

The intuition behind (28) can be grasped by taking into account that an increase in the relative relevance of the N-sector in the production of the aggregate final good, $\frac{\chi_N}{\chi_R}$, increases the relative demand for output in this sector that leads to an increase in relative prices. Hence, equation (28), through M_N and M_R, shows that if either the technological-knowledge is highly N-biased or if there is a large relative supply of N, the output of the N-sector is large – see (27) –, which implies a low relative price of the N-sector. In this case, the demand for N-machines is low, which discourages R&D activities aimed at improving their quality, as we can see below. Thus, labor structure affects the direction of R&D through the price channel, which appears in various papers by Acemoglu (e.g., 2002), although always dominated by the market-size channel. In our case, this latter channel may or may not be removed, being eliminated or not in conducting the economic mechanisms.

Finally, in this Subsection, the question of wages for labor type L_s^i and L_s^h, and the differences in wages that can be established still needs to be addressed. Bearing in mind (17), the wages in (15) and (16) can be rewritten in the form:

$$w_s^h(t) = \left[P_s^h(t)\right]^{\frac{1}{\alpha}} \cdot \left(\frac{1-\alpha}{q}\right)^{\frac{1-\alpha}{\alpha}} \cdot Q_s^h(t) \cdot l_s^h; \tag{29}$$

$$w_s^i(t) \quad = \quad \left[P_s^i(t)\right]^{\frac{1}{\alpha}} \quad \cdot \quad \left(\frac{1-\alpha}{q}\right)^{\frac{1-\alpha}{\alpha}} \quad \cdot \quad Q_s^i(t) \quad \cdot \quad l_s^i. \tag{30}$$

Hence, there is the possibility that, for example, an increase in l_s^h (due to better productivity of unskilled and abroad labor) increases the wages of all workers in the sector since $\frac{P_s^i}{P_s^h}$ increases – see (21), (15) and (16), or (29) and (30); i.e., in the short-run or, in other words, for a given technological-knowledge level. Moreover, from (29), (30), (21), and (28), we can obtain wage differentials between types of labor in each sector s,

$$\frac{w_s^i}{w_s^h} = \left(\frac{P_s^i}{P_s^h}\right)^{\frac{1}{\alpha}} \frac{Q_s^i(t)\cdot l_s^i}{l_s^h} = \left(\frac{Q_s^i \cdot l_s^i}{Q_s^h \cdot l_s^h}\frac{L_s^h}{L_s^i}\right)^{\frac{1}{2}}. \tag{31}$$

Proposition 4. *For a given technological knowledge, a decrease in labor type* i*,* L_s^i*, over labor type* h*,* L_s^h*, and an increase in the absolute advantages of a labor type* i*,* l_s^i*, over labor type,* h*,* l_s^h*, improve the relative wage of* L_s^h*.*

Proof. Directly from 31. □

Bearing in mind 31, in order to take the (domestic) skill premium, the following wage ratio should be analyzed:

$$\frac{w_N^i}{w_N^h} = \left(\frac{Q_N^i \cdot l_N^i}{Q_N^h \cdot l_N^h}\frac{L_N^h}{L_N^i}\right)^{\frac{1}{2}}. \tag{32}$$

In turn, to evaluate inter-country wage inequality in favor of the domestic country, from 31, the following wage ratio should be computed:[18]

$$\frac{w_R^i}{w_R^h} = \left(\frac{Q_R^i \cdot l_R^i}{Q_R^h \cdot l_R^h}\frac{L_R^h}{L_R^i}\right)^{\frac{1}{2}}. \tag{33}$$

Finally, to analyze wage polarization, from (15), (16), (17), (21), and 18, the following wage ratios should be calculated

[18] Strictly speaking, however, to calculate inter-country wage inequality the $\frac{w_N^i}{w_R^h} = \left(\frac{\chi_N}{\chi_R}\right)^{\frac{\epsilon}{\epsilon\cdot\alpha+1}}\left(\frac{M_N}{M_R}\right)^{-\frac{\epsilon}{\alpha\cdot(\epsilon\cdot\alpha+1)}+\frac{1}{2}}\left(\frac{Q_N^i}{Q_R^h}\frac{l_N^i}{l_R^h}\frac{L_R^h}{L_N^i}\right)^{\frac{1}{2}}$ and $\frac{w_N^h}{w_R^h} = \left(\frac{\chi_N}{\chi_R}\right)^{\frac{\epsilon}{\epsilon\cdot\alpha+1}}\left(\frac{M_N}{M_R}\right)^{-\frac{\epsilon}{\alpha\cdot(\epsilon\cdot\alpha+1)}+\frac{1}{2}}\left(\frac{Q_N^h}{Q_R^h}\frac{l_N^h}{l_R^h}\frac{L_R^h}{L_N^h}\right)^{\frac{1}{2}}$ ratios should also be calculated,

$$\frac{w_N^i}{w_R^i} = \left(\frac{\chi_N}{\chi_R}\right)^{\frac{\epsilon}{\epsilon\cdot\alpha+1}} \left(\frac{M_N}{M_R}\right)^{-\frac{\epsilon}{\alpha\cdot(\epsilon\cdot\alpha+1)}+\frac{1}{2}} \left(\frac{Q_N^i}{Q_R^i}\frac{l_N^i}{l_R^i}\frac{L_R^i}{L_N^i}\right)^{\frac{1}{2}}, \tag{34}$$

$$\frac{w_N^h}{w_R^i} = \left(\frac{\chi_N}{\chi_R}\right)^{\frac{\epsilon}{\epsilon\cdot\alpha+1}} \left(\frac{M_N}{M_R}\right)^{-\frac{\epsilon}{\alpha\cdot(\epsilon\cdot\alpha+1)}+\frac{1}{2}} \left(\frac{Q_N^h}{Q_R^i}\frac{l_N^h}{l_R^i}\frac{L_R^i}{L_N^h}\right)^{\frac{1}{2}}. \tag{35}$$

Proposition 5. *In the short run, ceteris paribus or with everything else constant, changes in labor endowments have the following effects:*

A. *the skill-premium is positively affected by an unexpected increase in unskilled-labor L_N^h, over the skilled labor, L_N^i – see* (32).

B. *the inter-country wage inequality in favor of the domestic country is positively affected by an unexpected increase in countries available to host relocations of routine production, reflected in L_R^h, over the domestic medium-skilled labor L_R^i – see* (33).

C. *considering that $\frac{\epsilon}{\alpha\cdot(\epsilon\cdot\alpha+1)} > \frac{1}{2}$, the wage polarization is positively affected by an unexpected: (i) increase in countries available to host relocations of routine production, reflected in L_R^h and increase in domestic (routine) medium-skilled labor L_R^i; (ii) decrease in domestic labor allocated to non-routine tasks, reflected in L_N^i and L_N^h – see 34 and 35.*

Proof. **A** and **B** result directly from 32 and (33), respectively, while **C** results directly from 34 and 35. Table 1 summarizes the Proposition advantage and extends it to the consideration of absolute advantage of labor levels. □

According to Table 1, the following remark can be stated.

Remark 6. In the short run, the domestic skill premium is concomitant with wage polarization if: (i) $\frac{\epsilon}{\alpha\cdot(\epsilon\cdot\alpha+1)} < \frac{1}{2}$ and l_N^i increases; (ii) $\frac{\epsilon}{\alpha\cdot(\epsilon\cdot\alpha+1)} > \frac{1}{2}$ and L_N^i decreases. In turn, inter-country wage inequality in favor of the domestic country is concomitant with domestic wage polarization if: (i) $\frac{\epsilon}{\alpha\cdot(\epsilon\cdot\alpha+1)} < \frac{1}{2}$ and l_R^h decreases; (ii) $\frac{\epsilon}{\alpha\cdot(\epsilon\cdot\alpha+1)} > \frac{1}{2}$ and L_R^h increases. Thus, if there are changes in relocations that increase labor productivity in these types of tasks; i.e., l_R^h increases, then there will be: (i) decrease in both intra-sector R wage inequality and wage polarization, if

	Labor levels								Absolute advantage of labor levels							
	$\frac{\epsilon}{\alpha\cdot(\epsilon\cdot\alpha+1)} < \frac{1}{2}$				$\frac{\epsilon}{\alpha\cdot(\epsilon\cdot\alpha+1)} > \frac{1}{2}$				$\frac{\epsilon}{\alpha\cdot(\epsilon\cdot\alpha+1)} < \frac{1}{2}$				$\frac{\epsilon}{\alpha\cdot(\epsilon\cdot\alpha+1)} > \frac{1}{2}$			
	L_N^h	L_R^h	L_N^i	L_R^i	L_N^h	L_R^h	L_N^i	L_R^i	l_N^h	l_R^h	l_N^i	l_R^i	l_N^h	l_R^h	l_N^i	l_R^i
$\frac{\partial \frac{w_N^i}{w_N^h}}{\partial(.)}$	$+$		$-$		$+$		$-$		$-$		$+$		$-$		$+$	
$\frac{\partial \frac{w_R^i}{w_R^h}}{\partial(.)}$		$+$		$-$		$+$		$-$		$-$		$+$		$-$		$+$
$\frac{\partial \frac{w_N^i}{w_R^i}}{\partial(.)}$	$+$	$-$	$\pm$	$\pm$	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$-$	$-$	$+$	$\pm$	$\pm$
$\frac{\partial \frac{w_N^h}{w_R^i}}{\partial(.)}$	$\pm$	$-$	$+$	$\pm$	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$-$	$\pm$	$+$	$-$	$\pm$

Table 1: Summary of Proposition 5: the effect of labor changes in the skill-premiums.

$\frac{\epsilon}{\alpha\cdot(\epsilon\cdot\alpha+1)} < \frac{1}{2}$. (ii) decrease in intra-sector R wage inequality and increase in wage polarization, if $\frac{\epsilon}{\alpha\cdot(\epsilon\cdot\alpha+1)} > \frac{1}{2}$.

Bearing in mind that the traditional approach of the literature and established the usual values for $\alpha = 0.6$ and $\epsilon = 0.4$ (e.g., Jones and Williams 2000, Chu and Lai 2013, and Afonso and Pinho 2022), the most likely situation in Table 1 is that $\frac{\epsilon}{\alpha\cdot(\epsilon\cdot\alpha+1)} > \frac{1}{2}$ and thus – in line with, e.g., Autor et al. (2003), Acemoglu and Autor (2011), Autor and Dorn (2013), Hémous and Osborne (2014), and Lankisch et al. (2019):

- if, for example, there are changes in automation that improve routine tasks performed by machines/robots, labor in these tasks becomes less productive; i.e., l_R^i decreases, and decreases the relative demand for medium-skilled workers, thus decreasing inter-country wage inequality.
- Relocation and automation of tasks performed by medium-skilled workers, immediately increase the relative labor supply – *labor effect* – thus generating wage polarization.

4.5 R&D technology and values

By producing innovative designs, R&D activities drive the rate and the direction of technological knowledge, and thus wages, and economic growth. Innovative designs, for the manufacture of new qualities of the machines, are patented, and the leader firm in each industry – the one that produces according to the latest patent – uses limit pricing to assure monopoly. The value of the leading-edge patent relies on the profit-yields accruing during each time t to the monopolist, and on the duration of the monopoly power. The duration, in turn, depends on the probability of a new innovation, which creatively destroys the current leading-edge design (e.g., Aghion and Howitt 1992, Grossman and Helpman 1991, ch. 12, and Barro and Sala-i-Martin 2004, ch. 7). The probability of successful innovation is, thus, at the heart of the R&D activity. Let $\mathcal{I}_s^h(k,j,t)$ and $\mathcal{I}_s^i(k,j,t)$ denote the instantaneous probability at time t in sector $s = \{N, R\}$ for, respectively, h and i – a Poisson arrival rate – of successful innovation in the next higher quality $[k(j,t)+1]$ in machine j given current rung quality k. We define it as follows:

$$\mathcal{I}_s^h(k,j,t) = e_s^h(k,j,t) \cdot \beta q^{k(j,t)} \cdot \zeta^{-1} q^{-\alpha^{-1}k(j,t)} \cdot (L_s^h)^{-\xi} \tag{36}$$

$$\mathcal{I}_s^i(k,j,t) = e_s^i(k,j,t) \cdot \beta q^{k(j,t)} \cdot \zeta^{-1} q^{-\alpha^{-1}k(j,t)} \cdot (L_s^i)^{-\xi} \tag{37}$$

where, following Afonso (2012) and Afonso and Sequeira (forthcoming): (i) $e_s^h(k,j,t)$ and $e_s^i(k,j,t)$ are the flow of domestic final-good resources devoted to R&D in j belonging to $s = \{N, R\}$ for, respectively, h and i, which defines our framework as a lab-equipment model; (ii) $\beta q^{k(j,t)}$, $\beta > 0$, is the learning-by-past domestic R&D, as a positive learning effect of public knowledge accumulated from past successful R&D; (iii) $\zeta^{-1} q^{-\alpha^{-1}k(j,t)}$, $\zeta > 0$, is the adverse effect – cost of complexity – caused by the increasing complexity of quality improvements;[19] (iv) $(L_s^h)^{-\xi}$ and $(L_s^i)^{-\xi}$, with $\xi \geq 0$, is the adverse effect of market size, capturing the idea that the difficulty of introducing new quality machines and replacing old ones is proportional to the geometric average of the effective units of labor in sector s. The scale benefits on profits can be partially ($0 < \xi < 1$), totally ($\xi = 1$) or over counterbalance ($\xi > 1$) and thus allows us to remove (explicit) scale effects on the economic growth rate. That is, for reasons of simplicity, we reflect in R&D the costs of scale increasing, due to coordination among agents, processing of ideas, and informational, organizational, marketing, and transportation costs (e.g., Dinopoulos

[19]The complexity cost is modeled in such a way that, together with the positive learning effect (ii), it exactly offsets the positive effect of the quality rung on profits of each leader machine firm; this is the reason for the presence of the production function parameter α in (37) – e.g., Barro and Sala-i-Martin (2004, ch. 7).

and Thompson 1999).[20]

Without wishing to enter into the discussion of who conducts R&D, because it is not central to our analysis, we consider that the probability of innovation presented above is similar for incumbents and entrants, and thus R&D is conducted by entrants, as shown in Appendix A.4.

The value of the leading-edge patent for the producer of an intermediate good j belonging to $s = \{N, R\}$ and used by, respectively, h and i, with quality level k at time t is the expected present value of flow of profits given by the following equations:[21] $V_s^i(j, k, t, T(k)) = \int_t^{t+T(k)} \pi_s^i(j, s) \exp\left(-\int_t^s r(w)dw\right) ds$ and $V_s^h(j, k, t, T(k)) = \int_t^{t+T(k)} \pi_s^h(j, s) \exp\left(-\int_t^s r(w)dw\right) ds$, where $T(k)$ is the duration of the patent during which there is no innovation in the quality level of intermediate good j by another entrant.[22]

Given the functional forms (37) and (36) of the probabilities of success in R&D, which rely on the resources – composite final goods – allocated to it, free-entry equilibrium is defined by the equality between expected revenue, $\mathcal{I}_s^h(j,t) \cdot V_s^h(j,t)$ and $\mathcal{I}_s^i(j,t) \cdot V_s^i(j,t)$, and resources spent, $e_s^h(j,t)$ and $e_s^i(j,t)$. By considering free entry in R&D activities, free access to the R&D technology, and a proportional relationship between successful R&D and the share of R&D effort, the R&D spending aimed at, for example, improving j should equal the expected payoff generated by the innovation; i.e.,

$$\mathcal{I}_s^h(k,j,t) \cdot V_s^h(j,t) = e_s^h(j,t) \, and \, \mathcal{I}_s^i(k,j,t) \cdot V_s^i(j,t) = e_s^i(j,t). \tag{38}$$

Assuming that all the prices and quantities are fixed during the time in which there is no quality improvements (e.g., Aghion and Howitt 1992, Barro and Sala-i-Martin 2004, Gil et al. 2013), then we have that – see Appendix A5:

$$V_s^h(j,k,t) = \frac{\pi_s^h(j,k,t)}{r_S(t) + \mathcal{I}_s^h(j,t)} \, and \, V_s^i(j,k,t) = \frac{\pi_s^i(j,k,t)}{r_S(t) + \mathcal{I}_s^i(j,t)}, \tag{39}$$

[20]This term will be presented in the following section to ensure that market-size effects are removed from the model, as is common in the literature since Jones (1995).

[21]$V_s^i(j,t)$ and $V_s^h(j,t)$ are the expected current value of the flow of profits to the monopolist producer of intermediate good j belonging to $s = \{N, R\}$ and used by, respectively, h and i, the market value of the patent, or the value of the monopolist firm owned by domestic consumers.

[22]For a complete derivation and explanation of the value of the patent, see Appendix A.5 and references therein (Aghion and Howitt 1992, Barro and Sala-i-Martin 2004, Gil et al. 2013).

and can be seen as the no-arbitrage condition, where $V_s^h\left(k,j,t\right) \cdot r\left(t\right)$ and $V_s^i\left(k,j,t\right) \cdot r\left(t\right)$, the expected income generated by a successful innovation at time t on rung k, equals the profit flow, $\pi_s^h(j,k,t)$ and $\pi_s^i(j,k,t)$, minus the expected capital loss, $V_s^h\left(k,j,t\right) \cdot \mathcal{I}_s^h\left(j,\tau\right)$ and $V_s^i\left(k,j,t\right) \cdot \mathcal{I}_s^i\left(j,\tau\right)$. Then plugging (39) into (38) and (38), respectively, and solving for $\mathcal{I}_s^h$ and $\mathcal{I}_s^i$, the equilibrium probability of a successful innovation in sector $s = \{N, R\}$ and for h and i are respectively – given the interest rate and the price indexes of final goods:

$$\mathcal{I}_s^h(t) = \frac{\beta}{\zeta} \cdot \left(\frac{q-1}{q}\right) \cdot \exp\left(-1\right) \cdot \left[(1-\alpha) \cdot P_s^h\right]^{\frac{1}{\alpha}} \cdot l_s^h \cdot (L_s^h)^{1-\xi} - r(t), \tag{40}$$

$$\mathcal{I}_s^i(t) = \frac{\beta}{\zeta} \cdot \left(\frac{q-1}{q}\right) \cdot \exp\left(-1\right) \cdot \left[(1-\alpha) \cdot P_s^i\right]^{\frac{1}{\alpha}} \cdot l_s^i \cdot (L_s^i)^{1-\xi} - r(t). \tag{41}$$

The equilibrium $\mathcal{I}_s^i(t)$ and $\mathcal{I}_s^h(t)$ in (36) and (37) are, respectively, independent of j and k since the removal of scale of technological-knowledge effects – see the exponents of q in the demand of intermediate goods above, which impacts the expression of profits, and in equations (36) and (37).

Finally, from the definition of the probabilities of achieving higher quality rungs (36) and (37), and since, by definition, $\mathcal{I}_s^h(k,j,t)$ and $\mathcal{I}_s^i(k,j,t)$ do not differentiate between different machines belonging to the same sector, we have that:

$$E_s(t) = \underbrace{\int_0^J e_s^h(k,j,t)dj}_{E_s^h(t)} + \underbrace{\int_J^1 e_s^i(k,j,t)dj}_{E_s^i(t)} = \mathcal{I}_s^h(k,j,t) \cdot \frac{\zeta}{\beta} \cdot Q_s^h \cdot (L_s^h)^{\xi} + \mathcal{I}_s^i(k,j,t) \cdot \frac{\zeta}{\beta} \cdot Q_s^i \cdot (L_s^i)^{\xi}, \tag{42}$$

and thus more resources devoted to R&D are needed as Q_s^h and Q_s^i rise to offset the greater difficulty of R&D when Q_s^h and Q_s^i increase.

5 General equilibrium

As the economic structure has been characterized for given states of technological knowledge, Q_s^h and Q_s^i, $s = \{N, R\}$, h and i, we now proceed to characterize the general equilibrium, considering that firms, like households, are always rational and solve their problems, and markets clear. We derive the law of motion of the distinct technological-knowledge indexes, which drive the path of all macroeconomic aggregates – see (24), (25) and (42) –, including consumption, as will be clear after deriving the aggregate resource constraint. We also derive the technological-knowledge bias in sector $s = N$, which drives the skill premium, and the technological-knowledge bias in sector $s = R$, which directs inter-country wage inequality.

5.1 Technological-knowledge indexes and bias

If a new quality of machine j is introduced the rate of change in the quality index of sector $s = \{N, R\}$ used by, for example, labor type h will be the following: $\Delta Q_s^h = Q_s^h\,(k+1, t) - Q_s^h(k, t) = \int_0^{J_s} q^{[k(j,t)+1]\left(\frac{1-\alpha}{\alpha}\right)} - \int_0^{J_s} q^{k(j,t)\left(\frac{1-\alpha}{\alpha}\right)}$ and thus $\frac{\Delta Q_s^h}{Q_s^h} = \left[q^{\left(\frac{1-\alpha}{\alpha}\right)} - 1\right]$. Since the probability of this occurring per unit of time if given by $\mathcal{I}_s^h(t)$, we have that:

$$\frac{\dot{Q}_s^h(t)}{Q_s^h(t)} = \mathcal{I}_s^h(t)\cdot\left[q^{\left(\frac{1-\alpha}{\alpha}\right)} - 1\right] = \left[\frac{\beta}{\zeta}\cdot\left(\frac{q-1}{q}\right)\cdot\exp\left(-1\right)\cdot\left[(1-\alpha)\cdot P_s^h(t)\right]^{\frac{1}{\alpha}}\cdot l_s^h\cdot(L_s^h)^{1-\xi} - r(t)\right]\cdot\left[q^{\left(\frac{1-\alpha}{\alpha}\right)} - 1\right] \tag{43}$$

Thus, bearing in mind (43), from the emergence of a shock to a parameter or an exogenous variable to the steady state, the path of the technological-knowledge bias in sector $s = N$, which drives the skill premium in 32, and in sector $s = R$, which directs the inter-country wage inequality in (33), are

$$\frac{\dot{Q}_N^i}{Q_N^i} - \frac{\dot{Q}_N^h}{Q_N^h} = \left[q^{\left(\frac{1-\alpha}{\alpha}\right)} - 1\right]\cdot\frac{\beta}{\zeta}\cdot\frac{q-1}{q}\cdot\exp\left(-1\right)\cdot(1-\alpha)^{\frac{1}{\alpha}}\cdot\left[\left(P_N^i\right)^{\frac{1}{\alpha}}\cdot l_s^i\cdot(L_N^i)^{1-\xi} - \left(P_N^h\right)^{\frac{1}{\alpha}}\cdot l_s^h\cdot(L_N^h)^{1-\xi}\right]; \tag{44}$$

$$\frac{\dot{Q}_R^i}{Q_R^i}-\frac{\dot{Q}_R^h}{Q_R^h}=\left[q^{\left(\frac{1-\alpha}{\alpha}\right)}-1\right]\cdot\frac{\beta}{\zeta}\cdot\frac{q-1}{q}\cdot\exp\left(-1\right)\cdot(1-\alpha)^{\frac{1}{\alpha}}\cdot\left[\left(P_R^i\right)^{\frac{1}{\alpha}}\cdot l_R^i\cdot(L_R^i)^{1-\xi}-\left(P_R^h\right)^{\frac{1}{\alpha}}\cdot l_R^h\cdot(L_R^h)^{1-\xi}\right]. \tag{45}$$

In turn, the inter-sector technological knowledge, which drives the wage polarization in 34 and 35, are

$$\frac{\dot{Q}_N^i}{Q_N^i}-\frac{\dot{Q}_R^i}{Q_R^i}=\left[q^{\left(\frac{1-\alpha}{\alpha}\right)}-1\right]\cdot\frac{\beta}{\zeta}\cdot\frac{q-1}{q}\cdot\exp\left(-1\right)\cdot(1-\alpha)^{\frac{1}{\alpha}}\cdot\left[\left(P_N^i\right)^{\frac{1}{\alpha}}\cdot l_N^i\cdot(L_N^i)^{1-\xi}-\left(P_R^i\right)^{\frac{1}{\alpha}}\cdot l_R^i\cdot(L_R^i)^{1-\xi}\right]; \tag{46}$$

$$\frac{\dot{Q}_N^h}{Q_N^h}-\frac{\dot{Q}_R^h}{Q_R^h}=\left[q^{\left(\frac{1-\alpha}{\alpha}\right)}-1\right]\cdot\frac{\beta}{\zeta}\cdot\frac{q-1}{q}\cdot\exp\left(-1\right)\cdot(1-\alpha)^{\frac{1}{\alpha}}\cdot\left[\left(P_N^h\right)^{\frac{1}{\alpha}}\cdot l_N^h\cdot(L_N^h)^{1-\xi}-\left(P_R^h\right)^{\frac{1}{\alpha}}\cdot l_R^h\cdot(L_R^h)^{1-\xi}\right]. \tag{47}$$

5.2 Steady-state results

Taking into account the aggregate expenditures in the final good are given by $Y=P_NY_N+P_RY_R$ from the profit maximization problem of the producer of aggregate output, considering that aggregate expenditures in machines and R&D activities are the sum of aggregates in both sectors already derived in equilibrium in the previous sections, $X\equiv X_N+X_R$ and $E\equiv E_N+E_R$ and that assets in the economy are the present value of the patent of all producers of machines, i.e., that $a=\sum_{s=N,R}\int_0^J V_s^h(k,j,t)dj+\int_J^1 V_s^i(k,j,t)dj$ we can prove that in equilibrium the aggregate flow constraint of households can be expressed as $Y=C+X+E$ – see the respective proof in Appendix A.6. Therefore, since Y, X and E are all multiples of the quality indexes Q_N^h, Q_N^i, Q_R^h and Q_R^i, the aggregate flow constraint implies that consumption C is also a constant multiple of these variables, which implies that the path of all relevant variables outside the steady state depends on the path of the different quality indexes. At the end of transitional dynamics, the economy reaches the steady state, which is unique and stable, and all relevant macroeconomic variables grow at the same constant rate. The steady-state growth rate, g^*, is:

$$g^* \equiv \left(\frac{\dot{Q}_s^i}{Q_s^i}\right)^* = \left(\frac{\dot{Q}_s^h}{Q_s^h}\right)^* = \left(\frac{\dot{Y}}{Y}\right)^* = \left(\frac{\dot{X}}{X}\right)^* = \left(\frac{\dot{E}}{E}\right)^* = \left(\frac{\dot{C}}{C}\right)^* = \frac{r^* - \rho}{\theta}. \tag{48}$$

The uniqueness of the steady state is guaranteed by the uniqueness of the interest rate. In order to take into account the stability, it must be taken into account (43), 44, 45,46, and 47, we note that the dynamics of the economy can be characterized by a two-dimensional dynamic system in detrended variables such as $\frac{Q_s^i}{Q_s^h}$ (or $\frac{Q_N^i}{Q_R^i}$ or $\frac{Q_N^h}{Q_R^i}$ or $\frac{Q_N^i}{Q_R^h}$ or $\frac{Q_N^h}{Q_R^h}$) and $\frac{C}{Q_s^i}$ (or $\frac{C}{Q_s^h}$), that has a recursive structure since the dynamics of $\frac{Q_s^i}{Q_s^h}$ depends, exclusively, on itself. For example, considering an economy starting out of the steady state, where $\frac{\dot{Q}_s^i}{Q_s^i} > \frac{\dot{Q}_s^h}{Q_s^h}$, we will prove that, over t, $\frac{\dot{Q}_s^i}{Q_s^i} > 0$ and $\frac{\dot{Q}_s^h}{Q_s^h} = 0$. Bearing in mind this situation, it is easy to perceive that $\overline{v}_s > \overline{v}_s^*$ which, in turn, implies $\frac{\dot{P}_s^i}{P_s^I} < \frac{\dot{P}_s^h}{P_s^h}$. Thereby, $\frac{P_s^i}{P_s^h}$ is declining until $\left(\frac{P_s^i}{P_s^h}\right)^* = \left[\frac{l_s^i}{l_s^h} \cdot \left(\frac{L_s^i}{L_s^h}\right)^{1-\xi}\right]^{-\alpha}$,

$$\left(\frac{P_s^i}{P_s^h}\right)^* = \left[\frac{l_s^i}{l_s^h} \cdot \left(\frac{L_s^i}{L_s^h}\right)^{1-\xi}\right]^{-\alpha}, \tag{49}$$

attenuating the rate at which $\frac{Q_s^i}{Q_s^h}$ is increasing. In this sense, even with $\frac{\dot{Q}_s^i}{Q_s^i} > \frac{\dot{Q}_s^h}{Q_s^h}$ the difference between both equilibrium path of technological knowledge, $\frac{\dot{Q}_s^i}{Q_s^i} - \frac{\dot{Q}_s^h}{Q_s^h}$, is decreasing until approaches the steady state, where $\frac{\dot{Q}_s^i}{Q_s^i} = \frac{\dot{Q}_s^h}{Q_s^h}$; the argument to exhibit the convergence to the steady state if $\frac{\dot{Q}_s^i}{Q_s^i} < \frac{\dot{Q}_s^h}{Q_s^h}$ is similar – in line with Acemoglu and Zilibotti (2001). Thus, the economy converges and remains in steady state, if we consider that no other exogenous changes have occurred.

Lemma. *There exists a unique and stable steady state along this growth path Y, C, X, and E growth at rate g^*.*

We now show the calculation of the variables of interest in the steady state and, for each of the variables of interest, a Proposition is presented in the face of an eventual "shock". At the end of the presentation of the Propositions, the underlying economic intuition is exposed. From (21) and (49), we have that:

$$\left(\frac{Q_s^i}{Q_s^h}\right)^* = \frac{l_s^i}{l_s^h} \cdot \left(\frac{L_s^i}{L_s^h}\right)^{1-2\xi}. \tag{50}$$

Proposition 7. *From (50), the steady-state intra-sector technological-knowledge gap increases when (i) the absolute advantages of a labor type* i*,* l_s^h*, over labor type,* h*,* l_s^h*, increases, (ii) the labor type* i*,* L_s^h*, over labor type* h*,* L_s^h*, increases and scale effects are strongly removed* $\xi > \frac{1}{2}$*. By looking specifically at the routine sector,* $s = R$*, an improvement in automation such that* $\frac{l_s^i}{l_s^h}$ *increases redirects the intra-sector technological-knowledge bias that penalizes relocations.*

Proof. Directly from (50).

□

In possession of (50), one can use 18 to determine:

$$\overline{v}_s^* = \left[1 + \frac{l_s^i}{l_s^h} \cdot \left(\frac{L_s^i}{L_s^h}\right)^{1-\xi}\right]^{-1}. \tag{51}$$

Proposition 8. *From (51), the steady-state threshold tasks* $\overline{v}_R$ *and* $\overline{v}_N$ *are small, implying that the number of: (i) automated routine tasks produced with medium-skilled labor is large when both the relative medium-skilled-labor supply and the respective relative absolute advantage are high in face of the foreign-labor supply and the respective absolute advantage; in other words, relocations are more intense the higher the productivity and availability of labor abroad; (ii) non-routine tasks produced with skilled labor is large when both the relative skilled-labor supply and the respective relative absolute advantage are high in face of the unskilled-labor supply and the respective absolute advantage. Labor levels only cease to have the stated impact when scale effects are completely removed.*

Proof. Directly from (51).

□

Hence, in terms of competitiveness, the race between robotization and relocation is won by robotization if the level of domestic medium-skilled workers exceeds the number existing abroad – that is, if the level of robotization exceeds the level of relocations – and if the absolute advantage of domestic medium-skilled workers outweighs the absolute advantage of the same type of workers abroad.

Therefore, from (32), (33), and (50), the (domestic) steady-state skill premium and the inter-country wage inequality in favor of the domestic country are, respectively:

$$\frac{w_N^i}{w_N^h} = \frac{l_N^i}{l_N^h} \cdot \left(\frac{L_N^i}{L_N^h}\right)^{-\xi} \; and \; \frac{w_R^i}{w_R^h} = \frac{l_R^i}{l_R^h} \cdot \left(\frac{L_R^i}{L_R^h}\right)^{-\xi}. \tag{52}$$

To compute the steady-state wage polarization, we start noting that in (46) and (47) $\left(\frac{\dot{Q}_N^i}{Q_N^i}\right)^* = \left(\frac{\dot{Q}_R^i}{Q_R^i}\right)^*$ and $\left(\frac{\dot{Q}_N^h}{Q_N^h}\right)^* = \left(\frac{\dot{Q}_R^i}{Q_R^i}\right)^*$, which implies that

$$\left(\frac{P_N^i}{P_R^i}\right)^* = \left[\frac{l_N^i}{l_R^i} \cdot \left(\frac{L_N^i}{L_R^i}\right)^{1-\xi}\right]^{-\alpha} \text{ and } \left(\frac{P_N^h}{P_R^i}\right)^* = \left[\frac{l_N^h}{l_R^i} \cdot \left(\frac{L_N^h}{L_R^i}\right)^{1-\xi}\right]^{-\alpha}, \tag{53}$$

which, after taking (21) into account, makes it possible to obtain $\left(\frac{Q_N^i}{Q_R^i}\right)^*$ and $\left(\frac{Q_N^h}{Q_R^i}\right)^*$, as well as to determine:

$$\frac{w_N^i}{w_R^i} = \left(\frac{\chi_N}{\chi_R}\right)^{\frac{2\epsilon}{\epsilon \cdot \alpha + 1}} \left(\frac{M_N}{M_R}\right)^{-\frac{2\epsilon}{\alpha \cdot (\epsilon \cdot \alpha + 1)} + 1} \left(\frac{l_N^i}{l_R^i}\right)^{\frac{1}{2}} \left(\frac{L_N^i}{L_R^i}\right)^{-\xi}, \tag{54}$$

$$\frac{w_N^h}{w_R^i} = \left(\frac{\chi_N}{\chi_R}\right)^{\frac{2\epsilon}{\epsilon \cdot \alpha + 1}} \left(\frac{M_N}{M_R}\right)^{-\frac{2\epsilon}{\alpha \cdot (\epsilon \cdot \alpha + 1)} + 1} \left(\frac{l_N^h}{l_R^i}\right)^{\frac{1}{2}} \left(\frac{L_N^h}{L_R^i}\right)^{-\xi}, \tag{55}$$

by using 34 and 35, allows us to state the following Proposition.

Proposition 9. *In the long run, provided there are some scale effects, changes in labor endowments have the following effects:*

A. *the skill-premium is positively affected by an unexpected increase in unskilled-labor* L_N^h*, over the skilled labor,* L_N^i *– see* (52) *and in line with Figure 1.*

B. *the inter-country wage inequality in favor of the domestic country is positively affected by an unexpected increase in countries available to host relocations of routine production, reflected in* L_R^h*, over the domestic medium-skilled labor* L_R^i*; i.e., inter-country wage inequality depends positively on relocation and negatively on automation – see* (52) *and Figure 2.*

	Labor levels								Absolute advantage of labor levels							
	$\frac{\epsilon}{\alpha\cdot(\epsilon\cdot\alpha+1)} < \frac{1}{2}, \xi > 0$				$\frac{\epsilon}{\alpha\cdot(\epsilon\cdot\alpha+1)} > \frac{1}{2}, \xi > 0$				$\frac{\epsilon}{\alpha\cdot(\epsilon\cdot\alpha+1)} < \frac{1}{2}$				$\frac{\epsilon}{\alpha\cdot(\epsilon\cdot\alpha+1)} > \frac{1}{2}$			
	L_N^h	L_R^h	L_N^i	L_R^i	L_N^h	L_R^h	L_N^i	L_R^i	l_N^h	l_R^h	l_N^i	l_R^i	l_N^h	l_R^h	l_N^i	l_R^i
$\frac{\partial \frac{w_N^i}{w_N^h}}{\partial(.)}$	+		–		+		–		–		+		–		+	
$\frac{\partial \frac{w_R^i}{w_R^h}}{\partial(.)}$		+		–		+		–		–		+		–		+
$\frac{\partial \frac{w_N^i}{w_R^i}}{\partial(.)}$	+	−	±	±	−	+	−	+	+	−	+	−	−	+	±	±
$\frac{\partial \frac{w_N^h}{w_R^i}}{\partial(.)}$	±	−	+	±	−	+	–	+	+	−	+	–	±	+	–	±

Table 2: Summary of Proposition 9: the effect of labor changes in the skill-premiums.

C. *considering that* $\frac{2\epsilon}{\alpha\cdot(\epsilon\cdot\alpha+1)} > 1$*, as follows from the literature, the wage polarization in favor of non-routine labor is positively affected by an unexpected: (i) increase in countries available to host relocations of routine production, reflected in* L_R^h *and domestic (routine) medium-skilled labor* L_R^i*; i.e., wage polarization increases relocations and with robotization – see 54, 55, and Figure 5;*[23] *(ii) increase in domestic medium-skilled labor – see 54 and 55 and Figure 4; (iii) decrease in domestic labor allocated to non-routine tasks, reflected in* L_N^i *and* L_N^h *– see 34 and 35 and Figures 6 and 7.*

Proof. **A** and **B** result directly from (52), while **C** results directly from 54 and 55. Table 2 summarizes the Proposition and extends it to the consideration of absolute advantage of labor levels. □

According to Table 2 the following Remark can be stated.

Remark 10. Remark 6 relative to a context immediately after the occurrence of a shock in productivity and/or labor availability, is maintained in the long run, after the impact on technological knowledge; that is, the immediate effects of a shock are enhanced by the effects that the shock induces in technological knowledge along the path to the new steady state.

Bearing in mind 43 and 48, the stable and unique steady-state economic growth rate is – considering, as example, the case in which labor is type h:

[23]The increase in wage polarization due to robotization is in line with, e.g., Acemoglu and Restrepo (2017), Graetz and Michaels (2018), and Lankisch et al. (2019), while the increase in wage polarization due to relocations in accordance with, e.g., Li and Liu (2005), Baharumshah and Almasaied (2009), Wang (2007), Kramer (2010), Cuadros and Alguacil (2014), and Su and Liu (2016).

$$g^{*}=\frac{\frac{\beta}{\zeta}\cdot\left(\frac{q-1}{q}\right)\cdot\exp\left(-1\right)\cdot\left[\left(1-\alpha\right)\cdot\left(P_{s}^{h}\right)^{*}\right]^{\frac{1}{\alpha}}\cdot l_{s}^{h}\cdot\left(L_{s}^{h}\right)^{1-\xi}-\rho}{\left(q^{\frac{1-\alpha}{\alpha}}-1\right)^{-1}+\theta}, \tag{56}$$

where $\left(P_{s}^{h}\right)^{*}=\left(P_{s}\right)^{*}\cdot\exp\left(-\alpha\right)\cdot\left(\overline{v}_{s}^{-\alpha}\right)^{*}$ and $\left(P_{s}\right)^{*}$ is determined in Appendix A.7.

Proposition 11. *From 56, the economic growth rate increases when the productive structure is improved – e.g., l_s^h, l_s^i, L_s^h, L_s^i, β increase and ς decrease; therefore, improvements in automation and in relocations positively affects economic growth. An increase of α decreases the value of patents, making R&D less productive, thus penalizing the economic growth rate. Finally, the more patient – i.e., the smaller the value of ρ – and the less keen the individuals are on consumption – i.e., the smaller the value of θ – the higher the steady-state growth rate.*

Proof. Results directly from 56, and 66 and 67. □

Hence, from 56, under some scale effects, economic growth depends positively on robotization – in line with, e.g., Zeira (1998), Acemoglu and Restrepo (2017), and Graetz and Michaels (2018) – and also on relocations – in line with the empirical studies performed by, e.g., Li and Liu (2005), Baharumshah and Almasaied (2009), Wang (2007), Kramer (2010), Cuadros and Alguacil (2014), and Su and Liu (2016).

The economic intuition underlying the Propositions set out in this section is essentially as follows: a shock that stimulates R&D activity in a set of tasks, increases the respective technological knowledge, and thus improves the relative competitiveness. This process improves the relative productivity of the set of tasks reflected in the technological-knowledge bias in its favor, which, in turn, affects wage inequality in favor of the set of the tasks. As this promotes R&D activity, it also increases the economic growth rate.

6 Concluding remarks

The rise of the skill premium since the 1980s gave rise to the development of the Direct Technical Change (DTC) literature. This literature links the increase in the relative supply of skilled workers with the technological-knowledge bias towards those workers, which induces a higher relative demand for this labor type. However, more recent and more detailed data point to a polarization of wages concerning the distribution of skills, requiring the literature to address modeling approaches focused on automating and/or relocating different types of tasks and considering more than two labor types.

Hence, in the theoretical model developed, three types of workers were considered and, as suggested by the literature, it was assumed that medium-skilled workers are employed in routine tasks and may be penalized by automation and/or by relocation of tasks abroad (hence by international trade), while unskilled and skilled workers are employed mainly in, respectively, "purely manual" and "abstract/cognitive" routine tasks. In this context, the impact of automation and relocations on competitiveness, wages, and economic growth was analyzed.

Relocations immediately affect the competitiveness of the country by decreasing the number of tasks produced in the developed country, in contrast to automation. Both – relocations and automation – affect wages through the labor effect, the market-size effect, and the price effect. The latter two effects operate through the bias of the progress of technological knowledge. Relocation and automation of tasks performed by medium-skilled workers immediately increase the relative supply of labor – the labor effect – thus generating wage polarization. In addition, the technological-knowledge bias, generated by the dynamics of market-size and price effects, also decisively influence wages; for example, if the bias of the technological knowledge is towards the non-routine sector, then it contributes to the emergence of wage polarization. In particular, we find that for the expected values of the parameters, wage polarization increases with automation and relocation, and decreases with the rise of skilled and unskilled domestic workers. Moreover, the observed technological-knowledge progress and thus economic growth is positively affected by both automation and relocations. Economic growth, in turn, frees up resources that become partially available for investment in R&D activities, thus increasing the probability of research success, which, in turn, accelerates technological knowledge. It should also be emphasized that: the skill premium is positively affected by an increase in unskilled-labor over the skilled labor; the inter-country wage inequality depends positively on relocation and negatively on automation; in terms of competitiveness, the race between automation and relocation is won by automation if exceeds relocations.

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A. Appendix

A.1 Impact of global firms on wages – Table

Table 3: Impact of global firms and/or trade on wage inequality – source: Brito (2017).

	Authors (year)	Positive effect	Negative effect	Positive & Negative effect	No effect
Traditional Theory	Stolper and Samuelson (1941)			X	
Location	Krugman (1991)			X	
Technology	Katz and Murphy (1992)	X			
	Feenstra and Hanson (1996)	X			
	Feenstra and Hanson (1997)	X			
	Leamer (2000)				X
	Krugman (2000)	X			
	Autor et al. (2003)	X			
	Autor et al. (2006)	X			
	Grossman and Rossi-Hansberg (2006)		X		
	Grossman and Rossi-Hansberg (2008)		X		
	Grossman and Rossi-Hansberg (2012)		X		
Assortative matching	Becker (1973)			X	
	Helpman (2016)			X	
Firms' heterogeneity	Melitz (2003)				X
	Bernard et al. (2007)				X
Labor market frictions	Diamond (1982a)			X	X
	Diamond (1982b)			X	X
	Mortensen and Pissarides (1994)			X	X
Observable attributes	Antrás et al. (2006)			X	
	Costinot and Vogel (2010)			X	
	Sampson (2014)	X			
	Grossman and Helpman (2018)	X			
Observable attributes and technology choice	Yeaple (2005)	X			
	Bustos (2011)	X			
Residual inequality	Helpman et al. (2010)	X			
	Amiti and Davis (2012)	X			
Total	27	12	3	9	6

A.2 Threshold task and labor units

From the definition of price indexes in (17) we can have that:

$$\frac{P_s^i}{P_s^h} = \frac{P_{v_s}^i}{P_{v_s}^h}\left(\frac{v_s}{1-v_s}\right)^\alpha \Leftrightarrow \frac{P_{v_s}^i}{P_{v_s}^h} = \frac{P_s^i}{P_s^h}\left(\frac{1-v_s}{v_s}\right)^\alpha. \tag{57}$$

We have that: (i) $\frac{P_{v_s}^i}{P_{v_s}^h}$ is a continuous function of v_s; (ii) Since $\frac{P_s^i}{P_s^h}$ is assumed to be a positive constant, $\frac{P_{v_s}^i}{P_{v_s}^h}$ varies negatively with v_s, *ceteris paribus*; (iii) $\lim_{v_s \to 1} \frac{P_{v_s}^i}{P_{v_s}^h} = 0$; and (iv) $\lim_{v_s \to 0} \frac{P_{v_s}^i}{P_{v_s}^h} = \infty$. Using (i)-(iv) by the Intermediate Value Theorem there is a $\overline{v}_s \in [0,1]$ such that $\frac{P_{v_s}^i}{P_{v_s}^h} = 1 \Leftrightarrow P_{v_s}^i = P_{v_s}^h$. Moreover by (i) for $v_s > \overline{v}_s$, $P_{v_s}^i < P_{v_s}^h$ and for $v_s < \overline{v}_s$ we have that $P_{v_s}^i > P_{v_s}^h$. Since the output of each variety v_s is produced in perfect competition, firms opt for producing task v_s with the lowest price. Therefore, for $v_s = \overline{v}_s$ they are indifferent between labor types, but for $v_s < \overline{v}_s$ $(v_s > \overline{v}_s)$ they choose L_s^h (L_s^i). From here, we can also establish that:

$$\frac{P_s^i}{P_s^h} = \left(\frac{\overline{v}_s}{1-\overline{v}_s}\right)^\alpha. \tag{58}$$

From the profit maximization problems of the producers of output in sector $s = \{N, R\}$ and task v_s we have that – see (13) and (14) – $P_{v_s}^h(t) \cdot Y_{v_s}^h(t) = \left(P_{v_s}^h(t)\right)^{\frac{1}{\alpha}} \cdot \left[\frac{1-\alpha}{p(k,j,t)}\right]^{\frac{1-\alpha}{\alpha}} \cdot Q_s^h(t) \cdot (1-v_s(t)) \cdot l_s^h \cdot L_{v_s}^h$ and $P_{v_s}^i(t) \cdot Y_{v_s}^i(t) = \left(P_{v_s}^i(t)\right)^{\frac{1}{\alpha}} \cdot \left[\frac{1-\alpha}{p(k,j,t)}\right]^{\frac{1-\alpha}{\alpha}} \cdot Q_s^i(t) \cdot v_s(t) \cdot l_s^i \cdot L_{v_s}^i$, which bearing in mind (17) allow us to write $L_{v_s}^h = \frac{P_{v_s}^h(t) \cdot Y_{v_s}^h(t)}{(P_s^h(t))^{\frac{1}{\alpha}} \cdot \left[\frac{1-\alpha}{p(k,j,t)}\right]^{\frac{1-\alpha}{\alpha}} \cdot Q_s^h(t) \cdot l_s^h}$ for $v_s \in [0, \overline{v}_s)$ and $L_{v_s}^i = \frac{P_{v_s}^i(t) \cdot Y_{v_s}^i(t)}{(P_s^i(t))^{\frac{1}{\alpha}} \cdot \left[\frac{1-\alpha}{p(k,j,t)}\right]^{\frac{1-\alpha}{\alpha}} \cdot Q_s^i(t) \cdot l_s^i}$ for $v_s \in (\overline{v}_N, 1]$. Since $P_{v_s}^i(t) \cdot Y_{v_s}^i(t)$ is constants for all $v_s \in [0,1]$, $p(k,j,t) = p(k,j,t) = q$ – as it will be shown in Section 3.3 –, and $\left(P_s^h\right)^{\frac{1}{\alpha}}$ and $\left(P_s^i\right)^{\frac{1}{\alpha}}$ are also constants, it becomes clear that both $L_{v_s}^h$ and $L_{v_s}^i$ are constants, implying that:

$$L_s^- \equiv \int_0^{\overline{v}_s} L_{v_s}^- \cdot dv_s = L_{v_s}^h \cdot \overline{v}_s \; and \; L_s^h \equiv \int_{\overline{v}_s}^1 L_{v_s}^i \cdot dv_s = L_{v_s}^i \cdot (1-\overline{v}_s). \tag{59}$$

Finally, using (58) and (59), we can determine $\overline{v}_N$, by solving the equation $P_{\overline{v}_s}^h \cdot Y_{\overline{v}_s}^h = P_{\overline{v}_s}^i \cdot Y_{\overline{v}_s}^i$, from which we obtain (18); i.e., $\overline{v}_s = \left[1 + \left(\frac{Q_s^i}{Q_s^h}\frac{l_s^i}{l_s^h}\frac{L_s^i}{L_s^h}\right)^{\frac{1}{2}}\right]^{-1}$. Further, developing the expression for $\overline{v}_N$, we have that $\overline{v}_s = \frac{(Q_s^h l_s^h L_s^h)^{\frac{1}{2}}}{(Q_s^h l_s^h L_s^h)^{\frac{1}{2}} + (Q_s^h l_s^i L_s^i)^{\frac{1}{2}}}$. Therefore, the threshold task can be interpreted as the weight of effective unskilled labor units in total effective labor units used in sector $s = \{N, R\}$.

A.3 Price indexes of tasks and price of the output in each sector

In this appendix we determine the values for price indexes of tasks produced with each type of labor. We start from $P_s = \exp\left(\int_0^1 \ln P_{v_s} dv_s\right)$, to write $\ln P_s = \int_0^{\overline{v}_s} \ln P_{v_s}^h \cdot dv_s + \int_{\overline{v}_s}^1 \ln P_{v_s}^i \cdot dv_s$, which from ((17)) results that $\ln P_s = \int_0^{\overline{v}_s} \ln\left[P_s^h (1 - v_s)^{-\alpha}\right] \cdot dv_s + \int_{\overline{v}_s}^1 \ln\left[P_s^i v_s^{-\alpha}\right] \cdot dv_s$ or, in other words, $\ln P_s = \overline{v}_s \ln P_s^h + (1 - \overline{v}_s) \ln P_s^i - \alpha \left[\int_0^{\overline{v}_s} \ln(1 - v_s) \cdot dv_s + \int_{\overline{v}_s}^1 \ln v_s \cdot dv_s\right]$. Now, since $\int_0^{\overline{v}_s} \ln(1 - v_s) \cdot dv_s = (\overline{v}_s - 1)\ln(1 - \overline{v}_s) - \overline{v}_s$, $\int_{\overline{v}_s}^1 \ln v_s \cdot dv_s = -1 - \overline{v}_s \ln \overline{v}_s + \overline{v}_s$, and from the definition of price indexes $P_s^i = \left(\frac{\overline{v}_s}{1-\overline{v}_s}\right)^\alpha P_s^h$, we have that

$$P_s^h = P_s \cdot \exp(-\alpha) \cdot \overline{v}_s^{-\alpha},$$

and, replacing in the relation between price indexes, we also have:

$$P_s^i = P_s \cdot \exp(-\alpha) \cdot (1 - \overline{v}_s)^{-\alpha}.$$

Moreover, from the maximization problem of the producer of Y we have that: $P_Y = \left[\sum_{s=N,R} \chi_s^\varepsilon \cdot P_s^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$ and thus

$$P_N = \left[\frac{P_Y^{1-\varepsilon} - \chi_R^\varepsilon \cdot P_R^{1-\varepsilon}}{\chi_N^\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{60}$$

which replaced in the expression of the relative price of the N-sector (28) allows to obtain $\frac{\left[\frac{P_Y^{1-\varepsilon} - \chi_R^\varepsilon \cdot P_R^{1-\varepsilon}}{\chi_N^\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{P_R} = \left(\frac{\chi_N}{\chi_R}\right)^{\frac{\epsilon\cdot\alpha}{\epsilon\cdot\alpha+1}} \left(\frac{M_N}{M_R}\right)^{-\frac{\epsilon}{\epsilon\cdot\alpha+1}}$ that is equivalent to $P_Y^{1-\varepsilon} = P_R^{1-\varepsilon}\left[\chi_N^\varepsilon \left(\frac{\chi_N}{\chi_R}\right)^{\frac{\epsilon\cdot\alpha\cdot(1-\epsilon\cdot)}{\epsilon\cdot\alpha+1}} \left(\frac{M_N}{M_R}\right)^{-\frac{\epsilon\cdot(1-\epsilon\cdot)}{\epsilon\cdot\alpha+1}} + \chi_R^\varepsilon\right]$. Solving the last expression in order P_R gives:

$$P_R = \left[\frac{\chi_R^{\frac{\epsilon\cdot\alpha\cdot(1-\epsilon\cdot)}{\epsilon\cdot\alpha+1}} M_R^{-\frac{\alpha\cdot(1-\epsilon\cdot)}{\epsilon\cdot\alpha+1}}}{\chi_R^{\epsilon+\frac{\epsilon\cdot\alpha\cdot(1-\epsilon\cdot)}{\epsilon\cdot\alpha+1}} M_R^{-\frac{\epsilon\cdot(1-\epsilon\cdot)}{\epsilon\cdot\alpha+1}} + \chi_N^{\epsilon+\frac{\epsilon\cdot\alpha\cdot(1-\epsilon\cdot)}{\epsilon\cdot\alpha+1}} (M_N)^{-\frac{\epsilon\cdot(1-\epsilon\cdot)}{\epsilon\cdot\alpha+1}}}\right]^{\frac{1}{1-\epsilon}} \cdot P_Y. \tag{61}$$

We can now use the price of the output in the R-sector to find the price of the output in the N-sector. For this purpose, it is sufficient to conjugate (61) and (60) to obtain:

$$P_N = \left[\frac{\chi_N^{\frac{\epsilon \cdot \alpha \cdot (1-\epsilon \cdot)}{\epsilon \cdot \alpha + 1}} (M_N)^{-\frac{\alpha \cdot (1-\epsilon \cdot)}{\epsilon \cdot \alpha + 1}}}{\chi_R^{\epsilon + \frac{\epsilon \cdot \alpha \cdot (1-\epsilon \cdot)}{\epsilon \cdot \alpha + 1}} M_R^{-\frac{\epsilon \cdot (1-\epsilon \cdot)}{\epsilon \cdot \alpha + 1}} + \chi_N^{\epsilon + \frac{\epsilon \cdot \alpha \cdot (1-\epsilon \cdot)}{\epsilon \cdot \alpha + 1}} (M_N)^{-\frac{\epsilon \cdot (1-\epsilon \cdot)}{\epsilon \cdot \alpha + 1}}} \right]^{\frac{1}{1-\epsilon}} \cdot P_Y. \tag{62}$$

A.4 Incentives to innovate

Improvements in quality can be achieved either by the incumbent firm or by a new entrant. In the first case, the incumbent firm was producing a machine j with quality $k - 1$ and practicing a price q. By improving the quality level to k, the incumbent also changes prices to q^2. Therefore, the change in profits for the incumbent is – for the case in which $h = \{-, B\}$:

$$\Delta\pi^h_{Incumbent}(j) = \pi^h(k, j) - \pi^h(k-1, j) = (q-1) \cdot \left[\frac{P_s^{L_s^h}(t) \cdot (1-\alpha)}{q} \right]^{\frac{1}{\alpha}} \cdot Q_s^h(t) \cdot l_s^h \cdot L_s^h \cdot \left[(q+1) \cdot q^{-\frac{1}{\alpha}} - q^{\frac{\alpha}{1-\alpha}} \right].$$

In the second case, the incumbent firm begins producing a machine j with quality k and practices the price q since it is new to the market. Therefore, the change in profits for the entrants is:

$$\Delta\pi^h_{Entrants}(j) = \pi^h(k, j) = (q-1) \cdot \left[\frac{P_s^{L_s^h}(t) \cdot (1-\alpha)}{q} \right]^{\frac{1}{\alpha}} \cdot Q_s^h(t) \cdot l_s^h \cdot L_s^h.$$

Comparing both we have that $\Delta\pi^h_{Incumbent}(j) = \left[(q+1) \cdot q^{-\frac{1}{\alpha}} - q^{\frac{\alpha}{1-\alpha}} \right] \Delta\pi^h_{Entrants}(j)$. Since $0 < \alpha < 1$ and $q = \frac{1}{1-\alpha}$, we have that $\left[(q+1) \cdot q^{-\frac{1}{\alpha}} - q^{\frac{\alpha}{1-\alpha}} \right] < 1$ and, therefore, $\Delta\pi^h_{Incumbent}(j) < \Delta\pi^h_{Entrants}(j)$, implying that the innovation effort will be carried out by the new entrant.

A.5 Market value of patents

Each moment in time in sector $s = \{N, R\}$ and for the case in which h, as example, there is a probability $\mathcal{I}_s^s(k, j, t)\, dt$ that the quality level improves by 1, *i.e.*, $k(j, t + dt) - k(j, t) = 1$, and a probability $\left(1 - \mathcal{I}_s^h(k, j, t)\right) dt$ that there is no improvement in the quality level, i.e., $k(j, t + dt) - k(j, t) = 0$. Bearing this in mind, if we consider each moment in time as a random experiment that can result in a success with probability $\mathcal{I}_s^h(k, j, t)$, we can characterize the time derivative of $k(j,t)$ as a random variable that follows a Binomial Distribution with an expected value of $\mathcal{I}_s^h(k,j,t)$, i.e., $\dot{k}(j,t) \sim B\left(1, \mathcal{I}_s^h(k,j,t)\right)$. Therefore, although $k(j,t)$ assumes only integer values, $k(j,t)$ and all the variables that depend on it can be differentiated in relation to time but, as a result of the derivative being stochastic, they are also random variables.

The value of the leading-edge patent for the producer of a machine j with quality level k at time t is the present value of flow of profits given by the following equation:

$$V_s^h(j,k,t,T(k)) = \int_t^{t+T(k)} \pi_s^h(j,s) \exp\left(-\int_t^v r(w)dw\right) dv,$$

where $T(k)$ is the duration of the patent during which there is no innovation in the quality level of machine j by another entrant. Since $k(j,t)$ is a random variable, $T(k)$ is also a random variable with a probability distribution that is equal to $B_s^h(T(k)=\tau) = \left(1 - \int_0^\tau B(T(k)=z)dz\right) \cdot \mathcal{I}_s^h(j,t+\tau)$. The intuition behind this formula is that the probability of no quality improvement of a machine j with quality level k being exactly equal to τ since time t, the time in which the monopoly was initiated, is the probability of no improvement occurring before $t+\tau$, times the probability of a successful innovation at time $t+\tau$ (Barro and Sala-i-Martin 2004) occurring in sector s. In the case of the value of a patented innovation, V_s^h, the challenge comes from a new innovation. Assuming that $\mathcal{I}_s^h(j,t+\tau) = \mathcal{I}_s^h(j,t)$, and the $B_s^h(T(k)=0)=0$, we have that $B_s^h(T(k)=\tau) = \mathcal{I}_s^h(j,t) \cdot \exp\left(-\mathcal{I}_s^h(j,t)\cdot\tau\right)$. Since $V_s^h(j,k,t,T(k))$ depends on $T(k)$, this is also a random variable with the same probability density function of $T(k)$, $B_s^h(T(k)=\tau)$. Assuming that the investors are risk-neutral implies that they only care about the expected value of $V_s^h(j,k,t,T(k))$ (Gil et al. 2013), which is equal to the following expression:

$$V_s^h(j,k,t) = \int_0^\infty \pi_s^h(j,s) \exp\left(-\left(\int_t^v r(w) + \mathcal{I}_s^h(j,t)\right) dw\right) dv.$$

Assuming that all the prices and quantities are fixed during the time in which there is no quality improvements (e.g., Aghion and Howitt 1992, Barro and Sala-i-Martin 2004, Gil et al. 2013), then we have that:

$$V_s^h(j,k,t) = \frac{\pi_s^h(j,k,t)}{r(t) + \mathcal{I}_s^h(j,t)}.$$

Notice that this always holds, even outside the steady state.

A.6 Aggregate resources constraint

Let $a = \sum_{s=N,R} \int_0^J V_s^h(k,j,t)dj + \int_J^1 V_s^i(k,j,t)dj$ be the total market value of all the firms that produce machines at time t. From the definition of market value of a firm and taking into account that in equilibrium $\mathcal{I}_s^h(k,j,t) = \mathcal{I}_s^h(t)$ and $\mathcal{I}_s^i(k,j,t) = \mathcal{I}_s^i(t)$, we can write that $V_s^h(k,j,t) = \frac{\pi_s^h(k,j,t)}{r(t)+\mathcal{I}_s^h(j,t)}$ and $V_s^i(k,j,t) = \frac{\pi_s^i(k,j,t)}{r(t)+\mathcal{I}_s^i(j,t)}$ – see Appendix A.5 –, which is equivalent to $r(t) \cdot V_s^h(k,j,t) = (q-1) \cdot x_s^h(k,j,t) - \mathcal{I}_s^h(j,t) \cdot V_s^h(k,j,t)$ and $r(t) \cdot V_s^i(k,j,t) = (q-1) \cdot x_s^i(k,j,t) - \mathcal{I}_s^i(j,t) \cdot V_s^i(k,j,t)$ since $\pi_s^h(k,j,t) = (q-1) \cdot x_s^h(k,j,t)$ and $\pi_s^i(k,j,t) = (q-1) \cdot x_s^i(k,j,t)$. Moreover, from the free-entry condition we have that $\mathcal{I}_s^h(j,t) \cdot V_s^h(k+1,j,t) = e_s^h(k,j,t)$ and $\mathcal{I}_s^i(j,t) \cdot V_s^i(k+1,j,t) = e_s^i(k,j,t)$, i.e., $e_s^h(k-1,j,t) = \mathcal{I}_s^h(j,t) \cdot V_s^h(k,j,t)$ and $e_s^i(k-1,j,t) = \mathcal{I}_s^i(j,t) \cdot V_s^i(k,j,t)$. From (36) and (37) we have $e_s^h(k-1,j,t) = \mathcal{I}_s^h(t) \cdot \frac{\beta}{\zeta} \cdot q^{[k(j,t)-1](\frac{\alpha-1}{\alpha})} \cdot (L_s^h)^\xi$ and $e_s^i(k-1,j,t) = \mathcal{I}_s^i(t) \cdot \frac{\beta}{\zeta} \cdot q^{[k(j,t)-1](\frac{\alpha-1}{\alpha})} \cdot (L_s^i)^\xi$, thus, $e_s^h(k-1,j,t) = q^{(\frac{\alpha-1}{\alpha})} \cdot e_s^h(k,j,t)$ and $e_s^i(k-1,j,t) = q^{(\frac{\alpha-1}{\alpha})} \cdot e_s^i(k,j,t)$. Using the prior information and integrating over j we have that $\int_0^J r(t) \cdot V_s^h(k,j,t)\,dj = \int_0^J (q-1) \cdot x_s^h(k,j,t) \cdot dj - \int_0^J e_s^h(k-1,j,t)$ and $\int_J^1 r(t) \cdot V_s^i(k,j,t)\,dj = \int_J^1 (q-1) \cdot x_s^i(k,j,t) \cdot dj - \int_J^1 e_s^i(k-1,j,t)$, which is equivalent to $r(t) \cdot a_s^h(t) = \left(q \cdot X_s^h(t) - X_s^h(t)\right) - q^{(\frac{\alpha-1}{\alpha})} \int_0^J e_s^h(k,j,t) \cdot dj$ and $r(t) \cdot a_s^i(t) = (q \cdot X_s^i(t) - X_s^i(t)) - q^{(\frac{\alpha-1}{\alpha})} \int_J^1 e_s^i(k,j,t) \cdot dj$. Therefore, we have that $r(t) \cdot a_s^h(t) = \left(q \cdot X_s^h(t) - X_s^h(t)\right) - q^{(\frac{\alpha-1}{\alpha})} \cdot E_s^h(t)$ and $r(t) \cdot a_s^i(t) = (q \cdot X_s^i(t) - X_s^i(t)) - q^{(\frac{\alpha-1}{\alpha})} \cdot E_s^i(t)$, which implies that $r(t) \cdot \underbrace{\left[a_s^h(t) + a_s^i(t)\right]}_{a_s} = \left\{ q \underbrace{\left[X_s^h + X_s^i\right]}_{X_s} - \left[X_s^h + X_s^i\right] \right\} - q^{\left(\frac{\alpha-1}{\alpha}\right)} \cdot \underbrace{\left[E_s^h(t) + E_s^i(t)\right]}_{E_s}$. In turn, from (24) and (25) we have $q \cdot X_s = (1-\alpha) \cdot P_s \cdot Y_s$ and therefore $r \cdot a_s = (1-\alpha) \cdot P_s \cdot Y_s - X_s - q^{(\frac{\alpha-1}{\alpha})} \cdot E_s$. Considering both sectors of the economy, the previous analysis can be summarized in the expression $r \cdot (a_N + a_R) = (1-\alpha) \cdot (P_N \cdot Y_N + P_R \cdot Y_R) - (X_N + X_R) - q^{(\frac{\alpha-1}{\alpha})} \cdot (E_N + E_R)$; i.e.,

$$r \cdot a = (1-\alpha) \cdot Y - X - q^{(\frac{\alpha-1}{\alpha})} \cdot E. \tag{63}$$

From (15) and (16) we have that

$$\begin{aligned}\sum_{s=N,R}\left(w_s^h\cdot L_s^h+w_s^i\cdot L_s^i\right)&=w_N^h\cdot L_N^h+w_N^i\cdot L_N^i+w_R^h\cdot L_R^h+w_R^i\cdot L_R^i\\&=\frac{\alpha\cdot P_N\cdot Y_N^h}{L_N^h}L_N^h+\frac{\alpha\cdot P_N\cdot Y_N^i}{L_N^i}L_N^i+\frac{\alpha\cdot P_R\cdot Y_R^h}{L_R^h}L_R^h+\frac{\alpha\cdot P_R\cdot Y_R^i}{L_R^i}L_R^i\\&=\alpha\cdot\left(P_N\cdot Y_N^h+P_N\cdot Y_N^i\right)+\alpha\cdot\left(P_R\cdot Y_R^h+P_R\cdot Y_R^i\right)=\alpha\cdot\left(P_N\cdot Y_N+P_R\cdot Y_R\right)\\&=\alpha\cdot Y,\end{aligned}$$

which replaced together with (63) in the flow budget constraint

$$\begin{aligned}\dot{a}&=r\cdot a+\sum_{s=N,R}\left(w_{L_s^h}\cdot L_s^h+w_{L_s^i}\cdot L_s^i\right)-C\\&=(1-\alpha)\cdot Y-X-q^{\left(\frac{\alpha-1}{\alpha}\right)}\cdot E+\alpha\cdot Y-C\\&=Y-X-q^{\left(\frac{\alpha-1}{\alpha}\right)}\cdot E-C.\end{aligned}\qquad(64)$$

Returning to the definition of market value of firms, $V_s^h\left(k,j,t\right)=\frac{\pi_s^h(k,j,t)}{r(t)+\mathcal{I}_s^h(j,t)}$ and $V_s^i\left(k,j,t\right)=\frac{\pi_s^i(k,j,t)}{r(t)+\mathcal{I}_s^i(j,t)}$, bearing in mind $\pi_{v_s}^h(k,j,t)$ and $\pi_{v_s}^i(k,j,t)$ in section 3.3, and (36) and (37) we have that $V_s^h\left(k,j,t\right)=\frac{\zeta}{\beta}\cdot q^{\left(\frac{\alpha-1}{\alpha}\right)}\cdot q^{k(j,t)\left(\frac{1-\alpha}{\alpha}\right)}\cdot\left(L_s^h\right)^{\xi}$ and $V_s^i\left(k,j,t\right)=\frac{\zeta}{\beta}\cdot q^{\left(\frac{\alpha-1}{\alpha}\right)}\cdot q^{k(j,t)\left(\frac{1-\alpha}{\alpha}\right)}\cdot\left(L_s^i\right)^{\xi}$. Therefore, the time derivative assets of producers of intermediate goods used in sector s are $\dot{a}_s^h=V_s^h\left(k,j,t\right)=\frac{\zeta}{\beta}\cdot q^{\left(\frac{\alpha-1}{\alpha}\right)}\cdot\left(L_s^h\right)^{\xi}\cdot\dot{Q}_s^h$ and $\dot{a}_s^i=V_s^i\left(k,j,t\right)=\frac{\zeta}{\beta}\cdot q^{\left(\frac{\alpha-1}{\alpha}\right)}\cdot\left(L_s^i\right)^{\xi}\cdot\dot{Q}_s^i$. Therefore, the time variation of total assets is as follows – bearing also in mind 43:

$$
\begin{aligned}
\dot{a} &= \dot{a}_N^h + \dot{a}_N^i + \dot{a}_R^h + \dot{a}_R^i \\
&= \frac{\zeta}{\beta} \cdot q^{\left(\frac{\alpha-1}{\alpha}\right)} \cdot \left\{ \left(L_N^h\right)^{\xi} \cdot \dot{Q}_N^h + \left(L_N^i\right)^{\xi} \cdot \dot{Q}_N^i + \left(L_R^h\right)^{\xi} \cdot \dot{Q}_R^h + \left(L_R^i\right)^{\xi} \cdot \dot{Q}_R^i \right\} \\
&= \frac{\zeta}{\beta} \cdot q^{\left(\frac{\alpha-1}{\alpha}\right)} \cdot \left\{ \left(L_N^h\right)^{\xi} \cdot \mathcal{I}_N^h + \left(L_N^i\right)^{\xi} \cdot \mathcal{I}_N^i + \left(L_R^h\right)^{\xi} \cdot \mathcal{I}_R^h + \left(L_R^i\right)^{\xi} \cdot \mathcal{I}_R^i \right\} \cdot \left[q^{\left(\frac{1-\alpha}{\alpha}\right)} - 1 \right] \\
&= \left[1 - q^{\left(\frac{\alpha-1}{\alpha}\right)} \right] \cdot \frac{\zeta}{\beta} \cdot \left\{ \left(L_N^h\right)^{\xi} \cdot \mathcal{I}_N^h \cdot Q_N^h + \left(L_N^i\right)^{\xi} \cdot \mathcal{I}_N^i \cdot Q_N^i + \left(L_R^h\right)^{\xi} \cdot \mathcal{I}_R^h \cdot Q_R^h + \left(L_R^i\right)^{\xi} \cdot \mathcal{I}_R^i \cdot Q_R^i \right\} \\
&= \left[1 - q^{\left(\frac{\alpha-1}{\alpha}\right)} \right] \cdot (E_N + E_R) \\
&= \left[1 - q^{\left(\frac{\alpha-1}{\alpha}\right)} \right] \cdot E \qquad (65)
\end{aligned}
$$

Finally, replacing (65) in the flow budget constraint (64) from the households, we have that $Y = C + X + E$.

A.7 Steady-state price of the output in each sector

From (61) and (60)), the steady-state price of the output in sector U is

$$
P_R^* = \left[\frac{\chi_R^{\frac{\epsilon \cdot \alpha \cdot (1-\epsilon \cdot)}{\epsilon \cdot \alpha + 1}} M_R^{*\left[-\frac{\epsilon \cdot (1-\epsilon \cdot)}{\epsilon \cdot \alpha + 1}\right]}}{\chi_R^{\epsilon + \frac{\epsilon \cdot \alpha \cdot (1-\epsilon \cdot)}{\epsilon \cdot \alpha + 1}} M_R^{*\left[-\frac{\epsilon \cdot (1-\epsilon \cdot)}{\epsilon \cdot \alpha + 1}\right]} + \chi_N^{\epsilon + \frac{\epsilon \cdot \alpha \cdot (1-\epsilon \cdot)}{\epsilon \cdot \alpha + 1}} M_N^{*\left[-\frac{\epsilon \cdot (1-\epsilon \cdot)}{\epsilon \cdot \alpha + 1}\right]}} \right]^{\frac{1}{1-\epsilon}} \cdot P_Y. \qquad (66)
$$

From (67) and (**??**), the steady-state price of the output in sector H is

$$
P_N^* = \left[\frac{\chi_N^{\frac{\epsilon \cdot \alpha \cdot (1-\epsilon \cdot)}{\epsilon \cdot \alpha + 1}} M_N^{*\left[-\frac{\epsilon \cdot (1-\epsilon \cdot)}{\epsilon \cdot \alpha + 1}\right]}}{\chi_R^{\epsilon + \frac{\epsilon \cdot \alpha \cdot (1-\epsilon \cdot)}{\epsilon \cdot \alpha + 1}} M_R^{*\left[-\frac{\epsilon \cdot (1-\epsilon \cdot)}{\epsilon \cdot \alpha + 1}\right]} + \chi_N^{\epsilon + \frac{\epsilon \cdot \alpha \cdot (1-\epsilon \cdot)}{\epsilon \cdot \alpha + 1}} M_N^{*\left[-\frac{\epsilon \cdot (1-\epsilon \cdot)}{\epsilon \cdot \alpha + 1}\right]}} \right]^{\frac{1}{1-\epsilon}} \cdot P_Y. \qquad (67)
$$