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Ex Ante Versus *Ex Post* Equality of Opportunity

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We study the difference between the *ex post* and *ex ante* perspectives in equality of opportunity. We show that the well documented conflicts between compensation and reward are but an aspect of a broader conflict between *ex ante* and *ex post* perspectives. The literature that takes the goal of providing equal opportunities as the guiding principle generally considers that this is implemented only when, *ex post*, all individuals with the same effort obtain equal success. It is easy to believe that *ex ante* compensation is another natural embodiment of the same idea. We show that this is not true.

INTRODUCTION

In the last two decades a new literature has flourished in the field of normative economics and distributional analysis: the equality of opportunity literature.[1] This literature has developed concepts of fairness for the context in which individual achievements are partly the outcome of morally arbitrary circumstances (such as inherited endowments, social background, etc.) and partly the outcome of individual effort or similar variables of personal responsibility.[2] Such concepts revolve around the idea that inequalities due to circumstances are unfair and should be eliminated as much as possible, while inequalities due to unequal effort should be considered acceptable. Some contributions in this literature have addressed the problem of designing fair allocation rules inspired by the equality of opportunity ideal,[3] while another part of the literature has been concerned with the characterization of social rankings and measures of inequality of opportunity.[4]

This literature has motivated a rapidly growing empirical literature interested in measuring the degree of opportunity inequality and in evaluating public policies in terms of equality of opportunity, in different countries and in different spheres of social life, such as, for instance, health and health care (Fleurbaey and Schokkaert 2009; Li Donni *et al.* 2011; Rosa Dias 2009; Trannoy *et al.* 2010), education (Betts and Roemer 2006; Peragine and Serlenga 2008) and income distributions (see, among others, Aaberge *et al.* 2011; Checchi and Peragine 2010; Lefranc *et al.* 2009; Roemer *et al.* 2003). Book-length collections of empirical analyses of equality of opportunity in developing countries can be found in World Bank (2006) and de Barros *et al.* (2009).

The growing interest for equality of opportunity, in addition to normative reasons, could also have an instrumental justification. First, studying the opportunity inequality in a given economy could help in understanding the mechanisms that generate inequalities in other more traditional spaces, such as income or welfare or other individual achievements. Second, as suggested by Bourguignon *et al.* (2007b) and World Bank (2006), the degree of circumstances-based inequalities, rather than income inequality, could be related to aggregate economic performance and economic growth. The idea is that the existence of 'inequality traps', which permanently exclude entire groups of the population from the participation into social and economic life, imposes strong constraints on growth and development. One of the reasons for the inconclusiveness of the empirical literature on inequality and growth, it is argued, is exactly the nature of the inequality concept used in that literature: overall inequality is used, without

distinguishing between inequalities due to exogenous circumstances and inequalities due to differential effort. The hypothesis is that these two types of inequality may affect growth in opposite ways.[5]

Finally, the interest for equality of opportunity might be motivated by the existing link between the perception of fairness (interpreted as equality of opportunities) and the individual attitudes toward redistribution. In particular, recent surveys show that most people judge income inequalities arising from different levels of effort as less objectionable than those due to exogenous circumstances such as race, family origin, etc. (see Inglehart *et al.* 2004). Moreover, as shown by Alesina and La Ferrara (2005) using US data, in general, people who believe that they live in a land of equal opportunities for all do not look favourably on government redistribution. Hence the evidence that a large amount of existing inequalities is due to unequal opportunities might increase the support for redistributive policies.

Now, although the opportunity egalitarian perspective has gained a consensus, both in the public debate and in the scientific literature, such consensus seems to be less robust when going from general concepts to more specific formulations.

In fact, the literature has clarified that the ideal of equal opportunities is multifaceted, which is the source of potential conflicts between various interpretations of the ideal and of its components. In particular, it has highlighted the distinction between the 'compensation' principle that 'inequalities due to circumstances should be eliminated' and the 'responsibility' principle that 'inequalities due to unequal effort should be considered acceptable', showing that the two principles actually tend to clash as soon as they are given precise expressions. Moreover, the latter principle can be applied in various ways, the two prominent ones being the 'liberal reward' principle that 'inequalities due to unequal effort should be left untouched'—prohibiting redistribution between individuals with identical circumstances—and the 'utilitarian reward' principle that 'inequalities due to unequal effort do not matter'—advocating a sum-maximizing policy among subgroups with identical circumstances.[6]

We argue in this paper that the clash between 'compensation' and 'responsibility' can be traced to a deeper divide between the *ex post* perspective and the *ex ante* perspective on opportunities. From the *ex ante* perspective, opportunities are evaluated by the circumstances and the outcome possibilities for various levels of effort that individuals can exert. From the *ex post* perspective, the actual level of effort of each individual can be used in the evaluation of unequal achievements. Our distinction between an *ex ante* situation in which circumstances are determined but not yet effort and an *ex post* situation in which all variables are determined does not always correspond to a real time sequence, but is convenient for an intuitive interpretation of the various approaches.

The compensation principle is usually formulated in the literature in terms of reducing inequality between individuals with the same level of effort but different circumstances, which implies adopting the *ex post* perspective. In contrast, responsibility or reward principles are usually formulated for subgroups with identical circumstances, which is more akin to the *ex ante* perspective. In order to show that the clash has to do with the *ex ante–ex post* divide more than the compensation–responsibility divide, we focus on the compensation principle and introduce alternative formulations of the principle that espouse the *ex ante* perspective. We show that the *ex post* and *ex ante* versions of the compensation principle are incompatible with each other, even though they seemingly emanate from the same ideal.

The second contribution of this paper is to show that the responsibility (or reward) principles can also be derived from *ex ante* compensation requirements applied to

individuals with identical circumstances. Indeed, once a particular measure of the *ex ante* value of opportunities is defined, it can be applied to the reduction of inequalities between individuals with different circumstances, but it can also be applied to individuals with identical circumstances. We show that the latter application makes it possible to derive the standard 'liberal' and 'utilitarian' reward requirements. We also introduce a very minimal reward requirement that is logically weaker than the usual reward axioms, and show that it is also incompatible with *ex post* compensation.

The *ex ante–ex post* distinction provides a convenient way to interpret the contributions of the literature. The contributions that focused on inequalities between social groups defined in terms of circumstances have naturally adopted the *ex ante* perspective. This is the approach proposed, in different frameworks, by Van de gaer (1993) and Kranich (1996), and used in empirical studies, among others, by Bourguignon *et al.* (2007a), Ferreira and Gignoux (2011), Lefranc *et al.* (2009), Peragine and Serlenga (2008) and World Bank (2006). Another part of the literature has been more directly interested in suppressing outcome inequalities between individuals having exerted the same effort, which corresponds to the *ex post* perspective. This is the approach proposed by Roemer (1993, 1998) and Fleurbaey (1995), and used by Aaberge *et al.* (2011) and Checchi and Peragine (2010) for an empirical analysis of opportunity inequality.

The difference between *ex post* and *ex ante* perspectives, and the possibility of a clash between them, has already been hinted at in Checchi and Peragine (2005, 2010), Fleurbaey (2008, ch. 9) and Ooghe *et al.* (2007). We attempt to bring this intuition to complete fruition in this paper, by providing clear and distinct formulations for the principles of '*ex post* compensation' and '*ex ante* compensation'. In summary, this paper is an exploration in the *ex ante* land, showing that its 'compensation' regions (focusing on opportunity inequalities) and its 'reward' regions (focusing on the shape of opportunity sets) are closely connected, and all display a basic incompatibility with *ex post* compensation.

In this paper we concentrate on comprehensive social rankings and leave the analysis of inequality rankings for future research. The paper is structured as follows. Section I introduces the formal framework. Section II shows the tension between *ex ante* and *ex post* perspectives on compensation. Section III examines the reward problem and analyses how it relates to the *ex ante–ex post* tension. Section IV concludes.

I. THE MODEL

Individual outcomes are determined by a function $u(r, c, e)$, where r (resources), c (circumstances), and e (effort) are real numbers.[7] The model could be generalized by assuming that r, c, e are vectors in an ordered set. This would not modify the substance of the analysis. For simplicity of the analysis that follows, effort e is allowed to take only a finite number of values (more than one) in a set $\mathcal{E}$. The function u is assumed to be continuous and strictly increasing in r and c, and not separable in (r, c), i.e. there exist $r, r', c, c', e, e' >$ *such that*

$$u(r, c, e) > u(r', c', e),$$
$$u(r, c, e') < u(r', c', e').$$

An *economy E* is composed of a population, i.e. a finite or infinite set of individuals, partitioned into a finite number of *types* and *cells*. A type is a set of individuals with the

same circumstances c. The set of types is $T(E) = \{1, \ldots, n\}$, with $n \geq 2$. Let c_t denote the circumstance of type t. A cell is a set of individuals with the same characteristics (c, e). The set of cells is $C(E) = \{1, \ldots, m\}$, with $m > n$. The size of cell i is denoted $p(i)$ (it can be an integer for a finite population, or a real number for a continuum of individuals). With an abuse of notation (but no ambiguity), c_i can also denote the circumstance of cell i. Similarly, $p(t)$ can denote the size of type t. We use the notation $t(i)$ to identify the type containing cell i. Obviously $c_i = c_{t(i)}$.

The transfer received by cell i is denoted r_i. Formally, an economy is defined as a vector describing the profile of circumstances and effort for each cell (which induces the definition of the sets $T(E)$ and $C(E)$), as well as the size of each cell $p(\cdot)$:

$$E = (((c_1, e_1), \ldots, (c_m, e_m)), p).$$

In this paper we consider only economies such that in every type t, the whole set $\mathcal{E}$ is spanned by the population effort levels: for all t,

$$\{e : \exists i, \ t(i) = t \text{ and } e_i = e\} = \mathcal{E}.$$

We restrict attention to anonymous transfer policies. With anonymous policies, the individuals with identical (c, e) get the same resource transfer, which defines a function $r(c, e)$ that we will call a transfer rule. Note that for every t, the function $r(c_t, \cdot)$ is unambiguously defined over the whole set $\mathcal{E}$ thanks to the restriction made in the previous paragraph. We can define the reduced outcome function that incorporates the transfer rule:

$$f(c, e) = u(r(c, e), c, e).$$

Let u_i denote the outcome of cell i: $u_i = f(c_i, e_i)$.

Individuals belonging to type t have an opportunity set defined as the possible combinations of effort and outcome that the outcome function f makes accessible to them:

$$O_t = \{(e, f(c_t, e)) : e \in \mathcal{E}\}.$$

An opportunity distribution for the n types is denoted $\mathbf{O} = (O_1, \ldots, O_n)$. For a given cell i, we can also denote $O_i = O_{t(i)}$. Let $\mu(O_t)$ be the average outcome of type t:

$$\mu(O_t) = \frac{1}{p(t)} \sum_{i:\, t(i) = t} p(i) u_i.$$

In view of the monotonicity of u with respect to r, there is a one-to-one mapping between transfer rules r and outcome functions f, as well as between any of these and opportunity distributions. In this paper we focus on the evaluation of outcome functions f, but it would be equivalent to study the evaluation of transfer rules or of opportunity distributions. A *social ordering* function defines, for every economy E in a domain $\mathcal{D}$, an ordering $\succeq(E)$ over all conceivable outcome functions, with $f \succeq (E) f'$ meaning that f is at least as good as f', and $f \succ (E) f'$ meaning that f is better than f'. The domain $\mathcal{D}$ over

which these social ordering functions $\succeq(E)$ are defined is the set of economies satisfying the above conditions.

Contrary to the literature on fair allocation rules (e.g. Fleurbaey (1994) and the subsequent literature), we deal with social orderings. Therefore we make no assumptions about whether individual characteristics are observable. If some of them are not, then the transfer rule r depends only on the observable ones. For instance, if e is not observed, then transfers depend only on c. Our focus being on a social ordering, we seek to rank all transfer rules and associated outcome functions, whether they are based on full observation of characteristics or on partial observation only.[8]

Another difference is that as we evaluate orderings, we do not restrict ourselves to transfer rules that satisfy a condition of budget balance. Typically, the budget constraint determines what transfer rules are feasible, and the selection of the best one is done by maximizing according to the social ordering. It may be worth stressing, though, that our results involve only very simple transfer rules and do not require a wide domain of rules.[9]

II. COMPENSATION: EX POST OR EX ANTE

The *ex post* approach to compensation tries to reduce inequalities between cells having the same level of effort but different levels of outcome. The goal is to achieve a situation in which circumstances are no longer the source of inequalities. This goal is embodied in the following axiom,[10] which says that it is good to reduce inequalities in outcomes between two cells sharing the same effort level but having unequal circumstances.

Ex Post Compensation. For all $E \in \mathcal{D}$, $f \succ (E)f'$ if there are $i, j \in C(E)$, such that $e_i = e_j$,

$$f'(c_i, e_i) > f(c_i, e_i) > f(c_j, e_j) > f'(c_j, e_j),$$

and $f(c_k, e_k) = f'(c_k, e_k)$ for all $k \in C(E) \setminus \{i,j\}$.

The *ex ante* approach to compensation seeks to identify situations of inequality based solely on information linked to the type to which individuals belong, ignoring their effort level. That is, one seeks situations in which two types are clearly unequal in terms of the perspectives offered by their circumstances and the respective transfer policies. This is the case when, as considered in the axiom below, individuals in type $t(i)$ have better circumstances than type $t(j)$, and are assured of receiving more resources. When this is observed, improving the situation of a cell i in the advantaged type while worsening that of a cell j in the disadvantaged type would worsen the situation.

Ex Ante Compensation. For all $E \in \mathcal{D}$, $f \succ (E)f'$ if there are $i, j \in C(E)$ such that $c_i > c_j$, $\min r(c_i, \cdot) > \max r(c_j, \cdot)$,

$$f'(c_i, e_i) > f(c_i, e_i) \quad \text{and} \quad f(c_j, e_j) > f'(c_j, e_j),$$

and $f(c_k, e_k) = f'(c_k, e_k)$ for all $k \in C(E) \setminus \{i,j\}$.

This axiom is very weak, and in particular is much weaker than the next axiom, which applies when the opportunities of a given type, as depicted by the outcome function $f(c, \cdot)$, dominate those of another. As domination of the outcome function can be due to the transfer rule rather than better circumstances, the following axiom[11] covers many more situations than *Ex Ante* Compensation.

Strong Ex Ante Compensation. For all $E \in \mathcal{D}$, $f \succ (E)f'$ if there are $i, j \in C(E)$ such that for all $e \in \mathcal{E}$, $f(c_i, e) > f(c_j, e)$,

$$f'(c_i, e_i) > f(c_i, e_i) \quad \text{and} \quad f(c_j, e_j) > f'(c_j, e_j),$$

and $f(c_k, e_k) = f'(c_k, e_k)$ for all $k \in C(E) \backslash \{i, j\}$.

The *ex ante* and *ex post* approaches to compensation, appealing though each of them may be, are incompatible.

Proposition 1 No social ordering function defined on $\mathcal{D}$ satisfies *Ex Ante* Compensation and *Ex Post* Compensation.

The proof is in the Appendix. The incompatibility between *Ex Post* Compensation and Strong *Ex Ante* Compensation was already shown[12] in Fleurbaey (2008, ch. 9). This result is stronger and shows that even when there is no ambiguity whatsoever about the fact that a cell is better off than another on all counts in terms of their *ex ante* situations (better circumstances, more resources at all effort levels), reducing inequality between them may go against the goal of giving all types the same outcome function, which is encapsulated in *Ex Post* Compensation. Also relevant for the purpose of this paper is the fact that, as we will see in the next section, *Ex Ante* Compensation is sufficiently weak to be compatible with *ex ante* evaluations of opportunity sets that can be related to the main reward principles.

The proof involves only very simple transfer rules and a modification of a transfer rule that does not waste resources or create a deficit in case the initial rule is budget balanced.

It is worth mentioning that the incompatibility would also hold[13] with versions of the compensation axioms that would involve only minimal aversion to inequality, relying on the Pigou–Dalton transfer principle.

Ex Post Pigou–Dalton Compensation. For all $E \in \mathcal{D}$, $f \succ (E)f'$ if there are $i, j \in C(E)$ such that $e_i = e_j$,

$$f'(c_i, e_i) > f(c_i, e_i) > f(c_j, e_j) > f'(c_j, e_j),$$

$$f'(c_i, e_i) - f(c_i, e_i) = f(c_j, e_j) - f'(c_j, e_j),$$

and $f(c_k, e_k) = f'(c_k, e_k)$ for all $k \in C(E) \backslash \{i, j\}$.

Ex Ante Pigou–Dalton Compensation. For all $E \in \mathcal{D}$, $f \succ (E)f'$ if there are $i, j \in C(E)$ such that $c_i > c_j$, $\min r(c_i, \cdot) > \max r(c_j, \cdot)$,

$$f'(c_i, e_i) > f(c_i, e_i) \quad \text{and} \quad f(c_j, e_j) > f'(c_j, e_j),$$

$$f'(c_i, e_i) - f(c_i, e_i) = f(c_j, e_j) - f'(c_j, e_j),$$

and $f(c_k, e_k) = f'(c_k, e_k)$ for all $k \in C(E) \backslash \{i, j\}$.

III. REWARD AND THE EVALUATION OF OPPORTUNITIES

Compensation axioms, whether they take the *ex post* or the *ex ante* standpoint, deal with the reduction of inequalities between individuals endowed with unequal circumstances. In contrast, reward principles are typically embodied in axioms that deal with individuals of the same type, in order to adjust the relationship between their outcome and their effort. We first introduce two axioms which represent the main reward principles that one finds in the literature. The Liberal Reward principle seeks to minimize redistribution related to differential effort levels, and therefore advocates submitting individuals with identical circumstances to equal transfers.[14] This idea is captured by the following axiom, which says that it is an improvement when the inequality in transfers received by two cells from the same type is reduced.

Liberal Reward. For all $E \in \mathcal{D}, f \succ (E) f'$ if there are $i, j \in C(E)$ such that $t(i) = t(j)$,

$$r'(c_i, e_i) > r(c_i, e_i) > r(c_j, e_j) > r'(c_j, e_j),$$

and $f(c_k, e_k) = f'(c_k, e_k)$ for all $k \in C(E) \backslash \{i,j\}$.

The utilitarian principle recommends an evaluation of outcome inequalities within types that is devoid of aversion to inequality, and therefore simply focuses on the sum of outcomes in order to evaluate a change affecting only one type.

Utilitarian Reward. For all $E \in \mathcal{D}, f \succ (E) f'$ if there are $i, j \in C(E)$ such that $t(i) = t(j)$,

$$p(i)f(c_i, e_i) + p(j)f(c_j, e_j) > p(i)f'(c_i, e_i) + p(j)f'(c_j, e_j),$$

and $f(c_k, e_k) = f'(c_k, e_k)$ for all $k \in C(E) \backslash \{i,j\}$.

The inequality in this axiom could equivalently be written as $\mu(O_i) > \mu(O_i')$. (Note that as $t(i) = t(j)$, one has $\mu(O_i) = \mu(O_j)$ and $\mu(O_i') = \mu(O_j')$.) The literature has shown that each of these axioms clashes with *Ex Post* Compensation. In contrast, each of them is compatible with *Ex Ante* Compensation. We now proceed to show that there is a more basic reward axiom that underlies the two axioms. Observe that Liberal Reward expresses a strong inequality aversion with respect to transfers, while Utilitarian Reward reflects zero inequality aversion with respect to outcomes. On the real line between $-\infty$ and $+\infty$, Liberal Reward picks a degree of inequality aversion (w.r.t. transfers) equal to $+\infty$, while Utilitarian Reward picks a degree of inequality aversion (w.r.t. outcomes) equal to 0.

This suggests that a minimal requirement would consist in excluding $-\infty$ for inequality aversion w.r.t. transfers, i.e. excluding the maximax criterion that gives absolute priority to the better-off, and in excluding both ∞ and $+\infty$ for inequality aversion w.r.t. outcomes, i.e. excluding both the maximax and maximin criteria. Such combination of requirements over inequality aversion w.r.t. transfers and outcomes is possible, by positing that when evaluating changes affecting two cells within a type, one should never give absolute priority to the cell that receives more resources. The judgment 'imposing a small sacrifice to the cell having more resources is worth doing if the less endowed cell receives a sufficient increment' is shared by all criteria with non-absolute preference for inequality in resources and by all criteria with non-absolute attitude (aversion or preference) toward inequality in outcomes, and it is rejected by all other criteria.[15]

One can then encapsulate this idea into the following axiom, which is very weak and is logically weaker than Liberal Reward and Utilitarian Reward.

Minimal Reward. For all $E \in \mathcal{D}$ and all $i, j \in C(E)$, if $t(i) = t(j)$ and $r'(c_{i,\ e_i}) > r'(c_j, e_j)$, then there exist $\delta, \delta' > 0$ such that if $r(c_i, e_i) = r'(c_{i,\ e_i}) - \delta$ and $r(c_j, e_j) = r'(c_j, e_j) + \delta'$, while $f(c_k, e_k) = f'(c_k, e_k)$ for all $k \in C(E)\backslash\{i,j\}$, then $f \succ (E)f'$.

Although very weak, this axiom is still excessively tied to the *ex ante* perspective, as shown in the following proposition.

Proposition 2 No social ordering function defined on $\mathcal{D}$ satisfies Minimal Reward and *Ex Post* Compensation.

The proof is in the Appendix. Note that this result no longer holds if *Ex Post* Pigou–Dalton Compensation is considered instead. Any ordinary social welfare function (applied to the distribution of outcomes) that incorporates a positive but finite aversion to inequality satisfies Minimal Reward and *Ex Post* Pigou–Dalton Compensation. This shows how weak Minimal Reward is.

It is interesting to look at the connection between *Ex Ante* Compensation and the reward axioms, as all are based on the *ex ante* approach. In fact, the structure of the proofs of the two propositions shows that a common underlying logic operates and can be uncovered. The Utilitarian Reward principle suggests that opportunities should be evaluated in terms of average outcome. One could then formulate a compensation axiom based on such evaluations, and requiring the opportunities of the least favoured cell to increase —when the two cells belong to the same type, this boils down to Utilitarian Reward.

Ex Ante Utilitarian Compensation. For all $E \in \mathcal{D}$, $f \succ (E)f'$ if there are $i, j \in C(E)$ such that

$$\min\{\mu(O_{t(i)}), \mu(O_{t(j)})\} > \min\{\mu(O_{t(i)}'), \mu(O_{t(j)}')\},$$

and $f(c_k, e_k) = f'(c_k, e_k)$ for all $k \in C(E)\backslash\{i,j\}$.

This axiom is logically stronger than both *Ex Ante* Compensation and Utilitarian Reward.

It is less obvious to see what kind of metric of opportunity the Liberal Reward principle suggests, as Liberal Reward does not tell us how to compare individuals endowed with unequal circumstances—it tells us only that resources can be used for the comparison of individuals of the same type. One possibility is to define opportunities as would be created by the resources actually received by cell i in the allocation under consideration:

$$O_i^r = \big\{\big(e, u(r_i, c_{t(i)}, e)\big) : e \in \mathcal{E}\big\}.$$

Indeed, if one applies this notion to cells belonging to the same type, the comparison of such opportunities is equivalent to comparing the resources that they receive. And one then obtains an axiom that embodies this metric and is logically stronger than both Liberal Reward and *Ex Ante* Compensation.

Ex Ante Liberal Compensation. For all $E \in \mathcal{D}$, $f \succ (E)f'$ if there are $i, j \in C(E)$ such that for all $e \in \mathcal{E}$, $u(r_i,\ c_{t(i)},\ e) > u(r_j, c_{t(j)}, e)$,

$$f'(c_i, e_i) > f(c_i, e_i) \quad \text{and} \quad f(c_j, e_j) > f'(c_j, e_j),$$

and $f(c_k, e_k) = f'(c_k, e_k)$ for all $k \in C(E)\backslash\{i,j\}$.
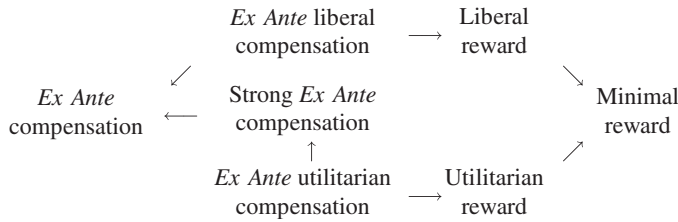
FIGURE 1. Relationships between the axioms.

Figure 1 summarizes the relations between the axioms, all of which are incompatible with *Ex Post* Compensation.

Figure 1 shows that the tension between compensation and reward, in all its variants, vanishes if one adopts an *ex ante* view of equality of opportunity.[16] On the other hand, all the axioms inspired by the *ex ante* view of equality of opportunity are incompatible with *Ex Post* Compensation.

IV. CONCLUSION

The ideal of equal opportunities is multifaceted, and this appears to be the source of potential conflicts between various interpretations of the ideal and of its components. In this paper we have shown that the well documented conflicts between the compensation principles and various reward principles are but an aspect of a broader conflict between *ex ante* and *ex post* perspectives. The compensation principle itself may be trapped in an internal tension between the *ex ante* and the *ex post* neutralization of inequalities in circumstances. We do not believe that such tensions and conflicts reveal an irredeemable inconsistency in the general idea of equalizing opportunity. But they do raise important ethical issues that any analyst or decision-maker interested in this approach must be aware of. The literature that takes the goal of providing equal opportunities as the guiding principle generally considers that this is clearly implemented only when, *ex post*, all individuals with the same effort obtain equal success. Along these lines, it is clear that *Ex Post* Compensation must then be given priority over the other axioms. In particular, we suspect that *Ex Ante* Compensation (or, similarly, Strong *Ex Ante* Compensation) is a misleading principle because it is easy to believe that it is another natural embodiment of the same idea. As we have shown, this is not true.

APPENDIX: PROOFS

Proof of Proposition 1

By assumption there exist r, r', c, c', e, e' such that

$$u(r, c, e) > u(r', c', e),$$

$$u(r, c, e') < u(r', c', e').$$

Let an economy be composed of four types with circumstances $c, c + \varepsilon, c', c' + \varepsilon$, where $\varepsilon > 0$ is sufficiently small so that $u(r, c, e) > u(r' + \varepsilon, c' + \varepsilon, e)$ and $u(r + \varepsilon, c + \varepsilon, e') < u(r', c', e')$. We will focus on eight cells:

$$(c_1, e_1) = (c, e), \quad (c_2, e_2) = (c, e'), \quad (c_3, e_3) = (c + \varepsilon, e),$$

$$(c_4, e_4) = (c + \varepsilon, e'), \quad (c_5, e_5) = (c', e), \quad (c_6, e_6) = (c', e'),$$

$$(c_7, e_7) = (c' + \varepsilon, e), \quad (c_8, e_8) = (c' + \varepsilon, e').$$

Consider an allocation such that $r(c, \cdot) \equiv r$, $r(c + \varepsilon, \cdot) \equiv r + \varepsilon$, $r(c', \cdot) \equiv r'$, $r(c' + \varepsilon, \cdot) \equiv r' + \varepsilon$. Let an alternative allocation $r^*(\cdot, \cdot)$ be derived from this one by modifying the resources given to the following cells:

$$r^*(c_1, e_1) = r + \varepsilon/3, \quad r^*(c_4, e_4) = r + 2\varepsilon/3,$$

$$r^*(c_6, e_6) = r' + \varepsilon/3, \quad r^*(c_7, e_7) = r' + 2\varepsilon/3.$$

Compare cells 1 and 4: $c_4 = c + \varepsilon > c_1 = c$ and

$$\min r^*(c_4, \cdot) = r + 2\varepsilon/3 > \max r^*(c_1, \cdot) = r + \varepsilon/3.$$

Therefore, by *Ex Ante* Compensation, the change from $r(\cdot, \cdot)$ to $r^*(\cdot, \cdot)$ for cells 1 and 4 (leaving all the others unaffected) is good. With the same argument, the change from $r(\cdot, \cdot)$ to $r^*(\cdot, \cdot)$ for cells 6 and 7 is good. By transitivity, the change from $r(\cdot, \cdot)$ to $r^*(\cdot, \cdot)$ is good.
   Compare cells 1 and 7: $e_1 = e_7 = e$ and

$$u_1^* > u_1 = u(r, c, e) > u(r' + \varepsilon, c' + \varepsilon, e) = u_7 > u_7^*.$$

Therefore, by *Ex Post* Compensation, the change from from $r(\cdot, \cdot)$ to $r^*(\cdot, \cdot)$ for cells 1 and 7 (leaving all the others unaffected) is bad. With the same argument, the change from $r(\cdot, \cdot)$ to $r^*(\cdot, \cdot)$ for cells 4 and 6 is bad. By transitivity, the change from $r(\cdot, \cdot)$ to $r^*(\cdot, \cdot)$ is bad.
   We have a contradiction. □

Proof of Proposition 2

By assumption there exist *r, r', c, c', e, e'* such that

$$u(r, c, e) > u(r', c', e),$$

$$u(r, c, e') < u(r', c', e').$$

Let an economy be composed of two types with circumstances *c, c'*, and let $\varepsilon > 0$ be sufficiently small so that $u(r, c, e) > u(r' + \varepsilon, c', e)$ and $u(r + \varepsilon, c, e') < u(r', c', e')$. We will focus on four cells:

$$(c_1, e_1) = (c', e), \quad (c_2, e_2) = (c', e'),$$

$$(c_3, e_3) = (c, e), \quad (c_4, e_4) = (c, e').$$

Consider an allocation such that $r_1 = r' + \varepsilon$, $r_2 = r'$, $r_3 = r$, $r_4 = r + \varepsilon$. Let an alternative allocation r^* be derived from this one by modifying the resources given to these cells as follows:

$$r_1^* = r_1 - \delta, \quad r_2^* = r_2 + \delta', \quad r_3^* = r_3 + \gamma', \quad r_4^* = r_4 - \gamma,$$

where δ, δ', γ, γ' are chosen so that changing r_1, r_2 into r_1^*, r_2^* is good for Minimal Reward, and changing r_3, r_4 into r_3^*, r_4^* is also good for Minimal Reward. By transitivity, changing r_1, r_2, r_3, r_4 into $r_1^*, r_2^*, r_3^*, r_4^*$ is good.

Compare cells 1 and 3: $e_1 = e_3 = e$ and

$$u_1^* < u_1 = u(r' + \varepsilon, c', e) < u(r, c, e) = u_3 < u_3^*.$$

Therefore, by *Ex Post* Compensation, the change from from r to r^* for cells 1 and 3 (leaving all the others unaffected) is bad. With the same argument, the change from r to r^* for cells 2 and 4 is bad. By transitivity, the change from r to r^* is bad.

We have a contradiction. □

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NOTES

1. See Fleurbaey (2008) for a general treatment of the issues discussed in this literature.
2. Seminal contributions are Roemer (1993, 1998). For the background philosophical literature, see Dworkin (1981a,b), Arneson (1989) and Cohen (1989).
3. See Bossert (1995) and Fleurbaey (1994) for initial contributions in this vein.
4. See, among others, Peragine (2002, 2004), Lefranc *et al.* (2009), Bourguignon *et al.* (2007a,b).
5. For a first investigation of the relationship between inequality of opportunity and growth, see Marrero and Rodriguez (2010).
6. These various distinctions are discussed in detail in Fleurbaey (2008).
7. For a different model where in addition to circumstances, effort and resources, also luck plays a role, see Lefranc *et al.* (2009). Here we put luck among circumstances, as in Fleurbaey (2008).
8. This is as in public economics in which a social welfare function is maximized under whatever informational constraints are prevailing—but the social welfare function itself is able to rank all allocations. Note that what we call a transfer rule is not an 'allocation rule' in the usual sense. The latter defines an allocation for every economy in the domain; the former just defines an allocation, by determining how transfers depend on individual characteristics.
9. The proofs invoke simple quantities for small numbers of cells.
10. *Ex Post* Compensation corresponds to the Compensation axiom in Ooghe *et al.* (2007).
11. Strong *Ex Ante* Compensation corresponds to the Dominance Compensation axiom in Ooghe *et al.* (2007).
12. The incompatibility between Strong *Ex Ante* Compensation and *Ex Post* Compensation was implicitly suggested in the discussion by Ooghe *et al.* (2007), although they focus on the axiomatic characterization of the minimum of means and mean of minimums rules (and of generalizations of them) rather than on the incompatibility between various axioms.
13. The adaptation of the proof is left to the reader.
14. There is a strong connection between liberal reward and incentive-compatibility (see Fleurbaey 2008), but the motivation is different. While incentive-compatibility is a constraint due to limitations in observability or verifiability, liberal reward is applicable even in absence of informational imperfections.
15. Obviously, the maximax criterion applied to resources rejects this judgment. The maximax criterion applied to outcomes rejects it when the cell with more resources has a greater outcome, and the maximin criterion applied to outcomes rejects it when the cell with more resources has a lower outcome.
16. This result is hinted at in Checchi and Peragine (2010). These authors in fact interpret the tension between *ex ante* and *ex post* as being due to the fact that the *ex ante* approach is linked to the principle of reward, while the *ex post* approach is inspired by the principle of compensation.

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