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Systematic analyses of the $t+t$ clustering effect in He isotopes

S. Aoyama,[1] N. Itagaki,[2,3] and M. Oi[4]
[1]*Integrated Information Processing Center, Niigata University, 950-2181 Niigata, Japan*
[2]*Hahn-Meitner-Institut Berlin, D-140109 Berlin, Germany*
[3]*Department of Physics, University of Tokyo, Hongo, 113-0033 Tokyo, Japan*
[4]*Department of Physics, University of Surrey, Guildford, GU2 7XH, Surrey, United Kingdom*
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A systematic study on the ground-state structure of He isotopes (including ^{10}He) is presented through a new method developed on the basis of antisymmetrized molecular dynamics (AMD), the generator coordinate method (GCM), and the stochastic variational method (SVM). In this approach, variational calculations are carried out by means of the GCM with the AMD wave functions produced by means of the SVM. A role of the $t+t$ cluster component is examined with the present method, allowing the wider configuration space containing simultaneously the "$t+t$+valence neutrons" structure and "^{4}He+valence neutrons" structure.

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Various intriguing properties of neutron-rich nuclei have been discovered as a result of developments in experimental techniques using radioactive beams [1]. Although study of these neutron-rich nuclei is challenging, it is important that it be done because it may bring us fundamental knowledge of the nature of nuclear forces, as well as of properties of nuclear matter in extreme conditions such as neutron stars. Recently, ^{10}He and ^{5}H were newly added to the nuclear chart [2,3] in the extremely neutron-rich section (the ratio of the neutron and proton numbers is $N/Z = 4$ for these nuclei). ^{10}He attracts particular interest because it is supposed to be a doubly closed-shell nucleus with the magic numbers $Z = 2$ and $N = 8$, and we might expect that the nucleus has a bound state. However, it was found that the nucleus has only a resonant state with the ^{8}He+n+n [2] structure. Therefore, and because ^{8}He has a relatively large two-neutron separation energy ($S_{2n} = 2.13$ MeV [4]), the magic number $N = 8$ is likely shifted to $N = 6$ in the neutron drip line and beyond. It is therefore interesting to systematically investigate the change in the shell structure for He-isotopes as the number of neutrons approaches $N = 8$ from $N = 2$.

It is widely accepted that ^{6}He is a typical case of a nuclear Borromean halo structure. The nucleus has been studied well theoretically by using the ^{4}He+n+n model. Various calculations based on this model were performed by many groups, and they could successfully explain the appearance of the halo structure [5–9]. The next level of study was a quantitative description of the halo structure, in particular, to precisely reproduce the binding energy by using reliable interactions for the subsystems (^{4}He+n, n+n). However, with such interactions and the above-mentioned ^{4}He+n+n models, the resultant binding energy of ^{6}He is accurate only up to 200–300 keV. To account for the discrepancy, Csótó suggested an influence of the $t+t$ component mixing in the ground state of ^{6}He, considering a possibility that the two valence neutrons can slightly distort the strongly bound α clustering configuration [10,11]. In his model, the distortion effect was taken into account through a coupled-channel model including the (^{4}He+n+n) and ($t+t$) structures. Surprisingly, the correct binding energy was successfully obtained through this model.

Such a mixing of the unexpected cluster (or distorted) component in the core part could be a key point in understanding the anomalous neutron orbits in light neutron-rich nuclei. There are some physical systems for which such a consideration has already been given. ^{10}Li is observed to be an unbound system with respect to its breakup into ^{9}Li+n. To explain the observed physical quantities of this nucleus, it is believed that the valence neutron needs to occupy an s state, but the emergence of this unexpectedly low-lying s-state orbit cannot be explained through a simple shell model picture [12]. The appearance of such low-lying s states has been discussed also for the Be isotopes. A possible mechanism to bring down the s orbit is suggested in terms of the $\alpha+\alpha$ clustering (or deformation) effect. This clustering effect is, nonetheless, not yet examined well in other isotopes. It is therefore interesting to extend the investigation to other systems. Systematic study of the competition between the shell structure and the cluster structure in light nuclear systems is worthwhile. In association with rearranging the single-particle levels, we may also be able to explain, through the investigation of the clustering effect, the disappearance of the $N = 8$ shell gap in light nuclei, which has been partly explained by the pairing blocking effect [13,14].

Thus, the aim of this paper is to study the $t+t$ clustering effect in He isotopes, as the first step in the systematic study. We attempt to show how additional neutrons can break the simple α-core structure to allow the $t+t$ clustering component in the simplest systems.

The recent development of numerical techniques with large computational power enables us to calculate few-body systems with more than three bodies [15–18]. Even with such methods, however, it is not easy, or almost impossible, to investigate the multichannel coupled systems beyond the three-body problem, such as (^{4}He+n+n)+($t+t$). In order to unify the pictures of the shell and cluster models, antisymmetrized molecular dynamics (AMD) was applied [19]. The method is proven to be useful in qualitative descriptions of both cluster and shell-model type of structures, in a single framework. Note, however, that the single AMD wave function, which represents each nucleon as a Gaussian wave packet, is not sufficient for quantitative descriptions of the mixed states of the cluster

and shell-model structures. Moreover, the single AMD wave function cannot describe the halo structure of neutrons with a long tail, which is particularly important in the He isotopes. Superposition of the AMD wave functions is therefore required for a more elaborate description of light nuclear systems [19,20].

Recently, we proposed an extended AMD approach [21,22] that combines the AMD with the generator coordinate method (GCM) [20,23]. In this method, generation of the GCM basis function is implemented in the procedure of the stochastic variational method (SVM) [15,24]. We named this new approach "AMD triple-S (superposition of selected snapshots)". It is demonstrated that coupled-channel cluster problems can be easily solved with our approach, even for seven-body systems such as ^{10}He (^{4}He+6n and t+t+4n). In this paper, we apply the AMD triple-S to the investigation of the t+t clustering effect in the He isotopes.

Let us briefly describe our framework. (For details, see our previous publications [21,22].) The AMD wave function is expressed as a fully antisymmetrized Slater determinant consisting of single-particle wave functions of the Gaussian form [19]

$$\Psi = \frac{1}{\sqrt{A!}}\det[\psi_i(j)\chi_i(j)], \tag{1}$$

$$\psi_i(j) \propto \exp\Big[-\nu(\vec{r}_j - \vec{z}_i/\sqrt{\nu})^2 + \vec{z}_i{}^2/2\Big], \tag{2}$$

where χ_i represents spinors for spin and isospin. Here, $\vec{z}_i$ is a complex variational parameter, whose real and imaginary parts represent the expectation values of the position and momentum of a single particle, respectively. The oscillator parameter ($b = 1/\sqrt{2\nu} = 1.46$ fm) is common for all nucleons so as to remove exactly the center-of-mass kinetic energy. In this framework, the nucleons forming a cluster (either an α or a triton) share the same value of $\vec{z}_i$. The values of $\vec{z}_i$ are randomly generated, and only the imaginary part of $\vec{z}_i$ is optimized so as to adequately handle the spin-orbit interaction. This optimization is carried out through the frictional cooling method proposed in the standard AMD framework [25]. During this cooling process, the parity of the Slater determinant is projected out. With these AMD wave functions, a nonorthogonal basis is prepared for the GCM calculation [23]. The GCM ansatz in our framework is therefore expressed as a superposition of the basis functions ($\Psi_k^{(\pm)}$), in the following manner:

$$\Phi_{MK}^{J(\pm)} = \sum_k c_k P_{MK}^J \Psi_k^{(\pm)}. \tag{3}$$

Projection is executed onto a good angular-momentum state by the operator P_{MK}^J. Coefficients c_k correspond to the variational parameters to be determined by diagonalization of the Hamiltonian matrix after the projection. To reduce the dimension of the diagonalization, the basis selection is performed for the GCM basis in the procedure of SVM [15]; that is, only the configurations making a sufficiently large energy gain are employed from the randomly produced GCM basis. The Volkov No. 2 potential [26] is used for the central-force part of the N-N interaction, and the G3RS potential [27] is employed for the spin-orbit part with $V_0 = 2000$ MeV.

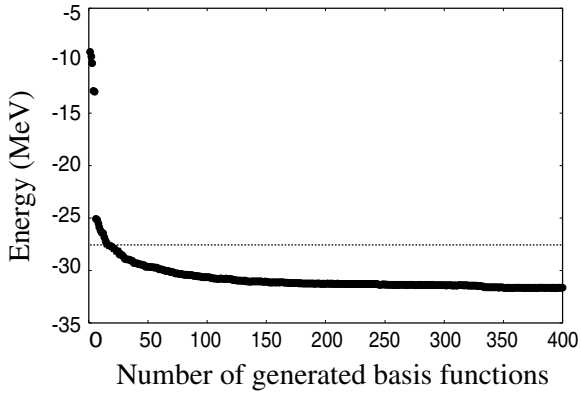

FIG. 1. Example of energy convergence of the ground 0^+ state of ^{8}He in the GCM diagonalization. Dotted line shows the energy of the ^{4}He+n+n+n+n threshold.

This choice of interactions is the same as in our previous studies [21,22].

In Fig. 1, an example of energy convergence is presented for the case of the ground state of ^{8}He, with the parameters of Volkov No. 2 set to be $M = 0.6$ and $B = H = 0$. The same value of $\vec{z}_i$ is chosen for the four nucleons forming the α core. The present model is therefore a simple five-body model with the configuration of ^{4}He+4n. In the figure, energy is plotted on the vertical axis, while the number of generated basis functions is on the horizontal axis. The dotted line shows the energy of the ^{4}He+4n threshold.

The dimension for diagonalization is increased until convergence is obtained. Each basis function is generated with the assistance of a stochastic variational algorithm. The real part of $\vec{z}_i$ is randomly given, whereas the imaginary part is determined through the frictional cooling method in the AMD framework. Such a basis function is applied to the diagonalization only when the resultant energy eigenvalues are lowered by the order of 10 keV. It is clearly seen in Fig. 1 that the ground state converges to -31.8 MeV with this method. The convergence is checked with several different random sequences, and it is confirmed that the same results are obtained to within several tens of keV. Binding energies calculated through our method are compared with resonating group method (RGM) calculations, in which the same interaction is used. For the α+α+α and ^{4}He+n+n systems, AMD triple-S, respectively, produces $E = -89.62$ [21] and -28.63 MeV, while RGM gives $E = -89.62$ [28] and -28.34 MeV [29]. (In the calculation of the latter system, our model space is slightly larger than the RGM model space.) Therefore, the present method is reasonably good in comparison with other sophisticated few-body calculations.

In Fig. 2, the systematics of the binding energies is shown for the He isotopes. The Volkov No. 2 potential ($M = 0.6$) is used with two different sets of B and H parameters, that is, $B = H = 0.125$ and $B = H = 0$. As seen from the figure, the second case ($B = H = 0$) gives rise to a better agreement with the experimental binding energies of ^{7}He and ^{8}He, although the first set ($B = H = 0.125$) has been known to reproduce the binding energies of ^{5}He and ^{6}He reasonably well. It is also known that the first set can describe the unbound

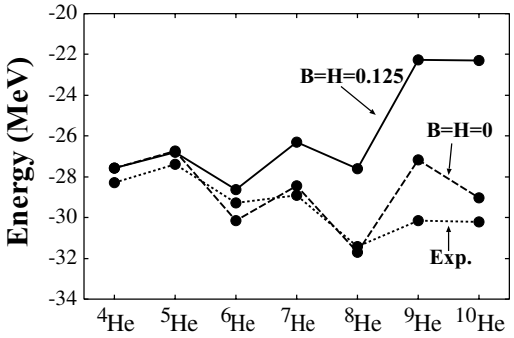

FIG. 2. Systematics of the calculated binding energies of the He isotopes compared with experimental data. The solid line corresponds to the case of $B = H = 0.125$, whereas the dashed line to the case of $B = H = 0$. The experimental values are shown as the dotted line.

nature of the di-neutron system. It turns out, however, that the parameters $B = H = 0.125$ make the interaction too repulsive when many-neutron systems are considered, particularly for ^{9}He and ^{10}He. Thus, it is difficult to consistently reproduce the trend of the binding energies for the He isotopes with the fixed parameters. This may be due to the limitation of the simple interaction we have employed. Another reason is that we calculated the binding energies with a bound-state approximation in this study, so that the calculated energies with the present method are expected to be slightly higher than the resonant energies calculated with the proper boundary condition. For these reasons, we hereafter use the parameters of $B = H = 0$ because this set reproduces the experimental total binding energies better, especially for ^{7}He, ^{8}He, ^{9}He, and ^{10}He. Our primary goal is to explain how the t+t clustering component contributes to the binding energy in neutron-rich isotopes heavier than ^{6}He. In this view, the interaction set of $B = H = 0$ seems to be adequate for our purpose. Further fine tuning of the interaction is therefore not attempted in this study.

In Fig. 3, the effect of the t+t clustering component is shown for the binding energies of He isotopes. The solid line indicates the binding energies calculated with the coupled-channel model, in which the {t+t+valence neutrons} channel is coupled to the {^{4}He+valence neutrons} channel. On the other hand, the dashed line represents the binding energies calculated with the single {^{4}He+valence neutrons} channel. Therefore, the difference between the two lines comes from the {t+t+valence neutrons} components, which implies that the α core is distorted. It is clearly seen that the binding energies are properly lowered by this distortion effect not only for ^{6}He, which was originally pointed out by Csótó [10] and Arai *et al*. [11], but also for the other He isotopes. In the figure, the energy gain ΔE due to the distortion effect is 0.35 MeV for ^{6}He. After the renormalization effect for the ^{4}He+n subsystem [10] is considered, ΔE reduces to 0.26 MeV. Our model is thus able to explain the missing part of the binding energy, which is of the order of 200–300 keV, with the coupled-channel approach. Many other models based on the ^{4}He+n+n structure could not explain this. As for ^{8}He, the energy gain ΔE is 0.64 MeV, which implies the two extra neutrons from ^{6}He enhance the t+t clustering effect. As for ^{10}He, the presence of many valence neutrons (six neutrons) may enhance further the t+t clustering effect on the basis of the previous discussions (i.e., on ^{6}He and ^{8}He), but at the same time the p-shell of neutrons is completely filled in this nucleus. This means that the core of the nucleus may favor the spherical shape; that is, the distortion of the α core may be strongly suppressed because of the shell effect. As seen from Fig. 3, the energy gain is calculated to be $\Delta E = 0.42$ MeV in the present study. This result suggests that the distortion effect is still present though it seems that the shell closure effect stops the increasing trend of the energy gain ΔE at ^{10}He. It should be noted, however, that the numerical error coming from the convergence criterion may be of an order of 10 keV.

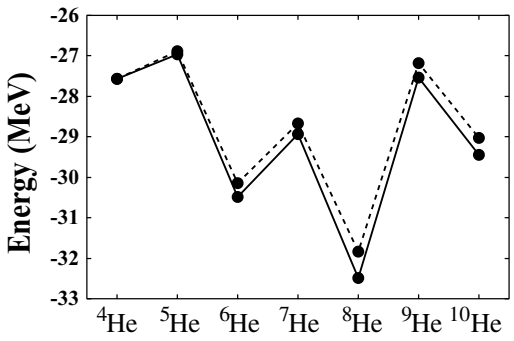

FIG. 3. Calculated binding energies of the He isotopes for the single-channel case and the coupled-channel case. Solid line corresponds to the coupled-channel model, in which the model space of {t+t+valence neutrons} is coupled to the model space of {^{4}He+valence neutrons}. The dashed line corresponds to the {^{4}He+valence neutrons} model.

In summary, by means of the AMD triple-S, we have systematically studied effect of the t+t clustering on the binding energies of He isotopes. The t+t clustering component is qualitatively demonstrated to be important, although the binding energies are not reproduced to a good accuracy, especially for the isotopes beyond the neutron drip line ($N = 7, 8$). It seems that this discrepancy is simply due to our choice of the two-body interaction. It should be noted that the current study is the first application of the newly developed method, so in order to test the method, our calculations were executed with the simple N-N interaction. From this viewpoint, the resultant calculations are satisfactory, but further development is certainly necessary. To improve the quality of the calculations, more realistic interactions should be used such as a full treatment of the tensor-force contribution, which is taken into account only through a renormalization of the central and spin-orbit parts of the interaction in the current framework. The use of more realistic interactions could be one of the key points to overcoming the difficulty we have encountered in the present study. The implementation of more elaborate interactions in the cluster model is now in progress [30].

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