Distributed Nonlinear Model Predictive Control for Heterogeneous Vehicle Platoons Under Uncertainty

Dan Shen, Jianhua Yin, Xiaoping Du, and Lingxi Li

Abstract— **This paper presents a novel distributed nonlinear model predictive control (DNMPC) for minimizing velocity tracking and spacing errors in heterogeneous vehicle platoon under uncertainty. The vehicle longitudinal dynamics and information flow in the platoon are established and analyzed. The algorithm of DNMPC with robustness and reliability considerations at each vehicle (or node) is developed based on the leading vehicle and reference information from nodes in its neighboring set. Together with the physical constraints on the control input, the nonlinear constraints on vehicle longitudinal dynamics, the terminal constraints on states, and the reliability constraints on both input and output, the objective function is defined to optimize the control accuracy and efficiency by penalizing the tracking errors between the predicted outputs and desirable outputs of the same node and neighboring nodes, respectively. Meanwhile, the robust design optimization model also minimizes the expected quality loss which consists of the mean and standard deviation of node inputs and outputs. The simulation results also demonstrate the accuracy and effectiveness of the proposed approach under two different traffic scenarios.**

I. INTRODUCTION

In recent years, the vigorous development in the automobile industry has brought great convenience to people's lives. However, the rapid increase in the number of vehicles has also led to back-of-queues, traffic accidents, and environmental pollution [1]–[3]. Comparing the means of widening and improving the road infrastructure, it is more efficient and economical to deal with the aforementioned issues by enhancing autonomous vehicles' technology and establishing intelligent transportation systems [4]–[7]. Currently, many advanced methodologies have been proposed to ameliorate the vehicle safety and fuel efficiency [8]. For instance, the authors in [9] proposed a minimum fuel control strategy in an automated vehicle-following scenario using the Pulse and Gliding method. The authors in [10] developed strategies to optimize the fuel consumption of vehicles during vehicle-following conditions.

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However, our ego vehicle is not a single participant on the road and there are many other road users in the surrounding environment, which forms a group of vehicles. In the vehicle group system, the road-driver-vehicles are mutually restricted, forming an extremely complex and strong nonlinear dynamic system, so that it is limited to improve the control accuracy and robustness of a single-vehicle. Recently, many research works have shown that vehicle platoon can alleviate traffic jams, enhance driving safety, and mitigate vehicle fuel emissions significantly [11]–[13]. The development of a control method for vehicle platoon started from the PATH project in California in the 1980s [14]. Vehicle platooning mainly composes several vehicles in the same lane as one single platoon by adjusting the inter-vehicle distance and maintaining the desired speed by both longitudinal and lateral control technologies based on the information from its neighboring vehicles or leading vehicle.

In the PATH project, the control task assignment in the platoon and the sensing and execution technologies were studied [15]. In addition, [16] investigated the string stability of lateral control solution for a homogeneous vehicle platoon. The lateral dynamics and motion equations have also been derived. The Cooperative Adaptive Cruise Control (CACC) was also designed to assess influences on the energy savings of heavy trucks [17]. For heterogeneous platoons, [18] proposed a robust coordinated control of nonlinear heterogeneous platoon under uncertain topology. Distributed nonlinear MPC and heterogeneous vehicle platooning Metric Learning with cut-in/cut-out maneuvers were explored in [19]. Although many problems in vehicle platooning have been investigated, there are still many open issues that remain to be explored and solved, such as relative single communication topology, comprehensive analysis of heterogeneous platooning, adaption to uncertainties, among others.

Uncertainty exists in vehicle platoon control. The performance of advanced vehicle platooning models and control methods will be affected if uncertainties in parameters and external environment are not properly addressed. Several robust control strategies, such as H_2-, H_∞- and μ-synthesis [20], were developed to find the optimal control parameters. However, such control strategies use deterministic parameters in the early vehicle design process. To optimize the control parameters and vehicle parameters simultaneously, the robust design approach [21], [22] is preferred over the robust control strategy. A Min-Max Model Predictive Control (MM-MPC) strategy was proposed in [17] to account for the uncertainty of time delays. The optimal control was obtained by minimizing the cost of the worst case [23]. Stochastic Model

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Predictive Control (SMPC) [24] is another powerful tool for optimal control under uncertainty. SMPC treats uncertainties with a probabilistic approach, which can provide more realistic solutions. In addition, chance constraints are applied to ensure that the system can maintain its intended state at a specific probability level. Data-based approaches were also used in SMPC. The data of the control object was directly obtained from sensors with noises following Gaussian distributions. Kalman Filter [25] was used to estimate the positions of target vehicles. This approach does not use the information of the vehicle dynamic model but accounts for all the parameters' uncertainty into measurement uncertainty.

To address the challenges mentioned above, this work proposes a novel distributed nonlinear model predictive control for heterogeneous vehicle platoons under uncertainty. The main contributions of this work are summarized as follows:

- Proposed a new distributed nonlinear model predictive control (DNMPC) method for heterogeneous vehicle platooning based on the existing work in [1]. Our approach is general for heterogeneous vehicle types and can keep a safe inter-vehicle spacing with the desired velocity under uncertainty.
- Applied robust design with reliability constraints to ensure the robustness and reliability of the vehicle control method given the existence of uncertainty.
- Designed two relatively complex traffic scenarios for evaluating the effectiveness of the proposed method under uncertainty.
- Demonstrated the effectiveness of the proposed approach via showing a good tracking performance related to both spacing error and speed tracking error.

The remainder of this paper can be organized as follows: Section II introduces the nonlinear platoon model and information flow model. The extension of DMPC using robust design and reliability-based design are presented in Section III. Simulation results of two designed scenarios on highways are presented and discussed in Section IV. Section V concludes the paper and points out several future research directions.

II. PLATOON MODELING

Different from most of the existing research, this paper mainly considers the heterogeneous vehicle platoon with the predecessor-following (PF) communication topology. There are one leading vehicle (indexed by 0) and seven following (indexed by 1 to 7) vehicles driving on a flat and straight road, and each vehicle is represented by a node so there are eight nodes in total in the platoon. We also assume the communication among all nodes is unidirectional and the information flow can only be delivered from the preceding vehicle to the downstream vehicles. Meanwhile, the heterogeneous platoon is actually a spatial formation, which consists of several different types of vehicles in terms of vehicle size, dynamics, and driving environments. There are two main components in the platoon model, one is the nonlinear dynamics for each following vehicle, and the other is the model of information flow. Note that only the longitudinal dynamics and control are considered in this paper.

A. Nonlinear Platoon Model

Each vehicle in the platoon also has its own moving dynamics with both state and control input constraints. To better understand and formulate the optimization problem proposed in the paper, we first need to study the vehicle dynamic model in the platoon.

Since the vehicle dynamics in the longitudinal direction is strongly nonlinear and includes many nonlinear terms such as engine, transmission, driveline, brake system, and aerodynamic drag, etc. Thus, it is impossible to establish an accurate model when the vehicle is running on the road by considering all above-mentioned elements in modeling. By taking both model accuracy and its feasibility, there are some simplifications we have made for vehicle longitudinal modeling as following:

1) Consider only the vehicle motion along the longitudinal direction. The lateral and vertical motions are neglected.
2) Focuses on the flat road condition and normal driving states, no side-slip angle is expected.
3) Vehicle is viewed as a rigid-body and left-right symmetric object.
4) The dynamics of the powertrain is simplified as a first-order inertial transfer function. There is only one control input for both driving torque and braking torque.

After applying the above-mentioned simplifications, we can derive a discrete-time (D-T) dynamic model of any node in the following vehicles [1] as following:

$$\begin{cases} p_i(k+i) = p_i(k) + v_i(k)\Delta t, \\ v_i(k+1) = v_i(k) + \frac{\Delta t}{m_i}(\frac{\eta_{T,i}}{R_i}T_i(k) - C_{A,i}v_i^2(k) \\ -m_i g f_i), \\ T_i(k+1) = T_i(k) + \frac{\Delta t}{\tau_i}(u_i(k) - T_i(k)) \end{cases} \tag{1}$$

where $N = 1, 2, ..., 7$ represents the set of all following vehicles. Variables p_i(k), v_i(k) and T_i(k) are the displacement, velocity and coupled torque of driving & braking of vehicle i at time k, respectively. m_i is the mass of vehicle i, $C_{A,i}$ is the lumped aerodynamic drag coefficient of node i, g is the constant of gravity, f_i is the rolling friction coefficient, Δt is the sampling time in the simulation, τ_i is the inertial time lag of drive line in vehicle i, η_i is the mechanical efficiency coefficient of drive line in node i, R_i is the tire radius of vehicle i, u_i is the control input of desired acceleration or torque, which is also subject to a box constraint as $u_i \in U_i$ = $\{u_{min,i} \leq u_i \leq u_{max,i}\}$, and where $u_{min,i}$ and $u_{max,i}$ are the lower bound and upper bound of control inputs. The state-space model of vehicle longitudinal dynamics is also built by defining the states as $x_i(t)$ = [$p_i(t)$, $v_i(t)$, T_i(t)] $\in \mathbb{R}^3$, and the system output denotes as $y_i(t)$ = [p_i(t), v_i(t)] $\in \mathbb{R}^2$. Thus, the system plant of each node can be rewritten in the following discrete-time compact form:

$$\begin{cases} x_i(k+1) = A_i(x_i(k)) + B_i u_i(k), \\ y_i(k+1) = C_i y_i(t), \end{cases} , i \in N \tag{2}$$

where $A_i(x_i(k))$ defined as

$$\begin{aligned} A_i(x_i(k)) &= \begin{bmatrix} p_i(k) + v_i(k)\Delta t \\ v_i(k) + \frac{\Delta t}{m_i}(\frac{\eta_{T,i}}{R_i}T_i(k) - F_i) \\ T_i(k) - \frac{\Delta t}{\tau_i}T_i(k) \end{bmatrix} \\ F_i(k) &= C_{A,i}v_i^2(k) + m_i g f_i \end{aligned} \tag{3}$$

and B_i =$[0 \quad 0 \quad \frac{1}{\tau}\Delta t]^T \in \mathbb{R}^{3\times 1}$, C_i = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \in \mathbb{R}^{2\times 3}$.

Furthermore, the vectors of vehicle state variables, outputs, and control inputs of all N nodes are defined below:

$$\begin{cases} X(k) = [x_1^T(k), x_2^T(k), \cdots, x_N^T(k)]^T \\ Y(k) = [y_1^T(k), y_2^T(k), \cdots, y_N^T(k)]^T \\ U(k) = [u_1^T(k), u_2^T(k), \cdots, u_N^T(k)]^T \end{cases} , i \in N \tag{4}$$

Then the overall discrete-time model of vehicle platoon for N following vehicles can be presented below:

$$\begin{cases} X(k+1) = \Theta(X(k)) + \Pi U(k), \\ Y(k+1) = \Upsilon y(K), \end{cases} , i \in N \tag{5}$$

where the new system matrix can be calculated as the following for all N nodes [1]:

$$\begin{cases} \Theta = [A_1^T(x_1), A_2^T(x_2), \cdots, A_N^T(x_N)]^T, \in \mathbb{R}^{3N\times 1} \\ \Pi = diag\{B_1, B_2, \cdots, B_N\}, \in \mathbb{R}^{3N\times N} \\ \Upsilon = I_N \bigotimes C_i, \in \mathbb{R}^{2N\times 3N} \end{cases} \tag{6}$$

where $\bigotimes$ denotes the Kronecker product in the computation.

B. Model of Information Flow

For the communication topology, an accurate model is significant to design the integrated objective function for distributed model predictive control [1], [11]. The directed graph theory was used to model the information flow in a platoon by interconnecting the allowable information flow between vehicles in a platoon, which is represented by G = (V,E). $V = \{\alpha_1, \alpha_2, ..., \alpha_N\}$ denotes the set of nodes and α_i is the i-th vehicle in the following vehicles, and $E = V \times V$ is the set of connection edges between nodes. The directed graph theory can model all aforementioned topologies, such as Predecessor Following (PF) topology, Predecessor-leader following (PLF) topology, and Bidirectional (BD) topology, and so on. To simplify the model, the communication model is continuously formulated as three matrices as adjacent matrix $\mathscr{A}$, Laplacian matrix $\mathscr{L}$, and pinning matrix $\mathscr{P}$.

The adjacent matrix associated with graph G is defined to represent the communication edge from node j to node i, which can be shown as $\mathscr{A} \in \mathbb{R}^{\mathbb{N}\times\mathbb{N}}$ with each entry defined as:

$$\begin{cases} \alpha_{ij} = 1, & \{\alpha_j, \alpha_i\} \in E \\ \alpha_{ij} = 0, & \{\alpha_j, \alpha_i\} \notin E \end{cases} , i, j \in N \tag{7}$$

where $\{\alpha_i, \alpha_j\} \in E$ means vehicle i can receive information from vehicle j, and there is no self-loop in the assumption, so α_{ij} = 0. Then the neighboring set of node i is given by N$_i$= $\{j|\alpha_{ij}$ = 1$\}$.

The Laplacian matrix $\mathscr{L}$ = $[l_{ij}] \in \mathbb{R}^{N\times N}$ associated with graph G can be defined as:

$$\mathscr{L} = \mathscr{D} - \mathscr{A} \tag{8}$$

where $\mathscr{D}$ is called an in-degree matrix that can be defined as

$$\mathscr{D} = \begin{bmatrix} deg_1 & & \\ & \ddots & \\ & & deg_N \end{bmatrix}, N \in 1, 2, \ldots, 7 \tag{9}$$

And the in-degree of vehicle i is defined as deg_N = $\sum_{j=1}^{N} \alpha_{ij}$, which represents the overall available communication edges between node i and other nodes in the neighboring set. Actually, the Laplacian matrix is an induced matrix from the adjacent matrix.

The pinning matrix P associated with graph G can be explained as the information flow from the leading vehicle to the following vehicles, which is denoted as:

$$\mathscr{P} = \begin{bmatrix} p_1 & & \\ & \ddots & \\ & & p_N \end{bmatrix} N \in 1, 2, \ldots, 7 \tag{10}$$

where $p_i = 1$ if communication edge $\{\alpha_0, \alpha_i\} \in E$, and otherwise $p_i = 0$. When $p_i = 1$, node i is called to be pinned to the leader, which indicates vehicle i can receive information from the leader. The leading vehicle (indexed by 0) accessible set of node i can also be defined:

$$\mathscr{P}_i = \begin{cases} \{0\}, & p_i = 1 \\ \emptyset, & p_i = 0, \end{cases} \tag{11}$$

III. CONTROL METHODS

A. Control Objective

The overall control objective of the proposed distributed MPC-based platoon control is to follow the leading vehicle's velocity while tracking and maintaining the desired gap between any two consecutive vehicles. Thus, the constant spacing policy ($d_{i-1,i}$ = d_0) is applied for the design of distributed MPC, and d_0 is the satisfied constant gap we need to regulate in vehicle platoon. The overall structure of the proposed DNMPC for a heterogeneous platoon with considering the robust design and reliability-based design under Predecessor Following (PF) topology is shown in Fig.1.

B. Control Problem Formulation

This section mainly introduce the formulation of the optimization problem for vehicle heterogeneous platoons. The initial position and velocity of leading vehicle (indexed by 0) can be represented as $p_0(t)$ and $v_0(t)$, respectively. For the leading speed, we made an assumption of constant speed at time t = 0 as some previous works [1], [11], such as p_0 = v_0 Δt. Then the desired state that the following vehicles aim to track and the expected control inputs are as the following:

$$\begin{aligned} x_{des,i}(k) &= [p_{des,i}(k), \quad v_{des,i}(k), \quad T_{des,i}(k)]^T \\ u_{des,i}(k) &= T_{des,i}(k) \end{aligned} \tag{12}$$

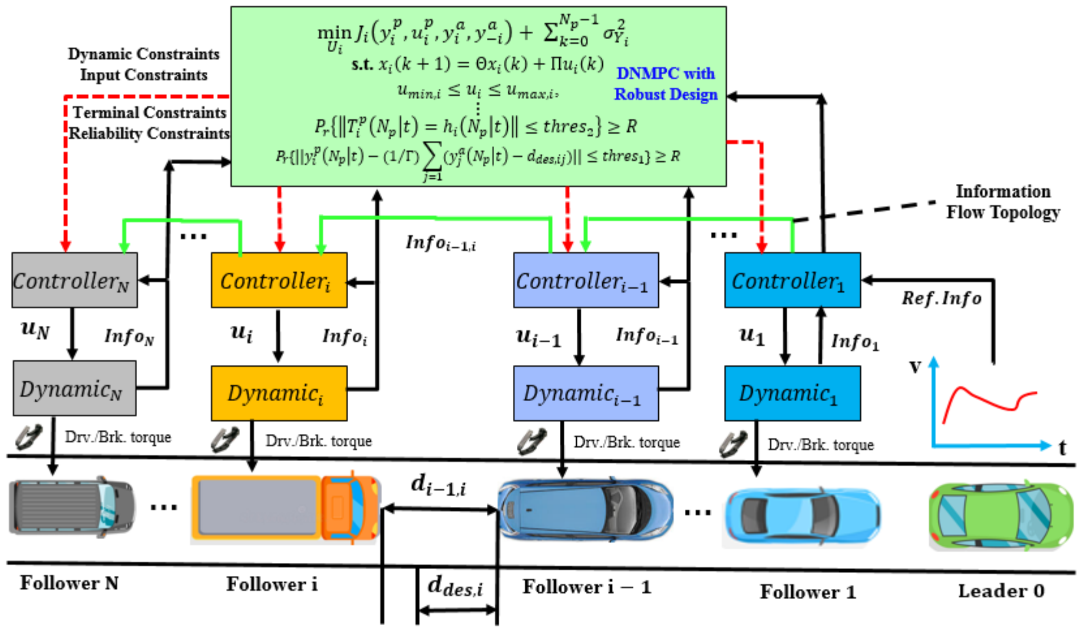

Fig. 1. Overall structure of DNMPC for heterogeneous platoon considering robust design and reliability-based design under PF topology.

where $p_{des,i}$(k) = $p_0(k)$ - d_0, $v_{des,i}$ = v_0. Note that the desired torque applied to the vehicle is $T_{des,i}$ = $C_{A,i}v_i{}^2$(k) + $m_i g f_i$, which is used to counterbalance the external drag.

For formulating the optimal control problem for each node i, all nodes that can send information to i will be used. In other words, the optimization for each node i will only utilize the information from its neighbor set N for obtaining the optimal control inputs at each iteration. Meanwhile, the vectors of outputs and inputs of N nodes are defined as the following [1], [11]:

$$\begin{cases} y_i(k) = [y_{j1}^T(k), \quad y_{j2}^T(k), \quad y_{j3}^T(k)]^T \\ u_i(k) = [u_{j1}(k), \quad u_{j2}(k), \quad u_{j3}(k)]^T \end{cases} \tag{13}$$

Furthermore, the prediction horizon in the distributed MPC is set as N_p, and three other types of control variables will be defined in the prediction horizon [1], [13]:

1) u_i^a(k): the assumed control inputs of node i;
2) u_i^p(k): the predicted control inputs of node i;
3) u_i^*(k): the desired control inputs of node i;
where N_p = 0,1,2,$\cdots$, $N_p - 1$

Similarly, three different types of system outputs can be defined based on the aforementioned three control inputs below:

1) y_i^a(k): the assumed system outputs of node i that is a shifted optimal plant outputs;
2) y_i^p(k): the predicted system outputs of node i in the local MPC problem;
3) y_i^*(k): the desired system outputs of node i after solving the local MPC problem;
where N_p = 0,1,2,$\cdots$, $N_p - 1$

And assume that the system outputs of each node i will be delivered to the nodes in its neighboring set for optimizing the spacing error of distance and tracking error of speed of each vehicle i. Therefore, we can formulate the platoon control problem for each following vehicle by the local optimization below [1], [11]:

$$\begin{aligned} &\min_{U_i} J_i(y_i^p(:|t), u_i^p(:|t), y_i^a(:|t), y_i^a(:|t)) \\ &= \min_{U_i} \sum_{k=0}^{N_p-1} \zeta(y_i^p(k|t), u_i^p(k|t), y_i^a(k|t), y_i^a(k|t)) \end{aligned} \tag{14}$$

s.t.

$$\begin{aligned} &x_i{}^p(k+1|t) = \Theta(x_i^p(k|t)) + \Pi u_i^p(k|t) \\ &y_i^p(k+1|t) = \Upsilon x_i^p(k|t) \\ &k = 0, 1, 2, \cdots, N_p - 1 \\ &u_{min,i} \le u_i^p(k|t) \le u_{max,i} \\ &y_i^p(N_p|t) = \tfrac{1}{\Gamma}\textstyle\sum_{j=\Gamma}(y_j^a(N_p|t) - d_{des,ij}) \\ &T_i^p(N_p|t) = h_i(N_p|t) \end{aligned}$$

where $\zeta(y_i^p(k|t)$, $u_i^p(k|t)$,$y_i^a(k|t)$, $y_i(k)(k|t))$ is the defined objective function, and it is presented below:

$$\begin{aligned} &\zeta(y_i^p(k|t), u_i^p(k|t), y_i^a(k|t), y_i^a(k|t)) \\ &= (\mathrm{y}_i^p(k|t) - y_{des,i}(k|t))^T Q_i((y_i^p(k|t) \\ &- \mathrm{y}_{des,i}(k|t)) \\ &+ (\mathrm{u}_i^p(k|t) - h_i(k|t))^T R_i(u_i^p(k|t) - h_i(k|t)) \\ &+ (\mathrm{y}_i^p(k|t) - y_i^a(k|t))^T W_{ai}(y_i^p(k|t) - y_i^a(k|t)) \\ &+ (\mathrm{y}_i^p(k|t) - y_j^a(k|t))^T W_{ni}(y_i^p(k|t) - y_j^a(k|t)) \end{aligned} \tag{15}$$

where Γ is the total number of nodes in the set of $N_i \cup \mathscr{P}_i$, which denotes the total number of nodes in the collection set of neighboring and the leader accessible set of node i. U_i = $[u_i^p(0|t), u_i^p(1|t),\cdots,\ u_i{}^p(N_p-1|t)]^T$ is the control sequence to be optimized in MPC. In the constraints, the first three equations are the equality constraints of vehicle nonlinear dynamics in the prediction horizon. The fourth one is the constraints on vehicle control inputs. The last two equations indicate the terminal constraints of the state variables. The first condition is neighboring average-based terminal constraints, which enforce the final states as close as possible to the average value of the known reference set points. The second condition of terminal constraint is to ensure that the vehicle state is around the equilibrium with the final control input, which indicates a smooth driving torque without sudden acceleration and braking. Note that the initial predicted system state $x_i^p(0|t)$ = x_i(t).

Meanwhile, there are four weighting matrices in the objective function, and they are all positive positive-definite matrices. Q_i is the weighting matrix that penalizes the output tracking error from the desired outputs, and it also indicates whether the node i can receive the reference information from the leading vehicle. When the node i is pinning to the leading vehicle, $Q_i > 0$, otherwise $Q_i = 0$. R_i is the weighting factor that penalizes the sudden change of control inputs. In other words, the vehicle prefers a constant and smooth driving speed in the end without jerk. W_{ai} means vehicle i tries to maintain the actual system output as close as possible to its assumed output (shifted optimal outputs of the same node). W_{ni} indicates that the node i tries to keep the plant outputs as close as to the assumed outputs of the nodes in its neighboring set N_i.

C. Robust design with reliability constraint

Robust design minimizes the effects of uncertainty in the design objective without eliminating the sources of uncertainty. High robustness is achieved by changing the nominal values of design variables.

Robustness is typically defined as insensitive to uncertainty. Using the notion of Taguchi's quality loss, we consider robustness in a broader sense: maximize both the motion responses' insensitivity to uncertainty and their average performance. We now use a nominal-the-best type performance as an example. Let $\mathbf{t} = (t_1, t_2, \ldots, t_m)$ be the targets we want to achieve for performance variables $\mathbf{Y} = (Y_1, Y_2, \ldots, Y_m)$. The traditional quality loss function [26] $L(\mathbf{Y})$ is illustrated in Fig. 2 (shown in only one dimension) and is defined by

$$L(\mathbf{Y}) = \sum_{i=1}^{m} k_i \left(Y_i - t_i\right)^2 \tag{16}$$

where k_i is a constant determined by the cost (see in Fig. 2) reaching the tolerance boundary, and m is the dimension of $\mathbf{Y}$. There is always a loss if $\mathbf{Y}$ deviates from their targets. We can change design variables to maximize the expected (average) quality loss, given by

$$E_L = \sum_{i=1}^{m} k_i \left[\left(\mu_{Y_i} - t_i\right)^2 + \sigma_{Y_i}^2\right] \tag{17}$$

where μ_{Y_i} and σ_{Y_i} are the mean and standard deviation of Y_i, respectively.

Minimizing E_L will reduce both $(\mu_{Y_i} - t_i^2)$ (bringing the average performance to the target) and $\sigma_{Y_i}^2$ (reducing variation of the performance). This gives a good trade-off between the performance and its variation.

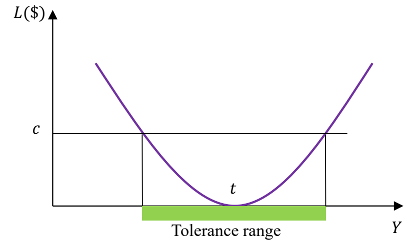

Fig. 2. Quality loss function.

The evaluation of the mean performance (μ_{Y_i}) is trivial by substituting the nominal values of random variables $(\boldsymbol{\mu}_X)$ into the performance function. The performance function is the vehicle dynamic model in this work. The vehicle state (position, velocity, torque) is the vehicle performance that we are interested in. Now we discuss how to approximate the standard deviation of Y_i. The First Order Second Moment (FOSM) [27] method is used to approximate the standard deviation. We denote the performance function by

$$\mathbf{Y} = g(\boldsymbol{X}) \tag{18}$$

FOSM employs the first-order Taylor expansion to linearize Eq. (18) at the means of input random variables. The approximation is given by

$$g(x) = g\left(\boldsymbol{\mu}_X\right) + \nabla g\left(\boldsymbol{\mu}_X\right) \cdot \left(\boldsymbol{X} - \boldsymbol{\mu}_X\right) \tag{19}$$

Since the random variables are assumed to be independent, the variance of $g(x)$ can be obtained by

$$\sigma_Y^2 = \sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i}\right)^2 \sigma_{X_i}^2 \tag{20}$$

In this preliminary study, we assume m_i, $C_{A,i}$, f_i, τ_i, η_i, R_i are independent random variables and follow normal distributions. The distributions are listed in Table 1. The control input u_i is the design variable.

Robust design minimizes the expected quality loss E_L which consists of the mean and standard deviation of Y_i as shown in Eq. (17). The first part $\left(\mu_{Y_i} - t_i\right)^2$ exactly is the same as Eq. (15) in the deterministic MPC. Now we just need to add the variance term $\sigma_{Y_i}^2$ into Eq. (14) to have the robustness of the objective. Using the longitudinal dynamic function (Eq. (1)) at $k+1$ time as an example, we can assume the output $\mathbf{Y}$ in Eq. (18) is $p_i(k+i)$, $v_i(k+1)$, and $T_i(k+1)$ in Eq. (1) at $k+1$ time. We have three performance functions that are the three equations in Eq. (1); namely, $g_1(\boldsymbol{X}) = p_i(k) + v_i(k)\Delta t$, $g_2(\boldsymbol{X}) = v_i(k) + \frac{\Delta t}{m_i}(\frac{\eta_{T,i}}{R_i}T_i(k) - C_{A,i}v_i^2(k) - m_i g f_i)$, and $g_3(\boldsymbol{X}) = T_i(k) + \frac{\Delta t}{\tau_i}(u_i(k) - T_i(k))$. In this work, we assume that the vehicle state (position, velocity, torque) at the previous time is deterministic, which means that the uncertainty from previous time is not propagated to current time point. The uncertainty propagation in terms of time will be further studied in the future. Therefore, taking the partial derivatives with respect to the random variables and substituting into

TABLE I

DETAILED DISTRIBUTION OF RANDOM VARIABLES.

Random Variables	Distribution	Mean	Standard deviation
$m_{1,\ldots,7}(kg)$	Normal	(2.5, 2.3, 2.0, 1.9, 1.7, 1.6, 1.5)$\times 10^3$	(375, 345, 300, 285, 255, 240, 225, 270)
$\tau_{1,\ldots,7}(s)$	Normal	(0.95, 0.89, 0.80, 0.77, 0.71, 0.68, 0.65)	(9.5, 8.9, 8.0, 7.7, 7.1, 6.8, 6.5)$\times 10^{-2}$
$C_{A,1,\ldots,7}(N \cdot s^2 m^{-2})$	Normal	(0.9, 0.9, 0.7, 0.7, 0.45, 0.45, 0.45)	(9.0, 9.0, 7.0, 7.0, 4.5, 4.5, 4.5)$\times 10^{-2}$
$R_{1,\ldots,7}(m)$	Normal	(0.45, 0.45, 0.3, 0.3, 0.27, 0.24, 0.23)	(2.25, 2.25, 1.5, 1.5, 1.2, 1.15, 1.35)$\times 10^{-3}$
$f_{1,\ldots,7}$	Normal	0.01	5×10^{-4}
$\eta_{1,\ldots,7}$	Normal	0.96	0.048

Eq.(20), we have

$$\begin{cases} \sigma^2_{p_i(k+i)} = 0, \\ \sigma^2_{v_i(k+1)} = \left(\frac{\partial g_2}{\partial m_i}\right)^2 \sigma^2_{m_i} + \left(\frac{\partial g_2}{\partial C_{A,i}}\right)^2 \sigma^2_{C_{A,i}} + \left(\frac{\partial g_2}{\partial f_i}\right)^2 \sigma^2_{f_i} \\ + \left(\frac{\partial g_2}{\partial \tau_i}\right)^2 \sigma^2_{\tau_i} + \left(\frac{\partial g_2}{\partial \eta_i}\right)^2 \sigma^2_{\eta_i} + \left(\frac{\partial g_2}{\partial R_i}\right)^2 \sigma^2_{R_i}, \\ \sigma^2_{T_i(k+1)} = \left(\frac{\partial g_3}{\partial \eta_i}\right)^2 \sigma^2_{\eta_i} \end{cases} \tag{21}$$

Then, the obtained standard deviations of p_i, v_i, and T_i at each time instant can be obtained by Eq. (21). As mentioned before, the objective function in Eq. (14) is the term of $(\mu_{Y_i} - t_i)^2$ in Eq. (16). The second term can be expressed as

$$\sum_{k=0}^{N_p-1} \sigma^2_{Y_i} = \sigma^2_{y^p_i(k|t)}{}^T (Q_i + W_{ai} + W_{ni}) + \sigma^2_{T_i(k+1)} R_i \tag{22}$$

where $\sigma^2_{y^p_i(k|t)} = [\sigma^2_{p_i(k+i)}, \sigma^2_{v_i(k+1)}]^T$. Therefore, the robust design objective can be formulated as

$$\begin{aligned} J_{N_P} = \min_{U_i} & J_i(y^p_i(:|t), u^p_i(:|t), y^a_i(:|t), y^a_i(:|t)) \\ & + \sigma^2_{y^p_i(k|t)}{}^T (Q_i + W_{ai} + W_{ni}) + \sigma^2_{T_i(k+1)} R_i \end{aligned} \tag{23}$$

In addition to the equality constraints in Eq. (14), the terminal constraints are replaced by reliability constraints to ensure the specific probability that vehicles maintain within the vicinity of the equilibrium state with pre-defined thresholds (Ts_1, Ts_2) under the influence of uncertainty. The reliability constraints can be expressed as

$$\begin{aligned} &\Pr\{\|y^p_i(N_p|t) - \frac{1}{\Gamma}\sum_{j-\Gamma}(y^a_j(N_p|t) - d_{des,ij})\| \ \le Ts_1\} \ge R \\ &\Pr\{\|T^p_i(N_p|t) = h_i(N_p|t)\| \ \le Ts_2\} \ge R \end{aligned} \tag{24}$$

Combined equality constraints in Eqs. (14), (23), and (24), we have the final robust design optimization model with reliability constraints, which is given by

$$\begin{aligned} J_{N_P} = \min_{U_i} & J_i(y^p_i(:|t), u^p_i(:|t), y^a_i(:|t), y^a_i(:|t)) \\ & + \sigma^2_{y^p_i(k|t)}{}^T (Q_i + W_{ai} + W_{ni}) + \sigma^2_{T_i(k+1)} R_i \end{aligned} \tag{25}$$

s.t.
$x^p_i(k+1|t) = \Theta(x^p_i(k|t)) + \Pi u^p_i(k|t)$
$y^p_i(k+1|t) = \Upsilon x^p_i(k|t)$
$k = 0, 1, 2, \cdots, N_p - 1$
$\Pr\{\|y^p_i(N_p|t) - \frac{1}{\Gamma}\sum_{j-\Gamma}(y^a_j(N_p|t) - d_{des,ij})\| \ \le Ts_1\} \ge \ R$
$\Pr\{\|T^p_i(N_p|t) = h_i(N_p|t)\| \le Ts_2\} \ge \ R$

Next, we use two scenarios to illustrate the control effects of the proposed method.

IV. SIMULATION RESULTS

To verify the control effects of the proposed DNMPC with robustness and reliability considerations under uncertainty, both the platoon model and controller are built and designed in MATLAB, and numerical simulations are conducted to demonstrate the main results of the paper. The simulated heterogeneous platoon contains one leading vehicle (vehicle ID 0) and seven following vehicles (Vehicle IDs from 1 to 7) under PF communication topology, and the desired spacing between each vehicle is 20 m, which is the distance from the rear end of the preceding vehicle to the front end of the following vehicle. The maximum and minimum accelerations are 6 m/s^2 and -6 m/s^2, respectively. Two different scenarios are designed to assess the control effects and accuracy of the distributed nonlinear model predictive control.

A. Scenario 1

In Scenario 1, there is no initial spacing error and the desired spacing between any consecutive vehicles is 20 m. All followers' initial velocity is 30 m/s which is different from the leading vehicle's speed of 28 m/s, and the leading vehicle has perturbations of acceleration of 3 m/s^2 from 1 sec to 2 sec, and -2 m/s^2 from 6 sec to 7 sec. The sampling time of simulation is 0.1 sec and the prediction horizon of DNMPC is 20.

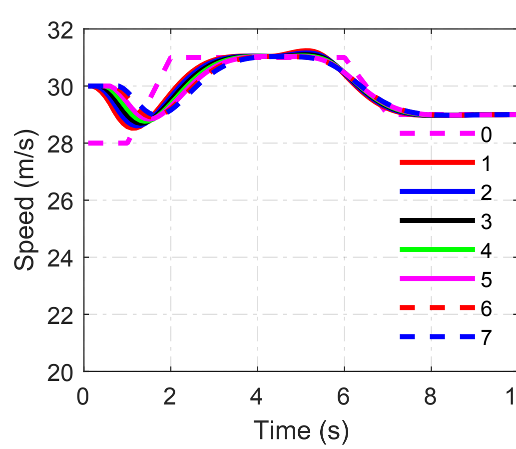

Fig. 3. Control effects of velocity for the platoon under PF topology in Scenario 1.

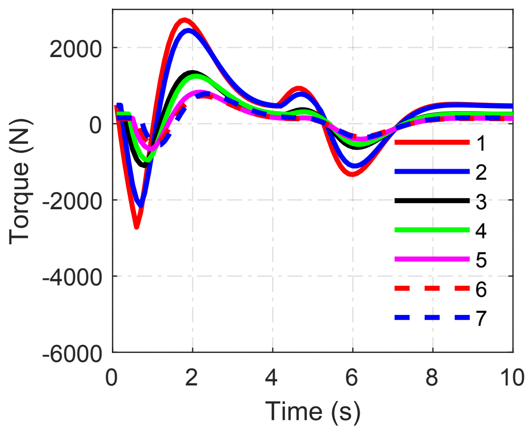

Fig. 4. Control inputs for the platoon under PF topology in Scenario 1.

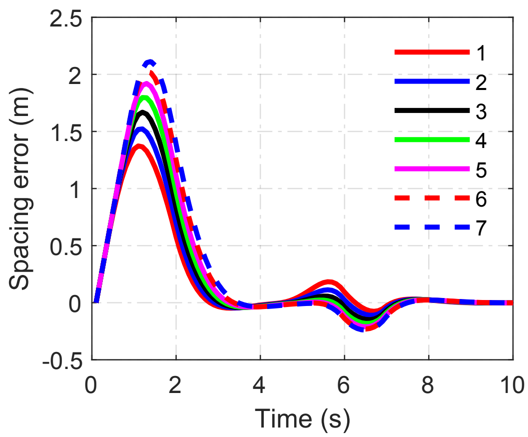

Fig. 5. Spacing errors for the platoon under PF topology in Scenario 1.

In the simulation, the initial speed of the leader and followers are different but with the same zero spacing error. As shown in Fig. 3, the leading speed is constant during the 1st sec and then increases from 28 m/s to around 31 m/s during the next one sec. After maintaining the constant speed for 5 seconds, node 0 decreases its speed from the 7th second to the 8th second until reaching 29 m/s. Fig. 3 and Fig. 5 indicate that both the speed tracking and spacing maintenance of all following vehicles are accurate and stable. The control inputs applied to each vehicle are also shown in Fig. 4.

B. Scenario 2

In Scenario 2, there is no initial spacing error and the desired spacing between any consecutive vehicles is still 20 m. All followers' initial velocity is 30 m/s which is different from the leading vehicle's speed of 32 m/s, and the leading vehicle has perturbations of acceleration of -3 m/s^2 from sec 1 to sec 2, and 2 m/s^2 from sec 6 to sec 7. The sampling time and prediction horizon of DNMPC are the same as Scenario 1.

In the simulation, the initial speed of the leader and followers are different with the same zero spacing error. As can be seen in Fig. 6, the leading speed is constant during the first second and then decreases from 32 m/s to around 29 m/s during the next second. After maintaining the constant speed for 5 seconds, node 0 increases its speed from the 7th second to the 8th second until reaching 31 m/s. It is easy to observe from the Fig. 6 and Fig. 8 that both the velocity tracking and spacing errors of all following vehicles are accurate and stable. The control inputs applied to each vehicle are also shown in Fig. 7.

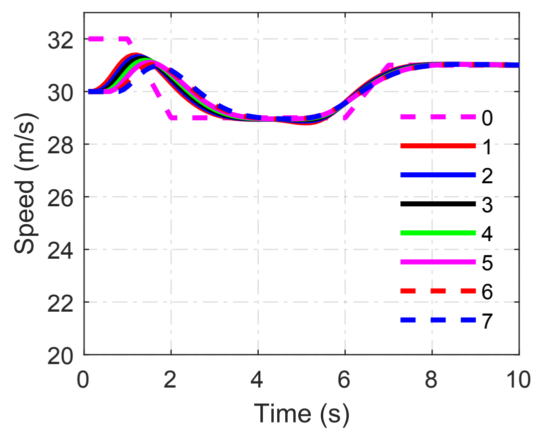

Fig. 6. Control effects of velocity for the platoon under PF topology in Scenario 2.

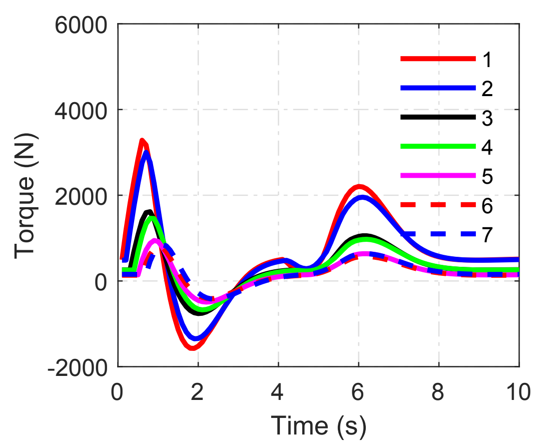

Fig. 7. Control inputs for the platoon under PF topology in Scenario 2.

V. CONCLUSIONS

This paper proposes a novel distributed nonlinear model predictive control technique with considering robustness and reliability to handle the potential spacing error in the heterogeneous vehicle platoon under unidirectional topologies. The platoon model consisting of vehicle longitudinal dynamics and information flow has been developed. An algorithm of the robust design optimization model with reliability constraints is derived and implemented. Under the proposed DNMPC framework, the control of heterogeneous platoon is well formulated and optimized by solving an online

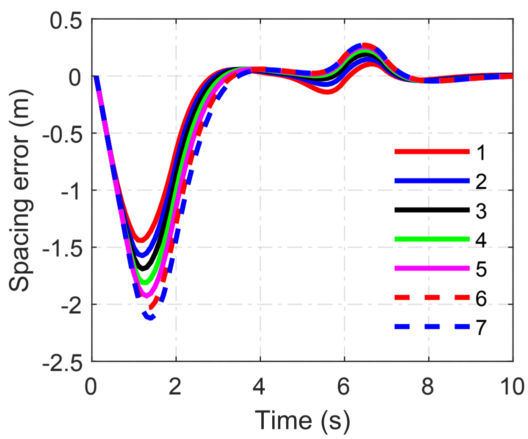

Fig. 8. Spacing errors for the platoon under PF topology in Scenario 2.

nonlinear programming problem. Two scenarios are devised to mimic the real highway scenarios in platooning with perturbations of accelerations. The simulation results illustrate no collisions and overshoot of system outputs during the transient process. The converging speeds and smoothness of the desired velocity tracking and spacing error are also satisfied.

One topic for future research is to extend the current platoon model to a more comprehensive one that includes longitudinal dynamics, lateral dynamics, and information flow. Another topic is to include the inherent time-dependent uncertainty when robustness and reliability are considered in DNMPC to handle more complex scenarios (cut-in or cut-out).

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