Textural characterization using P. Gy heterogeneity functions.

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Abstract

A solid can be regarded as a set of contiguous elementary units. The distribution within the solid of any properties, measurable within each elementary unit, can be characterized using two parameters. These parameters are built using the constitution and distribution heterogeneities of P. Gy (1982, 1988). The former account for the granularity of the elementary units, whereas the latter assess the spatial distribution of the property. A texture which definition involves several properties can be described using a diagram where both parameters work as variables. Potential applications encompass: (i) the textural classification of soils, ore, breccia and concrete and (ii) the monitoring of textural transformation during process like dolomitization, metamorphism, weathering, deformation or annealing.

Keywords: texture, homogeneity, characteristic distribution, constitution heterogeneity, distribution heterogeneity

1. Introduction

Hereinafter, the texture of a coherent solid is defined: (i) by the characterization of each component of the solid using various characteristics such as: the mineralogical composition, grain and pore size, shape, orientation, … and (ii) by the spatial distribution of these characteristics within the solid. The nine 2D solids in Figure 1 illustrate the purpose of this work (For convenience 2D solids are used, nevertheless this method can be applied to 3D

solids). These solids are composed of black and white squares or rectangles that could represent respectively: valuable and gangue minerals in an ore, pores and solid grains in a soil or aggregate and cement in a concrete. These nine solids differ in the proportion, size distribution and spatial distribution of black polygons. According to the definitions given above, the textures of the nine solids can be characterised by: (i) the amount and size of black and white polygons, (ii) the spatial distribution of black polygons. The size of polygons and the black and white polygons content are intrinsic properties of a given solid independent of their spatial distributions. In consequence, solids displaying the same intrinsic properties can differ by their spatial distribution. Then from the textural analysis point of view, a given characteristic, as the black polygon amount, operates: (i) as an intrinsic characteristic and (ii) through its spatial distribution. As delineated in Figure 1, it is proposed to characterize the textural contribution of a given characteristic using two parameters: (i) the granularity parameter (the reason for a such denomination is given in the next section) related to its intrinsic part, (ii) the distribution parameter which accounts for its spatial distribution.

The proposed method quantifies the granularity and distribution parameters of a characteristic by respectively the constitution heterogeneity (HC) and distribution heterogeneity (HD) of this characteristic. HC and HD are functions defined by Gy (1982, 1988) in his “Theory of particule material sampling”. We use these functions because: (i) the quantification of heterogeneities within a granular material as sand, is conceptually similar to the quantification of texture in coherent solids and (ii) the meaning of these functions are familiar to engineers working in mineral industry.

Let now consider a way to characterize granularity and distribution parameters of a characteristic ϕ using HC and HD.

Abbreviations and variables meaning are given in the appendix.

2. Evaluation of granularity parameter using constitution heterogeneity

The first step is to split up the “real solid” into elementary units (EU). EUs are the most little undividable compounds used to describe the "real solid". They are defined at convenience by the users according to the structure of the "real solid". For example, in an ore, it is convenient to consider individual grains of valuable mineral and gangue as EUs. The set of EUs constitutes the "model solid". According to the definition of the EU, a given "real solid" can be resolved in different "model solids". For instance, in a breccia it exists at least two definitions for EUs: EUs can correspond to each mono crystalline grains belonging either to the elements or the cement; but EUs can also match with each of the elements and each of the mono crystalline grains component of the cement.

The studied characteristic ϕ must be measurable within each EU. The model solid can be described as an array of points corresponding to the barycentre of the EUs, each of one bearing the following information: coordinates of the barycentre, volume V_i of the i^{th} EU and value $a_{\phi,i}$ of ϕ in the EU i.

The contribution of the i^{th} EU to the heterogeneity of the model solid with respect to ϕ can be evaluated (Gy 1982, 1986) by:

$$h_{\phi,i} = \left(\frac{a_{\phi,i} - a_\phi}{a_\phi}\right) \cdot \left(\frac{V_i}{\overline{V}}\right) = n \cdot \left(\frac{a_{\phi,i} - a_\phi}{a_\phi}\right) \cdot v_i \qquad (1)$$

where: n is the number of EU; V_i the volume of the i^{th} EU; $V_\Sigma = \sum_{i=1}^{n} V_i$ the volume of the solid;

$v_i = V_i / V_\Sigma$ the relative volume of i; $\overline{V} = (1/n) \cdot \sum_{i=1}^{n} V_i = V_\Sigma / n$, average of V_i; $a_{\phi,i}$ the measure of

ϕ in the i^{th} EU; $a_\phi = \sum_{i=1}^{n} (a_{\phi,i} \cdot v_i)$ the value of ϕ in the solid.

$h_{\phi,i}$ is an intrinsic characteristic of the i^{th} EU: the involvement of the i^{th} EU in the determination of ϕ in the whole solid. The measure of $h_{\phi,i}$ is the product of an intensive term

$\left(\frac{a_{\phi,i} - a_\phi}{a_\phi}\right)$ accounting for the relative difference with respect to ϕ between the i^{th} EU and the solid by a term $\left(\frac{V_i}{\overline{V}}\right)$ or $(n \cdot v_i)$ standing for the volume contribution of the i^{th} EU within the solid. Others expressions for the extensive term can be choose, for example v_i; $\left(\frac{V_i}{\overline{V}}\right)$ was preferred because: (i) it corresponds to a former definition (Gy, 1982, 1986) and (ii) it offers the possibility to treat the case of solids where n cannot be determined, as did Gy (1982, 1986) for granular materials. If $a_{\phi,i} = a_\phi$, the contribution of i to the heterogeneity of the solid is nil. If $a_{\phi,i}$ is high, but if the volume of the i^{th} EU is very small compared to the others, the contribution of this EU to the value of a_ϕ is negligible (the volume contribution is very little). On the other hand, a EU with high values for intensive term and volume contribution will have a considerable involvement in the determination of a_ϕ in the solid and consequently his contribution to the heterogeneity of ϕ will be important.

According to Gy (1982, 1988) the constitution heterogeneity of the solid with respect to ϕ (HC_ϕ) is the variance of $h_{\phi,i}$:

$$HC_\phi = n \cdot \sum_{i=1}^{n} \left(\frac{a_{\phi,i} - a_\phi}{a_\phi}\right)^2 \cdot v_i^2 . \qquad (2)$$

Note that if $\forall i, a_{\phi,i} = a_\phi$, then $HC_\phi = 0$, the solid is homogeneous with respect to ϕ. HC can be seen as a measure of the grade of similarity between the EU with regard to ϕ: for decreasing value of HC_ϕ the similarity between the EU increases, at least with respect to the characteristic ϕ.

HC_ϕ is an intrinsic characteristic of the solid taking into account the magnitude of ϕ in the solid and the size and ϕ content of each EU; therefore, as show the comparison between solids 1, 4 and 7 (or 2, 5 and 8, or 3, 6 and 9) in Figure 1, HC_ϕ can work as an estimate of the

granularity parameter. Now, the reason for such name appears more clearly. In physics, the granularity account for the smallest undividable unit used to describe a system: molecule in a gas or atom in a molecule. More over, this term is also used in civil engineering about the size distribution of particle making up an aggregate. These two meanings apply in our case because: (i) EUs are the smallest undividable units in the model solid and (ii) HC_ϕ takes into account the EU's volume.

There is a special case when 0 and 1 are the only permitted values for a_ϕ. This occurs for instance, if ϕ is the porosity, then EUs correspond to solid grains ($a_{\phi,i} = 0$) and pores ($a_{\phi,i} = 1$); equation 2 can be rewritten as:

$$HC_\phi = n \cdot \sum_{c=1}^{C} \left(\frac{1 - a_\phi}{a_\phi} \right)^2 \cdot v_c^2 + n \cdot \sum_{m=1}^{n-C} v_m^2 = HC_{\phi,c} + HC_{\phi,m} . \qquad (3)$$

The term $HC_{\phi,c}$ represents the contribution to the heterogeneity of the EUs bearing the studied parameter (i.e. if i is a pore, $a_{\phi,i} = 1$), the number of these EU is C and these EUs are indexed c in (3). The term $HC_{\phi,m}$ expresses the contribution to heterogeneity of the "matrix", i.e. the set of EU for which $a_{\phi,i} = 0$ (i.e. solid grains), they are (n – C) and indexed m in (3).

Values of constitution heterogeneity from 2D model solids are given in Figure 1 and Table 1. These solids display a binary mineralogical composition i.e. black (c phase) and white (m phase) polygons. The studied characteristic ϕ is the volumetric "black polygons" content. If all EU have the same size, HC_ϕ decreases as a_ϕ increases, i.e. as the "black polygons" content raises (compare solids 1 and 4 in Figure 1 or the solids which data appear in the three lines at the top of Table 1), more EU are similar, the solid becomes more homogeneous and HC_ϕ tends to zero. Obviously this variation is more effective for $HC_{\phi c}$ than for $HC_{\phi m}$. At constant a_ϕ, granulometric scattering within black or white polygons produces an increase, respectively

in $HC_{\phi c}$ and $HC_{\phi m}$ (compare solids 4 and 7 in Figure 1 or the solids which data appear in the three lower lines of Table 1).

Note that if the studied characteristic is different, for example if ϕ is the EU's volume, solid 1 to 6 (Fig. 1) are homogeneous and the corresponding HC_{ϕ} is nil.

3. Evaluation of distribution parameter using distribution heterogeneity

HC_{ϕ} is independent of the spatial distribution of ϕ in the solid. On account for this factor, let consider a network devoted to gather information on the spatial distribution of ϕ. This network would be designed in a way insuring homogeneity during collection of data. This could be done with a shape cell more isotropic as possible. For convenience we use a cubic periodic lattice characterised by: (i) the volume V_{MO} of the cell and (ii) the scale α of observation ($\alpha = V_{\Sigma}/V_{MO}$). Note that α is the number of cell in a network of scale α. Unbiased condition is realized if the lattice strictly fits the solid.

We can apply (1) and (2) at this new problem, considering the cell content as the EU and V_i as the volume U_j of matter assigned to the cell j. If n_j is the number of EU assigned to the cell j, then: $U_j = \sum_{i=1}^{n_j} V_i$. As previously, the contribution of a cell j to the heterogeneity of ϕ in the model solid can be evaluated by:

$$h_{\phi,\alpha,j} = \left(\frac{a_{\phi,\alpha,j} - a_{\phi}}{a_{\phi}} \right) \cdot \left(\frac{U_j}{\overline{U}(\alpha)} \right) \qquad (4)$$

where $a_{\phi,\alpha,j}$ is the value of ϕ within the matter assigned to the j^{th} cell belonging to a lattice of scale α: $a_{\phi,\alpha,j} = \frac{1}{U_j} \cdot \sum_{i=1}^{n_j} a_{\phi,i} \cdot V_i$ and $\overline{U}(\alpha)$ average of U_j depending on α. Note that, if $V_{MO} = V_{\Sigma}$ (i.e. if the lattice contains only one cell to which the total volume of the solid is assigned) $\Rightarrow \alpha = 1$, $n_j = n$ and $U_j = V_{\Sigma}$, then $a_{\phi,\alpha=1,j} = a_{\phi}$.

To distinguish an aggregate of EU identical in ϕ from one EU displaying the same value for ϕ and a volume equal to that of the aggregate (Fig. 2), the following rule is adopted: all EU which barycentre belong to the j^{th} cell are assigned to this cell. As the sum of U_j is equal to V_Σ, a result of this rule is that U_j can be greater than V_{MO} and consequently some cells can be empty (Fig. 2). This procedure could be seen complicate, but it insures that HC and HD are evaluated using strictly the same set of EU, i.e. the same model solid. It can be seen as a consequence of the indivisible character of the EU with respect to the limits of the cells.

Let $\beta(\alpha)$ the number of cell devoid of matter, the number of cell bearing matter is $[\alpha - \beta(\alpha)]$ and the average of matter volume assigned to the $[\alpha - \beta(\alpha)]$ "filled" cells is: $\overline{U}(\alpha) = \frac{1}{\alpha - \beta(\alpha)} \cdot \sum_{j=1}^{\alpha-\beta(\alpha)} U_j$. As $\sum_{j=1}^{\alpha-\beta(\alpha)} U_j = V_\Sigma$, $\overline{U}(\alpha) = \frac{V_\Sigma}{\alpha - \beta(\alpha)}$. Note that $\overline{U}(\alpha)$ is calculated using only the "filled" cells, i.e. the cells bearing information about the spatial distribution of ϕ. Our purpose is to distinguish a "filled" cell where $a_{\phi,j} = 0$ from a cell devoid of matter (cells coded $\varnothing$ in Figure 2). In the line of Gy (1982, 1988), at the scale α, the distribution heterogeneity of the solid with respect to ϕ (i.e. $HD_{\phi,\alpha}$) is the variance of $h_{\phi,\alpha,j}$.

$$HD_{\phi,\alpha} = \frac{1}{(\alpha - \beta(\alpha))} \cdot \sum_{j=1}^{\alpha-\beta(\alpha)} \left[\left(\frac{a_{\phi,\alpha,j} - a_\phi}{a_\phi} \right) \cdot \left(\frac{U_j}{\overline{U}(\alpha)} \right) \right]^2 = (\alpha - \beta(\alpha)) \cdot \sum_{j=1}^{\alpha-\beta(\alpha)} \left[\left(\frac{a_{\phi,\alpha,j} - a_\phi}{a_\phi} \right) \cdot u_j \right]^2 \quad (5)$$

where $u_j = U_j / V_\Sigma$ is the relative volume of matter assigned to the j^{th} cell, $a_{\phi,\alpha,j}$ the value of ϕ within the matter assigned to the j^{th} cell of a α-scale lattice. $a_{\phi,\alpha,j} = \frac{1}{U_j} \cdot \sum_{i=1}^{n_j} a_{\phi,i} \cdot V_i$, where n_j is number of EU assigned to the cell j. Note that: $a_\phi = \sum_{i=1}^{n} (a_{\phi,i} \cdot v_i) = \sum_{j=1}^{\alpha-\beta(\alpha)} (a_{\phi,\alpha,j} \cdot u_j)$.

$HD_{\phi,\alpha}$ is a function of: (i) the scale α of the lattice, (ii) the spatial distribution of ϕ.

Let first consider a model solid where: $\forall i$, $a_{\phi,i} = a_\phi$. Obviously in this case $HC_\phi = 0$. As all EU display the same value for ϕ, the spatial distribution of ϕ in the model solid is homogeneous, i.e. $\forall\alpha$, $HD_{\phi,\alpha} = 0$.

For $HC_\phi > 0$, $HD_{\phi,\alpha}$ displays two useful properties:

(i) If $V_{MO} = V_\Sigma \Rightarrow \alpha = 1 \Rightarrow a_{\phi,\alpha,j} = a_\phi \Rightarrow HD_{\phi,\alpha} = 0$.

(ii) Let n_j be the number of EU assigned to the cell j. There is a value α_c of α such as $\forall j$, $n_j = 0$ or 1. If $n_j = 1$, the cell j is "filled" by one and only one EU labelled i, then $a_{\phi,\alpha,j} = a_{\phi,i}$, where $a_{\phi,i}$ is the value of ϕ in the EU i bellowing to cell j. If $n_j = 0$, the cell is devoid of matter. Then for $\alpha = \alpha_c$, $HD_{\phi,\alpha} = HC_\phi$. For $\alpha > \alpha_c$, the number of cells for which $n_j = 1$ is the same as the former case (i.e. $\alpha = \alpha_c$) and the number of cells for which $n_j = 0$ increase. In these conditions, for $\alpha \geq \alpha_c$, $HD_{\phi,\alpha} = HC_\phi$. Note that, in this case: $\alpha - \beta(\alpha) = n$. This latter relation justifies the way we use to calculate $\overline{U}(\alpha)$.

Therefore a diagram $HD_{\phi,\alpha}/HC_\phi$ versus α (distribution curve) can be used to compare the spatial distributions of ϕ in distinct solids (Figures 3 and 4). If $\forall\alpha < \alpha_c$ $HD_{\phi,\alpha} = 0$, the solid is homogeneous with respect to the spatial distribution of ϕ (Figure 3b). If $HC_\phi = 0 \Rightarrow HD_{\phi,\alpha} = 0$. If $HD_{\phi,\alpha}/HC_\phi$ increases continuously with α, several model functions can be proposed (Table 2). The purpose of these model functions is to fit the experimental distribution curve as geostatisticians do for experimental variograms. The form of the proposed model function (first and second column of Table 2) is drawn from the classical model variograms (see for example, Isaaks and Srivastava 1989). Note that variograms (γ =f(h)) and distribution curves ($HD\phi,\alpha/HC\phi = f(\alpha)$) present some similarities: they display a variance and a variance ratio (γ and $HD\phi,\alpha/HC\phi$ respectively) versus a scale of observation (h and α respectively). Distribution curves consider a discretized volume and then take into account the

morphological properties of the components through the EU volume, while variogram describes the spatial variations of a characteristic in a continuous media.

Distribution curves for different sets of squares and rectangles are given in Figure 4.

Note that, if V_{min} is the volume of the smallest unit cell that can be used to describe the structure of a periodically ordered solid displaying a cubic symmetry, then $HD_{\phi,\alpha} = 0$ if $V_{\sum}/V_{min}$ is an integer.

For a given model solid, the integral $A_\phi = \int_1^{\alpha_c} \frac{HD_{\phi,\alpha}}{HC_\phi} \cdot d\alpha$ is defined without ambiguity (Figure 3a), so A_ϕ can work as an estimate of the distribution parameter. A_ϕ can be seen as the departure from a homogeneous spatial distribution (Figure 3). For each model function in Table 2, the value of the integral between $\alpha = 1$ and $\alpha = \alpha_c$ (i.e. A_ϕ) has been calculated. The results are given in the third column of Table 2.

Note that an unbiased comparison of A_ϕ between several solids requires the use of the same network. This condition is realised if samples exhibit equal volume $V_{\sum}$ and identical shape.

4. Representation of texture using HC_ϕ and A_ϕ

According to the definition given at the beginning of section 1, if a texture can be characterized with just one characteristic ϕ, a representation of this texture can be done in a diagram HC_ϕ (granularity parameter) versus A_ϕ (distribution parameter). Examples are given in Figure 5 for the nine solids drawn in Figure 1. A textural description involves generally several properties. Let consider a texture characterised by K properties. In a HC_ϕ versus A_ϕ diagram the set of K points represents the texture and a textural change appears as K different translations working on the initial K points.

If HC_ϕ and A_ϕ could be used to assess a texture, other parameters defined above, as α_c and $\beta(\alpha)$, characterise the geometry of the EUs. As showed in Figure 6, $\beta(\alpha)$ is a function of morphology, orientation, volume V_i, size distribution of V_i and α. So, a diagram $\beta(\alpha)$ versus α can give granularity and/or morphology and/or orientation information on the components of the model solid.

5 Conclusion

Based on the concept of heterogeneity defined by Gy (1982, 1988), two parameters, HC_ϕ and A_ϕ have been defined to characterize the distribution of a given characteristic ϕ in a coherent solid. The first is an intrinsic characteristic of the solid, independent of the spatial distribution of the characteristic, but taking into account the granularity of the EUs making up the model solid. The second parameter measures the spatial distribution of the characteristic. These two parameters are defined without ambiguity for any characteristic measurable within each constituent of the model solid. To compare different model solids, unbiased conditions are realized if the volume and the shape of the samples are identical. Then, HC_ϕ and A_ϕ appear as suitable tools to describe the texture and textural changes.

The possibility to calculate HC_ϕ and A_ϕ for numerous properties confers to the method a high grade of adaptability, allowing the use of a large number of characteristics to compare textures.

A_ϕ and HC_ϕ parameters can be used for two main types of applications: textural classifications and monitoring of textural transformation during processes like dolomitization, metamorphism, deformation, weathering or annealing. Note that the texture of any composite such as concrete or breccia can be assessed by the proposed method.

The field of textural classification encompasses a great diversity of solids and problems. Some of them are: concrete (through, for example, the granularity and distribution of

aggregate); porosity in coherent solids or soils (for each EUi, $a_{\phi,i} = 1$ if the i^{th} EU is a pore and $a_{\phi,i} = 0$ if the i^{th} EU is a solid grain); soils (in this case, at least, three characteristics ϕ_1, ϕ_2, and ϕ_3, can work: the sand (ϕ_1), silt (ϕ_2) and clay (ϕ_3) size fractions used by pedologists to characterize a soil); a quantized classification of breccia based on the granularity and distribution of the characteristics (for example: mineralogical composition, size, shape) of elements and cement; classification of ore texture where granularity and distribution refer to the valuable mineral ($a_{\phi,i} = 1$ if the i^{th} EU is a grain of valuable mineral and $a_{\phi,i} = 0$ if the i^{th} EU is a gangue mineral). In the latter case, as HC_ϕ refers to the liberation size, a classification of ore texture would also be a classification of the cost of mineral processing.

At last, using the Shannon theory of information in the same way as Martin and Rey (2000) and Martin and al. (2005), it is likely to give to A_ϕ a more relevant significance in term of relative entropy.

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Appendix: definitions of parameters follow the text

§2. Evaluation of granularity parameter using constitution heterogeneity

EU, elementary unit, undividable component of the model solid.

n, number of EU.

ϕ, a characteristic measurable in each EU.

HC, constitution heterogeneity.

HD, distribution heterogeneity.

V_i , volume of the i^{th} EU.

V_Σ solid volume, $V_\Sigma = \sum_{i=1}^{n} V_i$.

$\overline{V}$ average of V_i , $\overline{V} = \frac{1}{n} \cdot \sum_{i=1}^{n} V_i$.

v_i , relative volume of the i^{th} EU, $v_i = \frac{V_i}{V_\Sigma}$.

$a_{\phi,i}$, measure of ϕ in the i^{th} EU.

a_ϕ , measure of ϕ in the solid, $a_\phi = \sum_{i=1}^{n} a_{\phi,i} \cdot v_i$.

$h_{\phi,i}$, contribution of i^{th} EU to the scattering of ϕ within the solid,

$$h_{\phi,i} = \left(\frac{a_{\phi,i} - a_\phi}{a_\phi} \right) \cdot \frac{V_i}{\overline{V}} = n \cdot \left(\frac{a_{\phi,i} - a_\phi}{a_\phi} \right) \cdot v_i$$

HC_ϕ, constitution heterogeneity in the solid with respect to the characteristic ϕ,

$$HC_\phi = \mathrm{var}(h_{\phi,i}) = \frac{1}{n} \cdot \sum_{i=1}^{n} h_{\phi,i}^2 = n \cdot \sum_{i=1}^{n} \left(\frac{a_{\phi,j} - a_\phi}{a_\phi} \right)^2 \cdot v_i^2 .$$

$HC_{\phi,c} = n \cdot \sum_{c=1}^{C} \left(\frac{1 - a_\phi}{a_\phi} \right)^2 \cdot v_c^2$ If $a_{\phi,i} = 0 \text{ or } 1$, $HC_{\phi,c}$ represents the contribution to heterogeneity of the EUs bearing the studied characteristic (i.e. these for which $a_{\phi,i} = 1$). These EUs are C and indexed c.

$HC_{\phi,m} = n \cdot \sum_{m=1}^{n-C} v_m^2$ If $a_{\phi,i} = 0 \text{ or } 1$, $HC_{\phi,m}$ represents the contribution to heterogeneity of the EUs belonging to the "matrix" (i.e. these for which $a_{\phi,i} = 0$). These EUs are (n-C) and indexed m.

$HC_\phi = HC_{\phi,c} + HC_{\phi,m}$

§3. Evaluation of distribution parameter using distribution heterogeneity

V_{MO} , volume of the cell of the network devoted to gather information on the distribution of ϕ.

$\alpha = V_\Sigma / V_{MO}$, observation scale, i.e. the scale of the network devoted to gather information on the distribution of ϕ, α is also the number of cells.

n_j , number of EU in the cell j.

$a_{\phi,\alpha,j}$, measure of ϕ in the j^{th} cell belonging to a lattice of scale α, $a_{\phi,\alpha,j} = \frac{1}{U_j} \cdot \sum_{i=1}^{n_j} a_{\phi,i} \cdot V_i$.

U_j , volume of matter assigned to the j^{th} cell.

$\beta(\alpha)$, number of cell (network at scale α) devoid of matter.

$\overline{U}(\alpha) = \frac{1}{\alpha - \beta(\alpha)} \cdot \sum_{j=1}^{\alpha-\beta(\alpha)} U_j = \frac{V_\Sigma}{\alpha - \beta(\alpha)}$, average of U_j depending on α.

$u_j = U_j / V_\Sigma$, relative volume of matter assigned to the cell j.

$h_{\phi,\alpha,j} = \left(\frac{a_{\phi,\alpha,j} - a_\phi}{a_\phi} \right) \cdot \left(\frac{U_j}{\overline{U}(\alpha)} \right)$, contribution of j^{th} cell to the scattering of ϕ within the solid.

$HD_{\phi,\alpha}$, distribution heterogeneity at the scale α,

$$HD_{\phi,\alpha} = \mathrm{var}\left(h_{\phi,\alpha,j}\right) = \left(\alpha - \beta(\alpha)\right) \cdot \sum_{j=1}^{\alpha-\beta(\alpha)} \left[\left(\frac{a_{\phi,\alpha,j} - a_{\phi}}{a_{\phi}} \right) \cdot u_j \right]^2 .$$

α_c , value of α such as $\forall j$, $n_j = 0$ or 1. For $\alpha \geq \alpha_c$, $HD_{\phi,\alpha} = HC_{\phi}$.

$A_{\phi} = \int_1^{\alpha} \frac{HD_{\phi,\alpha}}{HC_{\phi}} \cdot d\alpha$ measure of the spatial distribution of ϕ. A_{ϕ} can be seen as the departure from a homogeneous spatial distribution of ϕ.

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Captions of figures and tables

Figure 1.

Definition of granularity and distribution parameters illustrated by nine 2D solids labelled by encircled numbers. The studied characteristic ϕ is the black polygon content (i.e. a_ϕ). Granulometric information about solids is given in the table. In all solids, white EUs have the same shape (square) and size. The three solids 1, 2 and 3 possess the same granularity, value for a_ϕ and consequently the same heterogeneity of constitution with respect to ϕ (i.e. HC_ϕ). The same remark is valid for group 4, 5 and 6 and group 7, 8 and 9. Distribution heterogeneity increases from 1 to 3, 4 to 6 and 7 to 9.

Figure 2.

Attribution's rule of EUs to a given cell. Bold and double lines represent respectively the limit of EUs and the boundary of cells. Left column show a special case where the volume of cell is equal to the volume of EUs. Then, inside each cell there is one and only one barycentre of a EU and all cells are "filled" (i.e. $\beta(\alpha) = 0$). In this case, $HD_\phi = HC_\phi$, so $\alpha=\alpha_c$. The "grey content" in solids draw in the right and left column is equal, but in the right column there is only one grey EU with a volume four times higher than the volume of grey EUs in the left column. The barycentre of this unique EU belong to one cell to which all the matter of this EU is assigned. This implies that three cells are devoid of matter (i.e. cells coded by $\varnothing$, thus $\beta(\alpha) = 3$). Note that: $\sum_{j=1}^{\alpha-\beta(\alpha)} U_j = V_\Sigma$ and thus $\overline{U}(\alpha) = \frac{1}{\alpha-\beta(\alpha)} \cdot \sum_{j=1}^{\alpha-\beta(\alpha)} U_j = \frac{V_\Sigma}{\alpha-\beta(\alpha)}$.

Figure 3.

Distribution curves and definition of A_ϕ, (a): general case, (b): homogeneous distribution case.

Figure 4.

Distribution curves for "2D solids" depicted on the right of the diagrams. The studied variable is the black polygons content. Characteristics of the nine solids are given in Figure 1 and Table 1. Bracketed numbers are the values of A_ϕ. Note that A_ϕ increases with the clustering of EUs (i.e. when spatial homogeneity decreases).

Figure 5.

Textural representation using HC_ϕ versus A_ϕ diagram. Numbers refer to the nine solids depicted in Figures 1 and 4 and Table 1.

Figure 6.

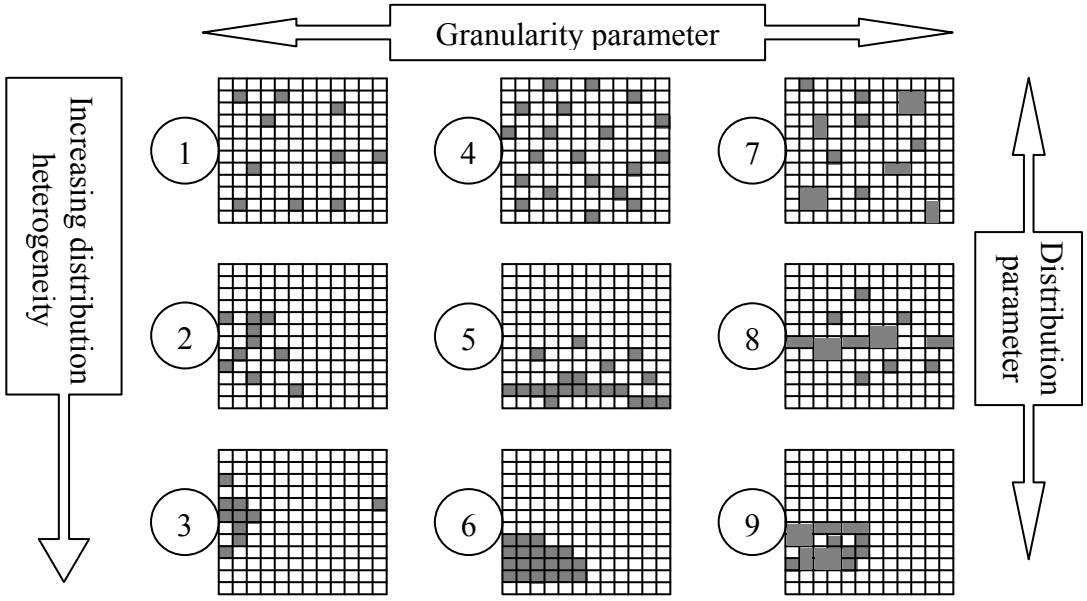
Sketches highlight factors controlling the value of $\beta(\alpha)$. Simple and double lines represent respectively the limit of EUs and the boundary of cells. First column: solids; second column: solids and lattices; third column: lattice, numbers correspond to the volume of matter assigned to each cell; cells coded $\varnothing$ are devoid of matter.

Table 1.

Constitution heterogeneity. Data from the three lines at the top of the table emphasize the effect of a_ϕ variation on HC. Data in the three lines at the foot of the table point to the influence of granularity on HC. The last column refers to the solids depicted in Figure 1. The solid depicted in the first row is similar to solids 1 to 6 in Figure 1, but contains only 5 black squares. For the significance of $HC_{\phi,c}$, $HC_{\phi,m}$ and HC_ϕ see text or appendix.

Table 2

Models for distribution curve (in the case where $HD_{\phi,\alpha}/HC_{\phi}$ increases continuously with α) with the corresponding values of A_{ϕ}, a is a fitting parameter, for the significance of α_c, see text or appendix. "erf" is the error function: $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-u^2) \cdot du$. "erf" is classically used to calculate integrals such as $\int_s^t \exp(r \cdot x^2) \cdot dx$ through the change of variable u. For example, in the case considered: $u = \frac{\sqrt{a}}{\alpha_c} \cdot \alpha$.



Solids	1, 2, 3		4, 5, 6		7, 8, 9
Proportion of black area = a_ϕ	0.07		0.14		0.14
Distribution size of black squares	10 black squares of 1 unit area		20 black squares of 1 unit area		2 black squares of 4 units area 3 black rectangles of 2 units area 6 black squares of 1 unit area
HC_ϕ	13.4		6.20		13.32

Figure 1.

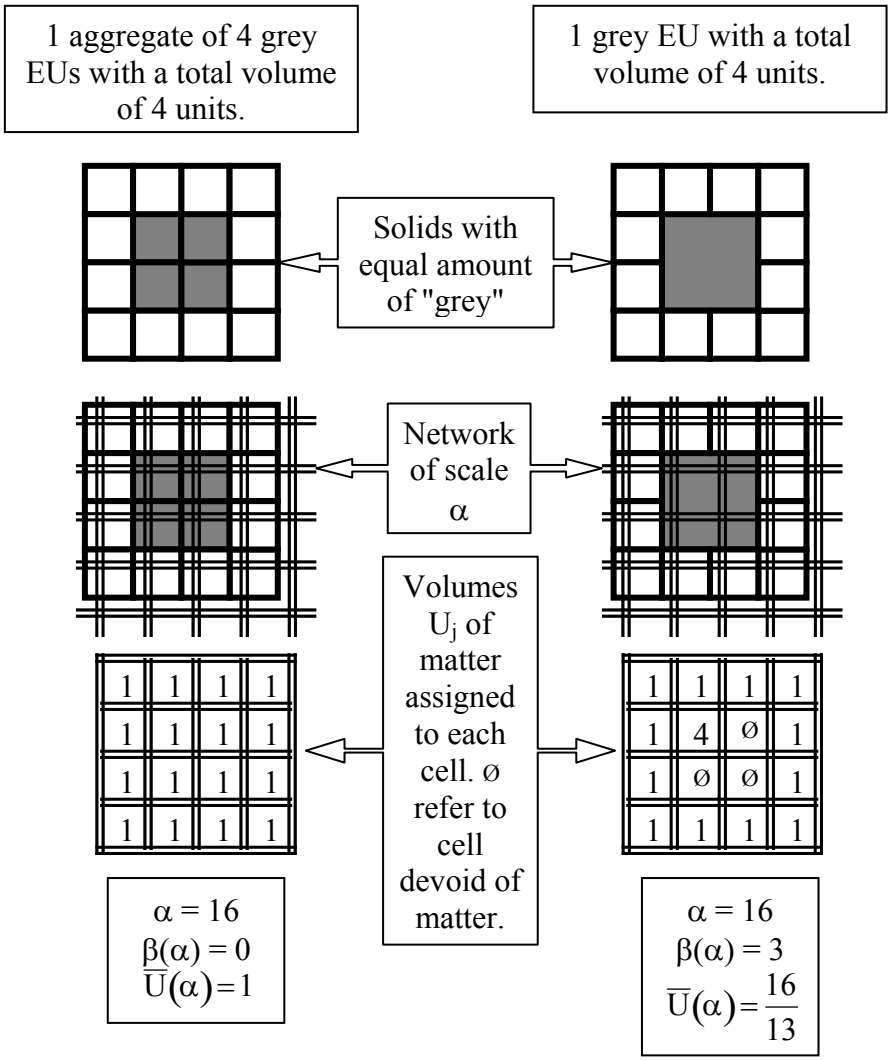

1 aggregate of 4 grey EUs with a total volume of 4 units.
1 grey EU with a total volume of 4 units.
Solids with equal amount of "grey"
Network of scale α
Volumes U_j of matter assigned to each cell. ø refer to cell devoid of matter.
1 1 1 1
1 1 1 1
1 1 1 1
1 1 1 1
α = 16
β(α) = 0
$\overline{U}(\alpha) = 1$
1 1 1 1
1 4 ø 1
1 ø ø 1
1 1 1 1
α = 16
β(α) = 3
$\overline{U}(\alpha) = \frac{16}{13}$

Figure 2

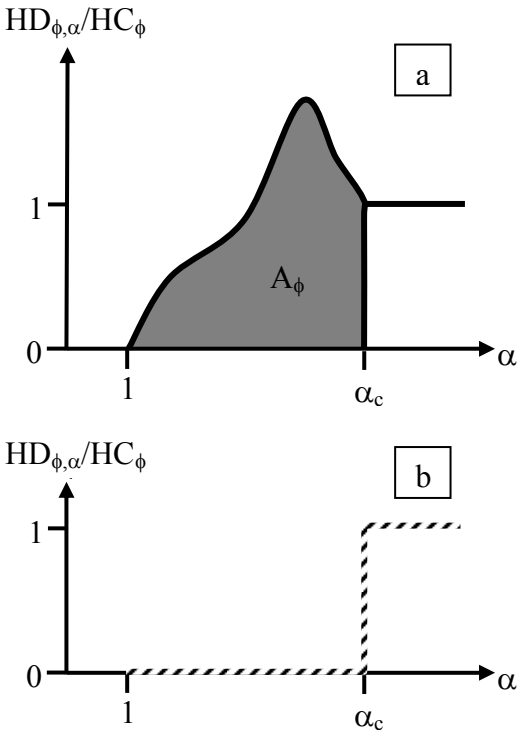

HD$_{\phi,\alpha}$/HC$_\phi$
a
1
A$_\phi$
0
1
α_c
α
HD$_{\phi,\alpha}$/HC$_\phi$
b
1
0
1
α_c
α

Figure 3

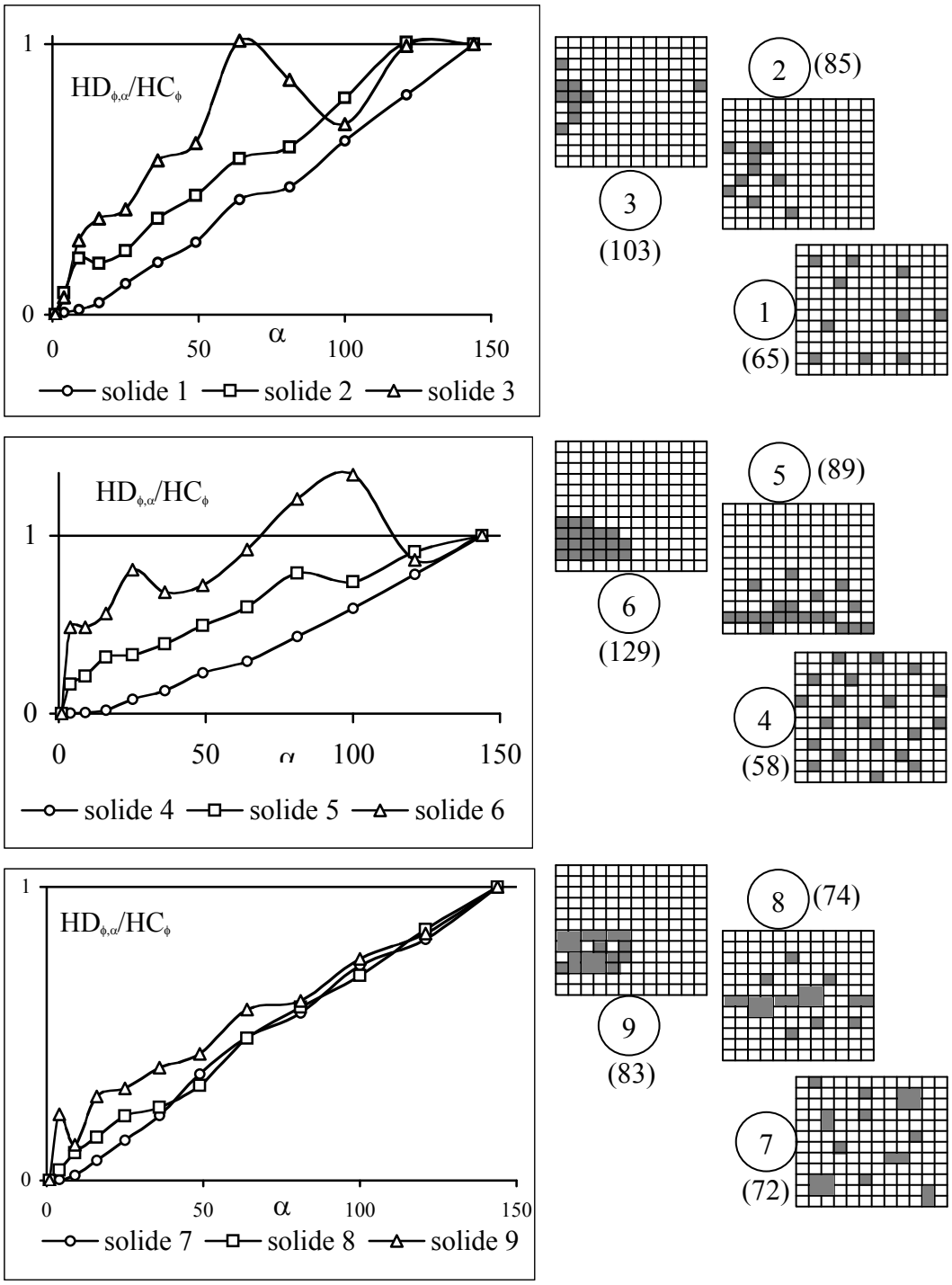

1
HD$_{\phi,\alpha}$/HC$_\phi$
0
0
50
α
100
150
solide 1
solide 2
solide 3
2
(85)
3
(103)
1
(65)
HD$_{\phi,\alpha}$/HC$_\phi$
1
0
0
50
100
150
solide 4
solide 5
solide 6
5
(89)
6
(129)
4
(58)
1
HD$_{\phi,\alpha}$/HC$_\phi$
0
0
50
α
100
150
solide 7
solide 8
solide 9
8
(74)
9
(83)
7
(72)

Figure 4

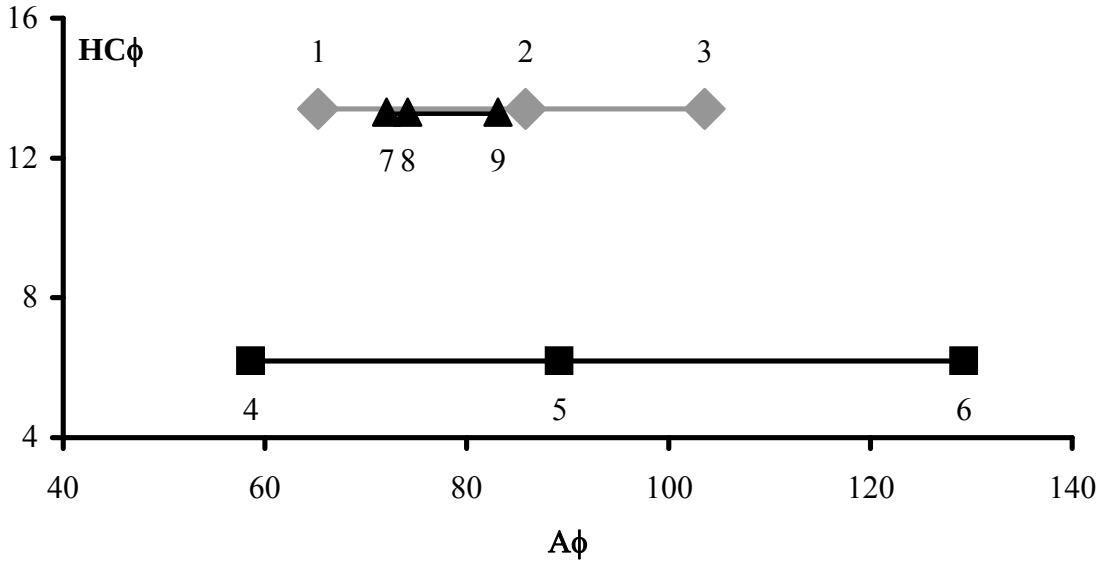


Figure 5

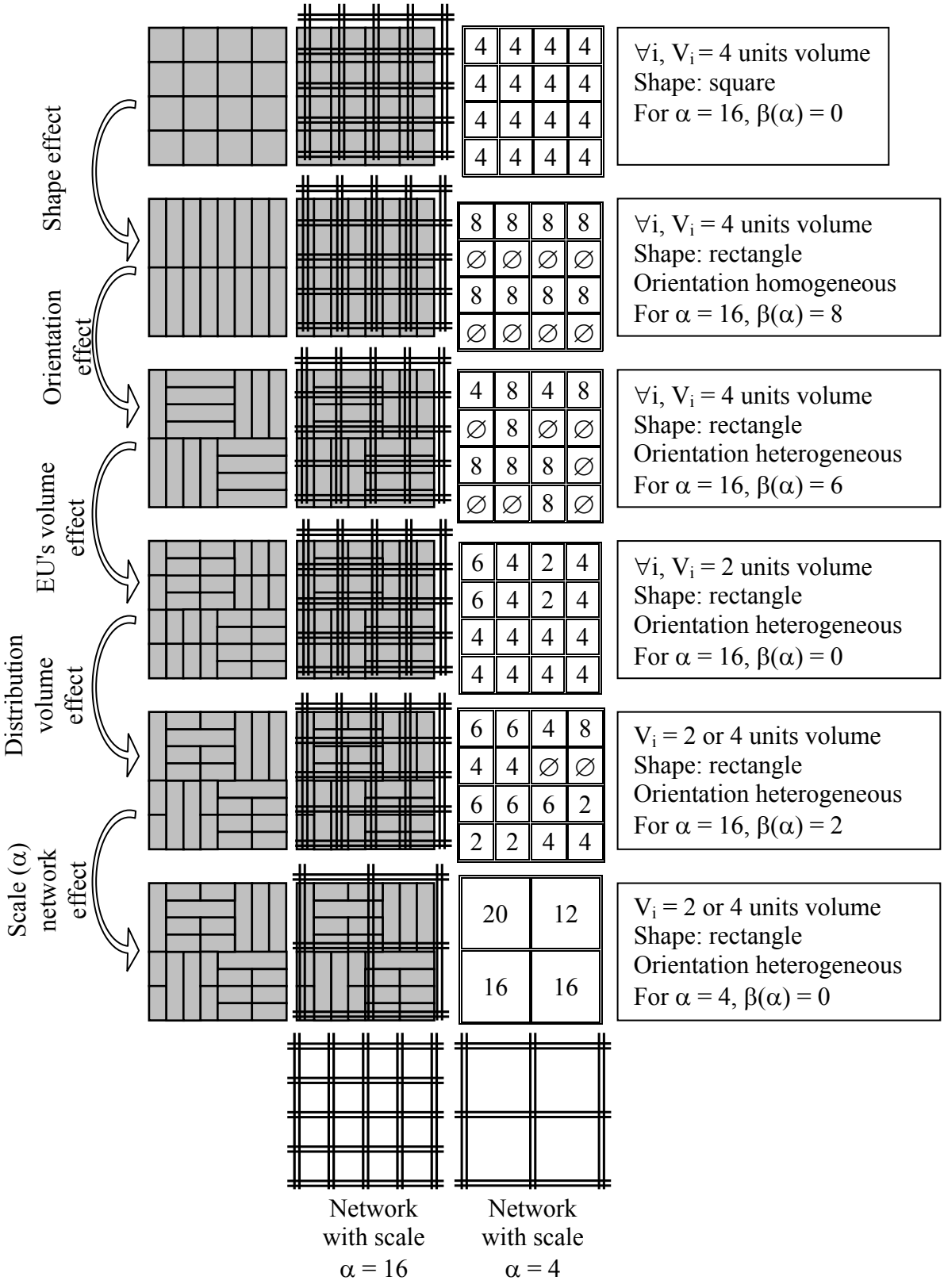

Shape effect
Orientation effect
EU's volume effect
Distribution volume effect
Scale (α) network effect
4 4 4 4
4 4 4 4
4 4 4 4
4 4 4 4
∀i, Vi = 4 units volume
Shape: square
For α = 16, β(α) = 0
8 8 8 8
∅ ∅ ∅ ∅
8 8 8 8
∅ ∅ ∅ ∅
∀i, Vi = 4 units volume
Shape: rectangle
Orientation homogeneous
For α = 16, β(α) = 8
4 8 4 8
∅ 8 ∅ ∅
8 8 8 ∅
∅ ∅ 8 ∅
∀i, Vi = 4 units volume
Shape: rectangle
Orientation heterogeneous
For α = 16, β(α) = 6
6 4 2 4
6 4 2 4
4 4 4 4
4 4 4 4
∀i, Vi = 2 units volume
Shape: rectangle
Orientation heterogeneous
For α = 16, β(α) = 0
6 6 4 8
4 4 ∅ ∅
6 6 6 2
2 2 4 4
Vi = 2 or 4 units volume
Shape: rectangle
Orientation heterogeneous
For α = 16, β(α) = 2
20 12
16 16
Vi = 2 or 4 units volume
Shape: rectangle
Orientation heterogeneous
For α = 4, β(α) = 0
Network with scale α = 16
Network with scale α = 4

Figure 6

Constitution Heterogeneity							
n	a_ϕ	Volumetric distribution of v_c (EU with $a_{\phi,i} = 1$)	Volumetric distribution of v_m (EU with $a_{\phi,i} = 0$)	$HC_{\phi,c}$	$HC_{\phi,m}$	HC_ϕ	Solids showed in figures 1 and 4
144	0.03	5 EU c of 1 unit volume	All EU m have the same volume V_i = 1 unit volume	26.83	0.97	27.80	
144	0.07	10 EU c of 1 unit volume		12.47	0.93	13.40	1, 2, 3
144	0.14	20 EU c of 1 unit volume		5.34	0.86	6.20	4, 5, 6
135	0.14	• 2 EU c have a volume of 4 units volume • 3 EU c have a volume of 2 units volume • 6 EU c have a volume of 1 unit volume	All EU m have the same volume V_i = 1 unit volume	12.51	0.81	13.32	7, 8, 9
84	0.14	20 EU c have a volume of 1 unit volume	20 EU m have a volume of 4 units volume 44 EU m have a volume of 1 unit volume	3.11	1.47	4.58	

Table 1

Some models for distribution curve		
Models	$HD_{\phi,\alpha}/HC_{\phi}$	A_{ϕ}
Linear	$\frac{HD_{\phi,\alpha}}{HC_{\phi}}=\left(\frac{1}{\alpha_c-1}\right)\cdot\alpha-\left(\frac{1}{\alpha_c-1}\right)$	$\left(\frac{1}{\alpha_c-1}\right)\cdot\left(\frac{\alpha_c^2}{2}-\alpha_c-\frac{1}{2}\right)$
Power	$\frac{HD_{\phi,\alpha}}{HC_{\phi}}=\left(\frac{\alpha-1}{\alpha_c-1}\right)^a$	$\frac{(\alpha_c-1)^{a+1}}{(a+1)\cdot(\alpha_c-1)^a}$
Exponential	$\frac{HD_{\phi,\alpha}}{HC_{\phi}}=\frac{1-\exp\left[\frac{a}{\alpha_c}\cdot(1-\alpha)\right]}{1-\exp\left[a\cdot\left(\frac{1}{\alpha_c}-1\right)\right]}$	$\frac{\alpha_c-1}{1-\exp\left[a\cdot\left(\frac{1}{\alpha_c}-1\right)\right]}-\frac{\alpha_c}{a}$
Gaussian	$\frac{HD_{\phi,\alpha}}{HC_{\phi}}=\frac{1-\exp\left[\frac{a}{\alpha_c^2}\cdot(1-\alpha^2)\right]}{1-\exp\left[a\cdot\left(\frac{1}{\alpha_c^2}-1\right)\right]}$	$\frac{\alpha_c-1-\frac{\alpha_c}{2}\cdot\sqrt{\frac{\pi}{a}}\cdot\exp\left(\frac{a}{\alpha_c^2}\right)\cdot\left[\operatorname{erf}\left(\sqrt{a}\right)-\operatorname{erf}\left(\frac{\sqrt{a}}{\alpha_c}\right)\right]}{1-\exp\left[a\cdot\left(\frac{1}{\alpha_c^2}-1\right)\right]}$

Table 2