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1 Introduction

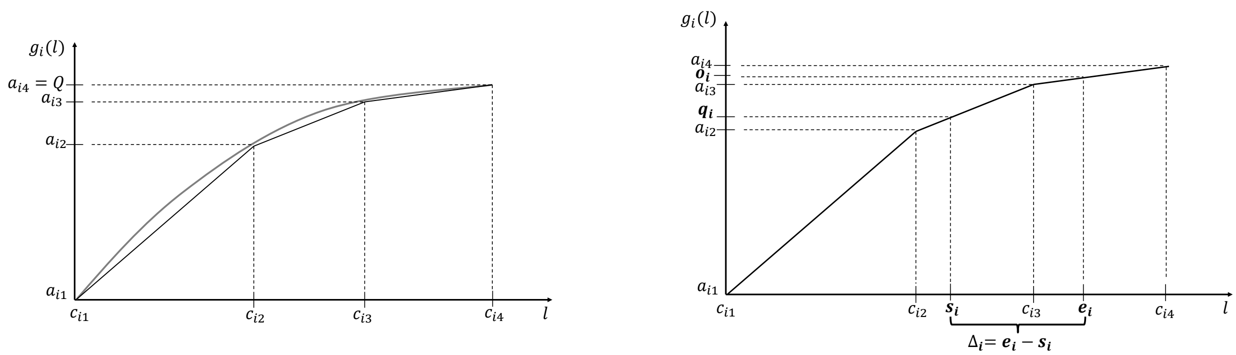
Electric vehicles (EVs) are one of the most promising technologies to reduce greenhouse gas emissions in the transportation sector [1]. Recently, the use of electric vehicles (EVs) in freight and passenger transportation gives birth to a new family of vehicle routing problems (VRPs), the so-called electric VRPs (e-VRPs). As their name suggests, e-VRPs extend classical VRPs to account (mainly) for two constraining EV features: the short driving range and the long battery charging time. As a matter of fact, routes performed by EVs usually need to include time-consuming detours to charging stations.

Most of the existing literature on e-VRPs relies on one of the following assumptions: i) vehicles recharge to their battery to its maximum level every time they reach a charging station or ii) the amount of battery charge is a linear function of the charging time. In practical situations, however, the amount of charge (and thus the time spent at each charging point) is a decision variable and battery charge levels are a concave function of the charging times. In this research we introduce the electric vehicle routing problem with non-linear charging functions (e-VRP-NLCF). We propose a mixed-integer linear programming (MILP) formulation that, running on a commercial solver, is able to solve small instances of the problem. To tackle large-scale instances we propose a metaheuristic that uses a MILP formulation to find the optimal charging policy. We report on extensive computational experiments evaluating the performance of the proposed methods and analyzing the impact on the solutions of different charging policy assumptions.

2 Problem description and mathematical model

Let I be the set of customers, F be the set of charging stations (CSs), and 0 be the depot where every route starts and ends. The e-VRP-NLCF is defined on an undirected and complete graph $G = (V, E)$, where $V = \{0\} \cup I \cup F'$ and F' contains copies of the CSs. For each CS $i \in F$, the number of copies in F' corresponds to the number of times that i can be visited in a solution.

Each CS $i \in F'$ has a charging mode (e.g., standard, fast, quick), which is associated to a charging function $g_i(l)$ that represents the battery charge level when the EV is charged over l time units. Function $g_i(l)$ is concave with an asymptote at battery capacity Q (expressed in KWh). According to experimental analysis this function can be approximated by a piecewise linear function. As Figure 1a shows a_{ik} and c_{ik} represent the battery level and the charging time, for the breakpoint $k \in B$ of the CS $i \in F'$, where $B = \{1, .., b\}$ is the set of breakpoints. The set $E = \{(i, j) : i, j \in V, i \neq j\}$ corresponds to edges connecting vertices of V. Each edge (i, j) has two associated nonnegative values: a travel time t_{ij} and a distance d_{ij}. The customers are served using an unlimited fleet of EVs with a consumption rate cr (expressed in KWh/km). The EV driving-range constraint is dictated by Q and a tour duration constraint T_{max}. It is assumed that the EVs leave the depot with a fully charged battery, and that all CSs can handle an unlimited number of EVs simultaneously.



(a) Function parameters

(b) Battery levels and recharging times $i \in F'$

Figure 1: Piecewise linear approximation for the charging function

In the e-VRP-NLCF the objective is to find a set of routes of minimum total time, which is defined as the sum of the travel times and the recharging times, such that each customer is visited exactly once; the level of the battery when the EV arrives at any vertex is nonnegative; each route satisfies the maximum-duration limit; and, each route starts and ends at the depot.

In the formulation we use the following decision variables: let x_{ij} be a binary variable, equal to 1 if an EV travels from vertex i to j and 0 otherwise. Let y_j be the battery level upon departure from vertex $j \in V$. Let τ_j be the departure time at vertex $j \in V$. Let q_i and o_i be the battery levels when an EV arrives to and depart from CS $i \in F'$, and s_i and e_i be their associated charging times (see Figure 1b). Let $\Delta_i = e_i - s_i$ be the time spent at CS $i \in F'$. Finally, let α_{ik}, λ_{ik}, w_{ik} and z_{ik} be auxiliary variables for the piecewise linear approximation. The MILP formulation for the e-VRP-NLCF follows:

$$\min \sum_{i,j \in V} t_{ij} x_{ij} + \sum_{i \in F'} \Delta_i \tag{1}$$

s.t

$$\sum_{j\in V, i\neq j} x_{ij} = 1 \qquad \forall i \in I \tag{2}$$

$$\sum_{j\in V, i\neq j} x_{ij} \leq 1 \qquad \forall i \in F' \tag{3}$$

$$\sum_{j\in V, i\neq j} x_{ji} - \sum_{j\in V, i\neq j} x_{ij} = 0 \qquad \forall i \in V \tag{4}$$

$$cr \cdot d_{ij}x_{ij} - (1 - x_{ij})Q \leq y_i - y_j \leq cr \cdot d_{ij}x_{ij} + (1 - x_{ij})Q \qquad \forall i \in V, \forall j \in I \tag{5}$$

$$cr \cdot d_{ij}x_{ij} - (1 - x_{ij})Q \leq y_i - q_j \leq cr \cdot d_{ij}x_{ij} + (1 - x_{ij})Q \qquad \forall i \in V, \forall j \in F' \tag{6}$$

$$y_i \geq cr \cdot d_{i0}x_{i0} \qquad \forall i \in V \tag{7}$$

$$y_i = o_i \qquad \forall i \in F' \tag{8}$$

$$y_0 = Q \tag{9}$$

$$q_i \leq o_i \qquad \forall i \in F' \tag{10}$$

$$q_i = \sum_{k\in B} \alpha_{ik}a_{ik} \qquad \forall i \in F' \tag{11}$$

$$s_i = \sum_{k\in B} \alpha_{ik}c_{ik} \qquad \forall i \in F' \tag{12}$$

$$\sum_{k\in B} \alpha_{ik} = \sum_{k\in B} z_{ik} \qquad \forall i \in F' \tag{13}$$

$$\sum_{k\in B} z_{ik} = \sum_{j\in V} x_{ij} \qquad \forall i \in F' \tag{14}$$

$$\alpha_{ik} \leq z_{ik} + z_{i,k+1} \qquad \forall i \in F', \forall k \in B \setminus b \tag{15}$$

$$\alpha_{ib} \leq z_{ib} \qquad \forall i \in F' \tag{16}$$

$$o_i = \sum_{k\in B} \lambda_{ik}a_{ik} \qquad \forall i \in F' \tag{17}$$

$$e_i = \sum_{k\in B} \lambda_{ik}c_{ik} \qquad \forall i \in F' \tag{18}$$

$$\sum_{k\in B} \lambda_{ik} = \sum_{k\in B} w_{ik} \qquad \forall i \in F' \tag{19}$$

$$\sum_{k\in B} w_{ik} = \sum_{j\in V} x_{ij} \qquad \forall i \in F' \tag{20}$$

$$\lambda_{ik} \leq w_{ik} + w_{i,k+1} \qquad \forall i \in F,' \forall k \in B \setminus b \tag{21}$$

$$\lambda_{ib} \leq w_{ib} \qquad \forall i \in F' \tag{22}$$

$$\Delta_i = e_i - s_i \qquad \forall i \in F' \tag{23}$$

$$\tau_i + (t_{ij} + p_j)x_{ij} - T_{max}(1 - x_{ij}) \leq \tau_j \qquad \forall i \in V, \forall j \in I \tag{24}$$

$$\tau_i + \Delta_j + t_{ij}x_{ij} - (T_{max} + S_{max})(1 - x_{ij}) \leq \tau_j \qquad \forall i \in V, \forall j \in F' \tag{25}$$

$$\tau_j + t_{j,0} \leq T_{max} \qquad \forall j \in V \tag{26}$$

$$\tau_0 \leq T_{max} \tag{27}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i, j \in V \tag{28}$$

$$\tau_i \geq 0, y_i \geq 0 \qquad \forall i \in V \tag{29}$$

$$z_{ik} \in \{0, 1\}, w_{ik} \in \{0, 1\}, \alpha_{ik} \geq 0, \lambda_{ik} \geq 0 \qquad \forall i \in F', \forall k \in B \tag{30}$$

$$q_i \geq 0, o_i \geq 0, s_i \geq 0, e_i \geq 0, \Delta_i \geq 0 \qquad \forall i \in F' \tag{31}$$

The objective function (1) seeks to minimize the total time (travel times and charging times). Constraints (2) ensure that each customer has exactly one successor. Constraints (3) ensure that each CS will have at most one successor. Constraints (4) impose the flow conservation. Constraints

(5) and (6) track the battery level at each vertex. Constraints (7) ensure that, when leaving a vertex, the EV has enough remaining energy to reach either the depot or a CS. Constraints (8) reset the battery tracking to o_i upon departure from CS $i \in F'$. Constraints (9) ensure that the battery level is Q at the depot. Constraints (10) relate the battery level when an EV arrives to and departs from any CS. Constraints (11-16) define the battery level (and its corresponding charging time) when an EV arrives to station $i \in F'$ based on the piecewise linear charging function of $g_i(l)$. Similarly, constraints (17-22) define the battery level (and its corresponding charging time) when an EV leaves the CS. Constraints (23) define the time spent at any CS. Constraints (24) and (25) track the departure time at each vertex, where $S_{max} = max_{i \in F'}\{c_{ib}\}$. Constraints (26) and (27) ensure that the EVs return to the depot no later than T_{max}. Finally, constraints (28-31) define the domain of the decision variables.

3 A modified multi-space sampling heuristic

To solve the e-VRP-NLCF, we extend the modified multi-space sampling heuristic (mMSH) introduced by Montoya et al. [2] for the Green VRP. This method has two phases: sampling and assembling. In the sampling phase the algorithm uses a set of randomized TSP heuristics to draw a biased sample from the set $\mathcal{K}$ of TSP-like tours (i.e., giant tours visiting all customers). Each TSP-like tour is split onto a feasible e-VRP-NLCF solution following the route-first cluster-second principle. To repair the energy infeasible routes, the splitting procedure uses a two phases heuristic an MILP formulation. The routes in the best e-VRP-NLCF solutions are stored in a set Ω. In the assembling phase mMSH maps set Ω to a solution s by solving a set partitioning formulation of the e-VRP-NLCF.

4 Computational results

MILP solves to optimality 10 instances with at most 10 customers and obtains feasible solutions for others 58 instances with at most 80 customers within 10,800 seconds. For the 10 instances with proven optimal solution, mMSH has an average gap of 0.62%, and an average CPU time of 2.89 seconds. For the other 58 instances mMSH has an average gap with respect to the best integer solution found with MILP of -1.88% and an average CPU time of 224.22 seconds. For the remaining 52 instances mMSH obtained feasible solutions on an average CPU time of 9,903.03 seconds.

We evaluated the impact of different charging policy assumptions (i.e., full charging, linear charging, among others). Preliminary experiments show that the objective function increases in average 8.4% and some instances are infeasible.

References

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