# The electric vehicle routing problem with partial charging and nonlinear charging function

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## Abstract

Electric vehicle routing problems (eVRPs) extend classical routing problems to consider the limited driving range of electric vehicles. In general, this limitation is overcome by introducing planned detours to battery charging stations. Most existing eVRP models rely on one (or both) of the following assumptions: (i) the vehicles fully charge their batteries every time they reach a charging station, and (ii) the battery charge level is a linear function of the charging time. In practical situations, however, the amount of charge is a decision variable, and the battery charge level is a concave function of the charging time. In this paper we extend current eVRP models to consider partial charging and nonlinear charging functions. We present a computational study comparing our assumptions with those commonly made in the literature. Our results suggest that neglecting partial and nonlinear charging may lead to infeasible or overly expensive solutions.

**Keywords:** Vehicle routing problem, Electric vehicles, Partial charging, Nonlinear charging function

# 1    Introduction

Electric vehicles are one of the most promising technologies for reducing petroleum dependency and greenhouse gas emissions. The use of electric vehicles in freight and passenger transportation introduces a new family of vehicle routing problems (VRPs), the so-called electric VRPs (eVRPs) [11]. As their name suggests, eVRPs extend classical VRPs to account for the technical features of electric vehicles. Two of these features are the short driving range and the relatively long battery charging time.

Because of the short driving range, eVRP solutions frequently include routes with planned detours to charging stations (CSs) where the vehicles recharge. To model the battery charging process eVRP models make assumptions about the *charging policy* and the *charging function approximation*. The former defines how much of the battery capacity can be (or must be) restored when a vehicle visits a CS, and the latter models the relationship between battery

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charging time and charging level. From the point of view of charging policies, the existing eVRP literature can be classified into two groups: studies assuming *full* and *partial* charging policies. As the name suggests, in full charging policies, the battery capacity is fully restored every time a vehicle reaches a CS. Some studies in this group assume that there is no charging function but rather a constant charging time [3, 5, 10]. This is a plausible assumption in applications where the CSs do not actually charge the batteries but rather replace a (partially) depleted battery with a fully charged one. On the other hand, [4, 7, 8, 14, 15] consider full charging policies with a linear charging function approximation (i.e., the battery charge level is assumed to be a linear function of the charging time). In their models, the time spent at each CS depends on the charge level when the vehicle arrives and on the (constant) charging rate of the CS. In partial charging policies, the amount of charge (and thus the time spent at each charging point) is a decision variable. To the best of our knowledge, all existing eVRP models with partial charging policies consider linear charging function approximations [2, 4, 6, 13].

In practice, using linear approximations to model the charging function may not be realistic: it is well documented that the charging level is a concave function of the charging time [1, 9, 12]. To the best of our knowledge, only [17] deals with nonlinear charging functions in electric vehicle routing-related problems. In that paper the author solves a shortest path problem with battery capacity constraints, and some of the nodes in the graph represent CSs with piecewise-linear charging functions.

In this paper, we extend existing eVRP models to consider partial charging and nonlinear charging functions. Our main goal is to compare these more realistic assumptions to those commonly used in the literature. We first define what we call the vehicle routing problem with partial charging and nonlinear charging functions (eVRP-PNL). Then, we propose a mixed-integer linear programming (MILP) formulation that, with a few modifications, can be adapted to other charging policies and charging function approximations. We use our MILP and a commercial solver to study the impact of different charging assumptions on the quality and feasibility of eVRP solutions. Our results suggest that disallowing partial charging and neglecting the nonlinear nature of the charging functions leads to solutions that may be infeasible or overly expensive.

# 2 Vehicle routing problem with partial charging and nonlinear charging function

## 2.1 Problem description

Let $I$ be the set of nodes representing the customers, $F$ be the set of CSs, and 0 be a node representing the depot. Each customer $i \in I$ has a service time $p_i$. The eVRP-PNL is defined on an undirected and complete graph $G = (V, E)$, where $V = \{0\} \cup I \cup F'$ and $F'$ contains the set $F$ and $\beta$ copies of each CS (i.e., $|F'| = |F| \times (1 + \beta)$). The value of $1 + \beta$ corresponds to the number of times that each CS can be visited. Let $E = \{(i, j) : i, j \in V, i \neq j\}$ be the set of edges connecting vertices of $V$. Each edge $(i, j)$ has two associated nonnegative values: a travel time $t_{ij}$ and a distance $d_{ij}$. The customers are served using an unlimited fleet of electric vehicles with a consumption rate $cr$ (expressed in kWh/km). All the vehicles have a battery with capacity $Q$ (expressed in kWh) and a maximum tour duration limit $T_{max}$. It is assumed that the vehicles leave the depot with a fully charged battery, and that all the CSs can handle an unlimited number of vehicles simultaneously.

In the eVRP-PNL the objective is to find a set of routes of minimum total time, defined as the sum of the travel times and the charging times, satisfying the following conditions: each customer is visited exactly once; the level of the battery when the vehicle arrives at any vertex

is nonnegative; each route satisfies the maximum-duration limit; and each route starts and ends at the depot.

## 2.2 Modeling of battery charging functions

Each CS $i \in F'$ has a charging mode (e.g., slow, moderate, fast), which is associated with a charging function $g_i(q_i, \Delta_i)$. This function maps the charge level when the vehicle arrives at $i$ ($q_i$), and the time spent charging at $i$ ($\Delta_i$) to the charge level when the vehicle leaves $i$. To avoid handling a two-dimensional function, we use the transformation proposed in [17]. Let $\hat{g}_i(l)$ be the charging function when $q_i = 0$ and the battery is charged for $l$ time units; $g_i(q_i, \Delta_i)$ is estimated as $\hat{g}_i(\Delta_i + \hat{g}^{-1}(q_i))$. Note that $\Delta_i = \hat{g}_i^{-1}(o_i) - \hat{g}_i^{-1}(q_i)$, where $o_i$ is the charge level when the vehicle departs from $i$.

The function $\hat{g}_i(l)$ is concave [1, 9, 12] with an asymptote at $Q$. Similarly to Zundorf in [17], we argue that $\hat{g}_i(l)$ can be accurately approximated using piecewise linear functions. We support our claim using the data provided by Uhrig and Leibfried in [16]. These authors conducted experiments to estimate the charging time for different charge levels with two types of electric vehicles and three types of CSs. We fit piecewise linear functions to their data and obtain approximations with an average relative absolute error of 0.90%, 1.24%, and 1.90% for CSs of 11, 22, and 44 kW, respectively. Figure 1(a) shows the piecewise linear approximation for a CS $i$ of 22 kW charging a vehicle equipped with a battery of 16 kWh. In the plot, $c_{ik}$ and $a_{ik}$ represent the charging time and the charge level for the breakpoint $k \in B$ of the CS $i \in F'$, where $B = \{0, .., b\}$ is the set of breakpoints of the piecewise linear approximation.
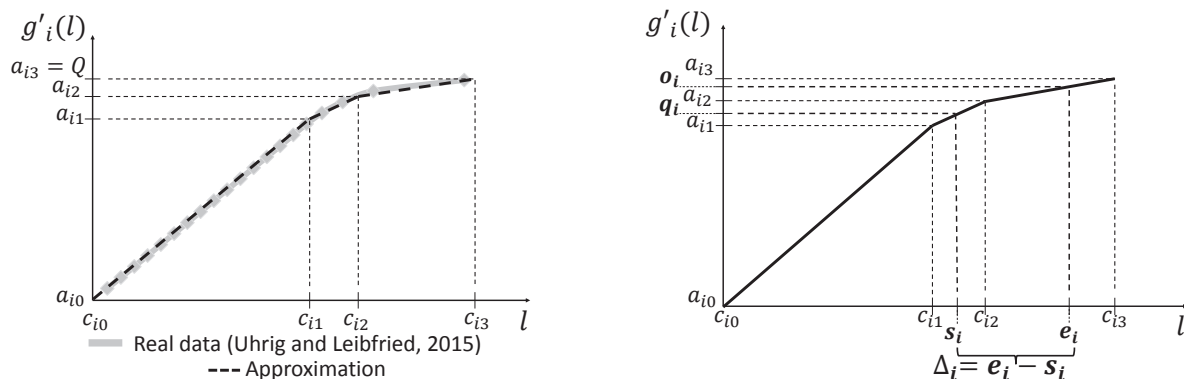


(a) Real data vs. piecewise linear approximation for a CS with 22 kW charging a battery of 16 kWh.

(b) Battery charge levels and charging times $i \in F'$

Figure 1: Piecewise linear approximation for the charging function.

## 2.3 Mixed-integer linear program

We formulate the eVRP-PNL as an MILP with the following decision variables: variable $x_{ij}$ is equal to 1 if a vehicle travels from vertex $i$ to $j$, and 0 otherwise. Variables $\tau_j$ and $y_j$ track the time and charge level when the vehicle departs from vertex $j \in V$. Variables $q_i$ and $o_i$ specify the charge levels when a vehicle arrives at and departs from CS $i \in F'$, and $s_i$ and $e_i$ are the associated charging times (see Figure 1(b)). Variable $\Delta_i = e_i - s_i$ represents the time spent at CS $i \in F'$. Variables $z_{ik}$ and $w_{ik}$ are equal to 1 if the charge level is between $a_{i,k-1}$ and $a_{ik}$, with $k \in B \setminus \{0\}$, when the vehicle arrives at and departs from CS $i \in F'$ respectively. Finally, variables $\alpha_{ik}$ and $\lambda_{ik}$ are the coefficients of the breakpoint $k \in B$ in the piecewise linear

approximation, when the vehicle arrives at and departs from CS $i \in F'$ respectively. The MILP follows:

$$\min \sum_{i,j \in V} t_{ij} x_{ij} + \sum_{i \in F'} \Delta_i \tag{1}$$

subject to

$$\sum_{j \in V, i \neq j} x_{ij} = 1, \qquad \forall i \in I \tag{2}$$

$$\sum_{j \in V, i \neq j} x_{ij} \leq 1, \qquad \forall i \in F' \tag{3}$$

$$\sum_{j \in V, i \neq j} x_{ji} - \sum_{j \in V, i \neq j} x_{ij} = 0, \qquad \forall i \in V \tag{4}$$

$$d_{ij} x_{ij} cr - (1 - x_{ij})Q \leq y_i - y_j \leq d_{ij} x_{ij} cr + (1 - x_{ij})Q, \qquad \forall i \in V, \forall j \in I \tag{5}$$

$$d_{ij} x_{ij} cr - (1 - x_{ij})Q \leq y_i - q_j \leq d_{ij} x_{ij} cr + (1 - x_{ij})Q, \qquad \forall i \in V, \forall j \in F' \tag{6}$$

$$y_i \geq d_{i0} x_{i0} cr, \qquad \forall i \in V \tag{7}$$

$$y_i = o_i, \qquad \forall i \in F' \tag{8}$$

$$y_0 = Q \tag{9}$$

$$q_i \leq o_i, \qquad \forall i \in F' \tag{10}$$

$$q_i = \sum_{k \in B} \alpha_{ik} a_{ik}, \qquad \forall i \in F' \tag{11}$$

$$s_i = \sum_{k \in B} \alpha_{ik} c_{ik}, \qquad \forall i \in F' \tag{12}$$

$$\sum_{k \in B} \alpha_{ik} = \sum_{k \in B} z_{ik}, \qquad \forall i \in F' \tag{13}$$

$$\sum_{k \in B} z_{ik} = \sum_{j \in V} x_{ij}, \qquad \forall i \in F' \tag{14}$$

$$\alpha_{ik} \leq z_{ik} + z_{i,k+1}, \qquad \forall i \in F', \forall k \in B \setminus \{b\} \tag{15}$$

$$\alpha_{ib} \leq z_{ib}, \qquad \forall i \in F' \tag{16}$$

$$o_i = \sum_{k \in B} \lambda_{ik} a_{ik}, \qquad \forall i \in F' \tag{17}$$

$$e_i = \sum_{k \in B} \lambda_{ik} c_{ik}, \qquad \forall i \in F' \tag{18}$$

$$\sum_{k \in B} \lambda_{ik} = \sum_{k \in B} w_{ik}, \qquad \forall i \in F' \tag{19}$$

$$\sum_{k \in B} w_{ik} = \sum_{j \in V} x_{ij}, \qquad \forall i \in F' \tag{20}$$

$$\lambda_{ik} \leq w_{ik} + w_{i,k+1}, \qquad \forall i \in F', \forall k \in B \setminus \{b\} \tag{21}$$

$$\lambda_{ib} \leq w_{ib}, \qquad \forall i \in F' \tag{22}$$

$$\Delta_i = e_i - s_i, \qquad \forall i \in F' \tag{23}$$

$$\tau_i + (t_{ij} + p_j)x_{ij} - T_{max}(1 - x_{ij}) \leq \tau_j, \qquad \forall i \in V, \forall j \in I \tag{24}$$

$$\tau_i + \Delta_j + t_{ij} x_{ij} - (S_{max} + T_{max})(1 - x_{ij}) \leq \tau_j, \qquad \forall i \in V, \forall j \in F' \tag{25}$$

$$\tau_j + t_{j0} \leq T_{max}, \qquad \forall j \in V \tag{26}$$

$$\tau_0 \leq T_{max} \tag{27}$$

$$x_{ij} \in \{0,1\}, \qquad \forall i,j \in V \tag{28}$$

$$\tau_i \geq 0, y_i \geq 0 \qquad \forall i \in V \tag{29}$$

$$z_{ik} \in \{0,1\}, w_{ik} \in \{0,1\}, \alpha_{ik} \geq 0, \lambda_{ik} \geq 0, \qquad \forall i \in F', \forall k \in B \tag{30}$$

$$q_i \geq 0, o_i \geq 0, s_i \geq 0, e_i \geq 0, \Delta_i \geq 0, \qquad \forall i \in F' \tag{31}$$

The objective function (1) seeks to minimize the total time (travel times plus charging times). Constraints (2) ensure that each customer is visited once. Constraints (3) ensure that each CS is visited at most once. Constraints (4) impose the flow conservation. Constraints (5) and (6) track the battery charge level at each vertex. Constraints (7) ensure that, if the vehicle travels between a vertex and the depot, it has sufficient energy to reach its destination. Constraints (8) reset the battery tracking to $o_i$ upon departure from CS $i \in F'$. Constraint (9) ensures that the battery charge level is $Q$ at the depot. Constraints (10) couple the charge levels when a vehicle arrives at and departs from any CS. Constraints (11–16) define the charge level (and its corresponding charging time) when a vehicle arrives at CS $i \in F'$ (based on the piecewise linear approximation of the charging function). Similarly, constraints (17–22) define the charge level (and its corresponding charging time) when a vehicle departs from CS $i \in F'$. Constraints (23) define the time spent at any CS. Constraints (24) and (25) track the departure time at each vertex, where $S_{max} = max_{i \in F}\{c_{ib}\}$. Constraints (26) and (27) ensure that the vehicles return to the depot no later than $T_{max}$. Finally, constraints (28–31) define the domain of the decision variables.

## 2.4  Valid inequalities

To help avoid the symmetry generated by the copies of the CSs, we added to our model the following set of valid inequalities:

$$x_{ij} = 0, \qquad\qquad \forall i, j \in F' : m_{ij} = 1 \qquad\qquad (32)$$

$$\tau_j \le \tau_i, \qquad\qquad \forall i, j \in F' : m_{ij} = 1, j \le i \qquad\qquad (33)$$

$$\tau_j \le T_{max} \sum_{i \in V} x_{ij}, \qquad\qquad \forall j \in F' \qquad\qquad (34)$$

$$\sum_{i \in V} x_{ih} \le \sum_{j \in V} x_{jf}, \qquad\qquad \forall h, f \in F' : m_{hf} = 1, h \le f \qquad\qquad (35)$$

where $m_{ij}$ is a parameter equal to 1 if $i$ and $j \in F'$ represent the same CS.

# 3  Computational study: Comparison of different charging assumptions

To assess the value of considering partial charging and nonlinear charging function approximations in eVRPs, we conducted an experiment to compare our battery charging assumptions with assumptions commonly used in the literature. For the sake of completeness we briefly explain here the charging assumptions considered in our study and how our MILP or input data can be adapted to work with those assumptions.

**Partial charge and piecewise linear approximation (PR-PL)** : These are the assumptions proposed in this paper (Figure 2(a)).

**Full charge and piecewise linear approximations (FR-PL)** : As mentioned earlier, the full charge policy is widely used in the literature [3, 5, 7, 8, 10, 14, 15]. We combined this policy with our piecewise linear approximation. To adapt our MILP to the full charge policy, we replace constraints (17) and (18) by

$$o_i = \sum_{k \in B} \lambda_{ik} a_{ib}, \forall i \in F \qquad (36)$$

$$e_i = \sum_{k \in B} \lambda_{ik} c_{ib}, \forall i \in F' \qquad (37)$$

**Partial charge using the first segment (PR-FS)** : According to Pelletier et al. [12], the actual battery charging function has two segments: a linear segment from 0 to (around) $0.8Q$, and a nonlinear segment from (around) $0.8Q$ to $Q$. To avoid dealing with the nonlinear segment, Bruglieri et al. [1] use a linear approximation that considers only the first segment (Figure 2(b)). To run our MILP with this assumption, we modify the input data to include only the first segment.

**Partial charge and linear approximations (PR-L)** : Although several authors assume a linear approximation [2, 4, 6, 13], they do not explain how the approximation is estimated. We implemented two linear approximations. For the first (PR-L1) we assume that the charging rate of the function corresponds to the slope of the first segment of the piecewise linear approximation (see Figure 2(c)). This approximation is optimistic, because it assumes that batteries charge up to $Q$ faster than they do in reality. For the second approximation (PR-L2) we assume that the charging rate is the slope of the line connecting the first and last observations (see Figure 2(d)). This approximation tends to be pessimistic, because over a large portion of the curve, the charging rate is slower than in reality. To run our MILP with PR-L1 and PR-L2, we modify the input data so that in the piecewise linear approximation there is a single segment with the corresponding charging rate.

We implemented our MILP (and its variant) in Gurobi 5.6. We ran the experiments on a set of 20 randomly generated instances of the eVRP-PNL. Because of the complexity of the problem, we restricted our instances to 10 customers and 2 or 3 CSs. We believe, however, that our conclusions hold for any instance size. We consider three types of CSs with different charging rates: slow (11 kW), moderate (22 kW), and fast (44 kW). For each CS type we build the charging function approximation by fitting a piecewise linear function to the Uhrig and Leibfried data [16]. To allow comparison with our results, our instances are publicly available at www.vrp-rep.org.[1]

As mentioned in Section 2, our MILP uses $\beta$ copies of the CSs to model multiple visits to the same CS. Although several authors have followed this strategy [3, 5, 7, 8, 13, 14], they do not explain how the value of $\beta$ is set. It is worth noting that $\beta$ plays an important role in the definition of the solution space, and therefore it restricts the optimal solution of the model. For instance, an optimal solution found with $\beta = 3$ may not be optimal for $\beta = 4$. In practice, there is no restriction on the number of times that a CS can be visited, but large values of $\beta$ result in models that are computationally intractable. To overcome this difficulty, we designed an iterative procedure in which our MILP is solved for increasing values of $\beta$. Starting with $\beta = 0$, at each iteration, our procedure (i) tries to solve the MILP to optimality with a time limit of 100 h, and (ii) sets $\beta = \beta + 1$. The procedure stops when the time limit is reached or an iteration ends with a solution $s_\beta$ satisfying $f(s_\beta) = f(s_{\beta-1}^*)$, where $f(\cdot)$ denotes the objective function and $*$ an optimal solution.

Table 1 presents the results. For each charging assumption, we give the objective function value ($of$), the percentage gap between $of$ and the PR-PL solution ($G$), the number of routes in the solution ($r$), and the value of $\beta$. Since in practice the charging time is controlled by

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[1]The instances will be released after the completion of the reviewing process.

(a) Piecewise linear approximation (PR-PL and FR-PL)



(b) First segment approximation (PR-FS)



(c) Linear approximation 1 (PR-L1)



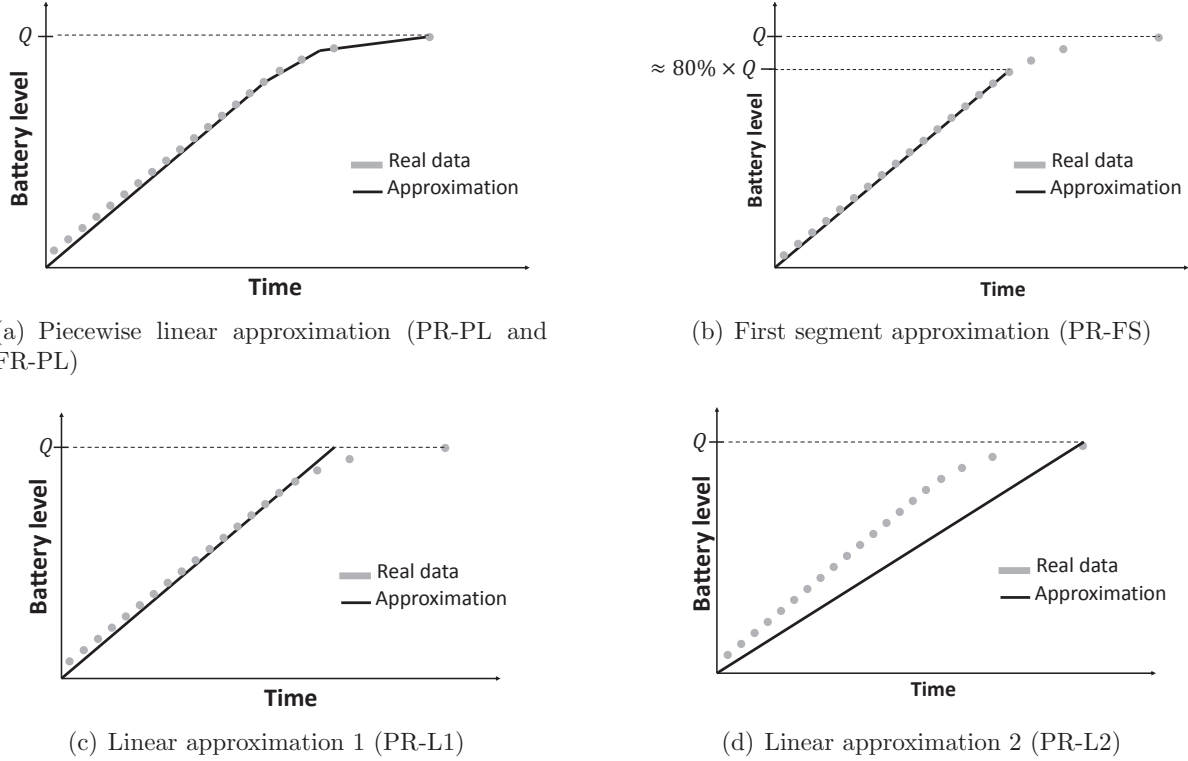(d) Linear approximation 2 (PR-L2)

Figure 2: Assumptions for charging function.

the nonlinear charging function, the charging decisions of the PR-L solutions are evaluated a posteriori using the piecewise linear approximation. The last rows of Table 1 summarize the results. We present, for each assumption, the average and maximum percentage gap, the number of solutions employing more vehicles than in the PR-PL solution, and the number of infeasible solutions.

The results show that solutions based on the full charging policy perform poorly in terms of both objective function (+20.11% on average) and number of routes (8 solutions use a larger fleet) with respect to those based on the partial charging policy. This is because the vehicles spend more time than necessary at the CSs. The main motivation for a full charging policy is to avoid complex charging-quantity decisions. However, according to our results, the gain in simplicity does not offset the loss of solution quality.

The main implication of the PR-FS assumption is that vehicles can charge their batteries up to only around 80% of the actual capacity. Artificially constraining the capacity may force vehicles to detour to CSs more often than necessary when traveling to distant customers. Because the maximum route duration is limited, the time spent detouring and recharging the battery reduces the number of customers that can be visited. Consequently, more routes may be needed to service the same number of customers. Our results confirm this intuition: in 3 of the 20 instances the PR-FS assumption increases the number of routes. Furthermore, in practice some distant customers may not be included in routes unless the vehicles can fully use their battery capacity. In our experiments, 9 instances become infeasible under PR-FS. In conclusion, although PR-FS simplifies the problem (avoiding the nonlinear segment of the charging function) it may lead to solutions that are infeasible or overly expensive.

As mentioned before, PR-L1 assumes that batteries charge faster than they do in reality (Figure 2(c)). As a consequence, routes based on PR-L1 may in practice need more time to reach the planned charge levels. The extra time may make a route infeasible if there is little

slack in the duration constraint. Indeed, a post-hoc evaluation shows that for 14 instances, the PR-L1 solutions are infeasible in practice. On the other hand, PR-L2 assumes that batteries charge slower than in reality (Figure 2(d)). Overestimating the charging times does not lead to feasibility issues, but the resulting routes may be overly conservative. For instance, in our experiments PR-L2 leads to solutions that are (on average) 1.45% more expensive, and it increases the number of routes in 2 instances. Although an extra 1.45% may seem reasonable, in the VRP context it represents the performance difference between state-of-the-art and regular algorithms.

Table 1: Comparison of our charging assumptions with charging assumptions from the literature

| Instance | PR-PL | | | FR-PL | | | | PR-FS | | | | PR-L1 Solution | | | PR-L1 Evaluation | | | | PR-L2 Solution | | | PR-L2 Evaluation | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $of$ | $r$ | $\beta$ | $of$ | G(%) | $r$ | $\beta$ | $of$ | G(%) | $r$ | $\beta$ | $of$ | $r$ | $\beta$ | $of$ | G(%) | $r$ | $\beta$ | $of$ | $r$ | $\beta$ | $of$ | G(%) | $r$ | $\beta$ |
| tc0c10s2cf1 | **19.75** | 3 | 2 | 20.82 | 5.42 | 3 | 2 | NF | NF | NF | NF | 19.61 | 3 | 2 | NF | NF | NF | NF | 20.50 | 3 | 2 | 20.22 | 2.38 | 3 | 2 |
| tc0c10s2ct1 | **12.30** | 2 | 0 | 12.53 | 1.87 | 2 | 0 | 12.61 | 2.52 | 3 | 0 | 12.22 | 2 | 0 | 12.42 | 0.98 | 2 | 0 | 12.46 | 2 | 0 | **12.30** | 0.00 | 2 | 0 |
| tc0c10s3cf1 | **19.75** | 3 | 2 | 20.82 | 5.42 | 3 | 2 | NF | NF | NF | NF | 19.61 | 3 | 2 | NF | NF | NF | NF | 20.50 | 3 | 2 | 20.22 | 2.38 | 3 | 2 |
| tc0c10s3ct1 | **10.80** | 2 | 0 | 11.10 | 2.78 | 2 | 0 | **10.80** | 0.00 | 2 | 0 | 10.79 | 2 | 0 | 11.03 | 2.13 | 2 | 0 | 10.97 | 2 | 0 | **10.80** | 0.00 | 2 | 0 |
| tc1c10s2cf2 | **9.03** | 3 | 0 | 9.19 | 1.77 | 3 | 0 | **9.03** | 0.00 | 3 | 0 | 9.03 | 3 | 0 | 9.12 | 1.00 | 3 | 0 | 9.14 | 3 | 0 | **9.03** | 0.00 | 3 | 0 |
| tc1c10s2cf3 | **16.37** | 3 | 2 | 21.33 | 30.30 | 3 | 2 | NF | NF | NF | NF | 15.99 | 3 | 1 | NF | NF | NF | NF | 16.89 | 3 | 2 | **16.37** | 0.00 | 3 | 2 |
| tc1c10s2cf4 | **16.10** | 3 | 2 | 25.31 | 57.20 | 4 | 3 | NF | NF | NF | NF | 15.66 | 3 | 2 | NF | NF | NF | NF | 16.43 | 3 | 2 | 16.23 | 0.81 | 3 | 2 |
| tc1c10s2ct2 | **10.75** | 3 | 1 | 11.14 | 3.63 | 3 | 0 | **10.75** | 0.00 | 3 | 1 | 10.75 | 3 | 0 | 10.76 | 0.09 | 3 | 0 | 10.94 | 3 | 0 | 10.78 | 0.28 | 3 | 0 |
| tc1c10s2ct3 | **13.17** | 2 | 2 | 22.76 | 72.82 | 3 | 3 | 15.98 | 21.34 | 3 | 2 | 13.06 | 2 | 2 | NF | NF | NF | NF | 13.60 | 2 | 2 | **13.17** | 0.00 | 2 | 2 |
| tc1c10s2ct4 | **13.83** | 2 | 1 | 17.61 | 27.33 | 3 | 1 | NF | NF | NF | NF | 13.34 | 2 | 1 | NF | NF | NF | NF | 14.17 | 2 | 1 | 14.17 | 2.46 | 2 | 1 |
| tc1c10s3cf2 | **9.03** | 3 | 0 | 9.19 | 1.77 | 3 | 0 | **9.03** | 0.00 | 3 | 0 | 9.03 | 3 | 0 | 9.12 | 1.00 | 3 | 0 | 9.14 | 3 | 0 | **9.03** | 0.00 | 3 | 0 |
| tc1c10s3cf3 | **16.37** | 3 | 1 | 21.33 | 30.30 | 3 | 2 | NF | NF | NF | NF | 15.99 | 3 | 1 | NF | NF | NF | NF | 16.89 | 3 | 2 | **16.37** | 0.00 | 3 | 2 |
| tc1c10s3cf4 | **14.90** | 3 | 1 | 18.43 | 23.69 | 4 | 0 | NF | NF | NF | NF | 14.56 | 2 | 1 | NF | NF | NF | NF | 15.18 | 3 | 0 | 15.18 | 1.88 | 3 | 0 |
| tc1c10s3ct2 | **9.20** | 3 | 0 | 11.14 | 21.09 | 3 | 0 | **9.20** | 0.00 | 3 | 0 | 9.19 | 3 | 0 | NF | NF | NF | NF | 10.80 | 3 | 0 | 10.57 | 14.89 | 3 | 0 |
| tc1c10s3ct3 | **13.02** | 2 | 0 | 17.06 | 31.03 | 3 | 0 | 13.07 | 0.38 | 2 | 1 | 12.98 | 2 | 0 | 13.16 | 1.08 | 2 | 0 | 13.60 | 2 | 0 | **13.02** | 0.00 | 2 | 0 |
| tc1c10s3ct4 | **13.21** | 2 | 0 | 15.54 | 17.64 | 3 | 1 | 13.58 | 2.80 | 3 | 1 | 12.92 | 2 | 1 | NF | NF | NF | NF | 13.71 | 2 | 0 | **13.21** | 0.00 | 2 | 0 |
| tc2c10s2cf0 | **21.77** | 3 | 3 | 25.24 | 15.94 | 4 | 2 | NF | NF | NF | NF | 14.53 | 2 | 2 | NF | NF | NF | NF | 22.78 | 4 | 4 | 22.15 | 1.75 | 4 | 4 |
| tc2c10s2ct0 | **12.45** | 3 | 2 | 15.05 | 20.88 | 3 | 3 | **12.45** | 0.00 | 3 | 2 | 12.44 | 3 | 3 | NF | NF | NF | NF | 12.93 | 3 | 2 | **12.45** | 0.00 | 3 | 2 |
| tc2c10s3cf0 | **21.77** | 3 | 2 | 25.24 | 15.94 | 4 | 2 | NF | NF | NF | NF | 14.53 | 2 | 2 | NF | NF | NF | NF | 23.02 | 4 | 3 | 22.20 | 1.98 | 4 | 3 |
| tc2c10s3ct0 | **11.51** | 3 | 0 | 13.27 | 15.29 | 2 | 0 | **11.51** | 0.00 | 3 | 0 | 11.50 | 3 | 0 | NF | NF | NF | NF | 11.92 | 3 | 0 | 11.54 | 0.26 | 3 | 0 |
| **Avg. Difference (%)** | | | | 20.11 | | | | 2.70 | | | | | | | 1.04 | | | | | | | 1.45 | | | |
| **Max. Difference (%)** | | | | 72.82 | | | | 21.34 | | | | | | | 2.13 | | | | | | | 14.89 | | | |
| **Solutions with larger fleet** | | | | 8 | | | | 3 | | | | | | | 0 | | | | | | | 2 | | | |
| **Infeasible solutions** | 0 | | | | | | | 9 | | | | | | | 14 | | | | | | | 0 | | | |

NF: Infeasible, $G(\%) = (of - of_{PR-PL})/of_{PR-PL} \times 100$

# 4    Conclusion and future work

We have extended existing eVRP models to allow more realistic assumptions regarding the battery charging policy and functions. We introduced the eVRP with partial charging and nonlinear charging functions (eVRP-PNL) and proposed an MILP to solve it. We then adapted our MILP to run with charging policies and functions commonly used in the literature. Using 20 instances that we built based on real charging data, we ran an experiment comparing our charging hypotheses to those in the literature. Our results show that disallowing partial charging and neglecting the nonlinear nature of the charging function may lead to infeasible or overly expensive solutions. Ongoing research includes the development of solution methods tailored for the eVRP-PNL and experimentation with industrial-scale instances.

## Acknowledgement

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