

# Beating phenomenon and energy localization in Single-Walled Carbon Nanotubes

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## Abstract

In this paper, the low-frequency nonlinear oscillations and energy localization of Single-Walled Carbon Nanotubes (SWNTs) are analysed. The SWNTs dynamics is studied in the framework of the Sanders-Koiter nonlinear shell theory. The circumferential flexure vibration modes (CFMs) are considered. Simply supported, clamped and free boundary conditions are analysed. Two different approaches are compared, based on numerical and analytical models. The numerical model uses a double mixed series expansion for the displacement fields based on the Chebyshev polynomials and harmonic functions. The Lagrange equations are considered to obtain a set of nonlinear ordinary differential equations of motion which are solved using the implicit Runge-Kutta numerical method. The analytical model considers a reduced form of the shell theory assuming small circumferential and tangential shear deformations. The Galerkin procedure is used to get the nonlinear ordinary differential equations of motion, which are then solved using the multiple scales analytical method. The natural frequencies of SWNTs obtained by considering the analytical and numerical approaches are compared for different boundary conditions. A convergence analysis in the nonlinear field is carried out for the numerical method in order to select the correct number of the axisymmetric and asymmetric modes providing the actual localization threshold. The effect of the aspect ratio on the analytical and numerical values of the localization threshold for SWNTs with different boundary conditions is investigated in the nonlinear field.

## Keywords

Single-Walled Carbon Nanotubes; nonlinear vibrations; energy localization.