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Higher spin fields from a worldline perspective

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Abstract: Higher spin fields in four dimensions, and more generally conformal fields in arbitrary dimensions, can be described by spinning particle models with a gauged SO(N) extended supergravity on the worldline. We consider here the one-loop quantization of these models by studying the corresponding partition function on the circle. After gauge fixing the supergravity multiplet, the partition function reduces to an integral over the corresponding moduli space which is computed using orthogonal polynomial techniques. We obtain a compact formula which gives the number of physical degrees of freedom for all N in all dimensions. As an aside we compute the physical degrees of freedom of the SO(4) = SU(2) × SU(2) model with only a SU(2) factor gauged, which has attracted some interest in the literature.

Keywords: Sigma Models, Gauge Symmetry, Supergravity Models.

Contents

1. Introduction

The study of higher spin fields has attracted a great deal of attention in the search for generalizations of the known gauge theories of fields of spin 1, 3/2 and 2. This search has proved to be quite difficult, and several no-go theorems have been discovered restricting the possible form of such generalizations. Positive results have been achieved as well, the most notorious being perhaps the Vasiliev's interacting field equations, which involve an infinite number of fields with higher spin [1], but an action principle for them is still lacking. Also, many studies of free higher spin fields have been carried out, trying to elucidate the problem further [2 − 4]. Additional interest in higher spin fields arises from the study of the AdS/CFT correspondence in the limit of high AdS curvature, where string theory seems to reduce to a tensionless string model with an infinite tower of massless higher spin fields [5, 6]. For a review on related topics and additional references see [7].

We consider here a different perspective by studying the higher spin fields from a first quantized point of view. It is known that spinning particles with a SO(N) extended local supersymmetry on the worldline, constructed and analyzed in [8, 9], describe the propagation of particles of spin $N/2$ in four dimensions. In fact, a canonical analysis produces the massless Bargmann-Wigner equations as constraints for the physical sector of the Hilbert space, and these equations are known to describe massless particles of arbitrary spin [10]. More generally, the SO(N) spinning particles are conformally invariant and describe all possible conformal free particles in arbitrary dimensions, as shown by Siegel [11].

We study here the one-loop quantization of the free spinning particles. Our purpose is to obtain the correct measure on the moduli space of the supergravity multiplet on the circle, which is necessary for computing more general quantum corrections arising when couplings to background fields are introduced. As mentioned before, the introduction of couplings for such higher spin fields is a rather delicate matter. Nevertheless a positive result for the SO(N) spinning particles has been obtained in [12], where the couplings to de Sitter or anti de Sitter backgrounds are constructed.

In this paper we restrict ourselves to flat space, and calculate the path integral on the circle to obtain compact formulas which give the number of physical degrees of freedom of the spinning particles for all N in all dimensions. In addition, we look at the SO(4) model introduced by Pashnev and Sorokin in [13], where only a SU(2) subgroup is gauged. We compute the corresponding physical degrees of freedom and resolve an ambiguity described there. The particular cases of $N = 0, 1, 2$ coupled to a curved target space have been discussed in [14–16], respectively. Some aspects of the path integral approach to the SO(N) spinning particles have also been studied in [17–20].

We structure our paper as follows. In section 2 we review the classical action of the SO(N) spinning particle and remind the reader of the gauge invariances that must be gauge fixed. In section 3 we describe the gauge fixing on the circle, and obtain the measure on the moduli space of the SO(N) extended supergravity fields on the circle. In section 4 we compute the integrals over the SO(N) moduli space using orthogonal polynomial techniques, and obtain the formulas for the number of physical degrees of freedom. In section 5 we apply our techniques to the Pashnev-Sorokin model and find that in $D = 4$ the model has five degrees of freedom, corresponding to a graviton and three scalars. We present our conclusions and future perspectives in section 6. We include in three appendices a brief discussion on the gauge fixing of the SO(N) gauge fields on the circle, a review of the relation between the Van der Monde determinant and orthogonal polynomials, and some technical details on the gauge fixing of the Pashnev-Sorokin model.

2. Action and gauge symmetries

The minkowskian action for the SO(N) spinning particle in flat target spacetime is given by

$$S_M[X,G] = \int_0^1 dt \left[\frac{1}{2}e^{-1}(\dot{x}^\mu - i\chi_i\psi_i^\mu)^2 + \frac{i}{2}\psi_i^\mu(\delta_{ij}\partial_t - a_{ij})\psi_{j\mu}\right] \tag{2.1}$$

where $X = (x^\mu, \psi_i^\mu)$ collectively describes the coordinates x^μ and the extra fermionic degrees of freedom ψ_i^μ of the spinning particle, and $G = (e, \chi_i, a_{ij})$ represents the set of gauge fields of the N-extended worldline supergravity, containing the einbein, gravitinos and SO(N) gauge fields. The index $\mu = 0, \dots, D-1$ is a spacetime index while $i, j = 1, \dots, N$ are internal SO(N) indices. The gauge transformations on the supergravity multiplet G

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are described by the gauge parameters $(\xi, \epsilon_i, \alpha_{ij})$ and read

$$\begin{aligned}\delta e &= \dot{\xi} + 2i\chi_i\epsilon_i \\ \delta\chi_i &= \dot{\epsilon}_i - a_{ij}\epsilon_j + \alpha_{ij}\chi_j \\ \delta a_{ij} &= \dot{\alpha}_{ij} + \alpha_{im}a_{mj} + \alpha_{jm}a_{im}\ .\end{aligned} \tag{2.2}$$

In the following we prefer to use euclidean conventions, and perform a Wick rotation to euclidean time $t \to -i\tau$, accompanied by the Wick rotations of the SO(N) gauge fields $a_{ij} \to ia_{ij}$, just as done in [16] for the $N = 2$ model. We obtain the euclidean action

$$S[X,G] = \int_0^1 d\tau \left[\frac{1}{2}e^{-1}(\dot{x}^\mu - \chi_i\psi_i^\mu)^2 + \frac{1}{2}\psi_i^\mu(\delta_{ij}\partial_\tau - a_{ij})\psi_{j\mu}\right] \tag{2.3}$$

with the gauge symmetries on the supergravity multiplet given by

$$\begin{aligned}\delta e &= \dot{\xi} + 2\chi_i\epsilon_i \\ \delta\chi_i &= \dot{\epsilon}_i - a_{ij}\epsilon_j + \alpha_{ij}\chi_j \\ \delta a_{ij} &= \dot{\alpha}_{ij} + \alpha_{im}a_{mj} + \alpha_{jm}a_{im}\end{aligned} \tag{2.4}$$

where we have also Wick rotated the gauge parameters $\epsilon_i \to -i\epsilon_i$, $\xi \to -i\xi$. These are the gauge symmetries that will be fixed on the circle in the next section.

3. Gauge fixing on the circle

Here we study the partition function on the circle S^1

$$Z \sim \int_{S^1} \frac{\mathcal{D}X\mathcal{D}G}{\text{Vol (Gauge)}}\, e^{-S[X,G]}\ . \tag{3.1}$$

First we need to gauge fix the local symmetries. We use the Faddeev-Popov method to extract the volume of the gauge group and select a gauge which fixes completely the supergravity multiplet up to some moduli. In particular, we specify a gauge where $(e, \chi_i, a_{ij}) = (\beta, 0, \hat{a}_{ij})$ are constants. The gauge on the einbein is rather standard, and produces an integral over the proper time β [21]. The fermions and the gravitinos are taken with antiperiodic boundary conditions. This implies that the gravitinos can be completely gauged away as there are no zero modes for the differential operator that relates the gauge parameters ϵ_i to the gravitinos, see eq. (2.4). As for the SO(N) gauge fields, the gauge conditions $a_{ij} = \hat{a}_{ij}(\theta_k)$ can be chosen to depend on a set of constant angles θ_k, with $k = 1, \ldots, r$, where r is the rank of group SO(N). This is reviewed in appendix A. These angles are the moduli of the gauge fields on the circle and must be integrated over a fundamental region. Thus, taking into account the ghost determinants, we find that the gauge fixed partition function reads as

$$\begin{aligned}Z = &-\frac{1}{2}\int_0^\infty \frac{d\beta}{\beta}\int \frac{d^Dx}{(2\pi\beta)^{\frac{D}{2}}} \\ &\times K_N\left[\prod_{k=1}^r \int_0^{2\pi}\frac{d\theta_k}{2\pi}\right]\left(\text{Det}\,(\partial_\tau - \hat{a}_{\text{vec}})_{ABC}\right)^{\frac{D}{2}-1}\text{Det}'\,(\partial_\tau - \hat{a}_{adj})_{PBC}\end{aligned} \tag{3.2}$$

where K_N is a normalization factor that implements the reduction to a fundamental region of moduli space and will be discussed shortly. This formula contains the well-known proper time integral with the appropriate measure for one-loop amplitudes, and the spacetime volume integral with the standard free particle measure $((2\pi\beta)^{-\frac{D}{2}})$. In addition, it contains the integrals over the SO(N) moduli θ_k and the determinants of the ghosts and of the remaining fermion fields. In particular, the second line contains the determinants of the susy ghosts and of the Majorana fermions ψ_i^μ which all have antiperiodic boundary conditions (ABC) and transform in the vector representation of SO(N). The last determinant instead is due to the ghosts for the SO(N) gauge symmetry. They transform in the adjoint representation and have periodic boundary conditions (PBC), so they have zero modes (corresponding to the moduli directions) which are excluded from the determinant (this is indicated by the prime on Det$'$). The whole second line computes the number of physical degrees of freedom, normalized to one for a real scalar field,

$$Dof(D,N) = K_N \left[\prod_{k=1}^{r} \int_0^{2\pi} \frac{d\theta_k}{2\pi}\right] \Big(\mathrm{Det}\,(\partial_\tau - \hat{a}_{\mathrm{vec}})_{ABC}\Big)^{\frac{D}{2}-1} \mathrm{Det}'\,(\partial_\tau - \hat{a}_{adj})_{PBC} \qquad (3.3)$$

In fact, for $N = 0$ there are neither gravitinos nor gauge fields, $K_0 = 1$, and all other terms in the formula are absent [14], so that

$$Dof(D,0) = 1 \qquad (3.4)$$

as it should, since the $N = 0$ model describes a real scalar field in target spacetime. We now present separate discussions for even N and odd N, as typical for the orthogonal groups, and explicitate further the previous general formula.

3.1 Even case: $N = 2r$

To get a flavor of the general formula let us briefly review the $N = 2$ case treated in [16]. We have a SO(2) = U(1) gauge field a_{ij} which can be gauge fixed to the constant value

$$\hat{a}_{ij} = \begin{pmatrix} 0 & \theta \\ -\theta & 0 \end{pmatrix} \qquad (3.5)$$

where θ is an angle that corresponds to the SO(2) modulus. A fundamental region of gauge inequivalent configurations is given by $\theta \in [0, 2\pi]$ with identified boundary values and corresponds to a one-dimensional torus (a circle). The factor $K_2 = 1$ because there are no further identifications on moduli space, and the formula reads

$$\begin{aligned} Dof(D,2) &= \int_0^{2\pi} \frac{d\theta}{2\pi} \Big(\underbrace{\mathrm{Det}\,(\partial_\tau - \hat{a}_{\mathrm{vec}})_{ABC}}_{(2\cos\frac{\theta}{2})^2}\Big)^{\frac{D}{2}-1} \underbrace{\mathrm{Det}'\,(\partial_\tau)_{PBC}}_{1} \\ &= \begin{cases} \frac{(D-2)!}{[(\frac{D}{2}-1)!]^2} & \text{even } D \\ 0 & \text{odd } D \end{cases} . \end{aligned} \qquad (3.6)$$

This formula correctly reproduces the number of physical degrees of freedoms of a gauge $(\frac{D}{2}-1)$-form in even dimensions D. Instead, for odd D, the above integral vanishes and one has no degrees of freedom left. This may be interpreted as due to the anomalous behavior of an odd number of Majorana fermions under large gauge transformations [22]. In this formula the first determinant is due to the D Majorana fermions, responsible for a power $\frac{D}{2}$ of the first determinant, and to the bosonic susy ghosts, i.e. the Faddeev-Popov determinant for local susy, responsible for the power -1 of the first determinant. This determinant is more easily computed using the U(1) basis which diagonalizes the gauge field in (3.5). The second determinant is due the SO(2) ghosts which of course do not couple to the gauge field in the abelian case. A zero mode is present since these ghosts have periodic boundary conditions and is excluded from the determinant. This last determinant does not contribute to the SO(2) modular measure.

In the general case, the rank of SO(N) is $r = \frac{N}{2}$ for even N, and by constant gauge transformations one can always put a constant field a_{ij} in a skew diagonal form

$$\hat{a}_{ij} = \begin{pmatrix} 0 & \theta_1 & 0 & 0 & . & 0 & 0 \\ -\theta_1 & 0 & 0 & 0 & . & 0 & 0 \\ 0 & 0 & 0 & \theta_2 & . & 0 & 0 \\ 0 & 0 & -\theta_2 & 0 & . & 0 & 0 \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & . & 0 & \theta_r \\ 0 & 0 & 0 & 0 & . & -\theta_r & 0 \end{pmatrix} . \tag{3.7}$$

The θ_k are angles since large gauge transformations can be used to identify $\theta_k \sim \theta_k + 2\pi n$ with integer n. The determinants are easily computed pairing up coordinates into complex variables that diagonalize the matrix (3.7). Then

$$\mathrm{Det}\,(\partial_\tau - \hat{a}_{\rm vec}) = \prod_{k=1}^{r} \mathrm{Det}\,(\partial_\tau + i\theta_r)\,\mathrm{Det}\,(\partial_\tau - i\theta_r) \tag{3.8}$$

and thus

$$\Big(\mathrm{Det}\,(\partial_\tau - \hat{a}_{\rm vec})_{ABC}\Big)^{\frac{D}{2}-1} = \prod_{k=1}^{r} \left(2\cos\frac{\theta_k}{2}\right)^{D-2} . \tag{3.9}$$

Similarly

$$\begin{aligned} \mathrm{Det}'\,(\partial_\tau - \hat{a}_{adj})_{PBC} &= \prod_{k=1}^{r} \mathrm{Det}'\,(\partial_\tau) \\ &\quad \times \prod_{k<l} \mathrm{Det}\,(\partial_\tau + i(\theta_k + \theta_l))\,\mathrm{Det}\,(\partial_\tau - i(\theta_k + \theta_l)) \\ &\quad \times \prod_{k<l} \mathrm{Det}\,(\partial_\tau + i(\theta_k - \theta_l))\,\mathrm{Det}\,(\partial_\tau - i(\theta_k - \theta_l)) \\ &= \prod_{k<l} \left(2\sin\frac{\theta_k + \theta_l}{2}\right)^2 \left(2\sin\frac{\theta_k - \theta_l}{2}\right)^2 . \end{aligned} \tag{3.10}$$

Thus, with the normalization factor $K_N = \frac{2}{2^r r!}$ one obtains the final formula

$$Dof(D,N) = \frac{2}{2^r r!}\Bigg[\prod_{k=1}^{r}\int_0^{2\pi}\frac{d\theta_k}{2\pi}\left(2\cos\frac{\theta_k}{2}\right)^{D-2}\Bigg] \times \prod_{k<l}\left(2\sin\frac{\theta_k+\theta_l}{2}\right)^2\left(2\sin\frac{\theta_k-\theta_l}{2}\right)^2 . \tag{3.11}$$

The normalization $K_N = \frac{2}{2^r r!}$ can be understood as follows. A factor $\frac{1}{r!}$ is due to the fact that with a SO(N) constant gauge transformation one can permute the angles θ_k and there are r angles in total. The remaining factor $\frac{2}{2^r}$ can be understood as follows. One could change any angle θ_k to $-\theta_k$ if parity would be allowed (i.e. reflections of a single coordinate) and this would give the factor $\frac{1}{2^r}$. Thus we introduce parity transformations, which is an invariance of (3.11), by enlarging the gauge group by a Z_2 factor and obtain the group $O(N)$. This justifies the identification of θ_k with $-\theta_k$ and explains the remaining factor 2; equivalently, within SO(N) gauge transformations one can only change signs to pairs of angles simultaneously. It is perhaps more convenient to use some trigonometric identities and write the number of degrees of freedom as

$$Dof(D,N) = \frac{2}{2^r r!}\prod_{k=1}^{r}\int_0^{2\pi}\frac{d\theta_k}{2\pi}\left(2\cos\frac{\theta_k}{2}\right)^{D-2} \times \prod_{k<l}\left[\left(2\cos\frac{\theta_k}{2}\right)^2-\left(2\cos\frac{\theta_l}{2}\right)^2\right]^2 . \tag{3.12}$$

3.2 Odd case: $N = 2r + 1$

The case of odd N describes a fermionic system in target space. In fact, the simplest example is for $N = 1$, which gives a spin 1/2 fermion. It has been treated in [15] on a general curved background, but there are no worldline gauge fields in this case. For odd $N > 1$ the rank of the gauge group is $r = \frac{N-1}{2}$ and the gauge field in the vector representation a_{ij} can be gauge fixed to a constant matrix of the form

$$\hat{a}_{ij} = \begin{pmatrix} 0 & \theta_1 & 0 & 0 & . & 0 & 0 & 0 \\ -\theta_1 & 0 & 0 & 0 & . & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta_2 & . & 0 & 0 & 0 \\ 0 & 0 & -\theta_2 & 0 & . & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & . & 0 & \theta_r & 0 \\ 0 & 0 & 0 & 0 & . & -\theta_r & 0 & 0 \\ 0 & 0 & 0 & 0 & . & 0 & 0 & 0 \end{pmatrix} . \tag{3.13}$$

Then, in a way somewhat similar to the even case, one gets

$$\mathrm{Det}\,(\partial_\tau - \hat{a}_{\mathrm{vec}}) = \mathrm{Det}\,(\partial_\tau)\prod_{k=1}^{r}\mathrm{Det}\,(\partial_\tau + i\theta_k)\,\mathrm{Det}\,(\partial_\tau - i\theta_k) \tag{3.14}$$

and thus

$$\left(\operatorname{Det}\left(\partial_\tau-\hat{a}_{\mathrm{vec}}\right)_{ABC}\right)^{\frac{D}{2}-1}=2^{\frac{D}{2}-1}\prod_{k=1}^{r}\left(2\cos\frac{\theta_k}{2}\right)^{D-2}. \tag{3.15}$$

Similarly for the determinant in the adjoint representation

$$\begin{aligned}\operatorname{Det}'\left(\partial_\tau-\hat{a}_{adj}\right)_{PBC} &= \prod_{k=1}^{r}\operatorname{Det}'\left(\partial_\tau\right)\operatorname{Det}\left(\partial_\tau+i\theta_k\right)\operatorname{Det}\left(\partial_\tau-i\theta_k\right)\\ &\times\prod_{k<l}\operatorname{Det}\left(\partial_\tau+i(\theta_k+\theta_l)\right)\operatorname{Det}\left(\partial_\tau-i(\theta_k+\theta_l)\right)\\ &\times\prod_{k<l}\operatorname{Det}\left(\partial_\tau+i(\theta_k-\theta_l)\right)\operatorname{Det}\left(\partial_\tau-i(\theta_k-\theta_l)\right)\end{aligned} \tag{3.16}$$

which gives

$$\begin{aligned}\operatorname{Det}'\left(\partial_\tau-\hat{a}_{adj}\right)_{PBC} &= \prod_{k=1}^{r}\left(2\sin\frac{\theta_k}{2}\right)^2\\ &\times\prod_{k<l}\left(2\sin\frac{\theta_k+\theta_l}{2}\right)^2\left(2\sin\frac{\theta_k-\theta_l}{2}\right)^2.\end{aligned} \tag{3.17}$$

Thus, with a factor

$$K_N=\frac{1}{2^r r!} \tag{3.18}$$

one gets the formula

$$\begin{aligned}Dof(D,N) &= \frac{2^{\frac{D}{2}-1}}{2^r r!}\prod_{k=1}^{r}\int_0^{2\pi}\frac{d\theta_k}{2\pi}\left(2\cos\frac{\theta_k}{2}\right)^{D-2}\left(2\sin\frac{\theta_k}{2}\right)^2\\ &\times\prod_{k<l}\left(2\sin\frac{\theta_k+\theta_l}{2}\right)^2\left(2\sin\frac{\theta_k-\theta_l}{2}\right)^2.\end{aligned} \tag{3.19}$$

In the expression for K_N the factor 2 that appeared in the even case is now not included, since in the gauge (3.13) one can always reflect the last coordinate to obtain a SO(N) transformation that changes θ_k into $-\theta_k$.

For explicit computations it is perhaps more convenient to write the number of degrees of freedom as

$$\begin{aligned}Dof(D,N) &= \frac{2^{\frac{D}{2}-1}}{2^r r!}\prod_{k=1}^{r}\int_0^{2\pi}\frac{d\theta_k}{2\pi}\left(2\cos\frac{\theta_k}{2}\right)^{D-2}\left(2\sin\frac{\theta_k}{2}\right)^2\\ &\times\prod_{k<l}\left[\left(2\cos\frac{\theta_k}{2}\right)^2-\left(2\cos\frac{\theta_l}{2}\right)^2\right]^2.\end{aligned} \tag{3.20}$$

4. Degrees of freedom

We now compute explicitly the number of physical degrees of freedom for the spinning particles propagating in arbitrary dimensions. In the previous section we have obtained the expressions which compute them, eqs. (3.12) and (3.20), which we rewrite here for commodity

$$
\begin{aligned}
Dof(D,2r) &= \frac{2}{2^r r!}\prod_{k=1}^{r}\int_0^{2\pi}\frac{d\theta_k}{2\pi}\left(2\cos\frac{\theta_k}{2}\right)^{D-2} \\
&\quad\times \prod_{1\le k<l\le r}\left[\left(2\cos\frac{\theta_l}{2}\right)^2-\left(2\cos\frac{\theta_k}{2}\right)^2\right]^2
\end{aligned}
\tag{4.1}
$$

$$
\begin{aligned}
Dof(D,2r+1) &= \frac{2^{\frac{D}{2}-1}}{2^r r!}\prod_{k=1}^{r}\int_0^{2\pi}\frac{d\theta_k}{2\pi}\left(2\cos\frac{\theta_k}{2}\right)^{D-2}\left(2\sin\frac{\theta_k}{2}\right)^2 \\
&\quad\times \prod_{1\le k<l\le r}\left[\left(2\cos\frac{\theta_l}{2}\right)^2-\left(2\cos\frac{\theta_k}{2}\right)^2\right]^2
\end{aligned}
\tag{4.2}
$$

with $N=2r$ and $N=2r+1$, respectively. It is obvious that $Dof(D,N)$ vanishes for an odd number of dimensions

$$Dof(2d+1,N)=0, \qquad \forall N>1 \tag{4.3}$$

as in such case the integrands are odd under the Z_2 symmetry $\frac{\theta}{2}\to\pi-\frac{\theta}{2}$. Only for $N=0,1$ these models have a non-vanishing number of degrees of freedom propagating in an odd-dimensional spacetime, as in such cases there are no constraints coming from the vector gauge fields. Also for $N=2$ these models can have degrees of freedom propagating in odd-dimensional target spaces, provided a suitable Chern-Simons term is added to the worldline action [9]. However, Chern-Simons couplings are not possible for $N>2$.

To compute (4.1) and (4.2) for an even-dimensional target space we make use of the orthogonal polynomials method reviewed in appendix B. In order to do that, we first observe that the integrands are even under the aforementioned Z_2 symmetry, and thus we can restrict the range of integration

$$
\begin{aligned}
Dof(D,2r) &= \frac{2}{r!}\prod_{k=1}^{r}\int_0^{\pi}\frac{d\theta_k}{2\pi}\left(2\cos\frac{\theta_k}{2}\right)^{D-2} \\
&\quad\times \prod_{1\le k<l\le r}\left[\left(2\cos\frac{\theta_l}{2}\right)^2-\left(2\cos\frac{\theta_k}{2}\right)^2\right]^2 ,
\end{aligned}
\tag{4.4}
$$

$$
\begin{aligned}
Dof(D,2r+1) &= \frac{2^{\frac{D}{2}-1}}{r!}\prod_{k=1}^{r}\int_0^{\pi}\frac{d\theta_k}{2\pi}\left(2\cos\frac{\theta_k}{2}\right)^{D-2}\left(2\sin\frac{\theta_k}{2}\right)^2 \\
&\quad\times \prod_{1\le k<l\le r}\left[\left(2\cos\frac{\theta_l}{2}\right)^2-\left(2\cos\frac{\theta_k}{2}\right)^2\right]^2 .
\end{aligned}
\tag{4.5}
$$

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Now, upon performing the transformations $x_k = \sin^2 \frac{\theta_k}{2}$, we get

$$\begin{aligned} Dof(2d, 2r) = & \frac{2^{2(d-1)r+(r-1)(2r-1)}}{\pi^r r!} \\ & \times \prod_{k=1}^{r} \int_0^1 dx_k \; x_k^{-1/2} (1-x_k)^{d-3/2} \prod_{k<l} (x_l - x_k)^2 \,, \end{aligned} \tag{4.6}$$

$$\begin{aligned} Dof(2d, 2r+1) = & \frac{2^{(d-1)+r(2r+2d-3)}}{\pi^r r!} \\ & \times \prod_{k=1}^{r} \int_0^1 dx_k \; x_k^{1/2} (1-x_k)^{d-3/2} \prod_{k<l} (x_l - x_k)^2 \,. \end{aligned} \tag{4.7}$$

We have made explicit in the integrands the square of the Van der Monde determinant: it is then possible to use the orthogonal polynomials method to perform the multiple integrals. Note in fact that in (4.6) and (4.7) the prefactors of the Van der Monde determinant have the correct form to be weights $w^{(p,q)}(x) = x^{q-1}(1-x)^{p-q}$ for the Jacobi polynomials $G_k^{(p,q)}$ with $(p, q) = (d-1, 1/2)$ and $(p, q) = (d, 3/2)$, respectively. The integration domain is also the correct one to set the orthogonality conditions

$$\int_0^1 dx \; w(x) G_k(x) G_l(x) = h_k(p, q)\, \delta_{kl} \tag{4.8}$$

with the normalizations given by

$$h_k(p, q) = \frac{k!\,\Gamma(k+q)\Gamma(k+p)\Gamma(k+p-q+1)}{(2k+p)\Gamma^2(2k+p)} \,, \tag{4.9}$$

see [23] for details about the known orthogonal polynomials. Since the Jacobi polynomials $G_k^{(p,q)}$ are all monic, the even$-N$ formula reduces to

$$\begin{aligned} Dof(2d, 2r) &= \frac{2^{2(d-1)r+(r-1)(2r-1)}}{\pi^r} \prod_{k=0}^{r-1} h_k(d-1, 1/2) \\ &= 2^{(r-1)(2r+2d-3)} \frac{\Gamma(2d-1)}{\Gamma^2(d)} \frac{1}{\pi^{r-1}} \prod_{k=1}^{r-1} h_k(d-1, 1/2) \end{aligned} \tag{4.10}$$

where in the second identity we have factored out the normalization of the zero-th order polynomial. It is straightforward algebra to get rid of all the irrational terms and reach the final expression

$$Dof(2d, 2r) = 2^{r-1} \frac{(2d-2)!}{[(d-1)!]^2} \prod_{k=1}^{r-1} \frac{k\;(2k-1)!\;(2k+2d-3)!}{(2k+d-2)!\;(2k+d-1)!} \,. \tag{4.11}$$

We have checked that these numbers correspond to the dimensions of the rectangular SO(N) Young tableaux with $(D-2)/2$ rows and $N/2$ columns.

For odd N we have instead

$$
\begin{aligned}
Dof(2d, 2r+1) &= \frac{2^{(d-1)+r(2r+2d-3)}}{\pi^r} \prod_{k=0}^{r-1} h_k(d, 3/2) \\
&= \frac{2^{(2-d)+r(2r+2d-3)}}{d} \frac{\Gamma(2d-1)}{\Gamma^2(d)} \frac{1}{\pi^{r-1}} \prod_{k=1}^{r-1} h_k(d, 3/2)
\end{aligned}
\tag{4.12}
$$

which can be reduced to

$$
Dof(2d, 2r+1) = \frac{2^{d-2+r}}{d} \frac{(2d-2)!}{[(d-1)!]^2} \prod_{k=1}^{r-1} \frac{(k+d-1)\ (2k+1)!\ (2k+2d-3)!}{(2k+d-1)!\ (2k+d)!} \tag{4.13}
$$

and again we have checked that these numbers correspond to the dimensions of the spinorial rectangular SO(N) Young tableaux with $(D-2)/2$ rows and $(N-1)/2$ columns.

From these final expressions we can single out a few interesting special cases

$$
\text{(i)} \quad Dof(2, N) = 1, \qquad \forall N \tag{4.14}
$$

$$
\text{(ii)} \quad Dof(4, N) = 2, \qquad \forall N \tag{4.15}
$$

$$
\text{(iii)} \quad Dof(2d, 2) = \frac{(2d-2)!}{[(d-1)!]^2} \tag{4.16}
$$

$$
\text{(iv)} \quad Dof(2d, 3) = \frac{2^{d-1}}{d} \frac{(2d-2)!}{[(d-1)!]^2} \tag{4.17}
$$

$$
\text{(v)} \quad Dof(2d, 4) = \frac{1}{(2d-1)(2d+2)} \left(\frac{(2d)!}{[d!]^2} \right)^2 \tag{4.18}
$$

$$
\text{(vi)} \quad Dof(2d, 5) = \frac{3 \cdot 2^{d-2}}{(2d-1)(2d+4)(2d+1)^2} \left(\frac{(2d+2)!}{[(d+1)!]^2} \right)^2 \tag{4.19}
$$

$$
\text{(vii)} \quad Dof(2d, 6) = \frac{12}{(2d-1)(2d+6)(2d+1)^2(2d+4)^2} \left(\frac{(2d+2)!}{[(d+1)!]^2} \right)^2 . \tag{4.20}
$$

In particular, in $D = 4$ one recognizes the two polarizations of massless particles of spin $N/2$. The cases of $N = 3$ and $N = 4$ correspond to free gravitino and graviton, respectively, but this is true only in $D = 4$. In other dimensions one has a different field content compatible with conformal invariance.

5. The case of $N = 4$ and the Pashnev-Sorokin model

For $N = 4$ the gauge group is SO(4) $=$ SU(2) $\times$ SU(2). Pashnev and Sorokin in [13] considered the model with a factor SU(2) gauged and the other SU(2) left as a global symmetry. In the analysis of Pashnev and Sorokin the model corresponds to a conformal gravitational multiplet, and it was left undecided if the field content in $D = 4$ is that of a graviton plus three scalars (five degrees of freedom) or that of a graviton plus two scalars (four degrees of freedom). Thus, we apply the previous techniques to compute the number of physical degrees of freedom to clarify the field content of the Pashnev-Sorokin model. As discussed in appendix C the number of degrees of freedom of the Pashnev-Sorokin model is given by

$$Dof(D,\mathrm{PS}) = \frac{1}{2}\int_0^{2\pi}\frac{d\theta}{2\pi}\left(2\cos\frac{\theta}{2}\right)^{2(D-2)}\left(2\sin\theta\right)^2 . \tag{5.1}$$

This can be cast in a form similar to those obtained in section 4, and computed explicitly

$$Dof(D,\mathrm{PS}) = \frac{2^{2D}}{2\pi}\int_0^1 dx\ (1-x)^{D-3/2}x^{1/2} = 2^{D-1}\frac{(2D-3)!!}{D!} \tag{5.2}$$

producing $Dof(D,\mathrm{PS}) = (1,2,5,14,42,132,429,\ldots)$ for $D = (2,3,4,5,6,7,8,\ldots)$. Thus in $D = 4$ one gets 5 degrees of freedom, which must correspond to a graviton plus three scalars. Notice that the Pashnev-Sorokin model contains physical degrees of freedom also in spacetimes of odd dimensions. Possible couplings of this model to curved backgrounds have been studied in [24].

6. Conclusions

We have studied the one-loop quantization of spinning particles with a gauged SO(N) extended supergravity on the worldline. These particles describe in first quantization all free conformal field equations in arbitrary dimensions and, in particular, massless fields of higher spin in $D = 4$.

We have considered propagation on a flat target spacetime and obtained the measure on the moduli space of the SO(N) supergravity on the circle. We have used it to compute the propagating physical degrees of freedom described by the spinning particles. These models can be coupled to de Sitter or anti de Sitter backgrounds, and it would be interesting to study their one-loop partition function on such spaces. Also, it would be interesting to study from the worldline point of view how one could introduce more general couplings, giving a different perspective on the problem of constructing consistent interactions for higher spin fields.

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A. Gauge fixing of the group SO(N)

Let us review how the SO(N) gauge fields on the circle can be gauge fixed to a set of constant angles taking values on the Cartan torus of the Lie algebra of SO(N). We parametrize the circle described by the worldline by $\tau \in [0,1]$ with periodic boundary conditions on τ.

Let us start with the simpler SO(2) = U(1) group. For this case the finite version of the gauge transformations (2.4) looks similar to the infinitesimal one

$$\begin{aligned} a' &= a + \dot{\alpha} \\ &= a + \frac{1}{i} g^{-1} \dot{g} \,, \qquad g = e^{i\alpha} \in \mathrm{U}(1) \,. \end{aligned} \tag{A.1}$$

One could try to fix the gauge field to zero by solving

$$a + \dot{\alpha} = 0 \qquad \Rightarrow \qquad \alpha(\tau) = -\int_0^\tau dt\, a(t) \,, \tag{A.2}$$

but this would not be correct as the gauge transformation

$$\tilde{g}(\tau) \equiv e^{-i\int_0^\tau dt\, a(t)} \tag{A.3}$$

is not periodic on the circle, $\tilde{g}(0) \neq \tilde{g}(1)$. In general this gauge transformation is not admissible as it modifies the boundary conditions of the fermions. Thus one introduces the constant

$$\theta = \int_0^1 dt\, a(t) \tag{A.4}$$

and uses it to construct a periodic gauge transformation connected to the identity ("small" gauge transformation)

$$g(\tau) \equiv e^{-i\int_0^\tau dt\, a(t)}\, e^{i\theta\tau} \,. \tag{A.5}$$

This transformation brings the gauge field to a constant value on the circle

$$a'(\tau) = \theta \,. \tag{A.6}$$

Now "large" gauge transformations $e^{i\alpha(\tau)}$ with $\alpha(\tau) = 2\pi n\tau$ are periodic and allow to identify

$$\theta \sim \theta + 2\pi n \,, \qquad n \text{ integer} \,. \tag{A.7}$$

Therefore θ is an angle, and one can take $\theta \in [0, 2\pi]$ as the fundamental region of the moduli space for the SO(2) gauge fields on the circle.

The general case of SO(N) can be treated similarly, using path ordering prescriptions to take into account the non-commutative character of the group. Finite gauge transformations can be written as

$$a' = g^{-1} a g + \frac{1}{i} g^{-1} \dot{g} \,, \qquad g = e^{i\alpha} \,, \qquad \alpha \in \mathrm{Lie}(\mathrm{SO}(N)) \,. \tag{A.8}$$

One can define the gauge transformation

$$\tilde{g}(\tau) = \mathrm{P} e^{-i\int_0^\tau dt\, a(t)} \tag{A.9}$$

where "P" stands for path ordering. This path ordered expression solves the equation

$$\partial_\tau \tilde{g}(\tau) = -i a(\tau) \tilde{g}(\tau) \tag{A.10}$$

and could be used to set a' to zero, but it is not periodic on the circle, $\tilde{g}(0) \neq \tilde{g}(1)$, and thus is not admissible. Therefore one identifies the Lie algebra valued constant A by

$$e^{-iA} = \mathrm{P}e^{-i\int_0^1 dt\, a(t)} \tag{A.11}$$

so that the gauge transformation given by

$$g(\tau) \equiv \mathrm{P}e^{-i\int_0^\tau dt\, a(t)}\, e^{iA\tau} \tag{A.12}$$

is periodic and brings the gauge potential equal to a constant

$$a'(\tau) = A\ . \tag{A.13}$$

Since the constant A is Lie algebra valued, it is given in the vector representation by an antisymmetric $N \times N$ matrix, which can always be skew diagonalized by an orthogonal transformation to produce eq. (3.7) or eq. (3.13), depending on whether N is even or odd. One can recognize that the parameters θ_i contained in the latter equations are angles, since one can use "large" U(1) gauge transformation contained in SO(N) to identify

$$\theta_i \sim \theta_i + 2\pi n_i\ , \qquad n_i \text{ integer}\ . \tag{A.14}$$

The range of these angles can be taken as $\theta_i \in [0, 2\pi]$ for $i = 1, \ldots, r$, with r the rank of the group. Further identifications restricting the range to a fundamental region are discussed in the main text.

B. The Van der Monde determinant and orthogonal polynomials

In this appendix we briefly review some properties of the Van der Monde determinant and the orthogonal polynomials method. Further details and applications of the method can be found in Mehta's book on random matrices [25].

The Van der Monde determinant is defined by

$$\Delta(x_i) = \prod_{1\leq k<l\leq r} (x_l - x_k) = \begin{vmatrix} {x_1}^0 & \cdots & {x_r}^0 \\ {x_1}^1 & \cdots & {x_r}^1 \\ \vdots & & \vdots \\ {x_1}^{r-1} & \cdots & {x_r}^{r-1} \end{vmatrix} \tag{B.1}$$

where the second identity can be easily proved by induction, observing that: (i) the determinant on the right hand side vanishes if $x_r = x_i,\ i = 1, \ldots, r-1$, and (ii) the coefficient of ${x_r}^{r-1}$ is the determinant of order $r-1$. Furthermore, using basic theorems of linear algebra the Van der Monde determinant can be written as

$$\Delta(x_i) = \begin{vmatrix} p_0(x_1) & \cdots & p_0(x_r) \\ p_1(x_1) & \cdots & p_1(x_r) \\ \vdots & & \vdots \\ p_{r-1}(x_1) & \cdots & p_{r-1}(x_r) \end{vmatrix} \tag{B.2}$$

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where $p_k(x)$ is an arbitrary, order$-k$ polynomial in the variable x, with the only constraint of being *monic*, that is $p_k(x) = x^k + a_{k-1}x^{k-1} + \cdots$.

Interesting properties are associated with the square of the Van der Monde determinant, which can be written as

$$\begin{aligned}\Delta^2(x_i) &= \det \begin{pmatrix} p_0(x_1) & \cdots & p_{r-1}(x_1) \\ p_0(x_2) & \cdots & p_{r-1}(x_2) \\ \vdots & & \vdots \\ p_0(x_r) & \cdots & p_{r-1}(x_r) \end{pmatrix} \begin{pmatrix} p_0(x_1) & \cdots & p_0(x_r) \\ p_1(x_1) & \cdots & p_1(x_r) \\ \vdots & & \vdots \\ p_{r-1}(x_1) & \cdots & p_{r-1}(x_r) \end{pmatrix} \\ &= \det K(x_i, x_j) \end{aligned} \tag{B.3}$$

where the kernel matrix K reads as

$$K(x_i, x_j) = \sum_{k=0}^{r-1} p_k(x_i) p_k(x_j) \ . \tag{B.4}$$

The above polynomials can be chosen to be orthogonal with respect to a certain positive weight $w(x)$ in a domain D

$$\int_D dx \ w(x) p_n(x) p_m(x) = h_n \, \delta_{nm} \ . \tag{B.5}$$

However, monic polynomials cannot in general be chosen to be ortho*normal*. Of course, one can relate them to a set of orthonormal polynomials $\tilde{p}_n(x)$

$$p_n(x) = \sqrt{h_n} \, \tilde{p}_n(x) \tag{B.6}$$

and the square of the Van der Monde determinant can be written in terms of a rescaled kernel

$$\Delta^2(x_i) = \prod_{k=0}^{r-1} h_k \det \tilde{K}(x_i, x_j) \tag{B.7}$$

with an obvious definition of the latter kernel in terms of the orthonormal polynomials. Thanks to the orthonormality condition, the rescaled kernel can be shown to satisfy the property

$$\int_D dz \ w(z) \tilde{K}(x, z) \tilde{K}(z, y) = \tilde{K}(x, y) \, , \tag{B.8}$$

that can be applied to prove (once again by induction) the following identity

$$\begin{aligned} &\int_D dx_r \ w(x_r) \int_D dx_{r-1} \ w(x_{r-1}) \cdots \int_D dx_{h+1} \ w(x_{h+1}) \det \tilde{K}(x_i, x_j) \\ &\qquad = (r-h)! \det \tilde{K}^{(h)}(x_i, x_j) \end{aligned}$$

where $\tilde{K}^{(h)}(x_i, x_j)$ is the order$-h$ minor obtained by removing from the kernel the last $r - h$ rows and columns. In particular

$$\int_D dx_r \ w(x_r) \cdots \int_D dx_1 \ w(x_1) \det \tilde{K}(x_i, x_j) = (r-1)! \int_D dx_1 \ w(x_1) \tilde{K}(x_1, x_1) = r! \tag{B.9}$$

and

$$\frac{1}{r!}\int_D dx_r\; w(x_r)\cdots\int_D dx_1\; w(x_1)\,\Delta^2(x_i)=\prod_{k=0}^{r-1}h_k\;. \tag{B.10}$$

C. Gauge fixing of the Pashnev-Sorokin model

To derive formula (5.1) for the physical degrees of freedom of the Pashnev-Sorokin model we take the action (2.3) and consider the gauging of a single SU(2) factor of the SO(4) = SU(2) × SU(2) symmetry group. In order to do that let us consider the change of variables

$$\psi^i=\psi^{\alpha\dot\alpha}\left(\sigma^i\right)_{\alpha\dot\alpha} \tag{C.1}$$

where

$$\left(\bar\sigma^i\right)^{\dot\alpha\alpha}=(-i1,\sigma)^{\dot\alpha\alpha}\,,\qquad \left(\sigma^i\right)_{\alpha\dot\alpha}=(i1,\sigma)_{\alpha\dot\alpha}=-\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}\left(\bar\sigma^i\right)^{\dot\beta\beta}\;. \tag{C.2}$$

The transformation (C.1) can be inverted as[1]

$$\psi^{\alpha\dot\alpha}=\frac{1}{2}\psi^i\left(\bar\sigma_i\right)^{\dot\alpha\alpha}\;. \tag{C.3}$$

The reality condition on ψ^i, along with the expressions (C.2), allows to write it also in the form

$$\psi^i=\bar\psi_{\alpha\dot\alpha}\left(\bar\sigma_i\right)^{\dot\alpha\alpha} \tag{C.4}$$

with

$$\bar\psi_{\alpha\dot\alpha}=-\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\psi^{\beta\dot\beta}\;. \tag{C.5}$$

Thus, the fermion part of the lagrangian can be written as

$$\frac{1}{2}\psi^i(\delta_{ij}\partial_\tau-a_{ij})\psi^j=\bar\psi_{\alpha\dot\alpha}\left(\delta^\alpha{}_\beta\delta^{\dot\alpha}{}_{\dot\beta}\partial_\tau-A^\alpha{}_\beta{}^{\dot\alpha}{}_{\dot\beta}\right)\psi^{\beta\dot\beta} \tag{C.6}$$

where

$$A^\alpha{}_\beta{}^{\dot\alpha}{}_{\dot\beta}=\frac{1}{2}a_{ij}\left(\bar\sigma^i\right)^{\dot\alpha\alpha}\left(\sigma^j\right)_{\beta\dot\beta} \tag{C.7}$$

and

$$a_{ij}=\frac{1}{2}\left(\sigma_i\right)_{\alpha\dot\alpha}\left(\bar\sigma_j\right)^{\dot\beta\beta}A^\alpha{}_\beta{}^{\dot\alpha}{}_{\dot\beta}\;. \tag{C.8}$$

The SU(2) × SU(2) gauge invariance of the action is now manifest. To gauge only a SU(2) subgroup one may choose

$$A^\alpha{}_\beta{}^{\dot\alpha}{}_{\dot\beta}=\delta^\alpha{}_\beta B^{\dot\alpha}{}_{\dot\beta}\quad\Rightarrow\quad a_{ij}=\frac{1}{2}\mathrm{tr}\left(\sigma_i B\bar\sigma_j\right) \tag{C.9}$$

[1] Here we make use of the well-known properties $\left(\sigma^i\bar\sigma^j+\sigma^j\bar\sigma^i\right)_\alpha{}^\beta=2\delta^{ij}\delta_\alpha{}^\beta$, $\left(\bar\sigma^i\sigma^j+\bar\sigma^j\sigma^i\right)^{\dot\alpha}{}_{\dot\beta}=2\delta^{ij}\delta^{\dot\alpha}{}_{\dot\beta}$, $\left(\sigma^i\right)_{\alpha\dot\alpha}\left(\bar\sigma_i\right)^{\dot\beta\beta}=2\delta_\alpha{}^\beta\delta_{\dot\alpha}{}^{\dot\beta}$.

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and gauge fix B to

$$B^{\dot\alpha}{}_{\dot\beta} = 2\theta\,(\frac{i}{2}\sigma^3)^{\dot\alpha}{}_{\dot\beta} = i\theta\,(\sigma^3)^{\dot\alpha}{}_{\dot\beta} \tag{C.10}$$

which gives

$$a_{ij} = \theta \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \tag{C.11}$$

so that

$$\begin{aligned} \int D\psi\ \exp\left(-\frac{1}{2}\int \psi^i_\mu(\partial_\tau\delta_{ij} - a_{ij})\psi^j_\mu\right) &= \mathrm{Det}^D(\partial_\tau + i\theta)_{ABC}\ \mathrm{Det}^D(\partial_\tau - i\theta)_{ABC} \\ &= \left(2\cos\frac{\theta}{2}\right)^{2D} . \end{aligned} \tag{C.12}$$

The Faddeev-Popov determinant associated to the gauge-fixing of the SU(2) gauge group reads

$$\mathrm{Det}\,(\partial_\tau 1_{adj} - B_{adj})_{PBC} = (2\sin\theta)^2 \tag{C.13}$$

since eq. (C.10) in the adjoint representation becomes

$$B_{adj} = 2\theta \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} . \tag{C.14}$$

Finally, the Faddeev-Popov determinant associated to gauge-fixing the local supersymmetry reads

$$\mathrm{Det}^{-1}(\partial_\tau\delta_{ij} - a_{ij})_{ABC} = \left(2\cos\frac{\theta}{2}\right)^{-4} . \tag{C.15}$$

Assembling all determinants one gets (5.1), where the factor 1/2 is due to the parity transformation $\theta \to -\theta$.

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