PATH-INDEPENDENT INTEGRALS AROUND TWO CIRCULAR HOLES IN A THERMOELASTIC MEDIUM

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Abstract: *An analytic solution in bipolar coordinates is presented for thermal stresses in an infinite thermoelastic medium with two unequal circular holes kept at different temperatures. The J_k-integral vector and M- and L- integrals are derived for steady thermoelasticity and calculated on a closed contour encircling one or both holes.*

1. Introduction

Thermal stress plays a significant role in a number of engineering problems ranging from the design of heat engines, nuclear plants and aircrafts to the reliability of electronic devices and MEMS. For instance, cracks usually nucleated at the cooling hole locations in ceramic thermal-barrier coatings of turbine blades due to mechanical and thermal stresses. The knowledge of stress concentration factor (SCF) due to thermal loadings is also a main issue for the accurate design of microelectronic packages, where several electric wires are embedded in a Silicon or ceramic matrix at small distance each other. In this case, the heat production due to the Joule effect may induce high enough thermal stresses to cause cracking and rupture of the insulating ligament between the wires, thus reducing the device performance.

The 2D problem of the stress distribution in an infinite elastic medium containing two circular holes under plane stress or plane strain conditions has attracted the interest of many researchers since Jeffery [1] found the most general form of a biharmonic stress function in bipolar coordinates. The Jeffrey approach was then used by Wu and Markenscoff [2] for finding the stress distribution in an infinite plate containing two holes under thermal loading conditions. Recently, Radi [3] studied the problem of stresses concentration induced in an infinite plate with two unequal circular holes by remote biaxial loading and arbitrary pressures in the holes. The latter author obtained closed form solution for the stress and displacement fields in the plate and investigated the variations of the SCF at the edge of the two holes for different geometries and loading conditions. He also calculated the J_k-integral vector and the *M*- and *L*-integrals on a closed contour encircling both holes under general remote loading conditions and traction free holes surfaces.

In this work, an analytical solution in bipolar coordinates is presented for the steady thermal stresses induced in a 2D thermoelastic solid with two unequal circular holes by a difference in the temperatures of the holes (Fig. 1), both for plane stress and plane strain conditions. The analysis is based on the decomposition of the stress field in the sum of a particular stress field due to the steady temperature distribution and an uxiliary isothermal stress field required for satisfying the vanishing of the tractions on the surface of the holes and the vanishing of the remote stress field. A Fourier series expansion of the total stress field is performed and the coefficients are analytically determined by imposing the boundary conditions on the surface of the holes and at infinity on each term of the series expansion. The solution for a hole in a half-plane and an eccentric annulus can be derived as special case. Moreover, the J_k-integral vector and the M- and L-integrals are first extended to thermoelastic solids under steady state heat flux and then calculated on a closed contour encircling one or both holes. The variations of these path-independent integrals with the holes geometry are then presented and discussed (Fig.2).

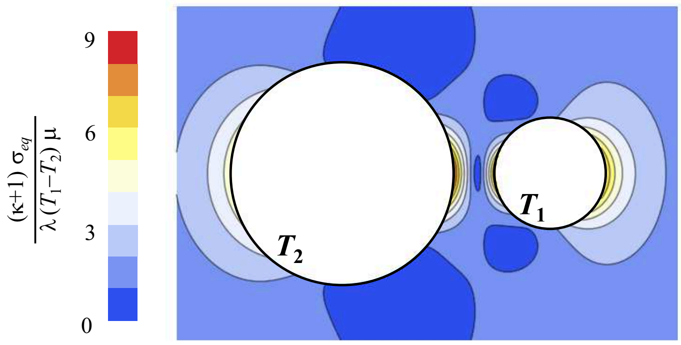

FIGURE 1: Distributions of von Mises stress near the holes under plane stress condition.

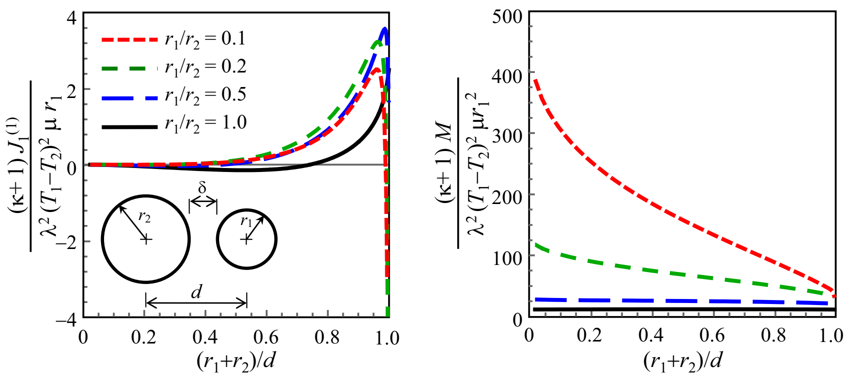

FIGURE 2: Variations of the normalized $J_1^{(1)}$- and M-integrals with the holes geometry.

2. References

[1] Jeffery, G.B., "Plane stress and plane strain in bipolar co-ordinates", *Philos. Trans. R. Soc. London*, Ser. A, Vol. 221, 1921, pp. 265-293.

[2] Wu, L., and Markenscoff, X., "Asymptotics for thin elastic ligaments with applications to body force and thermal loadings", *J. Mech. Phys. Solids*, Vol. 45, No. 1l/12, 1997, pp. 2033-2054.

[3] Radi, E., "Path-independent integrals around two circular holes in an infinite plate under biaxial loading conditions", *Int. J. Engng. Sci.*, Vol. 49, No. 9, 2011, pp. 893-914.