Thermal stress fields between two unequal circular holes in a ceramic medium

Enrico Radi[1], Lorenzo Morini[2]

[1] *DiSMI Università di Modena e Reggio Emilia, Reggio Emilia, Italy*
[2] *DICAM Università di Trento, Trento, Italy*

Email: enrico.radi@unimore.it

Thermal stresses play a significant role in a number of engineering problems ranging from the design of heat engines, nuclear plants and aircrafts to the enhancement of electronic devices and MEMS performance. In particular, the determination of stress concentrations due to thermal loadings is a main issue for the accurate design of many electronic devices, where a large number of conductive electric wires are embedded in a ceramic or Silicon matrix at a small distance from each other. In this case, the heat production due to the Joule effect may create high enough thermal stresses to cause cracking and rupture of the insulating ligament between the wires [1], thus reducing the performance of the device. Since cracks often initiate and propagate from the locations of stress concentration, such as holes and inclusions, then, an accurate evaluation of the stress concentration factor (SCF) in proximity of these defects is a prerequisite to assure the structural integrity of a number of ceramic components and to guarantee the proper functionality of many electronic devices.

An analytic solution is presented here for thermal stresses in an infinite thermoelastic medium with two unequal circular cylindrical holes held at different temperatures, under steady-state heat flux. The most general representation for a biharmonic function in bipolar coordinates [2] has been used. The stress field is decomposed in the sum of a particular stress field induced by the steady-state temperature distribution, which does not satisfy the conditions of vanishing tractions on the surfaces of the holes and vanishing remote stress field, and an auxiliary stress field required to satisfy these boundary conditions, which has been obtained for isothermal elasticity. The corresponding variations of the stress concentration factor, are determined in terms of the holes geometry and temperatures. Moreover, the J_k-integral vector and the *M*-integral are first generalized for steady state thermoelasticity and then calculated on a closed contour encircling one or both holes. Results are then presented for varying geometry of the holes (Fig. 1).

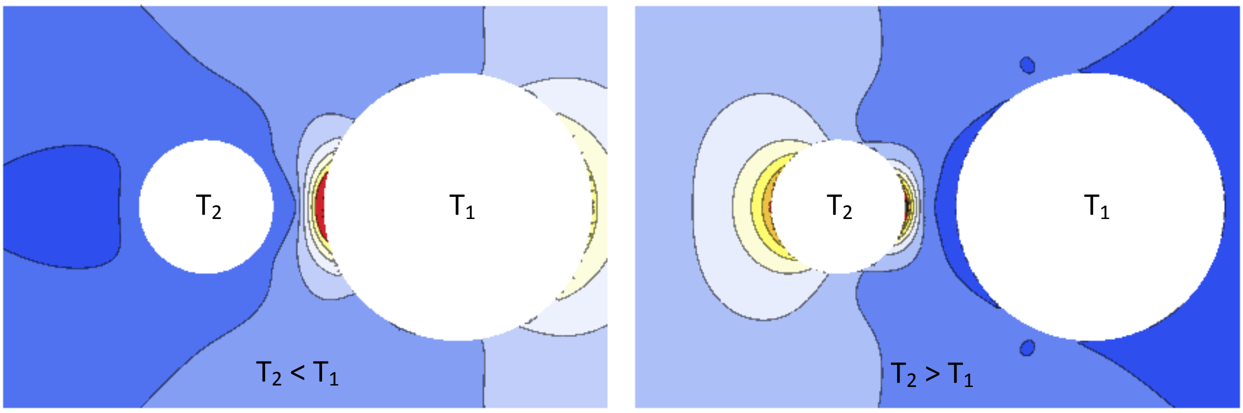

Fig. 1. Distribution of the maximum principal stress for $R_1 = 2R_2$, when $T_2 < T_1$ or $T_2 > T_1$.

Acknowledgements: Financial suopport from "Fondazione Cassa di Risparmio di Modena" is gratefully akowledged.

References:

[1] Wu L., Markenscoff X. 1997. Asymptotics for thin elastic ligaments with applications to body force and thermal loadings. J. Mech. Phys. Solids, 45 (11/12), 2033-2054.
[2] Radi E. 2011. Path-independent integrals around two circular holes in an infinite plate under biaxial loading conditions. Int. J. Engng. Scie. 49 (9), 893-914.