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Option implied trees and implied moments

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OPTION IMPLIED TREES AND IMPLIED MOMENTS

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Abstract

Implied trees are simple non-parametric discretizations of one- or two-dimension diffusions, aimed at introducing non-constant volatility in an option pricing model. The aim of the paper is twofold. First we investigate the ability of different option implied trees in pricing European options. Second, we compare the implied moments obtained with the use of option implied trees with the risk–neutral moments obtained with the use of Bakshi et al. (2003) formula and with realised physical moments. The comparison is pursued in the Italian market by analysing a data set which covers the years 2005-2009 and span both a relatively tranquil and a turmoil period.

Keywords: *option implied trees, risk neutral moments, financial turmoil.*

JEL classification: G13, G14.

1. Introduction

Given the stock-varying and time-varying volatility exhibited by financial data, several ways have been proposed in the literature in order to introduce non-constant volatility in an option pricing model. Deterministic smile consistent volatility models make volatility deterministically dependent on stock price and time. Among deterministic volatility models, some use forward induction in the derivation of the implied trees, others use backward induction (see Linaras and Skiadopoulos (2005) for a survey). The main disadvantage of deriving implied trees by forward induction is the occurrence of negative probabilities that indicate the presence of arbitrage opportunities. In this case, ad hoc methodologies to override the nodes that violate the no arbitrage condition have been proposed. Backward induction trees are by construction arbitrage free, however their main disadvantage is that they require a costly estimation of the ending risk neutral probabilities and they are not flexible enough to price path dependent options.

Recently, the literature has also focused on the information content of higher moments of the stock return distribution, which capture key aspects of the risk neutral density. Historical moments are usually poor estimates of expected future higher moments. Instead, the moments inferred from option prices contain the most recent information available and are thus of importance to both academics and practitioners. At the CBOE, a measure of the implied skewness of the distribution (SKEW index) has been recently introduced in order to capture the perceived risk of extreme market moves (tail risk). A number of recent papers show that moments are very useful and widely used in different empirical applications. For instance, investors could use the predictive power of these implied moments on future returns (Han (2008), Figlewski (2009)) or for settling volatility trading strategies (Bali and Murray (2011)). Besides, many papers investigate the variation of risk neutral implied moments across firms and identify the characteristics which influence them (Neumann and Skiadopoulos (2012)) and the dynamics of implied moments in an autoregressive model (Hansis, et al. (2010)). Moreover, Zhang e Xiang (2008) show that changes in risk neutral volatility, skewness and kurtosis are related to the level, slope and curvature of the implied volatility surface, respectively. Last, Kang et al. (2010), confirm the relationship between higher risk neutral moments and the variance risk premium.

The aim of the paper is twofold. First, we investigate the ability of different option implied trees in pricing European options. Second, we compare the implied moments obtained with the use of option implied trees with the risk–neutral moments obtained with the use of Bakshi et al. (2003) formula and with realised physical moments. The comparison is pursued in the Italian market by analysing a data set which covers the years 2005-2009 and span both a relatively tranquil and a turmoil period.

Given the limited empirical evidence on option implied trees, the paper contributes to the existing literature, by providing an updated literature review on the implied trees and providing new evidence on how the pricing performance of the latter models varies in different market conditions. Moreover, as far as we know, this is the first attempt of comparing option implied trees in order to extract risk neutral moments of the underlying asset distribution.

The paper proceeds as follows. In Section 2 we provide un updated literature review on option implied trees and briefly describe the implied trees used in the application. In Section 3 we recall the Bakshi et al. (2003) formula and the implied tree’s risk-neutral moments. In Section 4 we present the data-set and the methodology. Section 5 presents the results for the pricing performance and Section 6 the ones for the implied moments and the variance, skewness and kurtosis risk premia. The last section concludes.

2. Option implied trees: a literature review

Implied trees are simple non-parametric discretizations of one- or two-dimension diffusions, aimed at introducing non-constant volatility in an option pricing model. The purpose of the construction of smile consistent models is both to look at the distribution of the underlying asset in the future for risk management purposes and to price American and other exotic options consistently with traded European options. Two are the basic approaches for the construction of a deterministic volatility model: backward and forward induction. Among forward induction implied trees, Derman and Kani (1994) are the first to construct an option implied tree consistent with the observed smile; a drawback of the model is the occurrence of negative probabilities which denote arbitrage violations and negatively affect the performance of the implied tree. Barle and Cakici (1998) introduce some modifications to the Derman and Kani (1994) implied tree, but do not resolve the negative probability problem at the edge of the tree, in particular in case of high interest rates and pronounced smile functions. Moriggia et al. (2009) develop a no arbitrage check and a procedure in order to substitute the nodes which imply arbitrage violations throughout the entire tree. Derman, Kani and Chriss (1996) propose a trinomial tree which offers a better adaptation than a binomial model to the real data, but presents problems in the choice of the state space and is not free from arbitrage violations in case of high volatility. Charalambous et al. (2007) propose a non-recombining tree which is more flexible than a recombining one, but very complex from the computational point of view. Among backward induction models, the first is Rubinstein (1994), which propose a two-step procedure in order to derive the tree: estimate the ending nodes and probabilities and derive the tree backwardly by the hypothesis that the paths who lead to the same ending node are equally probable. An optimization problem is solved in order to minimize the distance between an a-priori density and the real one, under the constraint of correctly pricing by no-arbitrage the options and the underlying asset. The model has at least two drawbacks: it cannot price options which expire before the maturity of the tree and path-dependent options. In order to overcome these problems, Jackwerth (1997) proposes a function to weight the different paths in the tree, on the other hand, Brown and Toft (1999) and Herwig (2005) propose a methodology in order to calibrate the Rubinstein's tree to intermediate maturity options. Last, Tian (2012) proposes a methodology to diminish the number of unknowns in the optimization problem of Rubinstein (1994), by grouping the final nodes.

Form the empirical point of view, limited is the evidence about the pricing performance of the different implied trees. Dumas et al. (1998) find a poor performance of deterministic implied trees. Brandt e Wu (2002) and Linaras e Skiadopoulos (2005) compare in the UK and US market the performance of the Derman and Kani (1994) and Barle and Cakici (1998) implied trees with different smile estimation. Lim and Zhi (2002) and Kim and Park (2004) on the other hand, focus on the backward induction implied trees of Rubinstein (1994) and Jackwerth (1997) by looking at the UK and the Korean market respectively. The findings of these papers show an heterogeneous picture where it is difficult to assess the superiority of one model in absolute terms, given that the performance varies depending on moneyness, maturity and type of the options to be priced.

As for the use of option implied trees in order to estimate the risk neutral distribution of the underlying asset, Jackwerth and Rubinstein (1996) argue that if we observe a sufficient number of option prices, all the different methods tend to be rather similar, except in the modeling of the tails of the distribution. Campa et al. (1998) prefer the implied tree approach for its flexibility and good representation of the data, however they observed many outliers in the tails of the Rubinstein (1994) tree.

2.1. The Enhanced Derman and Kani implied tree

Derman and Kani (1994) construct an implied tree using forward induction. They build a recombining binomial tree which uses as inputs the market prices of European-style index options across all strikes and expirations. Their model has uniformly spaced levels Δt apart.

Let's assume that the tree has already been constructed up to the time level $j-1$, and let us see how to derive the next level stock prices. The known stock price $S_{i,j-1}$ can evolve in two states in the next level j: the up one, $S_{i+1,j}$, and the down one, $S_{i,j}$. The risk neutral probability of an up jump is $p_{i,j}$. Arrow-Debreu prices $\lambda_{i,j}$ are defined as the sum over all paths leading to node (i,j) of the product of the risk neutral probabilities discounted at the risk-free rate at each node. If the level is odd, the centering condition is given by equation (1), if the level is even, the two central nodes have to satisfy equation (2):

$$S_{\frac{j}{2}+1,j} = S_{0,0} \tag{1}$$

$$S_{j/2,j} = \frac{S_{0,0}}{S_{\frac{j}{2}+1,j}} \tag{2}$$

Let $C_{i,j-1}$ and $P_{i,j-1}$ be the price of a call and a put with strike $S_{i,j-1}$ and maturity j, respectively. These prices are computed using Black-Sholes formulas with constant volatility obtained from the smile function. In the upper part of the tree the recursive formula to compute $S_{i+1,j}$, given $S_{i,j}$, is:

$$S_{i+1,j} = \frac{S_{i,j}\left[e^{r\Delta t}C_{i,j-1} - \Sigma_C\right] - \lambda_{i,j-1}S_{i,j-1}(F_{i,j-1} - S_{i,j})}{\left[e^{r\Delta t}C_{i,j-1} - \Sigma_C\right] - \lambda_{i,j-1}(F_{i,j-1} - S_{i,j})} \tag{3}$$

In the lower part of the tree the recursive formula to compute $S_{i,j}$, given $S_{i+1,j}$, is:

$$S_{i,j} = \frac{S_{i+1,j}\left[e^{r\Delta t}P_{i,j-1} - \Sigma_P\right] - \lambda_{i,j-1}S_{i,j-1}(F_{i,j-1} - S_{i,j})}{\left[e^{r\Delta t}P_{i,j-1} - \Sigma_P\right] - \lambda_{i,j-1}(F_{i,j-1} - S_{i,j})} \tag{4}$$

where r is the risk-free rate, $\Sigma_C = \sum_{k=i+1}^{i} \lambda_{k,j-1}(F_{k,j-1} - S_{i,j})$, $\Sigma_P = \sum_{k=1}^{i-1} \lambda_{k,j-1}(F_{k,j-1} - S_{i,j})$ and $F_{i,j-1}$ is the forward value of $S_{i,j-1}$.

These equations can be used only if the level j is odd, when the starting central node is equal to the current spot price. If the number of nodes is even, by combining equations (2) and (3) we get:

$$S_{i+1,j} = \frac{S_{0,0}\left[e^{r\Delta t}C_{i,j-1} + \lambda_{i,j-1}S_{0,0} - \Sigma_C\right]}{\lambda_{i,j-1}F_{i,j-1} - e^{r\Delta t}C_{i,j-1} + \Sigma_C} \tag{5}$$

The transition probability $q_{i,j-1}$ of an up move is computed as:

$$q_{i,j-1} = \frac{F_{i,j-1} - S_{i,j}}{S_{i+1,j} - S_{i,j}} \tag{6}$$

The main problem in the derivation of the implied tree is the presence of riskless arbitrage opportunities, which are represented by a risk neutral probability which falls outside the (0,1) interval. The Derman and Kani's (1994) implied tree, even with the Barle and Cakici (1998) modifications, is not free from arbitrage, in particular at the boundary of the tree and may become numerically unstable, when the number of steps becomes large. Therefore we use the EDK methodology, which is aimed at ensuring the absence of no-arbitrage violations in the DK implied tree. The EDK methodology provides no-arbitrage checks and proposes no-arbitrage replacements for all the nodes in the tree (for more details see Moriggia et al. (2009)).

2.2. The Rubinstein's implied binomial tree

Rubinstein (1994) proposes an implied binomial tree using backward induction. The procedure can be split into two steps: first the risk-neutral probability distribution of underlying asset at the end of the tree is estimated; second, the tree is derived backwardly with a simple three step algorithm.

The Rubinstein's method consists of minimizing the square difference between prior and posterior risk-neutral probabilities, under some constraints. Let's define $Q_{i,n}$ and $Q'_{i,n}$ respectively the posterior and the prior probability at expiry date n, $Q_{i,n}$ is found as the solution of the following optimization problem:

$$\min \sum_{i=0}^{n} (Q_{i,n} - Q'_{i,n})^2 \tag{7}$$

subject to:

$$\sum_{i=0}^{n} Q_{i,n} = 1 \ \ and \ \ Q_{i,n} > 0 \qquad for\ i = 0, \dots, n$$

$$C_k^b \leq C_k \leq C_k^a \tag{8}$$

$S^b \leq S \leq S^a$

where C^b and C^a are respectively the option bid and ask price quotes observed for the European call with strike K_k with $k = 1, \ldots, m < n$, expiring at n and S^b and S^a are the bid and ask prices of the underlying asset, C_k is the price of a call with maturity n and strike price K_k:

$$C_k = e^{-rn} \sum_{i=0}^{n} Q_{i,n} \left(S_{i,n} - K_k\right)^+ \qquad (9)$$

and S is the value of underlying asset at time 0:

$$S = e^{-rn} \sum_{i=0}^{n} Q_{i,n} S_{i,n} \qquad (10)$$

The posterior implied risk-neutral probabilities are called nodal probabilities since $Q_{i,n}$ is the probability to reach node i at expiry n whatever the path to reach that node. Indeed, the rather arbitrary and restrictive assumption of equal path probabilities allows to build the tree in a very simple way with a three step procedure. First, calculate the nodal probabilities at the preceding nodes, as follows:

$$Q_{i-1,n-1} = \left(1 - w_{i,j}\right) * Q_{i-1,n} + w_{i,j} * Q_{i,n} \qquad (11)$$

Second, compute the probability of an up move over the next time interval:

$$q_{i-1,n-1} = w_{i,j} * \frac{Q_{i,n}}{Q_{i-1,n-1}} \qquad (12)$$

This rules out negative probabilities. Finally, compute the stock price at the preceding level with the risk-neutral valuation formula, as follows:

$$S_{i-1,n-1} = e^{-r\Delta t}[\left(1 - q_{i-1,n-1}\right) * S_{i-1,n} + q_{i-1,n-1} * S_{i,n}] \qquad (13)$$

The implied tree is derived by repeating this simple algorithm up to the first node, $S_{0,0}$.

3. Implied moments

In order to obtain implied moments, in this paper we follow two different methods: we use both the Bakshi et al. (2003) formula and the risk-neutral densities estimated with the implied trees described above.

Bakshi et al. (2003) developed a model-free method in order to extract volatility, skewness and kurtosis of the risk-neutral distribution on the expiry date from a cross section of call and put option prices. Their methodology is called model-free, since it is consistent with many underlying asset dynamics. Model-free variance, skewness and kurtosis are obtained from the following equations:

$$VAR(t,n) = e^{rn}V(t,n) - \mu(t,n)^2 \tag{17}$$

$$SKEW(t,n) = \frac{e^{rn}W(t,n) - 3e^{rn}\mu(t,n)V(t,n) + 2\mu(t,n)^3}{[e^{rn}V(t,n) - \mu(t,n)^2]^{3/2}} \tag{18}$$

$$KURT(t,n) = \frac{e^{rn}X(t,n) - 4e^{rn}\mu(t,n)W(t,n) + 6e^{rn}\mu(t,n)^2V(t,n) - 3\mu(t,n)^4}{[e^{rn}V(t,n) - \mu(t,n)^2]^2} \tag{19}$$

with

$$\mu(t,n) = e^{rn} - 1 - \frac{e^{rn}}{2}V(t,n) - \frac{e^{rn}}{6}W(t,n) - \frac{e^{rn}}{24}X(t,n) \tag{20}$$

$$V(t,n) = \int_{S_t}^{\infty} \frac{2\left(1 - ln\left[\frac{K}{S_t}\right]\right)}{K^2} C_{t,n,K}dK + \int_{0}^{S_t} \frac{2\left(1 - ln\left[\frac{S_t}{K}\right]\right)}{K^2} P_{t,n,K}dK \tag{21}$$

$$W(t,n) = \int_{S_t}^{\infty} \frac{6\ln\left[\frac{K}{S_t}\right] - 3\left(\ln\left[\frac{K}{S_t}\right]\right)^2}{K^2} C_{t,n,K}dK - \int_{o}^{S_t} \frac{6\ln\left[\frac{S_t}{K}\right] + 3\left(\ln\left[\frac{S_t}{K}\right]\right)^2}{K^2} P_{t,n,K}dK \tag{22}$$

$$X(t,n) = \int_{S_t}^{\infty} \frac{12\left(\ln\left[\frac{K}{S_t}\right]\right)^2 - 4\left(\ln\left[\frac{K}{S_t}\right]\right)^3}{K^2} C_{t,n,K}dK - \int_{o}^{S_t} \frac{12\left(\ln\left[\frac{S_t}{K}\right]\right)^2 + 4\left(\ln\left[\frac{S_t}{K}\right]\right)^3}{K^2} P_{t,n,K}dK \tag{23}$$

with $C_{t,n,K}$ and $P_{t,n,K}$ that are respectively the current price of a call and a put with maturity n and strike K.

The second approach is to calculate the moments m_α as integrals of the risk neutral density estimated from the implied trees as follows:

$$m_\alpha = \int_{-\infty}^{\infty} x^\alpha f(x) dx \tag{24}$$

with $\alpha = 1,2,3,4$, $x = ln\frac{S_n}{S_t}$ and $f(x)$ risk neutral density. As the implied tree yields a discrete cumulative distribution, a discrete summation over all nodes approximates the continuous integral in the formula (24).

With these moments variance, skewness and kurtosis are easily obtained as follows:

$$VAR(t,n) = m_2 - m_1^2 \tag{25}$$

$$SKEW(t,n) = \frac{m_3 - 3m_1m_2 + 2m_1^3}{(m_2 - m_1^2)^{3/2}} \tag{26}$$

$$KURT(t,n) = \frac{m_4 - 4m_1m_3 + 6m_1^2m_2 - 3m_1^4}{(m_2 - m_1^2)^2} \tag{27}$$

4. The data set and the methodology

The data set consists of closing prices on FTSE MIB-index options (MIBO), recorded from 1 January 2005 to 31 December 2009. MIBO are European options on the FTSE MIB index, which is a capital weighted index composed of 40 major stocks quoted on the Italian market. As for the underlying asset, closing prices of the FTSE MIB-index recorded in the same time period are used. The FTSE MIB is adjusted for dividends as follows:

$$\widehat{S}_t = S_t e^{-\delta_t \Delta t} \tag{28}$$

where S_t is the FTSE MIB value at time t, δ_t is the dividend yield at time t and Δt is the time to maturity of the option. As a proxy for the risk-free rate, Euribor rates with maturities one week, one, two and three months are used. Appropriate yields to maturity are computed by linear interpolation.

The data-set for the FTSE MIB index and the MIBO is kindly provided by Borsa Italiana S.p.A, Euribor rates and dividend yields are obtained from Datastream.

Several filters are applied to the option data set. First, we eliminate options near to expiry which may suffer from pricing anomalies that might occur close to expiration (in order to be consistent with the computation methodology of quoted volatility indexes, we choose to use the most conservative filter that eliminates options with time to maturity of less than eight days). Second, following Ait-Sahalia and Lo (1998) only at-the-money and out-of-the-money options are retained. Last, option prices violating the standard no-arbitrage constraints are eliminated.

In order to get implied moments based on implied trees, we follow the methodology described below and we reiterate the process for both near and next term option in each date of the sample. The benchmark tree is the Cox-Ross-Rubinstein's (1979) one, which is constructed by using a constant volatility equal to an average of at-the-money implied volatilities of a call and a put. The benchmark tree is also used as initial input for the Rubinstein's tree in order to have the prior estimate of the risk-neutral distribution which is used for the optimization process (equations (7-8)).

In order to derive the Enhanced Derman and Kani's tree, and to implement the Bakshi et al. (2003) formula, we first obtain the smile function by using an interpolation-extrapolation scheme. We recover Black-Scholes implied volatilities from traded option prices and interpolate between strikes by using cubic splines; we extrapolate volatilities outside the listed strike price range using a constant extrapolation scheme where the implied volatility is supposed equal to the volatility of the minimum or the maximum strike price respectively. We extrapolate outside the existing domain of strike prices by using a factor u=10 such that: $S/(1+u) \leq K \leq S(1+u)$; in order to have a sufficient discretization of the integration domain, we compute strikes spaced by an interval $\Delta K = 10$. The parameters u and ΔK have been chosen, accordingly to Muzzioli (2010), in order to have insignificant truncation and discretization errors. All implied trees have been derived with 100 steps.

In order to compare the pricing performance of the different implied trees, we resort to the following metrics widely used in the literature (see e.g. Moriggia et al. (2009)). In particular, we use the Mean Absolute Percentage Error (MAPE) and the Mispricing Index (MISP) defined as follows:

$$MAPE = \frac{1}{m}\sum_{k=1}^{m}\frac{|P_k^T - P_k^M|}{P_k^M} \tag{29}$$

$$MISP = \frac{\sum_{k=1}^{m}\left(\frac{P_k^T - P_k^M}{P_k^M}\right)}{\sum_{k=1}^{m}\left|\frac{P_k^T - P_k^M}{P_k^M}\right|} \tag{30}$$

with P^T and P^M which indicate respectively theoretical and market price of the options and m is the number of options in the class.

In order to have a constant 30-days measure for the implied moments, we use linear interpolation, with the same formula which is used for the computation of the VIX index:

$$\sigma_{30} = \sqrt{\left\{\frac{T_1}{365}\sigma_{T_1}^2\left[\frac{T_2 - 30}{T_2 - T_1}\right] + \frac{T_2}{365}\sigma_{T_2}^2\left[\frac{30 - T_1}{T_2 - T_1}\right]\right\} * 365/30} \tag{31}$$

where T_i is the number of calendar days to expiry of the *i*-th maturity index option, $i = 1,2$, i=1 for near term and i=2 for next term. Near and next term moments are derived by using the near and the next term options, with maturity closest to 30-days, either by the use of option implied trees, or with the Bakshi et al. (2003) formula.

The physical moments are obtained from daily log-returns of the underlying index by using a rolling window of about 22 working days (equivalent to a 30-day measure) and then annualised.

In order to gauge the ability of the implied trees in order to forecast the physical moments, we compute the RMSE metric:

$$RMSE = \sqrt{\frac{1}{m}\sum_{k=1}^{m}(M_k^P - M_k^{RP})^2} \tag{32}$$

where m is the number of observations in the sample, M^P are the physical moments and M^{RN} are the risk-neutral moments.

Chang et al. (2013) propose two new types of contracts: skewness and kurtosis swaps, with payoffs similar to the one of the well-known variance swap (Carr and Wu (2009)), i.e. the difference between physical and risk neutral variance. In line with their approach, we measure the payoff of a variance, skewness, and kurtosis swap in Euro terms. In particular, we compute the Euro payoff of a long position in a (variance, skewness or kurtosis) swap with notional amount N=1 Euro, held up to expiry:

$$\text{RP} = \frac{1}{m}\sum_{k=1}^{m}(M^P - M^{RN}) \tag{33}$$

Where m are the days in the sample, M^P are the physical moments and M^{RN} are the risk-neutral moments.

5. The pricing performance

In order to verify the precision of the implied binomial trees, we have used them to price options in sample, since the aim of this exercise is to verify the ability of the different trees in reproducing the underlying asset distribution. The benchmark is the standard Cox-Ross-Rubinstein model (CRR), which has been derived with a constant volatility equal to an average of at-the-money call and put implied volatility.

The results for the whole sample are reported in Table 1. The best model, according to the MAPE is the Enhanced Derman and Kani (EDK) one, followed by the Rubinstein (RUB) model. CRR obtains the worst performance. The mispricing index is negative in all the models, i.e. all the binomial trees substantially underprice options, the EDK model is the one with the highest underpricing, the CRR with the lowest one. Both EDK and RUB better price call than put options (the opposite holds for CRR) and the underpricing is higher for put options.

In order to see which option class is the best priced in each model, we have split options into three moneyness categories: in-the-money calls (out-of-the-money puts) if $K/S \leq 0.97$, at-the-money calls and puts if $0.97 < K/S < 1.03$, out-of-the-money calls (in-the-money puts) if $K/S \geq 1.03$, where K is the strike price and S is the underlying price. Differently from Moriggia et al. (2009), we have used a coarser partition, in order to have each day a homogeneous number of options in each class (around 5 per class). The results are reported in Table 2. According to the MAPE, the best performance of the three models is attained for in-the-money options and it gradually deteriorates with the decrease in moneyness level. The EDK model performs better than both CRR and RUB for all moneyness classes, but the difference is the highest for in-the-money options. The Rubinstein model performs better than CRR only for out-of-the-money options. In terms of mispricing, CRR overprices in-the-money options, while underprices at-the-money and out-of-the-money options. EDK and RUB underprice all options' categories. Overall, the better performance of option implied trees w.r.t. CRR is given mainly by the better pricing of out-of-the-money options, in particular call options. The highest underpricing for EDK is attained for out-of-the-money options, while for RUB for at-the-money options. Both EDK and RUB underprice most out-of-the-money puts, therefore attach a lower probability to the left tail of the risk neutral distribution.

The pricing performance in the two sub-periods is reported in Table 3. The low volatility period covers the years 2005-2007, while the high volatility period the years 2008-2009. Surprisingly, the pricing performance do not vary substantially across the two sub-periods, with the MAPE slightly lower for the CRR and the EDK models in the high volatility period. On the other hand, the RUB model obtains a better performance in the low volatility period. Both implied trees perform better than CRR in low and high volatility periods. EDK performs better than RUB in both sub-periods, but the difference is higher in the high volatility period. The underpricing is less severe for all the models in the high volatility period.

In Tables 4 and 5 we report how the pricing performance varies by moneyness in the two sub-periods. The pricing performance is quite similar across the two sub-periods. The best priced option class in both volatility periods, for all the models, remains the in-the-money one; the worst the out-of-the-money one. By looking at the MAPE, CRR and RUB models obtain a better performance in the high volatility period for all moneyness' classes, the EDK for at-the-money and out-of-the-money.

According to the MISP, CRR underprices (overprice) at-the-money and out-of-the-money (in-the-money) options more in the low volatility period. For the EDK model, the underpricing is more severe in the high volatility period for out-of-the-money options; for other options' classes it is more severe in the low volatility period. The RUB model underprices more severely in the low volatility period at-the-money and out-of-the-money options.

Therefore, we can conclude that better performance of option implied trees w.r.t. CRR is given mainly by the better pricing of out-of-the-money options. Implied trees underprice all options' categories. Among the two implied trees, the results points to a better performance of the EDK model w.r.t. other models in both sub-periods, which is mainly determined by a better pricing in the high volatility period and for in-the-money options. RUB performs fairly w.r.t. CRR for the pricing of out-of-the-money options.

6. The results for the moments

Risk neutral moments, computed both with the Bakshi et al. (2003) formula and the risk-neutral densities estimated with the implied trees described above, along with physical moments are reported in Table 6.

Variance and kurtosis are consistent across different estimation methods and with physical ones, whereas for skewness the results are fairly different in sign and magnitude. The variance estimated with the EDK model is much higher than model-free variance, and both measures overestimate realised variance. On the other hand, the variance computed with the RUB model is the smallest one, even smaller than physical variance. Risk neutral skewness is negative for the EDK model, while it is close to zero for the MF model, and positive for the RUB model. Differently from Conrad et al. (2013), we find that risk neutral skewness is less negative than physical one. The underpricing of out-of-the-money puts in both implied trees models could be a reason for this result, since they attach a lower probability to the left tail of the distribution. Risk-neutral kurtosis is the highest for the EDK method and the lowest for the RUB implied tree. Risk-neutral kurtosis is higher than physical one for all the estimation methods, pointing to the existence of a negative kurtosis risk premium. In fact, in the EDK model the underpricing is more severe for out-of-the-money calls and puts w.r.t. at-the-money. We report in Table 7 the forecasting ability, in terms of RMSE, of the risk neutral moments. The best forecasting method for realised moments is the MF one. EDK (RUB) is the second best for skewness (variance and kurtosis) respectively.

We report in Table 8 the variance, skewness and kurtosis risk premia for all the models. In line with the literature, the variance risk premium is negative for the EDK and MF models. Strikingly, it is positive for the RUB model. The skewness risk premium is negative for all models, it is the smallest for the EDK model. The kurtosis risk premium is negative for all models and it is smallest for the RUB model. Overall, we find significant risk neutral density deviations from physical counterparty, which could signal profitable volatility, skewness and kurtosis trades (Blaskowitz, Hardle and Schmidt (2003)).

In Table 9 we report the estimation of variance, skewness and kurtosis in the two sub-periods. The difference in physical variance among the two sub-periods is very high. Also risk neutral variance is much more higher in the high volatility period. Physical skewness is more negative in the low volatility period. On the other hand, risk neutral skewness tends to be more negative during turmoil periods (as noted by Dennis and Mayhew (2002)), the only exception being for the MF estimation method. Physical kurtosis is almost unvaried across the two periods, with a slightly higher value in the low volatility period. Accordingly, risk neutral kurtosis is a little higher in the low volatility period, for all the models.

The forecasting performance of risk neutral moments in the two sub-periods is reported in Table 10. Overall, all the models present a worst performance in the high volatility period. As for the variance estimation, all the methods obtain a worse performance in the high volatility period. For skewness the performance is worse in the low (high) volatility period for RUB and MF (DK). For kurtosis, the performance improves in the high volatility period for EDK and MF, while it deteriorates for RUB model. The best performance for variance, skewness and kurtosis is obtained by the MF method in both sub-periods. EDK (RUB) is the second best for skewness (variance and kurtosis) in both sub-periods. The variance, skewness and kurtosis risk premia in the two sub-periods are reported in Table 11. Variance risk premia are negative for all the models in the low volatility period and they remain negative in the high volatility period except for the RUB model. As expected, variance risk premia are higher in the high volatility period, where variance trades are more profitable. Moreover, the existence and the pricing of jump risk, which is higher in the high volatility period is a determinant of the variance risk premium. On the other hand, skewness risk premia are higher in absolute terms in the low volatility period, for all the methods. The EDK method finds a positive skewness risk premium in the high volatility period. Kurtosis risk premia are higher in the low volatility period, except for the RUB method. This means that both skewness and kurtosis trades are more profitable in low volatility periods. The analysis of the determinants of the variance, skewness and kurtosis risk premia is left for future research.

7. Conclusions

In this paper we have implemented in the Italian index market two types of implied trees, based on backward or forward induction. We have analysed the pricing performance of the various implied trees and compared it with the Cox-Ross-Rubinstein tree, which is used as a benchmark. Moreover, we have extracted the risk neutral moments of the distribution from the implied trees and compared them with the risk neutral moments obtained with the Bakshi et al. (2003) formula. Physical risk neutral moments have been computed in order to analyse the existence of a variance, skewness and kurtosis risk premium.

As for the pricing performance, the results prefer the Enhanced Derman and Kani model w.r.t. the Rubinstein's one in both sub-periods. The result is mainly determined by smaller errors in the high volatility period and a better pricing of in-the-money call options and out-of-the-money put options, therefore a better estimation of the left tail of the distribution.

Despite the good pricing performance in sample, the EDK model is not the best one for the moments estimation. The MF model remains one of the best for all moments. In fact, moments obtained from EDK implied tree are usually much higher than model-free ones. Moments obtained from the Rubinstein model are less reliable, since they are the less similar to physical ones.

Overall, the results suggest the potential profitability of variance, skewness and kurtosis trades. In particular, the variance and kurtosis risk premium is found to be negative in all models (except in the Rubinstein's one for variance), pointing to the evidence that investors are willing to pay a high fixed rate, in order to be hedged against peaks of variance and kurtosis, which are more present in turmoil periods and signal high perceived uncertainty and tail risk. More difficult is the interpretation of the skewness risk premium, since in the whole sample risk neutral skewness is found to be less negative than physical one: in the high volatility period physical skewness is much less than in the low volatility period. Notably, the EDK model is the only one which finds risk neutral skewness to be more negative than physical one in the high volatility period, where MF yields the opposite result. However, it has been noted in Conrad et al. (2013) that skewness estimates based on sample averages are prone to measurement errors more than other moments; therefore, we leave for future research the use of other asymmetry measures less sensitive to outliers.

The results are of practical importance for traders who may rely on implied trees in order to price other less liquid exotic options consistently with European ones and for settling profitable trades on risk neutral moments. The paper could be extended in many directions. The determinants of variance, skewness and kurtosis risk premia are the first to deserve attention in future research. Moreover, the dynamics and the relationship among implied moments and the importance of risk neutral moments in the forecasting of future returns merit careful investigation.

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Table 1. The pricing performance in the whole sample.

	CRR	***EDK***	***RUB***
MAPE	0,57785	0,19491	0,29391
MAPE Call	0,81140	0,14371	0,12666
MAPE Put	0,34430	0,24611	0,46117
MISP	-0,13398	-0,92463	-0,33374
MISP Call	0,71515	-0,89157	0,12332
MISP Put	-0,98311	-0,95769	-0,79079

Note: The Table shows the pricing errors measured by $MAPE = \frac{1}{m}\sum_{k=1}^{m}\frac{\left|P_k^T - P_k^M\right|}{P_k^M}$, $MISP = \frac{\sum_{k=1}^{m}\left(\frac{P_k^T - P_k^M}{P_k^M}\right)}{\sum_{k=1}^{m}\left|\frac{P_k^T - P_k^M}{P_k^M}\right|}$, *for the Cox-Ross-Rubinstein (CRR), Enhanced Derman and Kani (EDK), and Rubinstein (RUB) models.*

Table 2. The pricing performance by moneyness (whole sample).

Panel A: in-the-money	***CRR***	***EDK***	***RUB***
MAPE	0,0057	0,0016	0,0083
MAPE Call	0,0020	0,0007	0,0023
MAPE Put	0,0094	0,0026	0,0142
MISP	0,0223	-0,9183	-0,0284
MISP Call	0,9407	-0,9720	0,9307
MISP Put	-0,8960	-0,8845	-0,9874

Panel B: at-the-money	***CRR***	***EDK***	***RUB***
MAPE	0,0373	0,0226	0,0593
MAPE Call	0,0091	0,0117	0,0101
MAPE Put	0,0654	0,0334	0,1086
MISP	-0,2522	-0,7735	-0,8093
MISP Call	0,4835	-0,7545	-0,6422
MISP Put	-0,9878	-0,8005	-0,9764

Panel C: out-of-the-money	***CRR***	***EDK***	***RUB***
MAPE	0,1603	0,0767	0,0903
MAPE Call	0,1903	0,0497	0,0291
MAPE Put	0,1313	0,1042	0,1524
MISP	-0,1118	-0,9383	-0,5500
MISP Call	0,7691	-0,9045	-0,3806
MISP Put	-0,9928	-0,9720	-0,7195

Note: The Table shows the pricing performance for in-the-money options (Panel A), at-the-money options (Panel B) and out-of-the-money options (Panel C). The pricing errors measured by $MAPE = \frac{1}{m}\sum_{k=1}^{m}\frac{|P_k^T - P_k^M|}{P_k^M}$, $MISP = \frac{\sum_{k=1}^{m}\left(\frac{P_k^T - P_k^M}{P_k^M}\right)}{\sum_{k=1}^{m}\left|\frac{P_k^T - P_k^M}{P_k^M}\right|}$ *, for the Cox-Ross-Rubinstein (CRR), Enhanced Derman and Kani (EDK), and Rubinstein (RUB) models.*

Table 3. The pricing performance in the two sub-periods.

Panel A: Low volatility

	CRR	***EDK***	***RUB***
MAPE	0,5838	0,2073	0,2776
MISP	-0,1552	-0,9262	-0,4187

Panel B: High volatility

	CRR	***EDK***	***RUB***
MAPE	0,5689	0,1763	0,3185
MISP	-0,1021	-0,9222	-0,2061

Note: The Table shows the pricing errors measured by $MAPE = \frac{1}{m}\sum_{k=1}^{m}\frac{|P_k^T - P_k^M|}{P_k^M}$, $MISP = \frac{\sum_{k=1}^{m}\left(\frac{P_k^T - P_k^M}{P_k^M}\right)}{\sum_{k=1}^{m}\left|\frac{P_k^T - P_k^M}{P_k^M}\right|}$ *, for the Cox-Ross-Rubinstein (CRR), Enhanced Derman and Kani (EDK), and Rubinstein (RUB) models. Panel A of the Table shows the result for the low volatility period (2005-2007), Panel B of the Table for the high volatility period (2008-2009).*

Table 4. The pricing performance by moneyness (Low volatility period).

Panel A: in-the-money			
	CRR	***EDK***	***RUB***
MAPE	0,0068	0,0013	0,0096
MISP	0,0398	-0,7412	-0,0249
Panel B: at-the-money			
	CRR	***EDK***	***RUB***
MAPE	0,0418	0,0233	0,0675
MISP	-0,3018	-0,8039	-0,8911
Panel C: out-of-the money			
	CRR	***EDK***	***RUB***
MAPE	0,2109	0,0935	0,0914
MISP	-0,1508	-0,9247	-0,71

Note: The Table shows the pricing performance in the low volatility period (2005-2007) for in-the-money options (Panel A), at-the-money options (Panel B) and out-of-the-money options (Panel C). The pricing errors are measured by $MAPE = \frac{1}{m}\sum_{k=1}^{m}\frac{|P_k^T - P_k^M|}{P_k^M}$, $MISP = \frac{\sum_{k=1}^{m}\left(\frac{P_k^T - P_k^M}{P_k^M}\right)}{\sum_{k=1}^{m}\left|\frac{P_k^T - P_k^M}{P_k^M}\right|}$ *, for the Cox-Ross-Rubinstein (CRR), Enhanced Derman and Kani (EDK), and Rubinstein (RUB) models.*

Table 5. The pricing performance by moneyness (High volatility period).

Panel A: in-the-money	***CRR***	***EDK***	***RUB***
MAPE	0,004	0,0021	0,0063
MISP	-0,1775	-0,7278	-0,6864
Panel B: at-the-money	***CRR***	***EDK***	***RUB***
MAPE	0,0305	0,0214	0,047
MISP	-0,0039	-0,6432	-0,071
Panel C: out-of-the-money	***CRR***	***EDK***	***RUB***
MAPE	0,0843	0,0515	0,0886
MISP	-0,0533	-0,9586	-0,3097

The Table shows the pricing performance in the high volatility period (2008-2009) for in-the-money options (Panel A), at-the-money options (Panel B) and out-of-the-money options (Panel C). The pricing errors measured by $MAPE = \frac{1}{m}\sum_{k=1}^{m}\frac{|P_k^T - P_k^M|}{P_k^M}$, $MISP = \frac{\sum_{k=1}^{m}\left(\frac{P_k^T - P_k^M}{P_k^M}\right)}{\sum_{k=1}^{m}\left|\frac{P_k^T - P_k^M}{P_k^M}\right|}$ *, for the Cox-Ross-Rubinstein (CRR), Enhanced Derman and Kani (EDK), and Rubinstein (RUB) models.*

Table 6. Higher Moments (whole sample).

	EDK	***RUB***	***MF***	***PHYSICAL***
Variance	0,1318	0,0639	0,1030	0,0853
Skewness	-0,1148	0,0313	-0,0170	-0,1711
Kurtosis	3,2678	3,0847	3,1103	3,0366

Note: Risk-Neutral Moments are all calculated over a 30-days period by interpolating the moments of the near and next maturity for the Enhanced Derman and Kani (EDK), and Rubinstein (RUB) models. For model-free Moments (MF) we used the Bakshi et al. (2003) formula. Physical moments are obtained with a rolling analysis over a 22-working days period (equivalent to 30 days).

Table 7. The Forecasting performance of Higher Moments (whole sample).

	EDK	***RUB***	***MF***
RMSE Var	0,1288	0,1122	0,0812
RMSE Skew	0,6234	0,7008	0,5741
RMSE Kurt	3,5631	1,5197	0,9669

Note: The Table shows the forecasting errors for the Enhanced Derman and Kani (EDK), Rubinstein (RUB) and Bakshi et al. (2003) (MF) models, measured by $RMSE = \sqrt{\frac{1}{m}\sum_{k=1}^{m}(M_k^P - M_k^{RP})^2}$*, where* M^{RP} *is the risk neutral moment and* M^P *is the physical moment for variance, skewness and kurtosis.*

Table 8. Risk premia (whole sample).

	EDK	***RUB***	***MF***
Variance	-0,0465	0,0214	-0,0176
Skewness	-0,0563	-0,2023	-0,1540
Kurtosis	-0,2311	-0,0480	-0,0736

Note: The Table shows the risk premia in Euro terms, for Enhanced Derman and Kani (EDK), Rubinstein (RUB) and Bakshi et al. (2003) (MF) models. Risk premia are computed as the difference between physical and risk neutral moments.

Table 9. Higher Moments in the two sub-periods

Panel A: **Low volatility**	***EDK***	***RUB***	***MF***	***PHYSICAL***
Variance	0,0482	0,0393	0,0347	0,0253
Skewness	-0,0937	0,0552	-0,0381	-0,2631
Kurtosis	3,3247	3,0938	3,1470	3,0646

Panel B: **High volatility**	***EDK***	***RUB***	***MF***	***PHYSICAL***
Variance	0,2601	0,1015	0,2078	0,1774
Skewness	-0,1470	-0,0054	0,0154	-0,0295
Kurtosis	3,1804	3,0709	3,0537	2,9904

Note: The Table shows the moments in the low volatility period (2005-2007) in Panel A and in the high volatility period (2008-2009) in Panel B. Risk-neutral moments are calculated over a 30-days period by interpolating the moments of the near and next maturity for the Enhanced Derman and Kani (EDK), Rubinstein (RUB) and Bakshi et al. (2003) (MF) models. Physical moments are obtained with a rolling analysis over a 22-working days period (equivalent to 30 days).

Table 10. The Forecasting performance of Higher Moments in the two sub-periods.

Panel A:
Low volatility

	EDK	***RUB***	***MF***
RMSE Var	0,0379	0,0239	0,0191
RMSE Skew	0,6198	0,7440	0,5931
RMSE Kurt	4,3897	1,3659	0,9830

Panel B:
High volatility

	EDK	***RUB***	***MF***
RMSE Var	0,1997	0,1763	0,1272
RMSE Skew	0,6289	0,6285	0,5438
RMSE Kurt	1,6108	1,7296	0,9526

Note: The Table shows the result in the low volatility period (2005-2007) in Panel A, in the high volatility period (2008-2009) in Panel B. The Table shows the forecasting errors for the Enhanced Derman and Kani (EDK), Rubinstein (RUB) Bakshi et al. (2003) (MF) models, measured by $RMSE = \sqrt{\frac{1}{m}\sum_{k=1}^{m}(M_k^P - M_k^{RP})^2}$*, where* M^{RP} *is the risk neutral moment and* M^P *is the physical moment for variance, skewness and kurtosis.*

Table 11. Risk Premia in the two sub-periods.

Panel A: Low volatility	***EDK***	***RUB***	***MF***
Variance	-0,0229	-0,0139	-0,0094
Skewness	-0,1694	-0,3184	-0,2250
Kurtosis	-0,2600	-0,0292	-0,0823

Panel B: High volatility	***EDK***	***RUB***	***MF***
Variance	-0,0825	0,0759	-0,0303
Skewness	0,1175	-0,0240	-0,0449
Kurtosis	-0,1867	-0,0771	-0,0658

Note: The Table shows the risk premia in the low volatility period (2005-2007) in Panel A, in the high volatility period (2008-2009), in Panel B. Risk premia are computed as the difference between physical and risk neutral moments for the Enhanced Derman and Kani (EDK), Rubinstein (RUB) and Bakshi et al. (2003)(MF) models.