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THE OPTIMAL CORRIDOR FOR IMPLIED VOLATILITY: FROM CALM TO TURMOIL PERIODS

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Abstract

Corridor implied volatility is obtained from model-free implied volatility by truncating the integration domain between two barriers. Empirical evidence on volatility forecasting, in various markets, points to the utility of trimming the risk-neutral distribution of the underlying stock price, in order to obtain unbiased measures of future realised volatility (see e.g. [9], [3]). The aim of the paper is to investigate, both in a statistical and in an economic setting, the optimal corridor of strike prices to use for volatility forecasting in the Italian market, by analysing a data set which covers the years 2005-2010 and span both a relatively tranquil and a turmoil period.

Keywords: corridor implied volatility, model-free implied volatility, volatility forecasting, financial turmoil.

JEL classification: G13, G14.

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1. Introduction

How to extract a volatility forecast from the cross section of option prices is still an open debate. Model-free implied volatility, introduced by Britten-Jones and Neuberger [3], not only uses the whole cross-section of traded option prices, but requires interpolation and extrapolation of implied volatilities in order to reproduce a continuum of option prices in strikes, ranging from zero to infinity (a condition that cannot be met in the reality of financial markets). Corridor implied volatility, introduced in Carr and Madan [4], is obtained from model-free implied volatility by truncating the integration domain between two barriers. Corridor implied volatility can be convenient in at least two cases. First it can be used to cope with the problem of estimating the tails of risk-neutral distribution, due to the lack of liquid options for very high and very low strikes (see e.g. the computation of market volatility indexes, such as the VIX index at CBOE). Second it can be used in order to obtain a measure of volatility which focus either on a particular part of the risk neutral distribution of the underlying asset or on a given option class (e.g. call versus put), thereby incorporating the views of an hypothetical investor that can be more or less bullish or bearish.

Empirical evidence on volatility forecasting, in various markets, points to the utility of trimming the risk-neutral distribution of the underlying stock price, in order to obtain an unbiased measure of future realised volatility. However, the optimal choice of the corridor is still an open debate [see e.g. [1], [2], [8]]. In particular, in the Italian market, looking at different data-sets characterised by different volatility regimes, [9] and [10] find different corridors for the best volatility estimate.

The aim of the paper is twofold. First, we thoroughly investigate the optimal corridor of strike prices to use for volatility forecasting in the Italian market, on a data-set which spans the

years 2005-2010 and which covers both a relatively tranquil and a turmoil period. Second, we investigate different corridors with asymmetric cuts, and compare the results with the findings in [9], [10]. The different corridors are evaluated both in a statistical and an economic setting, by employing trading strategies based on delta neutral straddles as in [9].

The paper supplements existing literature by providing an answer to the choice of the optimal symmetric corridor to use in the Italian market, which is one of the most important European markets. Moreover, the results are important in order to assess if using asymmetric cuts or a pre-specified options' category (call or put) may yield superior forecasts. The answer to the above questions is provided in different volatility regimes (before and after the Lehman's collapse).

The results of the paper could be of practical importance both at the micro and macro level. Option traders and portfolio managers would benefit from the analysis of the economic significance of the different volatility forecasts, which shows how the different measures perform in settling a trading strategy and deeply analyse the usefulness of focussing on a prespecified option's category, depending on the current market conditions. Policy makers may rely on volatility forecasts as barometers for the vulnerability of financial markets and use the latter as an early warning in order to improve financial stability.

The paper proceeds as follows. Section 2 recalls the concept of corridor implied volatility. Section 3 presents the details for the computation of corridor implied volatility and the choice of the optimal interval of strikes. Sections 4 presents the results for the forecasting performance of corridor implied volatility in the two sub-periods characterized by low and high volatility respectively. Section 5 addresses the economic significance of the different forecasts and show how it varies in different market conditions. The final section concludes.

2. Corridor implied volatility measures

The notion of "corridor variance" has been introduced in [6]. A corridor variance contract pays realised variance only if the underlying asset lies between two specified barriers B_1 and B_2 , Therefore corridor integrated variance can be defined as follows:

$$\sigma_{RC}^{2} = \frac{1}{T} \int_{0}^{T} \sigma^{2}(t,...) I_{t}(B_{1}, B_{2}) dt$$
(1)

where $I(B_1, B_2)$ is the indicator function that is equal to 1 only when the underlying is inside the two barriers and determines if variance is accumulated or not.

Carr and Madan [4] show that it is possible to compute the expected value of corridor variance under the risk-neutral probability measure, by using a portfolio of options with strikes ranging from B_1 to B_2 , as follows:

$$\hat{E}(\sigma_{RC}^{2}) = \hat{E}\left[\frac{1}{T}\int_{0}^{T}\sigma^{2}(t,...)I_{t}(B_{1},B_{2})dt\right] = \frac{2e^{rT}}{T}\int_{B_{1}}^{B_{2}}\frac{M(K,T)}{K^{2}}dK$$
(2)

Equation (2) is known as corridor implied variance and its square root as corridor implied volatility (CIV). By choosing different levels for the barriers, we obtain CIV measures with wider or narrower corridors. CIV measures are implicitly linked with the concept that the tails of the risk-neutral distribution are estimated with less precision than central values, due to the lack of liquid options for very high and very low strikes. Upside and downside corridor implied variance is obtained by using formula (6) with barriers ($B_1=B$, $B_2=\infty$) and ($B_1=0$, $B_2=B$), respectively.

If $B_1=0$ and $B_2=\infty$, then corridor variance coincides with model-free variance ([3]), therefore in the following we refer to model-free implied volatility as the no-barriers corridor implied volatility.

3. The choice of the optimal cut

In this application, corridor implied volatility is computed as a discrete version of the square root of equation (2):

$$CIV = \sqrt{\frac{2e^{rT}}{T} \int_{B_{1}}^{B_{2}} \frac{M(K,T)}{K^{2}} dK} \approx \sqrt{\frac{e^{rT}}{T} \sum_{i=1}^{m} \left[g(T,K_{i}) + g(T,K_{i-1})\right] \Delta K}$$
(3)

where:

$$\Delta K = \left(K_{\max} - K_{\min}\right) / m , \qquad (4)$$

m is the number of strikes used,

$$K_i = K_{\min} + i\Delta K, \quad 0 \le i \le m$$

$$g(T, K_{i}) = [\min(C(T, K_{i}), P(T, K_{i})] / K_{i}^{2},$$
(6)

$$K_{\min} = B_1 = H^{-1}(p_1) \text{ and } K_{\max} = B_2 = H^{-1}(1-p_2)$$
 (7)

and the trapezoidal rule for integration is used.

The barriers B_1 and B_2 are computed by looking at the risk-neutral distribution obtained by fitting an implied binomial tree (with the Enhanced Derman and Kani model, see e.g. [7]).

How to optimally choose the corridor of strike prices to use for volatility forecasting is still an open question. Corridor implied volatility can be seen as a function of p (the probability that we cut symmetrically in the two tails) or more generally of p_1 (the probability that we cut in the left tail) and p_2 (the probability that we cut in the right tail). The problem boils down to the solution of:

$$\min_{p_1, p_2} \left(\sum_{i=1}^{m} CIV^{i}(p_1, p_2) - \sigma^{i}_{R} \right)^{2}$$
(8)
s.t. $p_1, p_2 \in [0, 1]$

where $CIV(p_1, p_2)$ is corridor implied volatility, σ_R is the subsequent realised volatility, and i=1,...,n is the i-th date in the sample. Unfortunately, given the complexity in computing corridor implied volatility (which requires spline interpolation of implied volatilities, the construction of option implied tree to determine the barriers, the computation of equation (3) for near and next term options and finally the linear interpolation in time in order to get a 30-day constant maturity estimate, see [10] for a detailed discussion), solving the problem (8) will be computationally infeasible.

Therefore in this paper we analyse different corridors in order to assess, by inspection of the ranking functions, a clear pattern that points to the optimal cut. Based on the preliminary results in [10], we add more corridors, which are used in [9] on a limited data-set of six months characterised by high volatility. We compute a total of eight corridor measures: four with symmetric cuts and four with asymmetric cuts. The four symmetric corridor measures are CIV0, CIV0.2, CIV0.3, CIV0.4, which correspond to $p_1 = p_2 = p$, with p = 0, 0.2 0.3, 0.4 respectively, and $B_1 = H^{-1}(p)$, $B_2 = H^{-1}(1 - p)$. CIV0 corresponds to the no-barriers implied volatility i.e. to model-free implied volatility, which is used as a benchmark. From CIV0.2 to CIV0.4 we explore narrower corridor implied volatility measures.

Moreover, in order to assess whether the lower part of the risk neutral distribution is more informative about future realized volatility than the upper part, we also compute corridor measures with asymmetric cuts of the risk neutral distribution: CIV0.1-0.3 cuts 0.1 in the upper part and 0.3 in the lower part ($p_1 = 0.1$, $p_2 = 0.3$, $B_1 = H_0^{-1}(0.3)$ and $B_2 = H_0^{-1}(0.9)$) while CIV0.3-0.1 cuts 0.3 in the upper part and 0.1 in the lower part ($p_1 = 0.1$, $p_2 = 0.3$, $B_1 = H_0^{-1}(0.1)$ and $B_2 = H_0^{-1}(0.7)$), therefore CIV0.1-0.3 relies more on call prices than on put prices, while CIV0.3-0.1 relies more on put prices than on call prices. Finally, in order to separate the effect of out of the money call and put prices, which are sensitive to increases or decreases in the underlying asset, we compute upside and downside corridor measures CIVUP (CIVDW) with barriers $B_I = F$ and $B_2 = K_{max} = H^{-1}(1)$ ($B_1 = K_{min} = H^{-1}(0)$ and $B_2 = F$) respectively, where F is the forward price.

To ease comparison with [10], we use the same data set made of closing prices on FTSE MIB-index options (MIBO), and FTSE MIB index recorded from 1 January 2005 to 31 December 2010 which has been kindly provided by Borsa Italiana S.p.A. Euribor rates and dividend yields are obtained from Datastream. Realised volatility (σ_R) is obtained from Datastream, and is computed, in annual terms, as the standard deviation of the returns over the next 30 days. The data set has been cleaned according to the same filtering constraints used in [10].

4. The results for the volatility measures

Descriptive statistics for the volatility series are reported in Table 1. The volatility series in our sample period are plotted in Figure 1 (symmetric-cut corridor implied volatilities) and Figure 2 (asymmetric-cut corridor implied volatilities). The no-barriers implied volatility is overwhelmingly higher than realized volatility. Symmetric CIV measures are lower than realised volatility and diminish on average as the corridor width shrinks from CIV0.2 to CIV0.4. Asymmetric cut CIV measures are lower on average than realized volatility; CIVUP is lower than CIVDW, reflecting the lower implied volatility of out of the money call options with respect to out of the money put options. All the volatility measures display positive skewness and excess kurtosis and the hypothesis of a normal distribution is rejected.

In order to compare the results with [9] and [10] we gauge the forecasting performance of the different volatility measures, by resorting to the same metrics widely used in the literature (see e.g. [12]). In particular, we use the MSE, the RMSE, the MAE, the MAPE and the

QLIKE, defined as follows: $MSE = \frac{1}{m} \sum_{k=1}^{m} (\sigma_i - \sigma_r)^2$, $RMSE = \sqrt{\frac{1}{m} \sum_{k=1}^{m} ((\sigma_i) - (\sigma_r))^2}$, $MAE = \sqrt{\frac{1}{m} \sum_{k=1}^{m} ((\sigma_i) - (\sigma_r))^2}$

$$\frac{1}{m}\sum_{k=1}^{m} \left| \sigma_{i} - \sigma_{r} \right|, MAPE = \frac{1}{m}\sum_{k=1}^{m} \left| \frac{\sigma_{i} - \sigma_{r}}{\sigma_{r}} \right| \quad , QLIKE = \frac{1}{m}\sum_{k=1}^{m} \left(\ln(\sigma_{i}) + \frac{\sigma_{r}}{\sigma_{i}} \right), \text{ where } \sigma_{i} \text{ is the } \sigma_{i}$$

volatility forecast (*i*=*CIV0*, *CIV0.2*, *CIV0.3*, *CIV0.4*, *CIV0.1-0.3*, *CIV0.3-0.1*, *CIVUP*, *CIVDW*), σ_{R} is the subsequent realized volatility, k=1,...,m, *m* is the number of observations.

The evaluation measures for the volatility forecasts are reported in Table 2. In order to see whether the differences in forecasting performance are significant from a statistical point of view, we compare the predictive accuracy of the forecasts by computing the Diebold and Mariano test statistic (for more details see [6]) by using the MSE function which is considered as robust to the presence of noise in the volatility proxy (see [11]). The pair-wise comparisons are reported in Table 3 (t-statistics along with the p-values). Note that a negative (positive) t-statistic indicates that the row model produced smaller (larger) average loss than the column model. The Diebold and Mariano test statistic under the null of equal predictive accuracy is distributed as a N(0,1).

According to all the indicators CIVDW obtains the best performance, closely followed by CIV0.2. Also CIV0.3-0.1, which favours put prices, according to the MSE and the MAE is one of the best measures. As for symmetric corridor measures, wide corridor measures obtain a better performance than narrow corridor measures: the performance improves when the corridor widens. Among asymmetric measures, the one which relies more on put prices (CIV0.3-0.1) performs better than the one which relies more on call prices (CIV0.1-0.3), but looking at the Diebold and Mariano tests of equal predictive accuracy, the two measures are not distinguishable. CIV0.3-0.1 performs better than narrow symmetric corridor measures. CIVDW, focusing on put option prices, performs fairly better than CIVUP, focusing on call option prices, and the difference is significant according to the Diebold and Mariano test. As a

result we can say that put prices which carry information on the probability of a downturn move of the underlying asset convey the best information about future realized volatility. Overall we can conclude that the results point to the utility of trimming the risk neutral distribution asymmetrically, by favouring put option prices, that account for downside risk. In order to better understand the behaviour of the measures in times of high or low volatility, we split the sample into two sub-periods: the first covers the period before the Lehman's collapse and is characterised by low volatility and stable market condition, the second covers the period after the Lehman's collapse and is characterised by high volatility and market turmoil. The descriptive statistics of the volatility series are reported in Table 4. Average realised volatility is 15% in the pre-Lehman period and doubles to 30% in the post-Lehman period. The average value of corridor measures more than double in the post-Lehman period. The no-barriers implied volatility and the upside and downside corridor measures are the ones which increase less if compared to the other corridor measures. In the post-Lehman period the series are more affected by positive skewness and excess kurtosis, which point to increased market tension (not only the volatility is higher, but extreme movements in volatility are more present). As a result, in the high volatility period the predictive ability of all the measures decreases substantially.

The predictive accuracy of the various corridor measures in the two subsamples is reported in Table 5. In the low volatility period, according to the MSE and the QLIKE the no-barriers implied volatility is one of the best measures. According to the MAE and the MAPE, CIV0.2 is the best one. Among the top performers we find also CIVDW and CIV0.1-0.3. In the high-volatility period, the no-barriers implied volatility performs poorly. CIVDW is the preferred one for all the indicators (except the MAPE, for which is at the second place after CIVUP). The other measures obtain a different place in the top list, depending on the performance measure. As for the MSE and the QLIKE, which are robust measures to the presence of noise,

CIV0.2 and CIV0.3-0.1 are the second best.

In Figure 3 is reported the pattern of the MSE in the whole sample and across the two subperiods. To ease the comparison with [10] and in order to assess the optimal symmetric cut, CIV0.1, CIV0.05 and CIV0.025 (which cut p=0.1, p=0.05, and p=0.025 in each tail, respectively) have been added to the picture. We can see that the optimal value of the symmetric cut remains around the 10% in each tail. However, both in the whole period and in the high volatility period, the MSE is minimal for CIVDW.

On the other hand in the low volatility period the optimal level is attained for the no-barriers implied volatility CIV0. Among asymmetric measures CIVDW is better than CIVUP in both sub-periods, CIV0.1-0.3 performs better (worse) than CIV0.3-0.1 in the low (high) volatility period. The Diebold and Mariano test has been computed w.r.t. the MSE and the results provided in Tables 6 and 7, confirm the ranking. To conclude, we can say that in both sub-periods CIVDW is the best. Overall symmetric corridor measures perform better (worse) than asymmetric corridor measures in the low (high) volatility period.

5. The economic significance of volatility forecasts

In this section we study the economic significance of the prediction ability of the different volatility forecasts by looking at the profitability of volatility trading strategies. In particular, we examine whether a hypothetical trader, who could go long or short (depending on the different volatility forecasts) on a delta neutral straddle on FTSE-MIB (see e.g. [5]), could gain positive and statistically significant trading profits. For ease of comparison, we follow the same methodology of [10], with the only difference that transaction costs on FTSEMIB options are taken into account (0,41 Euro per contract).

The summary statistics of average daily trading returns for each volatility forecasts are

reported in Table 8, for the whole sample and in Table 9, for the two sub-samples. Average daily returns are ascertained to be statistically different from zero by using t-statistics adjusted for serial dependence, according to Newey-West (lag 30 days).

The time evolution of the strategy is plotted in Figure 4. We can see that the no-barriers implied volatility end up in a negative total profit and the performance worsen in the high volatility period. On the other hand, all corridor measures obtain a positive profit which for some measures (CIV0.4 and CIVUP) starts stable and gains a lot in the high volatility period, for some other measures (CIV0.2, CIV0.1-0.3 and CIV0.3-0.1), gains in the low volatility period and loses in the high volatility period. In the whole period, the no-barriers implied volatility loses on average (statistically significant at the 5% level), while all corridor measures obtain a positive average return. CIV0.4 and CIV0.3 are among the best measures (statistically significant at the 10% level), followed by CIVUP and CIVDW.

The performance obtained in the high volatility period impact the most the overall performance, therefore the ranking remains almost the same in the high volatility period, where the profits/losses are very much higher than in the low volatility period. On the other hand, in the low volatility period (where neither measure is statistically significant), the nobarriers implied volatility obtains a positive return, which turns out negative in the high volatility period and therefore in the whole period. Other measures which gain in the low-volatility period and then lose in the high volatility period are CIV0.2, and the two asymmetric cut measures CIV0.1-0.3 and CIV0.3-0.1. In the low volatility period CIVDW is the best one, followed by CIV0.3, CIV0.1-0.3. In Figure 5 we report a visual comparison with the measures used in [10]: the strategies based on CIV0.025, CIV0.05, CIV0.1 (corridor implied volatility with symmetric cut of p=0.025, p=0.05 and p=0.1, in each tail respectively) and SHORT (a strategy which goes always short in volatility) have been recomputed with the methodology used in this paper for ease of comparison.

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We can see that neither strategy beats the naïve strategy of going short on volatility, which is the best one. Trading strategies based on the no-barriers implied volatility, along with wide corridor measures (from CIV0.025 to CIV0.2) are not profitable on average and should be preferred in low volatility periods. Narrow corridor measures (CIV0.4 and CIV0.3) are much more profitable on average. In the low volatility period, CIVDW and CIV0.1-0.3 are confirmed by profitable trading strategies to be among the best measures. On the other hand, in the high volatility period, corridor measures whit narrow corridor (CIV0.4 and CIV0.3) are among the best, along with CIVUP. Narrow corridor measures and CIVUP and CIVDW are more stable across high and low volatility periods, whereas slightly asymmetric corridor measures, and wide corridor measures change the sign and magnitude of the performance most across the two periods.

6. Conclusions

In this paper we have provided an answer to the problem of selecting an optimal cut of the risk neutral distribution, which corresponds to an optimal corridor of strike prices to use, in order to get an implied volatility forecast. We have thoroughly investigated the forecasting performance of measures which cut the risk neutral distribution of the underlying asset in order to focus on particular options' classes either symmetrically (by eliminating out-of-the-money options) or asymmetrically (focus more on call or put option prices). The forecasting performance of the volatility measures is evaluated both in a statistical and an economic setting.

The results highlight that, put prices, which carry information on the probability of a downturn move of the underlying asset, convey the best information about future realized volatility. Differently from [10], where only symmetric corridors were investigated, in the

whole sample, and in particular in the high volatility period, focussing on put prices (CIVDW) yields the best forecasting result. Put prices (CIVDW) convey better information than call prices (CIVUP) in both sub-periods, but in the high volatility period the superiority is more pronounced. Among symmetric corridors, we can safely say that the optimal cut is around 10% of the risk neutral distribution. The no-barriers implied volatility is on top of the list only if the period is characterised by low volatility, while in the high volatility it obtains a poor performance.

In assessing the economic significance of the different volatility forecasts, trading strategies based on CIVDW yield a positive but not significant trading profit. Neither measure obtains a significant profit in the low volatility period, and only the narrowest corridor measure CIV0.4 obtains a significant profit in the high volatility period.. Overall, narrow corridor measures obtain a positive return, whereas trading strategies based on the no-barriers implied volatility obtain on average a negative (and significant) return, along with wide corridor measures. Therefore, from an economic point of view, there is evidence for the preference of corridor measures with narrow corridor. However, neither strategy beats the naïve strategy of going short on volatility, which remains the best one.

The present paper lends itself to further development in many directions. High on the research agenda is the investigation of the determinants of the variance risk premium in the Italian market and the assessment of the unbiasedness and efficiency of the different volatility forecasts.

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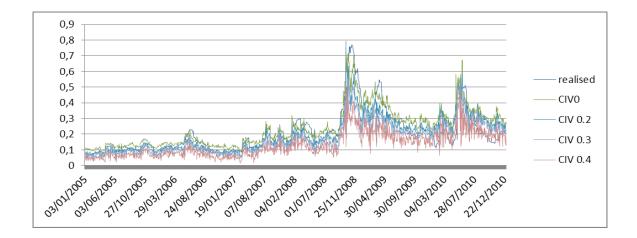


Figure 1. Realized volatility and corridor implied volatility with symmetric cuts.

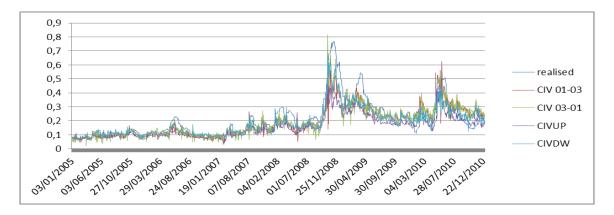


Figure 2. Realized volatility and corridor implied volatility with asymmetric cuts.

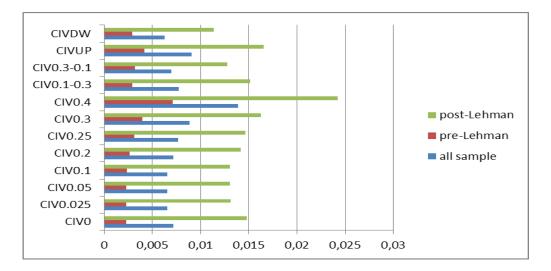


Figure 3. The Mean Squared Error of the forecasts in the whole sample and in the two subperiods (comparison with [10]).

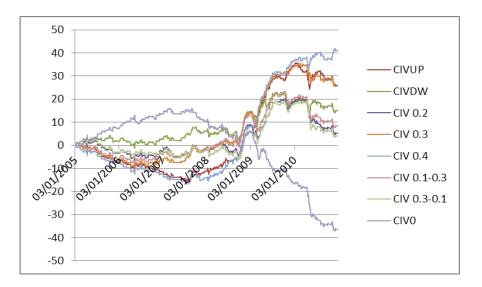


Figure 4. The time-evolution of the different strategies (total profit/loss).

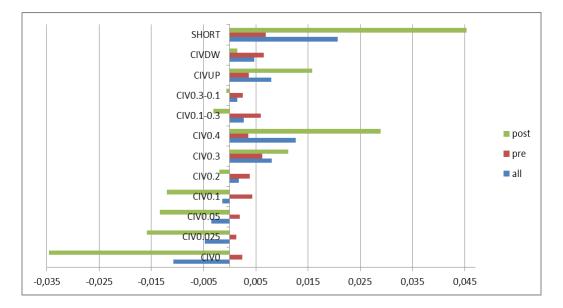


Figure 5. Average return of the different strategies (comparison with [10]).

Statistic	σ_{R}	σ _{CIV0.2}	σ _{CIV0.3}	σ _{CIV0.4}	σ _{CIV0}
Mean	0.21	0.20	0.17	0.13	0.24
Std dev	0.13	0.11	0.10	0.08	0.12
Skewness	1.91	1.09	1.05	1.15	1.20
Kurtosis	7.04	4.21	4.08	4.47	4.35
Jarque Bera	1804	365	326	435	441
p-value	0.00	0.00	0.00	0.00	0.00
Statistic	σ _{CIV0.1-0.3}	σ _{CIV0.3-0.1}	σ_{CIVUP}	σ _{civdw}	
Statistic Mean	σ _{CIV0.1-0.3} 0.19	σ _{CIV0.3-0.1} 0.20	σ _{CIVUP} 0.16	σ _{CIVDW} 0.19	
Mean	0.19	0.20	0.16	0.19	
Mean Std dev	0.19 0.10	0.20 0.11	0.16	0.19 0.10	
Mean Std dev Skewness	0.19 0.10 1.07	0.20 0.11 1.19	0.16 0.08 1.29	0.19 0.10 1.44	

Table 1. Descriptive statistics for the volatility series (all sample).

The Table presents the descriptive statistics for the volatility series used in the analysis: σ_R = realized volatility, σ_{CIV} = corridor implied volatility (p = 0.2, 0.3, 0.4 respectively for CIV0.2, CIV0.3, CIV0.4,), $\sigma_{CIV0.1-0.3}$ = corridor implied volatility with upper cut equal to 0.1 and lower cut equal to 0.3, $\sigma_{CIV0.3-0.1}$ = corridor implied volatility with upper cut equal to 0.3, and lower cut equal to 0.1, σ_{CIVUP} = upside corridor implied volatility, σ_{CIVDW} = downside corridor implied volatility, σ_{CIV0} = model-free implied volatility.

	$\sigma_{\text{CIV0.2}}$	σ _{CIV0.3}	$\sigma_{CIV0.4}$	$\sigma_{\rm CIV0}$
MSE	0.0072	0.0088	0.0139	0.0073
RMSE	0.0848	0.0940	0.1177	0.0849
MAE	0.0519	0.0589	0.0816	0.0620
MAPE	0.2371	0.2638	0.3732	0.3299
QLIKE	-0.6822	-0.6408	-0.4604	-0.6846
	σ _{CIV0.1-0.3}	σ _{CIV0.3-0.1}	σ _{civup}	σ_{CIVDW}
MSE	σ _{CIV0.1-0.3} 0.0078	σ _{CIV0.3-0.1} 0.0070	σ _{CIVUP} 0.0090	σ _{CIVDW} 0.0063
MSE RMSE				
	0.0078	0.0070	0.0090	0.0063
RMSE	0.0078 0.0881	0.0070 0.0835	0.0090 0.0951	0.0063 0.0791
RMSE MAE	0.0078 0.0881 0.0537	0.0070 0.0835 0.0535	0.0090 0.0951 0.0581	0.0063 0.0791 0.0484

Table 2. Predictive accuracy of the different volatility measures (all sample).

The Table presents the indicators of the goodness of fit of the volatility series used in the study for the whole

sample:
$$MSE = \frac{1}{m} \sum_{k=1}^{m} (\sigma_i - \sigma_r)^2$$
, $RMSE = \sqrt{\frac{1}{m} \sum_{k=1}^{m} ((\sigma_i) - (\sigma_r))^2}$, $MAE = \frac{1}{m} \sum_{k=1}^{m} |\sigma_i - \sigma_r|$, $MAPE = \frac{1}{m} \sum_{k=1}^{m} |\sigma_i - \sigma_r|$

$$\frac{1}{m}\sum_{k=1}^{m} \left| \frac{\sigma_{i} - \sigma_{r}}{\sigma_{r}} \right|, \text{ QLIKE} = \frac{1}{m}\sum_{k=1}^{m} \left(\ln(\sigma_{i}) + \frac{\sigma_{r}}{\sigma_{i}} \right), \text{ where } \sigma_{i} \text{ is the volatility forecast, } \sigma_{R} \text{ is the subsequent}$$

realized volatility, *m* is the number of observations, σ_{CIV} = corridor implied volatility (*p* = 0.2, 0.3, 0.4 respectively for CIV0.2, CIV0.3, CIV0.4,), $\sigma_{CIV0.1-0.3}$ = corridor implied volatility with upper cut equal to 0.1 and lower cut equal to 0.3, $\sigma_{CIV0.3-0.1}$ = corridor implied volatility with upper cut equal to 0.3, and lower cut equal to 0.1, σ_{CIVUP} = upside corridor implied volatility, σ_{CIVDW} = downside corridor implied volatility, σ_{CIV0} = model-free implied volatility.

	σ _{civ0.3}	σ _{civ0.4}	σ _{CIV0.1-0.3}	σ _{clV0.3-0.1}	σ _{сіνυρ}	σ _{сіνдw}	σ _{CIV0}
	01010	0.00.4				cirbit	
$\sigma_{CIV0.2}$	-3.45	-5.39	-1.79	0.51	-2.51	1.90	-0.01
	0.00	0.00	0.07	0.61	0.01	0.06	0.99
$\sigma_{CIV0.3}$		-6.26	3.04	2.45	-0.45	3.57	1.31
		0.00	0.00	0.01	0.66	0.00	0.19
$\sigma_{CIV0.4}$			5.55	4.90	6.91	5.86	3.46
			0.00	0.00	0.00	0.00	0.00
σ _{CIV0.1-0.3}				1.10	-2.02	2.11	0.53
				0.27	0.04	0.04	0.60
$\sigma_{\text{CIV0.3-0.1}}$					-2.28	2.37	-0.36
					0.02	0.02	0.72
σ_{civup}						3.50	1.33
						0.00	0.19
σ_{civdw}							-1.19
							0.23

Table 3. Diebold and Mariano tests: pair-wise comparisons (MSE, all sample).

The Table reports the t-statistic and associated p-value for the Diebold and Mariano test of equal predictive accuracy for each couple of forecasts, for the whole sample. The loss function used is the MSE, σ_{CIV} = corridor implied volatility (p = 0.2, 0.3, 0.4 respectively for CIV0.2, CIV0.3, CIV0.4,), $\sigma_{CIV0.1-0.3}$ = corridor implied volatility with upper cut equal to 0.1 and lower cut equal to 0.3, $\sigma_{CIV0.3-0.1}$ = corridor implied volatility with upper cut equal to 0.1, σ_{CIVUP} = upside corridor implied volatility, σ_{CIVDW} = downside corridor implied volatility, σ_{CIV0} = model-free implied volatility.

Table 4. Descriptive statistics for the volatility series in the two sub-periods.

	-								
Statistic	σ_{R}	$\sigma_{\text{CIV0.2}}$	$\sigma_{\text{CIV0.3}}$	$\sigma_{\text{CIV0.4}}$	$\sigma_{\text{CIV0.1-0.3}}$	$\sigma_{\text{CIV0.3-0.1}}$	σ_{CIVUP}	σ_{civdw}	σ_{CIV0}
mean	0.15	0.13	0.11	0.08	0.12	0.12	0.13	0.11	0.16
std dev	0.07	0.04	0.04	0.03	0.04	0.04	0.04	0.03	0.05
skewness	1.66	0.70	0.70	0.00	0.00	0.62	0.00	0.02	0.00
Sile (Tiless	1.66	0.78	0.79	0.82	0.89	0.62	0.99	0.92	0.80
kurtosis	7 08	2.91	3.14	3.47	3.13	2.97	4.02	3.60	2.85
	7.00	2.71	5.17	5.47	5.15	2.91	4.02	5.00	2.05
Jarque Bera	980	86	89	103	112	54	175	132	92
_									
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Pre-Lehman's collapse

Post-Lehman's collapse

Statistic	σ_{R}	$\sigma_{\text{CIV0.2}}$	$\sigma_{\text{CIV0.3}}$	$\sigma_{CIV0.4}$	σ _{CIV0.1-0.3}	σ _{CIV0.3-0.1}	σ_{CIVUP}	σ_{civdw}	σ _{civo}
mean	0.30	0.31	0.27	0.21	0.30	0.31	0.28	0.24	0.36
std dev	0.15	0.08	0.08	0.07	0.08	0.09	0.09	0.07	0.10
skewness	1.35	1.45	1.42	1.34	1.38	1.60	1.58	1.41	1.33
kurtosis	4.14	6.08	6.18	5.77	5.94	6.51	5.93	5.51	4.73
Jarque Bera	198	412	417	342	375	522	429	328	231
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

The Table presents the descriptive statistics for the volatility series in the two sub-periods (pre-Lehman and post-Lehman's collapse): σ_R = realized volatility, σ_{CIV} = corridor implied volatility (p = 0.2, 0.3, 0.4 respectively for CIV0.2, CIV0.3, CIV0.4,), $\sigma_{CIV0.1-0.3}$ = corridor implied volatility with upper cut equal to 0.1 and lower cut equal to 0.3, $\sigma_{CIV0.3-0.1}$ = corridor implied volatility with upper cut equal to 0.3, and lower cut equal to 0.1, σ_{CIVUP} = upside corridor implied volatility, σ_{CIVDW} = downside corridor implied volatility, σ_{CIV0} = model-free implied volatility.

Pre-Lehman's collapse				
	σ _{CIV0.2}	$\sigma_{CIV0.3}$	$\sigma_{CIV0.4}$	σ _{CIV0}
MSE	0.0026	0.0040	0.0071	0.0023
RMSE	0.0514	0.0632	0.0843	0.0475
MAE	0.0311	0.0422	0.0659	0.0347
MAPE	0.1891	0.2538	0.4203	0.2656
QLIKE	-0.9635	-0.9045	-0.6625	-0.9717
	σ _{CIV0.1-0.3}	σ _{CIV0.3-0.1}	σ _{CIVUP}	σ _{CIVDW}
MSE	0.0029	0.0032	0.0041	0.0029
RMSE	0.0542	0.0565	0.0644	0.0543
MAE	0.0342	0.0351	0.0432	0.0332
MAPE	0.2049	0.2176	0.2483	0.2002
QLIKE	-0.9528	-0.9307	-0.9186	-0.9608
Post-Lehman's collapse				
	σ _{CIV0.2}	$\sigma_{CIV0.3}$	$\sigma_{CIV0.4}$	σ_{CIV0}
MSE	0.0142	0.0163	0.0242	0.0148
RMSE	0.1191	0.1276	0.1557	0.1216
MAE	0.0840	0.0846	0.1056	0.1039
MAPE	0.3110	0.2791	0.3007	0.4288
QLIKE	-0.2494	-0.2354	-0.1496	-0.2431
	σ _{CIV0.1-0.3}	σ _{CIV0.3-0.1}	σ _{CIVUP}	σ_{CIVDW}
MSE	0.0152	0.0128	0.0166	0.0114
RMSE	0.1232	0.1131	0.1287	0.1066
MAE	0.0839	0.0817	0.0809	0.0717
MAPE	0.2990	0.3033	0.2263	0.2361
QLIKE	-0.2432	-0.2549	-0.2376	-0.2634

Table 5. Predictive accuracy of the different volatility measures in the two sub-periods.

The Table presents the indicators of the goodness of fit of the volatility series used in the study for the two sub-

periods (pre- and post-Lehman's collapse): $MSE = \frac{1}{m} \sum_{k=1}^{m} (\sigma_i - \sigma_r)^2$, $RMSE = \sqrt{\frac{1}{m} \sum_{k=1}^{m} ((\sigma_i) - (\sigma_r))^2}$, $MAE = \sqrt{\frac{1}{m} \sum_{k=1}^{m} ((\sigma_i) - (\sigma_r))^2}$

$$\frac{1}{m}\sum_{k=1}^{m} \left| \sigma_{i} - \sigma_{r} \right|, \quad MAPE = \frac{1}{m}\sum_{k=1}^{m} \left| \frac{\sigma_{i} - \sigma_{r}}{\sigma_{r}} \right|, \quad QLIKE = \frac{1}{m}\sum_{k=1}^{m} \left(\ln(\sigma_{i}) + \frac{\sigma_{r}}{\sigma_{i}} \right), \quad where \quad \sigma_{i} \quad \text{is the volatility}$$

forecast, σ_{R} is the subsequent realized volatility, *m* is the number of observations, σ_{CIV} = corridor implied volatility (p = 0.2, 0.3, 0.4 respectively for CIV0.2, CIV0.3, CIV0.4,), $\sigma_{CIV0.1-0.3}$ = corridor implied volatility with upper cut equal to 0.1 and lower cut equal to 0.3, $\sigma_{CIV0.3-0.1}$ = corridor implied volatility with upper cut equal to 0.1, σ_{CIVUP} = upside corridor implied volatility, σ_{CIVDW} = downside corridor implied volatility, σ_{CIVDW} = model-free implied volatility.

	$\sigma_{\text{CIV0.3}}$	$\sigma_{\text{CIV0.4}}$	$\sigma_{\text{CIV0.1-0.3}}$	$\sigma_{\text{CIV0.3-0.1}}$	σ_{civup}	σ_{civdw}	σ_{CIV0}	
$\sigma_{\text{CIV0.2}}$	-6.04	-8.58	-4.59	-3.84	-6.24	-3.09	1.34	
	0.00	0.00	0.00	0.00	0.00	0.00	0.18	
$\sigma_{\text{CIV0.3}}$		-9.98	4.85	5.73	-0.99	5.63	3.51	
		0.00	0.00	0.00	0.32	0.00	0.00	
$\sigma_{\text{CIV0.4}}$			8.28	9.11	8.95	8.94	6.14	
			0.00	0.00	0.00	0.00	0.00	
$\sigma_{\text{CIV0.1-0.3}}$				-1.49	-5.68	-0.09	2.15	
				0.14	0.00	0.93	0.03	
$\sigma_{\text{CIV0.3-0.1}}$					-4.18	1.66	2.42	
					0.00	0.10	0.02	
σ_{civup}						6.59	3.67	
						0.00	0.00	
σ_{civdw}							1.94	
							0.05	

Table 6. Diebold and Mariano tests: pair-wise comparisons (MSE) sub-period: pre-Lehman's collapse.

The Table reports the t-statistic and associated p-value for the Diebold and Mariano test of equal predictive accuracy for each couple of forecasts, for the pre-Lehman period. The loss function used is the MSE, σ_{CIV} = corridor implied volatility (p = 0.2, 0.3, 0.4 respectively for CIV0.2, CIV0.3, CIV0.4,), $\sigma_{CIV0.1-0.3}$ = corridor implied volatility with upper cut equal to 0.1 and lower cut equal to 0.3, $\sigma_{CIV0.3-0.1}$ = corridor implied volatility with upper cut equal to 0.1, σ_{CIVUP} = upside corridor implied volatility, σ_{CIVDW} = downside corridor implied volatility, σ_{CIV0} = model-free implied volatility.

	$\sigma_{\text{CIV0.3}}$	$\sigma_{\text{CIV0.4}}$	$\sigma_{\text{CIV0.1-0.3}}$	$\sigma_{\text{CIV0.3-0.1}}$	σ_{civup}	σ_{civdw}	σ_{CIV0}
$\sigma_{\text{CIV0.2}}$	-2.07	-3.85	-1.36	1.44	-1.46	2.55	-0.35
	0.04	0.00	0.17	0.15	0.14	0.01	0.73
$\sigma_{\text{CIV0.3}}$		-4.68	1.47	2.05	-0.26	3.08	0.55
		0.00	0.14	0.04	0.79	0.00	0.58
$\sigma_{\text{CIV0.4}}$			3.90	3.82	5.13	4.74	2.31
			0.00	0.00	0.00	0.00	0.02
σ _{CIV0.1-0.3}				1.47	-0.96	2.36	0.16
				0.14	0.34	0.02	0.87
$\sigma_{\text{CIV0.3-0.1}}$					-1.92	2.19	-1.58
					0.06	0.03	0.11
σ_{civup}						3.02	0.60
						0.00	0.55
σ_{civdw}							-2.10
							0.04

Table 7. Diebold and Mariano tests: pair-wise comparisons (MSE) sub-period: post-Lehman's collapse.

The Table reports the t-statistic and associated p-value for the Diebold and Mariano test of equal predictive accuracy for each couple of forecasts, for the post-Lehman period. The loss function used is the MSE, σ_{CIV} = corridor implied volatility (p = 0.2, 0.3, 0.4 respectively for CIV0.2, CIV0.3, CIV0.4,), $\sigma_{CIV0.1-0.3}$ = corridor implied volatility with upper cut equal to 0.1 and lower cut equal to 0.3, $\sigma_{CIV0.3-0.1}$ = corridor implied volatility with upper cut equal to 0.1, σ_{CIVUP} = upside corridor implied volatility, σ_{CIVDW} = downside corridor implied volatility, σ_{CIV0} = model-free implied volatility.

	mean	t-stat	p-value
$\sigma_{CIV0.2}$	0.002	0.342	0.732
σ _{CIV0.3}	0.008	1.832	0.067
σ _{CIV0.4}	0.012	2.798	0.005
σ _{CIV0.1-0.3}	0.003	0.572	0.568
σ _{CIV0.3-0.1}	0.001	0.261	0.794
σ	0.008	1.717	0.086
σ _{CIVDW}	0.005	0.968	0.333
σ _{CIV0}	-0.011	-2.382	0.017

Table 8. Economic significance of the different volatility forecasts (all sample).

The Table presents the average daily return for the volatility forecast, for the whole sample, σ_{CIV} = corridor implied volatility (p = 0.2, 0.3, 0.4 respectively for CIV0.2, CIV0.3, CIV0.4,), $\sigma_{CIV0.1-0.3}$ = corridor implied volatility with upper cut equal to 0.1 and lower cut equal to 0.3, $\sigma_{CIV0.3-0.1}$ = corridor implied volatility with upper cut equal to 0.1, σ_{CIVUP} = upside corridor implied volatility, σ_{CIVDW} = downside corridor implied volatility, σ_{CIV0} = model-free implied volatility.

	Pre-Lehman			Post-L		
	mean	t-stat	p-value	mean	t-stat	p-value
$\sigma_{CIV0.2}$	0.004	0.720	0.472	-0.002	-0.246	0.806
σ _{CIV0.3}	0.006	1.293	0.196	0.011	1.340	0.181
σ _{CIV0.4}	0.003	0.693	0.488	0.029	3.535	0.000
σ _{CIV0.1-0.3}	0.006	1.211	0.226	-0.003	-0.386	0.700
σ _{CIV0.3-0.1}	0.002	0.474	0.636	-0.001	-0.088	0.930
σ _{CIVUP}	0.003	0.730	0.465	0.016	1.723	0.085
σ _{CIVDW}	0.006	1.303	0.193	0.001	0.141	0.888
σ _{CIV0}	0.002	0.464	0.643	-0.035	-3.980	0.000

Table 9. Economic significance of the different volatility forecasts in the two sub-periods.

The Table presents the average daily return for the volatility forecast in the two sub-periods, σ_{CIV} = corridor implied volatility (p = 0.2, 0.3, 0.4 respectively for CIV0.2, CIV0.3, CIV0.4,), $\sigma_{CIV0.1-0.3}$ = corridor implied volatility with upper cut equal to 0.1 and lower cut equal to 0.3, $\sigma_{CIV0.3-0.1}$ = corridor implied volatility with upper cut equal to 0.3 ,and lower cut equal to 0.1, σ_{CIVUP} = upside corridor implied volatility, σ_{CIVDW} = downside corridor implied volatility, σ_{CIV0} = model-free implied volatility.