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## Materiali di discussione

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### The skew pattern of implied volatility in the DAX index options market

by

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# The skew pattern of implied volatility in the DAX index options market

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## Abstract

The aim of this paper is twofold: to investigate how the information content of implied volatility varies according to moneyness and option type and to compare the latter option based forecasts with historical volatility in order to see if they subsume all the information contained in the latter. We run a horse race of different implied volatility estimates: at the money and out of the money call and put implied volatilities and average implied that is a weighted average of at the money call and put implied volatilities with weights proportional to trading volume. Two hypotheses are tested: unbiasedness and efficiency of the different volatility forecasts. The investigation is pursued in the Dax index options market, by using synchronous prices matched in a one minute interval. The results highlight that the information content of implied volatility has a humped shape, with out of the money options being less informative than at the money ones. Overall, the best forecast is at the money put implied volatility: it is unbiased (after a constant adjustment) and efficient, in that it subsumes all the information contained in historical volatility.

*Keywords: Implied Volatility, Volatility Smile, Volatility forecasting, Option type.*

*JEL classification: G13, G14.*

## 1. Introduction.

Black-Scholes implied volatility is a forward looking measure of the expected volatility between now and the expiration of the option. Even if theoretically the Black-Scholes model postulates a constant volatility, empirically, implied volatility varies according to the option's strike price, describing a smile or skew, depending on the shape of the relation. As it is often necessary to have implied volatilities that correspond to strike prices that are not traded in the market, implied volatilities are usually interpolated (e.g. by cubic splines) in order to obtain a smile or skew function. The latter is fundamental both for the construction of option implied

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trees (see e.g. Derman and Kani (1994)) that are used to price and hedge exotic options and for the computation of many market volatility indexes (see e.g. CBOE VIX, for the Chicago Board Options Exchange, or the VDAX-New for the German Equity market).

The recent turmoil in the financial markets caused by the sub-prime crisis has clearly highlighted the important role of market volatility indexes in the detection and anticipation of market stress. These indexes are highly correlated with future market volatility and with risk factors embedded in credit spreads of sovereign debt, as such they are deemed to capture the so called market “fear”.

Numerous papers have investigated the forecasting power of Black-Scholes volatility versus a time series volatility forecast (we refer the interested reader to Poon (2005), that examines 93 studies on the issue of volatility forecasting and conclude that predictions based on implied volatility are on average superior to time series volatility models). However, as far as we know, little is the evidence about the different information content of implied volatilities extracted from options with different strike price and type (call or put), that are used in the computation of the smile function. As for the strike price dimension, Ederington and Guan (2005), in the S&P500 options market, highlights that the information content of implied volatilities varies roughly in a mirror image of the implied volatility smile. As for the option type dimension, Fleming (1998) and Christensen and Hansen (2002), in the S&P100 options market, find that at the money call implied volatility has slightly more predictive power than put implied volatility. The two latter studies use American type options on a dividend paying index: the early exercise feature and the dividend yield estimation influence in a different manner call and put option prices, and may have altered the comparison if not properly addressed.

Even if theoretically call and put implied volatilities extracted from an option with the very same strike price and time to maturity should be the same due to no arbitrage considerations, empirically there are many reasons that may cause call and put implied volatilities to differ. These reasons are amplified if call and put options are compared in a different strike price dimension. First of all converting option prices into implied volatilities leads to measurement errors (stemming from finite quote precision, bid-ask spreads, non-synchronous observations and other measurement errors): small errors in any of the input may produce large errors in the implied volatility (see e.g. Hentshle (2003)). This is also documented by Fleming (1999) that highlights that deviations of call and put option prices from no arbitrage values do not necessarily signal market inefficiency but are rather due to transaction costs and other market imperfections. Along the same line of reasoning, the no arbitrage replication of a put or a call through put-call parity implies to go short (long) on the underlying asset. Differently

from the long side, the short side usually requires an initial margin, and is exposed to margin calls if the underlying asset price begins to rise. Second, the demand for put options is inherently different from the one of the calls. Put options are used for portfolio insurance purposes, in particular by institutional investors. Rubinstein (1994) finds that out of the money put implied volatilities are usually higher than both in the money put and out of the money call implied ones due to the crash phobia developed after October 1987. This hedging pressure has been documented both along different moneyness classes and also in the same moneyness category and may lead the implied volatilities of options whose price is impacted by hedging pressure to be less informative about future market volatility. Last, call and put option volumes are very different: usually put options are traded for a wider strike price interval than call options and they are also more traded than call options if compared in the same moneyness class (see e.g. Buraschi and Jackwerth (2001). Bollen and Whaley (2004) document that the demand for at the money put options is much higher than the one for the very same at the money call options. As implied volatility is a forward looking estimate of future realised volatility, we expect actively traded options to be more informative of future realised volatility than less traded options. This has been documented in various papers that have analysed index options markets. As pointed out in Donaldson and Kamstra (2005), trading volume can be considered as an indicator of the amount of investors' information: they find that when trading volume is high, also the forecasting power of implied volatility is high. Sarwar (2005) finds a positive relation between trading volume and implied volatility, determined by the activity of informed traders that usually prefer options market rather than stock markets, in order to benefit of lower transaction costs and higher leverage.

The aim of this paper is twofold: to investigate how the information content of implied volatility varies according to moneyness and option type and to compare the latter option based forecasts with historical volatility in order to see if they subsume all the information contained in the latter. The different information content of implied volatility is examined for the most liquid at the money and out of the money options: put (call) options for strikes below (above) the current underlying asset, i.e. the ones that are usually used as inputs for the computation of the smile function. This is very important for the understanding of the role of the different ingredients of the smile function and can be seen as a preliminary exercise in order to choose different weights for each volatility input in a volatility index. In particular, for at the money volatilities, that are widely used by market participants and are usually inserted in the smile function by using some average of both call and put implied ones, we investigate if one option class better forecasts future realised volatility and if a combination of the two adds substantial

benefit. Two hypotheses are tested: unbiasedness and efficiency of the different volatility forecasts w.r.t. historical volatility. Historical volatility is measured by both lagged realised volatility and a GARCH(1,1) forecast. The investigation is pursued in the Dax index options market. The market is chosen for two main reasons: the options are European, therefore the estimation of the early exercise premium is not needed and can not influence the results; the Dax index is a capital weighted performance index composed of 30 major German stocks and is adjusted for dividends, stocks splits and changes in capital: dividends are assumed to be reinvested into the shares and they do not affect the index value. Differently from previous studies, that use settlement prices, we are using the more informative synchronous prices, matched in a one minute interval. This is very important to stress, since our implied volatilities are real “prices”, as determined by synchronous no-arbitrage relations.

The plan of the paper is the following. Section 2 illustrates the data set, the sampling procedure and the definition of the variables. Section 3 describes the methodology used in order to address the unbiasedness and efficiency of the different volatility forecasts. Sections 4 and 5 report the results of the univariate and augmented regressions respectively and assess the relative performance of the different volatility forecasts (at the money and out of the money call and put implied volatilities, lagged realised volatility and GARCH(1,1)). Section 6 investigates the forecasting performance of a combination of at the money call and put implied volatilities. The last section concludes.

## **2. The Data set and the definition of the variables.**

The data set<sup>1</sup> consists of intra-daily data on DAX-index options, recorded from 19 July 1999 to 31 December 2005. Each record reports the strike price, expiration month, transaction price, contract size, hour, minute, second and centisecond. As for the underlying asset we use intra-daily prices of the DAX-index recorded in the same time period. As a proxy for the risk-free rate we use the one month Euribor rate.

DAX-options started trading on the German Options and Futures Exchange (EUREX) in August 1991. They are European options on the DAX-index, which is a capital weighted performance index composed of 30 major German stocks and is adjusted for dividends, stocks splits and changes in capital. Since dividends are assumed to be reinvested into the shares, they do not affect the index value, therefore we do not have to estimate the dividend payments.

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<sup>1</sup> The data source for Dax-index options and Dax index is the Institute of Finance, Banking, and Insurance of the University of Karlsruhe (TH), the risk-free rate is available in Data-Stream.

Moreover the fact that the options are European avoids the estimation of the early exercise premium. This latter feature is very important since our data set is by construction less prone to estimation errors if compared to the majority of previous studies that use American style options.

Several filters are applied to the option data set. First, we eliminate option prices that are smaller than 1 Euro, since the closeness to the tick size may affect the true option value. Second, in order not to use stale quotes, we eliminate options with trading volume less than one contract. Third, as it is standard practice in the literature to estimate the smile by using only the more liquid at the money and out of the money options, following Jiang and Tian (2005) we eliminate in the money options (call options with moneyness<sup>2</sup>  $(X/S) < 0,97$  and put options with moneyness  $(X/S) > 1,03$ ). Fourth, we eliminate option prices violating the standard no arbitrage bounds. Finally, in order to reduce computational burden, we only retain options that are traded between 3.00 and 4.00 p.m, (the choice is motivated by the active trading activity during this hour).

As for the sampling procedure, in order to avoid the telescoping problem described in Christensen, Hansen and Prabhala (2001), we use monthly non-overlapping samples. In particular, we collect the prices recorded on the Wednesday that immediately follows the expiry of the option (third Saturday of the expiry month) since the week immediately following the expiration date is one of the most active. These options have a fixed maturity of almost one month (from 17 to 22 days to expiration). If the Wednesday is not a trading day we move to the trading day immediately following.

Implied volatility is computed separately for out of the money and at the money call and put prices. We start from the cleaned data set of option prices that is composed of at the money and out of the money call and put prices recorded from 3.00 to 4.00 p.m. We compute call and put implied volatilities by using synchronous prices, matched in a one minute interval, by inverting the Black-Scholes formula. Implied volatilities are grouped into four sets depending on the option's moneyness and type and averaged in order to obtain four implied volatility estimates: at the money call (ATMC) implied volatility ( $\sigma_{ATMC}$ ), at the money put (ATMP) implied volatility ( $\sigma_{ATMP}$ ), out of the money call (OTMC) implied volatility ( $\sigma_{OTMC}$ ), out of the money (OTMP) implied volatility ( $\sigma_{OTMP}$ ) (OTMC if  $(X/S) > 1,03$ , ATMC e ATMP if  $0,97 \leq (X/S) \leq 1,03$ , OTMP if  $(X/S) < 0,97$ ).

Differently from Ederington and Guan (2005), that use settlement prices, we are using the more informative synchronous prices, matched in a one minute interval. This is very important to stress, since our implied volatilities are real “prices”, as determined by no-arbitrage relations. As

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<sup>2</sup> Moneyness is defined as  $X/S$ , where  $X$  is the strike price and  $S$  is the underlying asset.

a consequence it is very unlikely to have for each day observations for all the twelve categories of moneyness that Ederington and Guan (2005) use in their paper, since most of the trading concentrates on at the money and close to the money options. Therefore, in order to avoid the case in which one option class is empty (that is faced in Ederington and Guan (2005)) and to have a simple and clear-cut comparison between at the money and out of the money options, we choose to examine much broader classes w.r.t. Ederington and Guan (2005).

Implied volatility is an ex-ante forecast of future realised volatility on the time period until the option expiration. Therefore we compute realised volatility ( $\sigma_R$ ) in month  $t$ , as the sample standard deviation of the daily index returns over the option's remaining life:

$$\sigma_R = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (R_i - \bar{R})^2}$$

where  $R_i$  is the return of the DAX-index on day  $i$  and  $\bar{R}$  is the mean return of the Dax-INDEX in month  $t$ . We annualize the standard deviation by multiplying it by  $\sqrt{252}$ .

In order to examine the predictive power of implied volatility versus historical volatility, following Christensen and Prabhala (1998) and Jorion (1995) we choose to use two different time series volatility forecasts: lagged realized (LR), i.e. one month before, volatility ( $\sigma_{LR}$ ) and a GARCH (1,1) (GAR) forecast ( $\sigma_{GAR}$ ). Using daily data on the Dax index, the GARCH(1,1) variance equation is defined as:  $\sigma_{t+1}^2 = a_0 + a_1 R_t^2 + b_1 \sigma_t^2$ , where  $R_t$  is the de-measured DAX-index return on day  $t$  (for more details see Bollerslev (1986)). As in Jorion (1995), the GARCH model has been estimated via maximum likelihood over the entire data set. Following Fleming (1998) the GARCH forecast ( $\sigma_{GAR}$ ) of the average volatility over the life of the option is defined as:

$$\sigma_{GAR} = \sqrt{\frac{1}{T-t-1} \sum_{j=1}^{T-t} \tilde{\sigma}_{t+j|t}^2}$$

where  $\tilde{\sigma}_{t+j|t}^2$  is the forecast at time  $t$  of the variance  $j$  days into the future, and  $T$  is the maturity of the option. We annualize the standard deviation by multiplying it by  $\sqrt{252}$ . The GARCH forecast, being estimated over the entire sample period, benefits from information that is not available to other forecasts.

Descriptive statistics for volatility and log volatility series are reported in Table 1. We can see that on average realized volatility is lower than the implied volatility estimates (except for out of the money call implied), with on average put implied volatility being higher than call implied volatility.

**Table 1. Descriptive statistics for volatility and log-volatility series.**

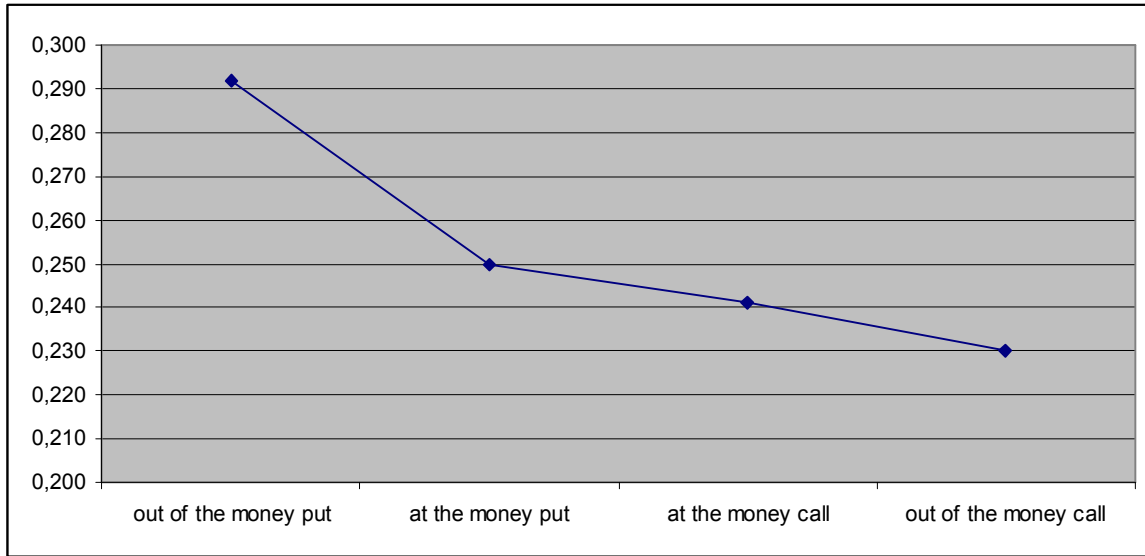
<b>Statistic</b>	$\sigma_{ATMC}$	$\sigma_{OTMC}$	$\sigma_{ATMP}$	$\sigma_{OTMP}$	$\sigma_R$	$\sigma_{LR}$	$\sigma_{GAR}$
<b>Mean</b>	0,241	0,230	0,250	0,292	0,238	0,239	0,240
<b>std dev</b>	0,111	0,100	0,109	0,134	0,127	0,125	0,110
<b>Skewness</b>	1,748	1,658	1,560	1,873	1,245	1,255	1,520
<b>Kurtosis</b>	6,137	5,590	5,560	6,440	3,976	4,003	4,740
<b>Jarque Bera</b>	70,770	56,800	52,300	83,050	23,250	23,450	39,190
<b>p-value</b>	0,000	0,000	0,000	0,000	0,000	0,000	0,000
	$\ln(\sigma_{ATMC})$	$\ln(\sigma_{OTMC})$	$\ln(\sigma_{ATMP})$	$\ln(\sigma_{OTMP})$	$\ln(\sigma_R)$	$\ln(\sigma_{LR})$	$\ln(\sigma_{GAR})$
<b>Mean</b>	-1,506	-1,565	-1,465	-1,311	-1,558	-1,550	-1,507
<b>std dev</b>	0,395	0,383	0,386	0,381	0,486	0,482	0,395
<b>Skewness</b>	0,644	0,692	0,543	0,873	0,376	0,357	0,693
<b>Kurtosis</b>	3,277	3,197	2,972	3,512	2,340	2,374	2,888
<b>Jarque Bera</b>	5,576	6,263	3,790	10,622	3,220	2,899	6,218
<b>p-value</b>	0,062	0,044	0,150	0,005	0,199	0,235	0,045

This skew pattern, depicted in Figure 1<sup>3</sup>, is typical for index options and is consistent with the crash-phobia explanation, since the demand for out of the money put options to hedge against downside risk pushes implied volatility to rise at low strikes. As for the standard deviation, realised volatility is slightly more volatile than the implied volatility estimates (except for out of the money put implied). The volatility series are highly skewed (long right tail) and leptokurtic. In line with the literature (see e.g. Jiang and Tian (2005)) we decided to use the natural logarithm of the volatility series instead of the volatility itself in the empirical analysis for the following reasons. First log-volatility series conform more closely to normality than pure volatility series, this is documented in various papers and it is the case in our sample (see Table 1). Second, natural logarithms are less likely to be affected by outliers in the regression analysis.

<sup>3</sup> In the graph ATMP implied volatility is to the left of ATMC implied volatility because at the money call (put) implied volatility is mainly obtained from options with  $1 < X/S \leq 1,03$  ( $0,97 \leq X/S < 1$ ), since these are the most traded strike price intervals).



**Figure 1. The skew pattern of implied volatility.**



### 3. The methodology.

The information content of implied volatility is examined both in univariate and in augmented regressions. In univariate regressions, realized volatility is regressed against one of the six volatility forecasts in order to examine the different predictive power of each forecast. The univariate regressions are the following:

$$\ln(\sigma_R) = \alpha + \beta \ln(\sigma_i) \quad (1)$$

where  $\sigma_R$  = realized volatility and  $\sigma_i$  = volatility forecast,  $i=ATMC, OTMC, ATMP, OTMP, LR, GAR$ .

In augmented regressions, realized volatility is regressed against two or more volatility forecasts in order to distinguish which one has the highest explanatory power. We choose to compare first pairwise one volatility forecast with a time series volatility forecast in order to see if implied volatility subsumes all the information contained in historical volatility. The augmented regressions used are the following:

$$\ln(\sigma_R) = \alpha + \beta \ln(\sigma_i) + \gamma \ln(\sigma_j) \quad (2)$$

where  $\sigma_R$  = realized volatility,  $\sigma_i$  = implied volatility,  $i= ATMC, OTMC, ATMP, OTMP$  and  $\sigma_j = LR, GAR$ .

Moreover, we compare pairwise the four implied volatility forecasts in order to understand if the information carried by one option class is more valuable than the information carried by the other:

$$\ln(\sigma_R) = \alpha + \beta \ln(\sigma_i) + \gamma \ln(\sigma_j) \quad (3)$$

where  $\sigma_R$  = realized volatility,  $\sigma_i = ATMC, OTMC, ATMP, OTMP$  and  $\sigma_j = ATMC, OTMC, ATMP, OTMP, i \neq j$ .

We also compare the two times series volatility forecasts, in order to see which one has the highest forecasting power on future realised volatility:

$$\ln(\sigma_R) = \alpha + \beta \ln(\sigma_{LR}) + \gamma \ln(\sigma_{GAR}) \quad (4)$$

Following Christensen and Prabhala (1998) three are the hypotheses tested in univariate regressions (1). The first hypothesis concerns the amount of information about future realized volatility contained in the volatility forecast. If the volatility forecast contains some information, then the slope coefficient should be different from zero. Therefore we test if  $\beta = 0$  and we see whether it can be rejected. The second hypothesis is about the unbiasedness of the volatility forecast. If the volatility forecast is an unbiased estimator of future realised volatility, then the intercept should be zero and the slope coefficient should be one ( $H_0: \alpha = 0$  and  $\beta = 1$ ). In case this latter hypothesis is rejected, we see if at least the slope coefficient is equal to one ( $H_0: \beta = 1$ ) and, if confirmed, following Jiang and Tian (2005) we interpret the volatility forecast as unbiased after a constant adjustment. Finally if implied volatility is efficient then the error term should be white noise and uncorrelated with the information set.

In augmented regressions (2) two are the hypotheses to be tested. The first is about the efficiency of the volatility forecast: we test whether the implied volatility (ATMC, OTMC, ATMP, OTMP) forecast subsumes all the information contained in historical volatility. In affirmative case the slope coefficient of historical volatility should be equal to zero, ( $H_0: \gamma = 0$ ). Moreover, as a joint test of information content and efficiency we test in equations (2) if the slope coefficients of historical volatility and implied volatility (ATMC, OTMC, ATMP, OTMP) are equal to zero and one respectively ( $H_0: \gamma = 0$  and  $\beta = 1$ ). Following Jiang and Tian (2005), we ignore the intercept in the latter null hypothesis, and if our null hypothesis is verified, we interpret the volatility forecast as unbiased after a constant adjustment.

Moreover we investigate the different information content of each option class w.r.t. the others. To this end we test, in augmented regressions (3), if  $\gamma = 0$  and  $\beta = 1$ , or  $\gamma = 1$  and  $\beta = 0$ , in order to see if the implied volatility of one option class subsumes all the information contained in the other. Finally we test, in augmented regression (4), if  $\gamma = 0$  and  $\beta = 1$ , or  $\gamma = 1$  and  $\beta = 0$ , in order to see if one time series volatility forecast subsumes all the information contained in the other.

Differently from other papers (see e.g. Christensen and Prabhala 1998, Christensen and Hansen (2002)) that use American options on dividend paying indexes, our data set of European

style options on a non-dividend paying index avoids measurement errors that may arise in the estimation of the dividend yield and the early exercise premium. Moreover, we have carefully cleaned the data set and we are using synchronous prices. Nonetheless, as we are averaging different implied volatilities in a single class, some measurement errors may still affect our estimates. Therefore we adopt an instrumental variable procedure (IV), we regress implied volatility in each class on an instrument (in univariate regressions) and on an instrument and any other exogenous variable (in augmented regressions) and replace fitted values in the original univariate and augmented regressions. As the instrument for implied volatility in each class we use both LR volatility, GAR, and past implied volatility in the same class as they are possibly correlated to the true implied volatility, but unrelated to the measurement error associated with implied volatility one month later. As an indicator of the presence of errors in variables we use the Hausman (1978) specification test statistic<sup>4</sup>.

#### 4. The results of univariate regressions.

The results of the OLS univariate regressions (equation (1)) are reported in Table 2 (p-values in parentheses). In all the regressions the residuals are normal, homoscedastic and not autocorrelated (the Durbin Watson statistic is not significantly different from two and the Breusch-Godfrey LM test confirms non autocorrelation up to lag 12<sup>5</sup>).

First of all, in all the univariate regressions all the beta coefficients are significantly different from zero: this means that all the six volatility forecasts contain some information about future realised volatility. Among the two time series volatility forecasts, GAR performs much better than LR volatility: this is not surprising, since GAR has been estimated on the entire data set and therefore uses information that is not available for other forecasts. Overall put implied volatility obtains a better performance than call implied one.

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<sup>4</sup> The Hausman specification test is defined as:  $m = \frac{(\hat{\beta}_{IV} - \hat{\beta}_{OLS})^2}{Var(\hat{\beta}_{IV}) - Var(\hat{\beta}_{OLS})}$  where:  $\hat{\beta}_{IV}$  is the beta obtained

through the TSLS procedure,  $\hat{\beta}_{OLS}$  is the beta obtained through the OLS procedure and  $Var(x)$  is the variance of the coefficient  $x$ . The Hausman specification test is distributed as a  $\chi^2(1)$ .

<sup>5</sup> In the regression that include as explanatory variable lagged realised volatility, the Durbin's alternative has been computed. The results have confirmed the non autocorrelation of the residuals. The results of the Durbin's alternative and of the Breusch-Godfrey LM test are available upon request.

**Table 2. OLS univariate regressions.**

Dependent variable: log realized volatility										
Independent variables										
Intercept	$\ln(\sigma_{ATMC})$	$\ln(\sigma_{OTMC})$	$\ln(\sigma_{ATMP})$	$\ln(\sigma_{OTMP})$	$\ln(\sigma_{LR})$	$\ln(\sigma_{GAR})$	Adj. R <sup>2</sup>	DW	$\chi^2$	Hausman test
-0,002	1,03***						0,70	1,97	3,16	8,50
(0,99)	(0,00)								(0,21)	
0,083		1,05***					0,68	1,95	0,40	10,59
(0,53)		(0,00)							(0,81)	
0,0569			1,10***				0,76	1,94	13,95	2,11
(0,60)			(0,00)						(0,00)	
-0,123				1,09***			0,73	1,74	75,84	5,78
(0,24)				(0,00)					(0,00)	
-0,3018					0,81		0,64	2,19	7,50	
(0,01)					(0,00)				(0,02)	
-0,002						1,03***	0,70	2,17	2,92	
(0,98)						(0,00)			(0,23)	

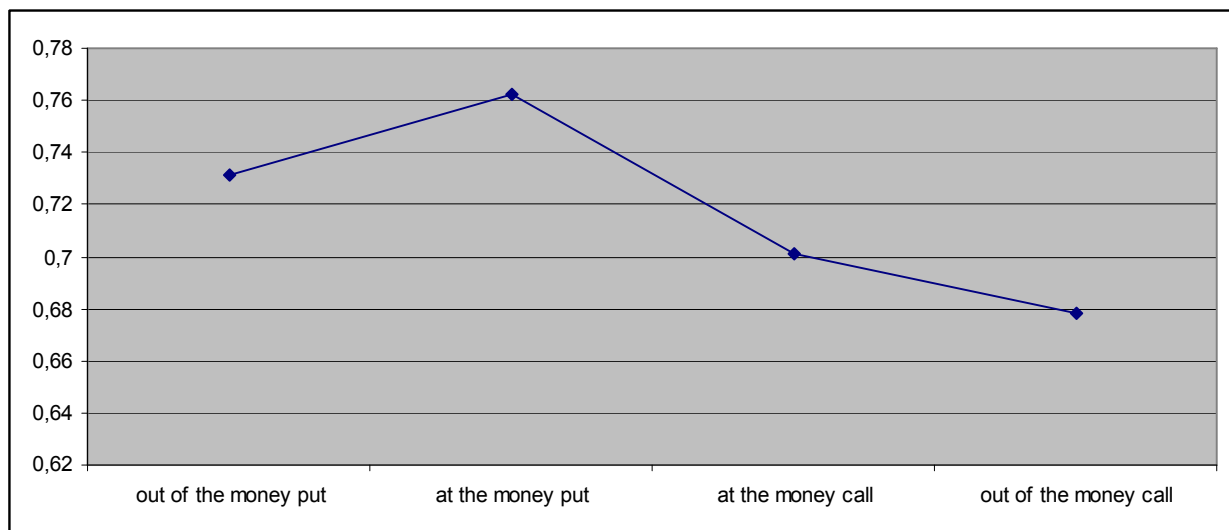
Note: The number in brackets are the p-values. The  $\chi^2$  report the statistic of a  $\chi^2$  test for the joint null hypothesis  $\alpha = 0$  and  $\beta = 1$  (p-values in parentheses) in the following univariate regressions  $\ln(\sigma_R) = \alpha + \beta \ln(\sigma_i)$ , where  $\sigma_R$  = realized volatility and  $\sigma_i$ = volatility forecast  $i=ATMC, OTMC, ATMP, OTMP, LR, GAR$ . The superscripts \*\*\*, \*\*, \* indicate that the slope coefficient is insignificantly different from one at the 10%, 5%, and 1% critical level respectively. The last column reports the Hausman (1978) specification test statistic (one degree of freedom) 5% critical level = 3,841.

The adjusted R<sup>2</sup> is the highest for ATMP implied volatility, followed by OTMP implied, and by ATMC and GAR, that obtain a similar performance. LR volatility and OTMC implied volatility have the lowest adjusted R<sup>2</sup>. If we plot the R<sup>2</sup> against the option moneyness (keeping in mind that ATMP (ATMC) implied volatility is mainly obtained from options with  $1 < X/S \leq 1,03$  ( $0,97 \leq X/S < 1$ ), since these are the most traded strike prices, we find the pattern depicted in Figure 2. The results highlight that the information content of implied volatility has a humped shape, with out of the money options being less informative than at the money ones. This is consistent with the hedging pressure argument documented in Bollen and Whaley (2004): out of the money options are less informative than at the money ones. Differently from the results in Ederington and Guan (2005) the forecasting power of implied volatility does not vary in a mirror image of the implied volatility smile: rather it exactly follows the volatility skew pattern, the only exception being OTMP implied volatility that has a smaller forecasting power than it should have by looking at the skew. The difference can be attributed to

the fact that, w.r.t. Ederington and Guan (2005) our option classes are broader<sup>6</sup> and our results are based on synchronous prices, matched in a one minute interval.

The null hypothesis that the volatility forecast is an unbiased estimate of future realized volatility is not rejected for both call implied volatility forecasts (ATMC and OTMC) and for GAR, however, it is rejected for both put implied volatility forecasts (ATMP and OTMP). This is probably due to the fact that, in our sample, realized volatility is on average much lower than both ATMP and OTMP implied volatility forecasts. However, the null hypothesis that  $\beta$  is insignificantly different from one can not be rejected at the 10% critical level for both put implied volatility forecasts. Therefore also ATMP and OTMP implied volatilities can be considered as unbiased after a constant adjustment given by the intercept of the regression. LR volatility obtains the worst performance: it is not unbiased even after a constant adjustment.

**Figure 2. The adjusted  $R^2$  for different moneyness classes.**



Finally, in order to test for robustness our results, and see if implied volatility has been measured with errors, we adopt an instrumental variable procedure and run a two stage least squares. The Hausman (1978) specification test reported in the last column of Table 2 indicates that the errors in variables problem is not significant only for ATMP. Therefore we report in Table 3 the TSLS regressions. As expected, the TSLS estimates of the beta coefficients are higher than the OLS ones. This causes the slope coefficients to be insignificantly different from one at a lower confidence level than in the OLS case. Nonetheless, the results are virtually the same of the OLS case, with ATMC and OTMC being unbiased and ATMP and OTMP being

<sup>6</sup> The choice has been made in order to avoid having samples of different length, caused by missing observations for some dates.

unbiased after a constant adjustment. Therefore, the forecasting power of each volatility forecast is not substantially changed w.r.t. the OLS case.

**Table 3. TSLS univariate regressions.**

Dependent variable: log realized volatility							
Independent variables							
Intercept	$\ln(\sigma_{ATMC})$	$\ln(\sigma_{OTMC})$	$\ln(\sigma_{ATMP})$	$\ln(\sigma_{OTMP})$	Adj. $R^2$	DW	$\chi^2$
0,185 (0,18)	1,157** (0,00)				0,69	2,11	6,08 (0,05)
0,328 (0,04)		1,205* (0,00)			0,66	2,14	4,62 (0,10)
0,118 (0,31)			1,145** (0,00)		0,76	1,97	15,36 (0,00)
-0,01 (0,93)				1,18* (0,00)	0,73	1,81	77,25 (0,00)

Note: The number in brackets are the p-values. The  $\chi^2$  report the statistic of a  $\chi^2$  test for the joint null hypothesis  $\alpha = 0$  and  $\beta = 1$  (p-values in parentheses) in the following univariate regressions  $\ln(\sigma_R) = \alpha + \beta \ln(\sigma_i)$ , where  $\sigma_R$  = realized volatility and  $\sigma_i$ = volatility forecast, i=ATMC, OTMC, ATMP, OTMP. The superscripts \*\*\*, \*\*, \* indicate that the slope coefficient is insignificantly different from one at the 10%, 5%, and 1% critical level respectively.

## 5. The results of augmented regressions.

The results of the OLS augmented regressions (equation (2), (3) and (4)) are reported in Table 4 (p-values in parentheses). In all the regressions the residuals are normal, homoscedastic and not autocorrelated (the Durbin Watson statistic is not significantly different from two and the Breusch-Godfrey LM test confirms non autocorrelation up to lag 12<sup>7</sup>).

In augmented regressions (2), we compare each implied volatility forecast with historical volatility in order to see if any of the implied volatility forecasts is efficient, i.e. it subsumes all the information contained in historical volatility. For historical volatility we use both LR volatility and GAR. As the results are very similar, in the following we use the term historical volatility, without mentioning which is the forecasting method. The results differ somehow across option type: overall put implied volatilities are more efficient than call implied ones. At the 5% level, only ATMP implied volatility is efficient. In fact, the slope coefficient of historical volatility is not significantly different from zero at the 10% level for both LR volatility and GAR, indicating that ATMP implied volatility subsumes all the information contained in

<sup>7</sup> In the regressions that include as explanatory variable lagged realised volatility, the Durbin's alternative has been computed but it was not possible to obtain a result. The results of the Durbin's alternative and of the Breusch-Godfrey LM test are available upon request.

historical volatility. Moreover, from the comparison of univariate and augmented regressions, the inclusion of historical volatility does not improve the goodness of fit according to the adjusted  $R^2$ .

**Table 4. Augmented regressions.**

Dependent variable: log realized volatility											
Independent variables											
Intercept	$\ln(\sigma_{ATMC})$	$\ln(\sigma_{OTMC})$	$\ln(\sigma_{ATMP})$	$\ln(\sigma_{OTMP})$	$\ln(\sigma_{LR})$	$\ln(\sigma_{GAR})$	Adj. $R^2$	DW	$\chi^2^a$	$\chi^2^b$	Hausman test
-0,009	0,70				0,32 <sup>+</sup>		0,73	2,26	7,39		2,907
(0,94)	(0,00)				(0,01)				(0,02)		
0,0447		0,65			0,37		0,72	2,28	11,21		3,019
(0,72)		(0,00)			(0,00)				(0,00)		
0,0477			0,95		0,14 <sup>+++</sup>		0,76	2,08	3,38		0,931
(0,66)			(0,00)		(0,26)				(0,18)		
-0,93				0,82	0,25 <sup>+</sup>		0,74	2,04	6,34		1,455
(0,36)				(0,00)	(0,03)				(0,04)		
0,088	0,56					0,53	0,74	2,23	11,42		0,016
(0,44)	(0,00)					(0,00)			(0,00)		
0,134		0,5				0,6	0,73	2,22	11,51		0,122
(0,28)		(0,00)				(0,00)			(0,00)		
0,09			0,86			0,26 <sup>+++</sup>	0,77	2,11	4,40		0,003
(0,40)			(0,00)			(0,14)			(0,11)		
0,0014				0,69		0,43 <sup>+</sup>	0,75	2,074	9,01		0,015
(0,99)				(0,00)		(0,01)			(0,01)		
-0,003					0,004	1,027	0,69	2,17	22,68	0,167	
(0,98)					(0,99)	(0,00)			(0,00)	(0,92)	
0	1,02 <sup>+</sup>	0,01 <sup>+++</sup>					0,69	1,97	0,19	6,07	3,512
(0,99)	(0,02)	(0,98)							(0,91)	(0,04)	
0,031	-0,91 <sup>+</sup>		2,02				0,78	1,90	26,12	7,77	0,949
(0,76)	(0,02)		(0,00)						(0,00)	(0,02)	
-0,014	0,42 <sup>+</sup>			0,7			0,75	1,83	14,77	7,29	2,288
(0,90)	(0,02)			(0,00)					(0,00)	(0,03)	
-0,001		-0,36 <sup>+++</sup>	1,45				0,76	1,89	28,94	3,89	1,671
(0,99)		(0,19)	(0,00)						(0,00)	(0,14)	
0,035		0,3965 <sup>+</sup>		0,74			0,75	1,84	22,80	8,13	2,101
(0,77)		(0,01)		(0,00)					(0,00)	(0,02)	
0,36			0,85	0,264 <sup>+++</sup>			0,77	1,89	3,12	12,47	1,766
(0,74)			(0,00)	(0,32)					(0,21)	(0,00)	

Note: The number in brackets are the p-values. The  $\chi^2^a$ ,  $\chi^2^b$  report the statistic of a  $\chi^2$  test for the joint null hypothesis  $\gamma = 0$  and  $\beta = 1$  or  $\gamma = 1$  and  $\beta = 0$  (p-values in parentheses) in the following regressions:  $\ln(\sigma_R) = \alpha + \beta \ln(\sigma_i) + \gamma \ln(\sigma_j)$ , where  $\sigma_R$  = realized volatility,  $\sigma_i$ = volatility forecast  $i$ = ATMC, OTMC, ATMP, OTMP, LR, GAR and  $\sigma_j$ = volatility forecast  $j$ ,  $j$ = ATMC, OTMC, ATMP, OTMP, LR, GAR,  $i \neq j$ . The superscripts <sup>+++</sup>, <sup>++</sup>, <sup>+</sup> indicate that the slope coefficient is insignificantly different from zero at the 10%, 5%, and 1% critical level respectively. The last column reports the Hausman (1978) specification test statistic (one degree of freedom) 5% critical level = 3,841.

The slope coefficient of ATMP implied volatility is not significantly different from one at the 10% level and the joint test of information content and efficiency  $\gamma = 0$  and  $\beta = 1$  does not reject the null hypothesis, indicating that ATMP implied volatility is efficient and unbiased after a constant adjustment. OTMP implied volatility is marginally inefficient, since the coefficient of historical volatility is not significantly different from zero only at the 1% level, and the joint test of information content and efficiency  $\gamma = 0$  and  $\beta = 1$  does not reject the null hypothesis only at the 1% level. For ATMC and OTMC implied volatilities the results are quite similar, with ATMC performing slightly better. In both cases, from the comparison of univariate and augmented regressions, the inclusion of historical volatility improves the goodness of fit according to the adjusted  $R^2$ . In fact, the slope coefficient of historical volatility is significantly different from zero and the joint test of information content and efficiency  $\gamma = 0$  and  $\beta = 1$  rejects the null hypothesis (the only exception being ATMC implied volatility w.r.t. LR volatility at the 1% level).

In order to see if any one of the implied volatilities subsumes all the information contained in the others, we test in augmented regressions (3) if  $\gamma = 0$  and  $\beta = 1$  or  $\gamma = 1$  and  $\beta = 0$ . By looking at the significance of the coefficients and at the results of the  $\chi^2$  test, we can see that ATMP implied volatility subsumes all the information contained in both OTMP and OTMC implied volatilities. ATMC implied volatility subsumes all the information contained only in OTMC implied volatility. The comparison of ATMP and ATMC implied volatilities is not straightforward since the coefficient of ATMC is statistically not different from zero only at the 1% level and the  $\chi^2$  test marginally rejects the null hypothesis for ATMP at the 5% level. In order to better understand the performance of the two at the money implied volatility forecasts, we compute the Diebold and Mariano test statistic (for more details see Diebold and Mariano (1995)). The loss function chosen is the absolute error loss. The Diebold and Mariano test statistic under the null of equal predictive accuracy is distributed as a  $N(0, I)$ , in our case the test statistic is -2,35, therefore we can reject the null of equal predictive accuracy at the 5% level. Based on these results we can say that ATMP implied volatility has a slightly better predictive power than ATMC implied one. Therefore, in our sample, at the money put options are priced more efficiently than at the money call ones, probably due to the larger trading volume, determined by a higher demand.

Finally, in order to distinguish among the time series forecasts which is the best one, we test in augmented regression (4) if  $\gamma = 0$  and  $\beta = 1$  or  $\gamma = 1$  and  $\beta = 0$ . The results highlight that GAR subsumes all the information contained in LR volatility. As a last step, in order to test for robustness our results, and see if implied volatility has been measured with errors, we adopt



an instrumental variable procedure and run a two stage least squares. The Hausman (1978) specification test reported in the last column of Table 4 indicates that the errors in variables problem is not significant neither in augmented regressions (2) nor in augmented regression (3)<sup>8</sup>. Therefore we can trust the OLS regressions results.

## 6. A combination of call and put at the money volatilities.

At the money volatilities are widely used by market participants. Call and put at the money volatilities are usually inserted in the smile function by using some average of both option classes. Given that prices are observed with measurement errors (stemming from finite quote precision, bid-ask spreads, non-synchronous observations and other measurement errors) small errors in any of the input may produce large errors in the implied volatility. Quoting Hentshle (2003): “Unfortunately many authors preclude the cancellation of errors across puts and calls by using only the more liquid out of the money options. Unless underlying asset prices and dividend rates are observed with high precision, this practice can result in a substantial loss of efficiency”. Moreover, as noted in Moriggia, Muzzioli and Torricelli (2007) the use of both call and put options in the volatility estimation, highly improves the pricing performance of option pricing models based on implied binomial trees.

Therefore, in this section we investigate how to combine at the money call and put implied volatilities in a single estimate, in order to convey the information from both call and put prices and cancel possible errors across option type. In the logarithmic specification, natural candidates for the weights that we may assign to call and put implied volatilities would be the estimated coefficients of augmented regression (3). However, as the beta coefficient of call implied volatility is not significantly different from zero, it is not possible to find an optimal combination of the two with constant weights through time.

In line with the approach by Christensen and Hansen (2002), that proposes to favour the most actively traded options, we construct a weighted average of ATMC and ATMP implied volatilities ( $\sigma_M$ ), where the weights are the relative trading volume of each option class on the total trading volume:

$$\sigma_M = \frac{\sigma_{ATMC} V_c + \sigma_{ATMP} V_p}{V_c + V_p}$$

where  $V_i$  is the trading volume of option in class  $i$ ,  $i=c,p$ . The weighting rule favours the most actively traded options, that in our sample are the put ones.

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<sup>8</sup> In augmented regressions (3) the instrumental variables procedure is used for the most significant variable in each regression.

Descriptive statistics of average implied volatility and log average implied volatility are reported in Table 5. Average implied volatility is slightly higher than realised volatility. Similarly to the results in Table 1, we can see that the natural logarithm of average implied volatility conforms more to normality than the plain average implied volatility series. Therefore it will be used as explanatory variable in univariate and augmented regressions.

In order to analyse the performance of average implied volatility, we run both univariate and augmented regressions (1), (2) and (3)<sup>9</sup> with  $\sigma_t = \sigma_M$ . Furthermore, in order to test for robustness our results, we look for possible errors in variables. The results are reported in Table 6. In all the regressions the residuals are normal, homoscedastic and not autocorrelated (the Durbin Watson statistic is not significantly different from two and the Breusch-Godfrey LM test confirms non autocorrelation up to lag 12<sup>10</sup>).

**Table 5. Descriptive statistics for average implied volatility.**

<b>Statistic</b>	$\sigma_M$	$\ln(\sigma_M)$
<b>mean</b>	0,246	-1,484
<b>std dev</b>	0,11	0,39
<b>skewness</b>	1,67	0,59
<b>kurtosis</b>	5,97	3,14
<b>Jarque Bera</b>	64,15	4,52
<b>p-value</b>	0,00	0,10

In univariate regression (1), the beta coefficient of average implied is significantly different from zero, but the null hypothesis that average implied is an unbiased estimate of future realized volatility is rejected at the 5% level. The null hypothesis that  $\beta$  is insignificantly different from one can not be rejected at the 10% critical level: therefore we can consider average implied volatility as unbiased after a constant adjustment given by the intercept of the regression.

In augmented regressions (2) we compare average implied volatility with historical volatility in order to understand if average implied volatility subsumes all the information contained in historical volatility. The results provide evidence for both the unbiasedness and efficiency of average implied volatility forecast w.r.t LR volatility, w.r.t GAR the evidence is less clear-cut since the joint test of information content and efficiency  $\gamma = 0$  and  $\beta = 1$

<sup>9</sup> In augmented regression 3 we compare average implied only with ATMP implied, since we are looking for an improvement over the best forecast.

<sup>10</sup> In the regression that include as explanatory variable the lagged realised volatility, the Durbin's alternative has been computed, but it was not possible to obtain a result. The results of the Durbin's alternative and of the Breusch-Godfrey LM test are available upon request.

marginally rejects the null hypothesis. If we compare the performance of average implied volatility with ATMP we see that the adjusted  $R^2$  is lower for average implied. Moreover from the results in augmented regression (3) we see that average implied does not subsume all the information of ATMP. Therefore we conclude that the attempt of combining at the money call and put implied volatilities in a single estimate does not improve the forecasting power over the simple use of ATMP.

**Table 6. OLS and TSLS regressions of realised volatility on average implied volatility.**

<b>PANEL A: OLS REGRESSIONS</b>										
Dependent variable: log realized volatility										
Independent variables										
Intercept	$\ln(\sigma_M)$	$\ln(\sigma_{ATMP})$	$\ln(\sigma_{LR})$	$\ln(\sigma_{GAR})$	Adj. $R^2$	DW	$\chi^2$	$\chi^{2a}$	$\chi^{2b}$	Hausman test
0,04	1,078***				0,74	1,97	7,87			5,280
(0,71)	(0,00)						(0,02)			
0,029	0,84		0,22 <sup>++</sup>		0,74	2,18		4,47		2,319
(0,79)	(0,00)		(0,07)					(0,11)		
0,096	0,713			0,396 <sup>+</sup>	0,75	2,19		6,72		0,433
(0,39)	(0,00)			(0,02)				(0,04)		
0,025	-1,75 <sup>+</sup>	2,852			0,78	1,89		15,27	7,52	0,019
(0,81)	(0,02)	(0,00)						(0,00)	(0,02)	
<b>PANEL B: TSLS REGRESSION</b>										
Dependent variable: log realized volatility										
Independent variables										
Intercept	$\ln(\sigma_M)$				Adj. $R^2$	DW	$\chi^2$			
0,16	1,157				0,7322	2,051	10,37			
(0,20)	(0,00)						(0,01)			

Note: The number in brackets are the p-values. The  $\chi^2$  report the statistic of a  $\chi^2$  test for the joint null hypothesis  $\alpha = 0$  and  $\beta = 1$  (p-values in parentheses) in the following univariate regression  $\ln(\sigma_R) = \alpha + \beta \ln(\sigma_M)$ , where  $\sigma_R$  = realized volatility and  $\sigma_M$  = average implied volatility. The  $\chi^{2a}$ ,  $\chi^{2b}$  report the statistic of a  $\chi^2$  test for the joint null hypothesis  $\gamma = 0$  and  $\beta = 1$  or  $\gamma = 1$  and  $\beta = 0$  (p-values in parentheses) in the following regressions:  $\ln(\sigma_R) = \alpha + \beta \ln(\sigma_M) + \gamma \ln(\sigma_j)$ , where  $\sigma_R$  = realized volatility,  $\sigma_M$  = average implied volatility  $\sigma_j$  = volatility forecast j, j = ATMP, LR, GAR. The superscripts \*\*\*, \*\*, \* indicate that the slope coefficient is insignificantly different from one at the 10%, 5%, and 1% critical level respectively. The superscripts <sup>+++</sup>, <sup>++</sup>, <sup>+</sup> indicate that the slope coefficient is insignificantly different from zero at the 10%, 5%, and 1% critical level respectively. The last column reports the Hausman (1978) specification test statistic (one degree of freedom): 5% critical level = 3,841.

Finally, we test for robustness our results by adopting an instrumental variable procedure. The Hausman (1978) specification test reported in the last column of Table 6 indicates that the errors in variables problem is significant only in univariate regression (1). We report in Panel B

the TSLS regression output, but the results do not change the conclusions based on the OLS regression.

## **7. Conclusions.**

In this paper we have investigated how the information content of implied volatility varies according to moneyness and option type and we have compared the latter option based forecasts with historical volatility. The information content of implied volatility has been examined for the most liquid at the money and out of the money call and put options i.e. the ones that are usually used as inputs for the computation of the smile function. Differently from previous studies, that use settlement prices, we have used synchronous prices, matched in a one minute interval.

The results highlight that the information content of implied volatility has a humped shape, with out of the money options being less informative than at the money ones. This is consistent with the hedging pressure argument documented in Bollen and Whaley (2004), that causes out of the money options to be less informative than at the money ones. All the implied volatility forecasts contain more information about future realised volatility than LR volatility. The GAR forecast obtains roughly the same performance of ATMC implied volatility and is superior to both OTMC implied volatility and LR volatility.

Two hypotheses have been tested: unbiasedness and efficiency of the different volatility forecasts. Overall, call implied volatilities forecasts are unbiased, while put implied volatilities are unbiased only after a constant adjustment given by the intercept of the regression. Efficiency has been evaluated by assessing whether the implied volatility forecast subsumes all the information contained in historical volatility. Only ATMP implied volatility is efficient, in that it subsumes all the information contained in historical volatility. Of the remaining three volatility forecasts, OTMP is marginally inefficient, while ATMC and OTMC are strongly inefficient.

By comparing pairwise the four implied volatility forecasts, it is clear that ATMC subsumes all the information contained in OTMC, ATMP subsumes all the information contained in both OTMP and OTMC. The comparison of ATMC and ATMP is less clear-cut, but we can conclude that ATMP obtains a slightly better performance than ATMC. Therefore, in our sample, at the money put options are priced more efficiently than at the money call options: ATMP options, being more heavily traded than ATMC options, are more informative of future realised volatility. This is an interesting result, different from previous research (see e.g. Christensen and Hansen (2002)), and is a warning against the a-priori choice of using call implied volatility. The attempt of combining ATMC and ATMP in a single forecast in order to

cancel possible errors across option type does not lead to an improvement over the simple use of ATMP implied volatility.

The present investigation is very important for the understanding of the role of the different ingredients of the smile function and can be seen as a preliminary exercise in order to choose different weights for each volatility input in a volatility index. The VDAX-New, the new volatility index of the German equity market, is based on an approximation of the so-called “model free” implied volatility, proposed by Britten-Jones and Neuberger (2000), and is derived by using the most liquid at the money and out of the money call and put options. The VDAX-New has replaced the old VDAX, that was computed by using only at the money options (pairs of calls and puts with the four strikes below and above the at the money point). The present investigation suggests some directions in order to improve the information content of the VDAX-New: overall put options are more informative than call options, ATMP are preferred to ATMC, OTMP predict future realised volatility better than both ATMC and OTMC. How these rules can be embedded in the index and the empirical comparison between the suggested modifications and the existent VDAX-New is left for future research.

### **References.**

1. Black, F., M. Scholes. 1973. The pricing of options and corporate liabilities. *Journal of Political Economy* 81, 637-654.
2. Bollen, N.P.B., Whaley R.E. 2004. Does net buying pressure affect the shape of implied volatility functions? *The Journal of Finance* 59, 711-753.
3. Bollerslev, T. 1986. Generalized Autoregressive Conditional Heteroscedasticity. *Journal of Econometrics*, 307-327.
4. Britten-Jones, M., A. Neuberger. 2000. Option prices, implied price processes and stochastic volatility. *Journal of Finance*, 55, 839-866.
5. Buraschi, A., J. Jackwerth. 2001. The price of a smile: hedging and spanning in option markets. *The Review of Financial Studies*, 14, 495-527.
6. Christensen B.J., N.R. Prabhala. 1998. The relation between implied and realized volatility. *Journal of Financial Economics*, 50, 125-150.
7. Christensen B.J., C. S. Hansen, N.R. Prabhala. 2001. The telescoping overlap problem in options data. *Working paper, University of Aarhus and University of Maryland*.
8. Christensen B.J., C. S. Hansen. 2002. New evidence on the implied-realized volatility relation. *The European Journal of Finance*, 8 (2), 187-205.

9. Derman, E., I. Kani. 1994. Riding on a smile. *Risk* 7 (2) 32-39.
10. Diebold F.X., R.S. Mariano. 1995. Comparing predictive accuracy. *Journal of Business & Economic Statistics*, 13 (3) 134-144.
11. Donaldson R.G., M. Kamstra. 2005. Volatility forecasts, trading volume and the ARCH vs. option-implied tradeoff. *Journal of Financial Research*, 28 (4), 519-538.
12. Ederington L. H., W. Guan. 2005. The information frown in option prices. *Journal of Banking and Finance*, 29 (6), 1429-1457.
13. Fleming, J. 1998. The quality of market volatility forecasts implied by S&P100 index option prices. *Journal of Empirical Finance*, 5 (4), 317-345.
14. Fleming, J. 1999. The economic significance of the forecast bias of S&P100 Index option implied volatility. *Advances in Futures and Option Research* 10, 219-251.
15. Godbey J.M., J.W. Mahar. 2007. Forecasting power of implied volatility: evidence from individual equities. *B>Quest, University of West Georgia*.
16. Hentschel, L. 2003. Errors in implied volatility estimation. *Journal of Financial and Quantitative analysis*, 38 (4), 779-810.
17. Jiang G. J., Y. S. Tian. 2005. Model free implied volatility and its information content. *The Review of Financial Studies*, 18 ( 4), 1305-1342.
18. Jorion, P. 1995. Predicting volatility in the foreign exchange market. *Journal of Finance* 50 (2) 507-528.
19. Moriggia V., S. Muzzioli, C. Torricelli. 2007. Call and put implied volatilities and the derivation of option implied trees. *Frontiers in Finance and Economics*, 4(1), 35-64.
20. Poon S., C.W. Granger. 2003. Forecasting volatility in financial markets: a review. *Journal of Economic Literature*, 41, 478-539.
21. Poon, S. 2005. *A practical guide to forecasting financial market volatility*. Wiley Finance, Chichester.
22. Rubinstein, M. 1994. Implied binomial trees. *The Journal of Finance*, 49, 771-818.
23. Sarwar, G. 2005. The informational role of option trading volume in equity index options markets. *Review of Quantitative Finance and Accounting*, 24, 159-176.

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- N. 491 - V. Moriggia, S. Muzzioli and C. Torricelli, *On the no arbitrage condition in option implied trees*, European Journal of Operational Research, forthcoming (doi: 10.1016/j.ejor.2007.10.017), **WP No. 491 (May 2005)**.
- N. 482 - G. Di Lorenzo and G. Marotta, *A less effective monetary transmission in the wake of EMU? Evidence from lending rates passthrough*, ICAFI Journal of Monetary Economics, Vol. 4, 2, pp. 6-31, **WP No. 482 (February 2005)**.
- N. 472 - M. Brunetti and C. Torricelli, *The internal and cross market efficiency in index option markets: an investigation of the Italian market*, Applied Financial Economics, Vol. 17, 1, pp. 25-33, **WP No. 472 (November 2004)**.
- N. 466 - G. Marotta, *La finanza del settore non profit tra ritardi nei pagamenti e Basilea 2*, Banca Impresa Società, Vol. XXIV, 1, pp. 35-51, **WP No. 466 (September 2004)**.



- N. 453 - Pederzoli and C. Torricelli, *Capital requirements and Business Cycle Regimes: Forward-looking modelling of Default Probabilities*, Journal of Banking and Finance, VI. 29, 12, 2005, pp. 3121-3140, **WP No. 453 (February 2004)**.
- N. 448 - V. Moriggia, S. Muzzioli, C. Torricelli, *Call and put implied volatilities and the derivation of option implied trees*, Frontiers In Finance and Economics, vol.4, 1, 2007, pp. 35-64, **WP No. 448 (November 2003)**.
- N. 436 - M. Brunetti and C. Torricelli, *Put-Call Parity and cross-market efficiency in the Index Options Markets: evidence from the Italian market*, International Review of Financial Analysis, VI.14, 5, pp. 508-532, **WP No. 436 (July 2003)**.
- N. 429 - G. Marotta, *When do trade credit discounts matter? Evidence from Italian Firm-Level Data*, Applied Economics, Vol. 37, 4, pp. 403-416, **WP No. 429 (February 2003)**.
- N. 426 - A. Rinaldi and M. Vasta, *The Structure of Italian Capitalism, 1952-1972: New Evidence Using the Interlocking Directorates Technique*, Financial History Review, vol, 12, 2, pp. 173-198, **WP No. 426 (January 2003)**.
- N. 417 - A. Rinaldi, *The Emilian Model Revisited: Twenty Years After*, Business History, vol. 47, 2, pp. 244-226, **WP No. 417 (September 2002)**.
- N. 375 - G. Marotta, *La direttiva comunitaria contro i ritardi nei pagamenti tra imprese. Alcune riflessioni sul caso italiano*, Banca, Impresa, Società, Vol. XX, 3, pp. 451-71, **WP No. 375 (September 2001)**.
- N. 303 - G. Marotta and M. Mazzoli, *Fattori di mutamento nella domanda di prestiti ed effetti sulla trasmissione della politica monetaria*, in P. ALESSANDRINI (ed.) *Il sistema finanziario italiano tra globalizzazione e localismo*, Bologna, Il Mulino, pp. 223-260, **WP No. 303 (April 2000)**.
- N. 131 - G. Marotta, *Does trade credit redistribution thwart monetary policy? Evidence from Italy*, Applied Economics, Vol. 29, December, pp. 1619-29, **WP No. 131 (1996)**.
- N. 121 - G. Marotta, *Il credito commerciale in Italia: una nota su alcuni aspetti strutturali e sulle implicazioni di politica monetaria*, L'Industria, Vol. XVIII, 1, pp. 193-210, **WP No. 121 (1995)**.
- N. 105 - G. Marotta, *Credito commerciale e "lending view"*, Giornale degli Economisti e Annali di Economia, Vol. LIV, 1-3, gennaio-marzo, pp. 79-102; anche in G. Vaciago (a cura di) *Moneta e finanza*, Bologna, Il Mulino, **WP No. 105 (1994)**.