THEORY OF DIDACTICAL SITUATIONS AND INSTRUMENTAL GENESIS FOR THE DESIGN OF A CABRI ELEM BOOK

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The contributions of two theoretical frameworks (Theory of Didactic Situations and Instrumental Genesis) to the design of a sequence of tasks in the Cabri Elem environment, where task and technology design are closely linked, are shown. Considering the potential for instrumental genesis as a theory of technology design reveals a fundamental difficulty in dealing with representations. It is hence suggested that the role of the artefact be broadened to include environments, tools, and entities.

INTRODUCTION

The first part of this paper consists of a summary of some aspects of our ICMI-22 submission on task design, in which we analyzed a particular sequence of tasks created using Cabri Elem in order to illustrate the interconnections between the affordances of the technology and the ability to implement particular didactic principles. Our choice of theoretical frameworks to use for this analysis was based on an analysis of tasks conducted as part of the Intergeo project (Trgalova, Soury-Lavergne & Jahn, 2011), which used Brousseau's theory of didactical situations (TDS) (Brousseau, 1998) together with instrumental genesis (IG) (Trouche, 2005).

In part 2 we begin a study of instrumental genesis as a theory of task/technology design with the aim of both creating greater links between TDS and IG and enabling IG to become a more effective framework for design.

PART 1: TECHNOLOGY AND TASK DESIGN

Cabri Elem technology was created to serve the needs of primary students and also to enable the creation of "applets" in order to support teachers to engage more confidently with technology (Laborde and Laborde, 2011). It is a task design environment in which "activity books" consisting of a succession of pages incorporating a sequence of tasks may be created, and a more restrictive task performance environment in which activity books may be used by teachers and students. Cabri Elem has the affordances of earlier Cabri technology for direct manipulation of geometrical objects and numbers, together with additional features such as 3D models and tools. A major difference between Cabri Elem technology and other dynamic geometry software, and also other generic technology, such as graphical calculators, CAS or spreadsheets is that the user interface of the task performance environment is under the control of the activity book designer, who must decide which objects (tool icons, images, text, geometric figures, etc.) to arrange on initially empty pages, and who may program control actions on these objects. Creating an activity book hence involves issues of both task and technology design.

The theory of didactic situations

In this theory (Brousseau, 1998), knowledge is a property of a system constituted by a subject and a "milieu" in interaction. Learning occurs through this interaction: the subject acts within and receives feedback from the milieu. Technology, or the part of technology relevant to the mathematics concerned, may form part of the milieu, and the milieu related to a student changes as student knowledge, both technical and mathematical, develops. With a learning task in a technology environment, the author determines the possible milieu and hence the potential for learning by creating all the elements the student will deal with: the objects the student will manipulate, the possibilities of actions on these objects and the feedback provided by the environment.

Key aspects of a didactical situation are the mathematical problem and the choice of didactical variable values to set for the task, where the task involves learning objectives and the mathematical problem. The teacher assumes that achieving the task will cause the student to learn. The goal of a task, whether teacher or student determined, should be clear, together with criteria for success or failure. A task is performed by concrete and conceptual student actions, with the existence of a space of uncertainty and freedom for the subject about appropriate action and strategy. This contrasts with the common dynamic geometry tasks such as "drag this point and observe" where the student has no choice of action and is uncertain about what is relevant to observe. The task corresponds to phases of the didactical situation and is related to different values of a set of didactical variables. Didactical variables are parameters of the situation, with values that affect solution strategies. The effects can be of three kinds: (i) a change in the validity of a strategy, where a strategy that produces a correct answer with a certain value of a didactical variable will produce an incorrect answer with another value, (ii) a change in the cost of the strategy (for example counting elements one by one is efficient for a small number but much more costly for a larger number) (iii) the impossibility of using the strategy. A combination of the different didactical variable values contributes to the task definition. The learning situation is a choice of different tasks that lead the students to construct the appropriate strategy. Thus task design will consist, for a part, in identifying the didactical variables of the situation and then choosing the succession of appropriate combinations of didactical variable values.

Instrumental genesis

Learning situations involving the use of technology may be modeled as instrument-mediated activity situations (Rabardel, 2002), wherein the subject acts upon an object either directly or with the mediation of an instrument, which consists of an artefact together with the subject's utilization schemes. Instrumental genesis (IG) is the main aspect of instrument-mediated activity situations to be considered in the literature, and is the process by which instruments are developed from artefacts, through instrumentation (the development of utilization schemes) and instrumentalization (using the artefact for new purposes). Instrumental genesis, originating in ergonomics (Rabardel, 2002), is well established as part of the instrumental approach (Artigue, 2002) dealing with the integration of technology in the mathematics classroom, but also, being derived from the work both of Vygotsky and of Piaget (Rabardel, 2002), has links to socio-cultural approaches and, less explicitly, to constructivist approaches and constructionism.

An issue with instrument-mediated activity situations is that the "*object*" can be interpreted either as the goal of activity, consistent with the activity theory of Leontiev (Kaptelinin & Nardi, 2006, p. 59), or as the "*thing*", not necessarily concrete, upon which the subject acts, which is the main sense in which Rabardel (2002) uses the word (e.g identifying object with wallpaper and ceiling (p. 43)). In considering learning tasks in technology environments, the *object-as-goal-of-activity* and the *object-as-thing-acted-on* are both of importance. We will refer to the former by the word *goal* and to the latter by the phrase. Note that the *object-as-thing-acted-on* will vary as the subject's actions change; nothing is intrinsically an *object-as-thing-acted-on*. We will use the word object without italics to refer to entities that may become *objects-as-things-acted-on* during the course of the activity: this is consistent with the way it has been used in describing TDS above.

The Cabri Elem Task

We will now look at an activity book and discuss the links between this sequence of tasks and the theoretical frameworks from which the tasks were generated. The "Target" activity book addresses the French primary school level CE1 (7 year old students) and the *goal* for the teacher is for students to learn about the representation of numbers using place value notation. The idea arose from comparing counters on a scoreboard, where the value of the counter depends on its position on the board, with the way that the value of a digit depends on its position in a written number. It was designed by a team of ten researchers (including two of the authors of this paper), teacher educators and teachers involved in a French national project [1] whose purpose is to create resources for the teaching of mathematics in kindergarten and primary school.

The process of creating the activity book involved elaborating the milieu by choosing appropriate objects, possible actions and resultant feedback. In our example, the objects are essentially the scoreboard with three different regions, the counters, the target number and the score, as shown below.

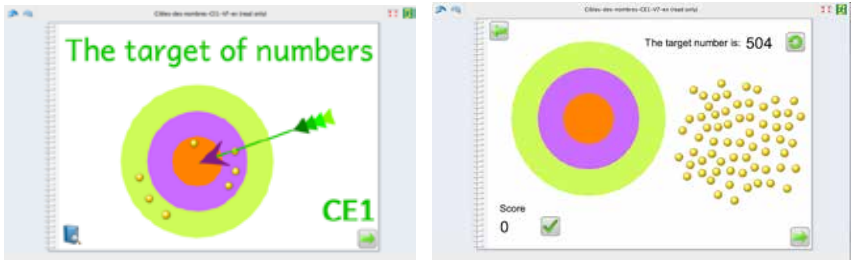

Figure 1. Title page and a task page from the "Target" activity book

The actions on the objects are simple: dragging the counters, clicking on a button to reset counters and get a new target number or to get an evaluation.

Didactical variables played an important role in the task design process. Some were identified a priori, while others emerged during the design process as the authors became more aware of what aspects of the situation could be changed. Once a potential variable was identified, an analysis of the ways in which this variable could be changed produced a better understanding of the possible tasks and their consequences. It also enabled the creation of strategy feedback.

Three kinds of feedback were essential to the activity book design. Evaluation feedback is related to the achievement of the task or part of the task. Strategy feedback aims to support the student in the course of task resolution, like scaffolding (Wood et. al., 1976). It is a response to the strategy used by the student. The authors needed to identify (i) configurations of objects that were typical of a strategy and hence enabled a diagnosis and (ii) new objects or actions that could be provided to help the student without changing the nature of the task. Such feedback could consist of help messages, or a graphic enlightening of contradictory elements. Another possibility is to modify the values of didactical variables in order to make the student aware of the current strategy limitations. Direct manipulation feedback is the response of the environment to student action, and may serve the function of either of the previous types of feedback.

The first page of the resultant activity book, shown in Figure 2, is a title page. In page 2, the main objects are presented. The student may interact with these objects, by dragging counters to different positions on the scoreboard and noticing how this affects the score. This is dynamically calculated: one, ten and one hundred for each counter in the green outside region, the purple intermediate region and the orange central region respectively. The aim of the page is to give time for instrumentation to both teachers and students. They can explore interactions with the objects that will constitute the milieu without the constraints of a particular task. It also contains a reset button which, when clicked, replaces counters in their initial positions, and a button which allows students to move on to the next page.

The changing score is direct manipulation feedback that shows students not only the effect of their action, but also that action on one object (moving a counter to a different region) will affect another object (the score). The score is always displayed in some pages, but displayed only after a specific sequence of actions in other pages.

On page 3 the student first receives evaluation feedback. A specific task is given: to reach a score equal to a target number, randomly generated between 1 and 999 (see Figure 1). Clicking on the reset button now in addition generates a new target number. Another new action is that the student may, in addition to comparing whether the score matches the scoreboard, click on a new button for evaluation feedback: a red frowning face if the answer is wrong, and a yellow smiling face if the answer is correct. In case of failure, the student can continue to drag counters and ask for a new evaluation: a new smiley will appear to the right of the previous one. It is important that new feedback is only generated at the student's request: otherwise a trial and error strategy not stemming from mathematical considerations could lead to success.

From page 4 to 7 students are no longer given the direct manipulation feedback of seeing the score. They hence need to take into account the value of the counters in the different regions of the scoreboard to determine the score. "Score" was identified a priori as a possible didactical variable, with two values: visible or hidden.

In page 5, the number of counters is reduced so that, if the target number is over 27, a strategy that consists in placing counters only in the green units region will fail. A strategy which takes into account that a single counter can have another value than 1, i.e. using the inside regions of the scoreboard, is necessary. Therefore, another potential didactical variable is identified: the number of available counters, with two values, 3x9=27 and >27. In page 6, the target number is a multiple of ten, between 10 and 990. As there are enough counters to either leave the green region empty or to fill it with multiples of ten counters, a change of strategy is not necessary. In page 7, however, a single counter is fixed in the green region. Therefore, new strategies are required, involving the placement of a multiple of ten counters into the units region of the scoreboard. The "fixed counter" didactical variable is identified, with four values: no fixed counters, or fixed counters in the units, tens, or hundreds region.

Page 8 contains input boxes for the student to enter the values of a counter in each region of the scoreboard. The aim of this task is to summarize the key idea of the activity book, i.e. that the value of a counter depends on the scoreboard region.

Other pages of the activity book are not devoted to student tasks. The first page is a title page showing an iconic representation of some of the main objects. Pages 9 and 10 contain commentaries for teachers, reporting the main aspect of the task, the evolution between pages, possible student strategies, and also the solution. The didactical variable analysis helps to determine what information is useful.

Trialing the Activity Book

This occurred in spring 2012 in two primary school classes: CE1 with the version presented here and CP (six year old students) with a version where the target number size was limited to 99. Teachers used the activity book as one resource for learning about place value and instrumentalized the book by printing pages to construct related paper and pencil tasks. They were enthusiastic about student engagement, mathematical reasoning and the evolution of strategies, but raised a number of issues.

It was expected that the strong metaphor between the task situation and real scoreboard situations would both provide a meaningful context and minimize the need for instrumentation. Students expected, however, that moving a counter would require tossing it in some way and were initially uncertain about how to do this using the software. Teachers also proposed that instrumentation would be enhanced by modifying page 2 to include a target number chosen either by the teacher according to the constraints of the class, or chosen by students in order to challenge each other.

Some students used the target number update not only to get a new number after finding a previous target but also, unexpectedly, to get a number they knew they were able to deal with, showing the ability to diagnose their level of expertise. This is an example of students' instrumentalization that has led the designers to modify the task in two ways: to provoke the task achievement for each target number, but also, in some pages, to enable the students to choose the target number. The possibility for students to adapt part of the task to their level of expertise is a new, generalizable element in activity book design.

The number of available counters was not a didactical variable for most CE1 students, who used each region of the scoreboard and limited the number of counters they needed to drag. Many of them did not notice the reduced number of counters on page 5 and were surprised to apparently have to solve the same task again. However, for many of the younger CP students who used only the units region of the scoreboard the number of available counters was indeed a didactical variable. The status of page 5 will hence be changed in further developments of the book. Instead of being automatically displayed to CE1 students, it will only be displayed as necessary, i.e. if the unit region is repeatedly filled with many more than 10 counters. The strategy feedback, resulting from our analysis in terms of didactic variables, will consist in reducing the number of counters to better fit the sum of digits of the target number and choosing a target number over 50.

Discussion

Both TDS and IG provide a useful lens to explore aspects of the design of the task. However, according to Prediger *et al.*(2008), there is not an integration between the two theories in our above analysis, but rather a coordination: for example student instrumentalization (described by IG) will affect solving strategies, and hence the milieu and the learning, as described by TDS. Identification of possible instrumental geneses should hence contribute to an a priori analysis in the framework of the TDS. A further issue is that technology design issues, crucial in the Cabri Elem environment, are not readily addressed within TDS.

PART 2: INSTRUMENTAL GENESIS TO DESIGN TECHNOLOGY

In this part, we will first address the potential for IG as a theory of technology design, and then show how an extension to the role of the artefact may both resolve some of the issues in its use in technology design and enable further integration with TDS.

In the field of human-computer interaction, IG is already recognised as a theory for the design of technology (Kaptelinin & Nardi, 2006). General design principles are that artefacts should be designed for efficient transformation into instruments through enabling flexible user modification and through taking into account the real needs of users while appropriating the artefact. It also explores user contribution to design, particularly through instrumentalization. An example of a design principle from Rabardel's (2002) analysis is that in a professional situation action should be easy, safe and reliable, but in a learning situation, action might be constrained in order to promote learning. This has connections to the use of constraint in TDS and is relevant in analyzing the types of action and feedback that should be enabled.

For fifteen years, IG studies concerning technology in the classroom have produced analyses of constraints and possibilities, that could provide a base for a constructive critique of the technologies being used. However, IG in mathematics education has primarily been used to describe the use of existing technologies and to contribute to the design of tasks, by using their constraints to promote student learning (e.g. Fuglestad, 2007, using spreadsheets). It was mainly used to analyze complex environments, like CAS on the TI-92 (Lagrange, 1999), where researchers had no control over the technology. Thus IG has not been much used to explore how such technology might be designed. For instance, Lagrange (2011) discusses the design of Casyopée with no connection to a framework that he was one of the first to use (Lagrange, 1999), and CERME 7 reports on the design and development of new technologies do not mention IG. An interesting exception is the current constructionist exploration of the significance of instrumentalization in design (Healy & Kynigos, 2010).

A means for connecting IG and technology design can be found in a list of artefact affordances that are perceived to enhance instrumentation: to constitute exploration spaces, mediate between formal and informal, provide executable representations, offer dynamic manipulation, evoke interplay between private and public expression and generate interdependent representations (Kynigos *et al.* 2007).

However, the model of the instrument-mediated activity situation has a major shortcoming: the *object-as-thing-acted-on* is typically taken as being independent of the instrumented artefact that mediates the action. Hence, with the focus on the development of the instrument, the *object-as-thing-acted-on* is not problematized. This is appropriate in the ergonomic context in which the model was developed, where typically the subject is construed as a worker using a machine to create or manipulate a product. It is also appropriate at the level of analysis of the teacher-as-subject, whose activity is directed toward the students rather than toward the technological artefact; the entire artefact is part of an instrument used in achieving the *goal* of facilitating student understanding of a particular concept. However, for the student-as-subject the *goal* is given by a task that involves using instruments to interact with objects that are screen representations - and both the means to perform the action and the *objects-as-things-acted-upon* are contained within the same technological artefact.

We will hence consider a technological artefact which provides representations (such as on a screen) as consisting of an environment within which are the means of action and also the objects which are acted upon. The need to consider the environment has been raised both by Hegedus *et al.* (2007) and Trouche (2005). The environment also gives feedback as to the way objects change through interaction.

There is a link to the TDS concept of milieu: milieu is the share of the environment that has a mathematical signification for the student. The counters, target and feedback within the "Target" activity book environment are clearly parts of the milieu, while the button to click to move to the next page is not. Note that the environment provided by an artefact does not constitute the entire environment within which student action takes place. Aspects of the wider environment may form part of the milieu, while aspects of the artefact environment may not be part of the milieu.

Identifying the object, the means of action and the feedback provided as distinct within IG clearly connects the theory more closely to TDS. A question is whether the unique contribution of IG, that of genesis (loosely considered as developing cognitive schemes in order to more effectively meet the goals of the activity) can usefully be applied to the artefact environment and objects as well as to potential instruments. We will address this question by again considering the "Target" activity book.

One aspect of "environmental genesis" is the ability to navigate to different parts of the environment, as required to be able to move between pages in the activity book The title page contains text and images aimed at enabling the student to connect to a familiar activity in a real-world environment: other aspects of environmental genesis might be assimilating the environment to previously encountered environments and developing expectations as to the task and the type of actions relevant to the task.

Objects have a number of roles in the activity book, with different associated geneses. Objects which are icons (such as the reset button) provide a means of action: clicking on such an object will cause a particular action to occur. This action does not affect the icon itself. The "genesis" of such an object is instrumental. A second type of object (such as the counters) may be manipulated directly by means of an instrument (such as dragging) with feedback resulting. Objects (such as the score) cannot be manipulated directly but give feedback as to the result of actions on other objects. Interpreting such feedback requires some form of "meaning" genesis. In contrast to icons, the counters and score will not immediately be experienced as a means of action. But such as these may be linked to form both instruments for action and at the same time representations of mathematical concepts.

During the course of using the activity book, the student progressively develops the instrument of "using counters and target together to change the score" (instrumental genesis). By doing so, the student develops the understanding of the way in which the counters, target and score together constitute a means of action on the representation of a number in place value notation and also develops an understanding of the place value representation of the score.

It should also be noted that the development of the counters, target and score instrument and understanding is specifically facilitated through forcing student strategies to evolve via different types of feedback and changes in the values of different didactical variables: perhaps a consideration of feedback and didactic variables could also form a more prominent part of instrumental genesis in general.

CONCLUSION

In part 1 of this paper we have shown that two theoretical frameworks, TDS and IG, even though coordinated rather than integrated, contributed effectively to the design of a sequence of tasks in the Cabri Elem environment where task and technology design are closely related.

In part 2, it was shown that a technology such as a computer is not only an artefact to be instrumented, but consists of an environment which contains objects, such as representations, which are acted upon and can become means of action. This enables closer links with the TDS concept of milieu, containing the objects which the student will manipulate, as well as the possibilities for action on these objects, when they generate a mathematical meaning.

Consideration of the geneses of the environment and of objects has also been shown to be useful in analyzing the original task and the learning outcomes.

Representation is a major concern in the use of technology in teaching mathematics. Expanding the focus of IG from potential instruments to include objects such as mathematical representations will enable useful links to be made.

The distinction is also important for the designer. Potential instruments have design considerations such as accessibility, whereas representations have different considerations, such as their appearance, behaviour and the feedback given when they are manipulated.

In most software there is much more scope for creating and designing the representations with which students will interact than creating and designing the tools which they will use; this expansion may also be empowering for many teachers and researchers who would like to more actively engage with the design of technology in mathematics education.

NOTES

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