23rd International Congress of Theoretical and Applied Mechanics,
August, 19-24, 2012, Beijing, China

Interfacial cracks in bi-material solids: Stroh formalism and skew-symmetric weight functions

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August 7, 2012

● Outline
● Interfacial cracks: Stroh formalism
● Riemann-Hilbert formulation
● Mirror traction-free problem
● Weight functions
● Decoupling plane and antiplane strain and stress
● Stress intensity factors evaluation
● Plane strain in orthotropic bimaterials I
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Outline

- Interface cracks in anisotropic materials: Stroh formalism;
- Riemann-Hilbert formulation;
- Symmetric and skew-symmetric weight functions;
- Stress intensity factors evaluation;
- Application: point forces applied at crack faces;
- Conclusions;

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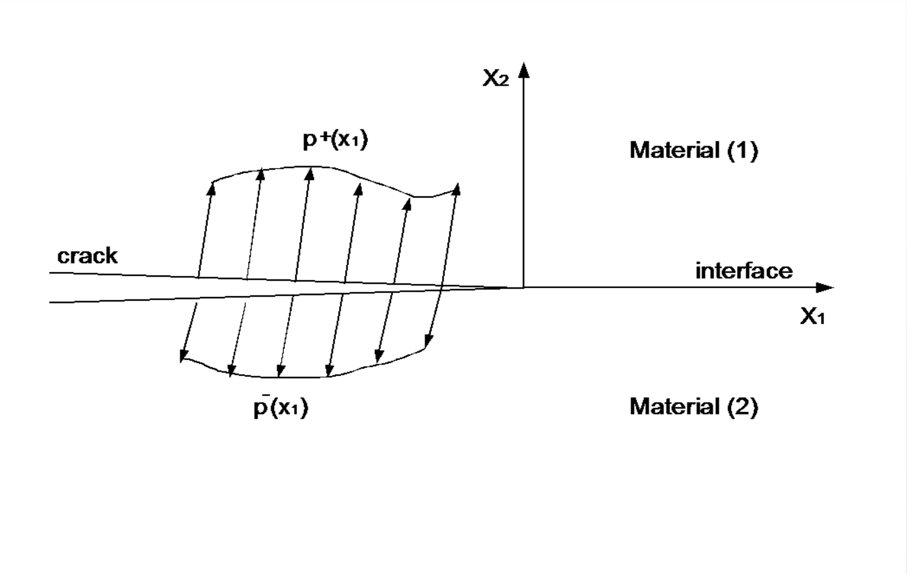
Interfacial cracks: Stroh formalism



- Quasi-static semi-infinite plane interfacial crack with general loading acting on the faces;
- Displacements and tractions in terms of functions of complex variable $z_j = x_1 + \mu_j x_2$:

$$\mathbf{u}_{,1}(x_1, x_2) = 2\text{Re}[\mathbf{A}\mathbf{g}(\mathbf{z})], \quad \mathcal{T}(x_1, x_2) = 2\text{Re}[\mathbf{B}\mathbf{g}(\mathbf{z})],$$

Assuming Stroh representation:

$$[Q_{ik} + (R_{ik} + R_{ki})\mu_j + T_{ik}\mu_j^2]A_{kj} = 0$$

$$B_{ij} = (R_{ki} + \mu_j T_{ik})A_{kj}$$

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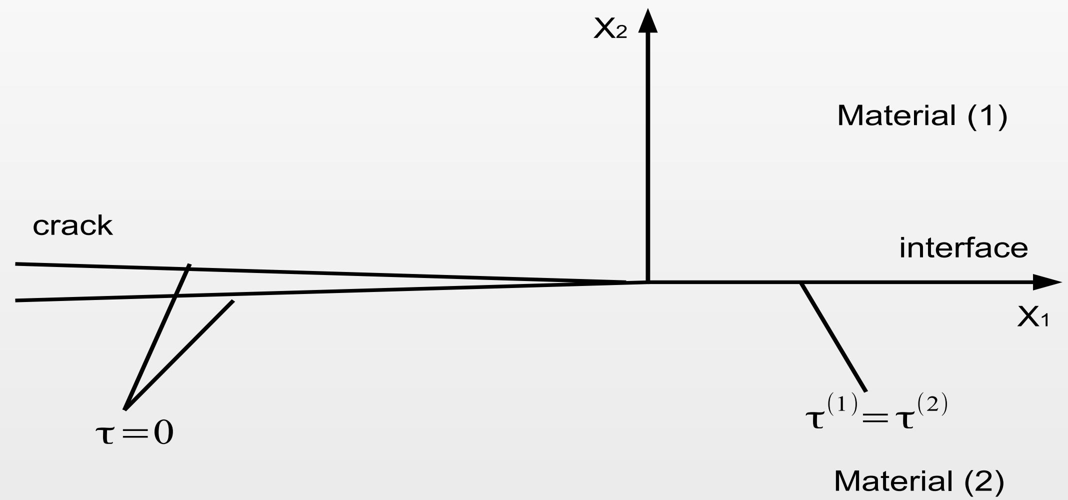
Riemann-Hilbert formulation

Traction-free crack problem:



-Free traction condition at $\boxed{x_1 < 0}$;

-Tractions and displacements continuity at $\boxed{x_1 > 0}$;

-Boundary conditions at the interface yields to a R-H problem:

$$\mathbf{h}^+(x_1) + \overline{\mathbf{H}}^{-1}\mathbf{H}\mathbf{h}^-(x_1) = \mathcal{T}(x_1) \quad \text{for} \quad x_1 > 0$$

$$\mathbf{h}^+(x_1) + \overline{\mathbf{H}}^{-1}\mathbf{H}\mathbf{h}^-(x_1) = 0 \qquad \text{for} \qquad x_1 < 0$$

Where $\mathbf{H} = \mathbf{Y}^{(1)} + \overline{\mathbf{Y}}^{(2)}$ and $\mathbf{Y} = i\mathbf{A}\mathbf{B}^{-1}$;

Mirror traction-free problem

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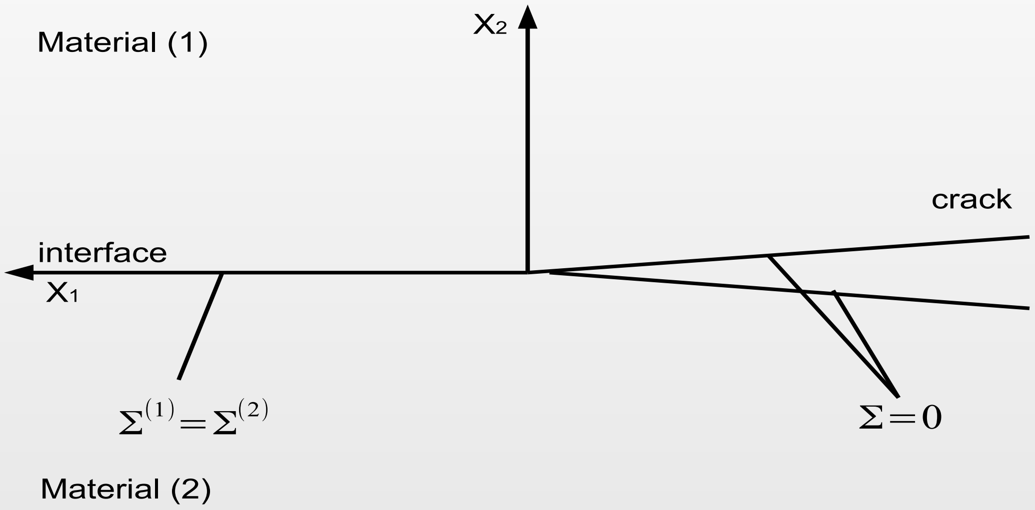
Mirror traction-free crack problem:



-Free traction at $\boxed{x_1 > 0}$;

-Tractions and displacements continuity at $\boxed{x_1 < 0}$;

-At the interface:

$$\mathbf{w}^+(x_1) + \overline{\mathbf{H}}^{-1}\mathbf{H}\mathbf{w}^-(x_1) = 0 \quad \text{for} \quad x_1 > 0$$
$$\mathbf{w}^+(x_1) + \overline{\mathbf{H}}^{-1}\mathbf{H}\mathbf{w}^-(x_1) = \boldsymbol{\Sigma}(x_1) \quad \text{for} \quad x_1 < 0$$

-$\mathbf{U}$ singular solutions of the mirror problem;

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Weight functions

- Symmetric weight functions:

$$[\mathbf{U}](x_1) = \mathbf{U}(x_1, x_2 = 0^+) - \mathbf{U}(x_1, x_2 = 0^-)$$

- Skew-symmetric weight functions:

$$\langle \mathbf{U} \rangle (x_1) = \frac{1}{2}(\mathbf{U}(x_1, x_2 = 0^+) + \mathbf{U}(x_1, x_2 = 0^-))$$

- Mirror traction-free problem is solved in Fourier space;
- A Wiener-Hopf-like equation is derived:

$$[\hat{\mathbf{U}}]^+(\xi) = -\frac{1}{|\xi|}\Big\{\text{Re}\mathbf{H} - i\,\text{sign}(\xi)\,\text{Im}\mathbf{H}\Big\}\hat{\mathbf{\Sigma}}^-(\xi),$$

- The skew-symmetric weight function become:

$$\langle \hat{\mathbf{U}} \rangle(\xi) = -\frac{1}{2|\xi|}\Big\{\text{Re}\mathbf{W} - i\,\text{sign}(\xi)\,\text{Im}\mathbf{W}\Big\}\hat{\mathbf{\Sigma}}^-(\xi), \quad \xi \in \text{R}.$$

Where $\mathbf{H} = \mathbf{Y}^{(1)} + \overline{\mathbf{Y}}^{(2)}$ and $\mathbf{W} = \mathbf{Y}^{(1)} - \overline{\mathbf{Y}}^{(2)}$;

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Decoupling plane and antiplane strain and stress

- Materials where $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{Y}$ and then $\mathbf{H}$ and $\mathbf{W}$ have the following structure are considered:

$$\begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}$$

- Uncoupled inplane and antiplane strain and stresses;
- Monoclinic and orthotropic materials have this property;
- Physical tractions:

$$\boldsymbol{\mathcal{T}}(x_1) = \frac{1}{\sqrt{2\pi x_1}} \mathrm{Re}\, \left(K x_1^{i\varepsilon} \mathbf{w}\right) \Rightarrow \boxed{\text{Mode I and II}}$$

$$\mathcal{T}_3(x_1) = \frac{K_3}{\sqrt{2\pi x_1}} \Rightarrow \boxed{\text{Mode III}}$$

- $K = K_I + iK_{II}$, and $\mathbf{w} = (w_1, w_2)$ is a complex vector;
- K_3 is a real scalar;
- Same behaviour for the singular solution $(\mathbf{\Sigma}, \mathbf{U})$;

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Stress intensity factors evaluation

Betti integral's theorem relates $(\mathbf{u}, \boldsymbol{\mathcal{T}}^{(+)})$ to $(\mathbf{U}, \boldsymbol{\Sigma}^{(-)})$:

- For plane strain:

$$[\hat{\mathbf{U}}]^{+T}\boldsymbol{\mathcal{R}}\hat{\boldsymbol{\mathcal{T}}}^{+} - \hat{\boldsymbol{\Sigma}}^{-T}\boldsymbol{\mathcal{R}}[\hat{\mathbf{u}}]^{-} = -[\hat{\mathbf{U}}]^{+T}\boldsymbol{\mathcal{R}}\langle\hat{\mathbf{p}}\rangle - \langle\hat{\mathbf{U}}\rangle^{T}\boldsymbol{\mathcal{R}}[\hat{\mathbf{p}}]$$

Where $\boldsymbol{\mathcal{R}}$ is the rotation matrix:

$$\boldsymbol{\mathcal{R}} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

- For antiplane strain:

$$[\hat{U}_3]\hat{\mathcal{T}}_3^{+} - \hat{\Sigma}_3[\hat{u}_3]^{-} = -[\hat{U}_3]\,\langle\hat{p}_3\rangle - \langle\hat{U}_3\rangle[\hat{p}_3]$$

- Integral formulas for stress intensity factors:

$$\mathbf{K} = \frac{\boldsymbol{\mathcal{M}}_1^{-1}}{2\pi i}\int_{-\infty}^{\infty}\left\{[\hat{\mathbf{U}}]^{+T}(\tau)\boldsymbol{\mathcal{R}}\langle\hat{\mathbf{p}}\rangle(\tau) + \langle\hat{\mathbf{U}}\rangle^{T}(\tau)\boldsymbol{\mathcal{R}}[\hat{\mathbf{p}}](\tau)\right\}d\tau$$

$$K_3 = \frac{1}{2\pi i\mathcal{K}_{33}}\int_{-\infty}^{\infty}\left\{[\hat{U}_3]^{+}(\tau)\langle\hat{p}_3\rangle(\tau) + \langle\hat{U}_3\rangle(\tau)[\hat{p}_3](\tau)\right\}d\tau$$

Where $\mathbf{K} = (K, \overline{K})^T$;

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- 2D vector problem in orthotropic bimaterials;
- Symmetric bimaterial matrix:

$$\mathbf{H} = \begin{pmatrix} H_{11} & -i\beta\sqrt{H_{11}H_{22}} \\ i\beta\sqrt{H_{11}H_{22}} & H_{22} \end{pmatrix}$$

- Bimaterial parameters:

$$\begin{aligned} H_{11} &= [2n\lambda^{\frac{1}{4}}(\tilde{s}_{11}\tilde{s}_{22})^{\frac{1}{2}}]^{(1)} + [2n\lambda^{\frac{1}{4}}(\tilde{s}_{11}\tilde{s}_{22})^{\frac{1}{2}}]^{(2)}, \\ H_{22} &= [2n\lambda^{-\frac{1}{4}}(\tilde{s}_{11}\tilde{s}_{22})^{\frac{1}{2}}]^{(1)} + [2n\lambda^{-\frac{1}{4}}(\tilde{s}_{11}\tilde{s}_{22})^{\frac{1}{2}}]^{(2)}, \\ \beta\sqrt{H_{11}H_{22}} &= [((\tilde{s}_{11}\tilde{s}_{22})^{\frac{1}{2}} + \tilde{s}_{12})]^{(2)} - [((\tilde{s}_{11}\tilde{s}_{22})^{\frac{1}{2}} + \tilde{s}_{12})]^{(1)}, \end{aligned}$$

Where: $\lambda = \frac{\tilde{s}_{11}}{\tilde{s}_{22}}, \quad \rho = \frac{1}{2}\frac{2\tilde{s}_{12}+\tilde{s}_{66}}{\sqrt{\tilde{s}_{11}\tilde{s}_{22}}}, \quad n = \left(\frac{1}{2}(1+\rho)\right)^{\frac{1}{2}},$

- Generalized Dundurs parameter connected to oscillatory index:

$$\varepsilon = \frac{1}{2\pi}\ln\left(\frac{1-\beta}{1+\beta}\right)$$

- Homogeneous material $\Rightarrow \beta, \varepsilon = 0$, no oscillations;

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- Skew-symmetric bimaterial matrix:

$$\mathbf{W} = \mathbf{Y}^{(1)} - \overline{\mathbf{Y}}^{(2)} = \begin{pmatrix} \delta_1 H_{11} & i\gamma\sqrt{H_{11}H_{22}} \\ -i\gamma\sqrt{H_{11}H_{22}} & \delta_2 H_{22} \end{pmatrix}$$

- Bimaterial parameters:

$$\delta_1 = \frac{[2n\lambda^{\frac{1}{4}}(\tilde{s}_{11}\tilde{s}_{22})^{\frac{1}{2}}]^{(1)} - [2n\lambda^{\frac{1}{4}}(\tilde{s}_{11}\tilde{s}_{22})^{\frac{1}{2}}]^{(2)}}{H_{11}},$$

$$\delta_2 = \frac{[2n\lambda^{-\frac{1}{4}}(\tilde{s}_{11}\tilde{s}_{22})^{\frac{1}{2}}]^{(1)} - [2n\lambda^{-\frac{1}{4}}(\tilde{s}_{11}\tilde{s}_{22})^{\frac{1}{2}}]^{(2)}}{H_{22}},$$

$$\gamma = \frac{[((\tilde{s}_{11}\tilde{s}_{22})^{\frac{1}{2}} + \tilde{s}_{12})]^{(1)} + [((\tilde{s}_{11}\tilde{s}_{22})^{\frac{1}{2}} + \tilde{s}_{12})]^{(2)}}{\sqrt{H_{11}H_{22}}},$$

- Homogeneous material $\Rightarrow \delta_1, \delta_2 = 0$, but $\boxed{\gamma \neq 0}$ then even in homogeneous case we have non-zero skew-symmetric weight functions;

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Plane strain in orthotropic bimaterials: weight functions

- Fourier transform symmetric and skew-symmetric weight functions:

$$[\hat{\mathbf{U}}]^+ = -\frac{\sqrt{H_{11}H_{22}}}{|\xi|}\begin{pmatrix} \sqrt{\frac{H_{11}}{H_{22}}} & i\beta\,\mathrm{sign}(\xi) \\ -i\beta\,\mathrm{sign}(\xi) & \sqrt{\frac{H_{22}}{H_{11}}} \end{pmatrix}\hat{\boldsymbol{\Sigma}}^-(\xi);$$

$$\langle\hat{\mathbf{U}}\rangle = -\frac{\sqrt{H_{11}H_{22}}}{2|\xi|}\begin{pmatrix} \delta_1\sqrt{\frac{H_{11}}{H_{22}}} & -i\gamma\,\mathrm{sign}(\xi) \\ +i\gamma\,\mathrm{sign}(\xi) & \delta_2\sqrt{\frac{H_{22}}{H_{11}}} \end{pmatrix}\hat{\boldsymbol{\Sigma}}^-(\xi);$$

- Inverting these expressions we get $[\mathbf{U}]$ and $\langle\mathbf{U}\rangle$.
- Since Mode I and II are coupled:

$$\mathbf{U} = \begin{pmatrix} U_1^1 & U_1^2 \\ U_2^1 & U_2^2 \end{pmatrix}, \boldsymbol{\Sigma} = \begin{pmatrix} \Sigma_1^1 & \Sigma_1^2 \\ \Sigma_2^1 & \Sigma_2^2 \end{pmatrix}$$

- $[\hat{\mathbf{U}}] \Rightarrow$ Wiener-Hopf equation;

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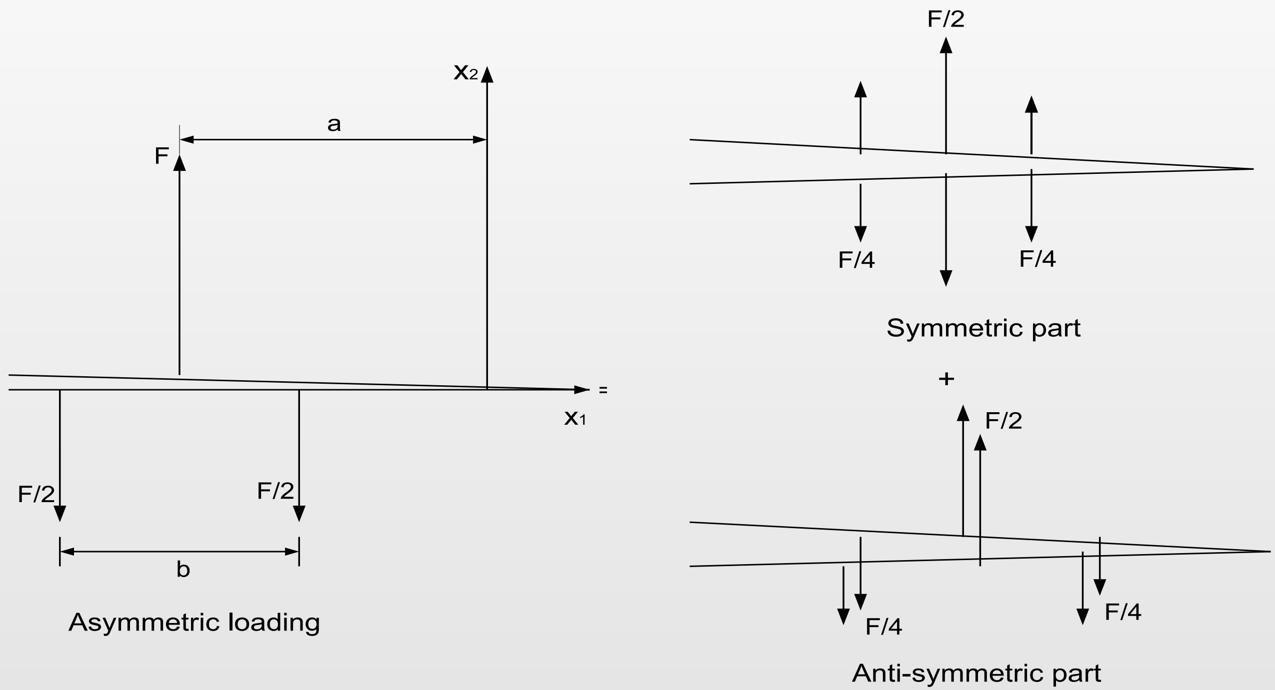
Asymmetric loading

Asymmetric point forces acting on the crack faces:



$$\langle p_2\rangle(x_1) = -\frac{F}{2}\delta(x_1+a) - \frac{F}{4}\delta(x_1+a+b) - \frac{F}{4}\delta(x_1+a-b)$$

$$[p_2]\,(x_1) = -F\delta(x_1+a) + \frac{F}{2}\delta(x_1+a+b) + \frac{F}{2}\delta(x_1+a-b)$$

Symmetric and skew-symmetric components $\Rightarrow K = K^S + K^A$;

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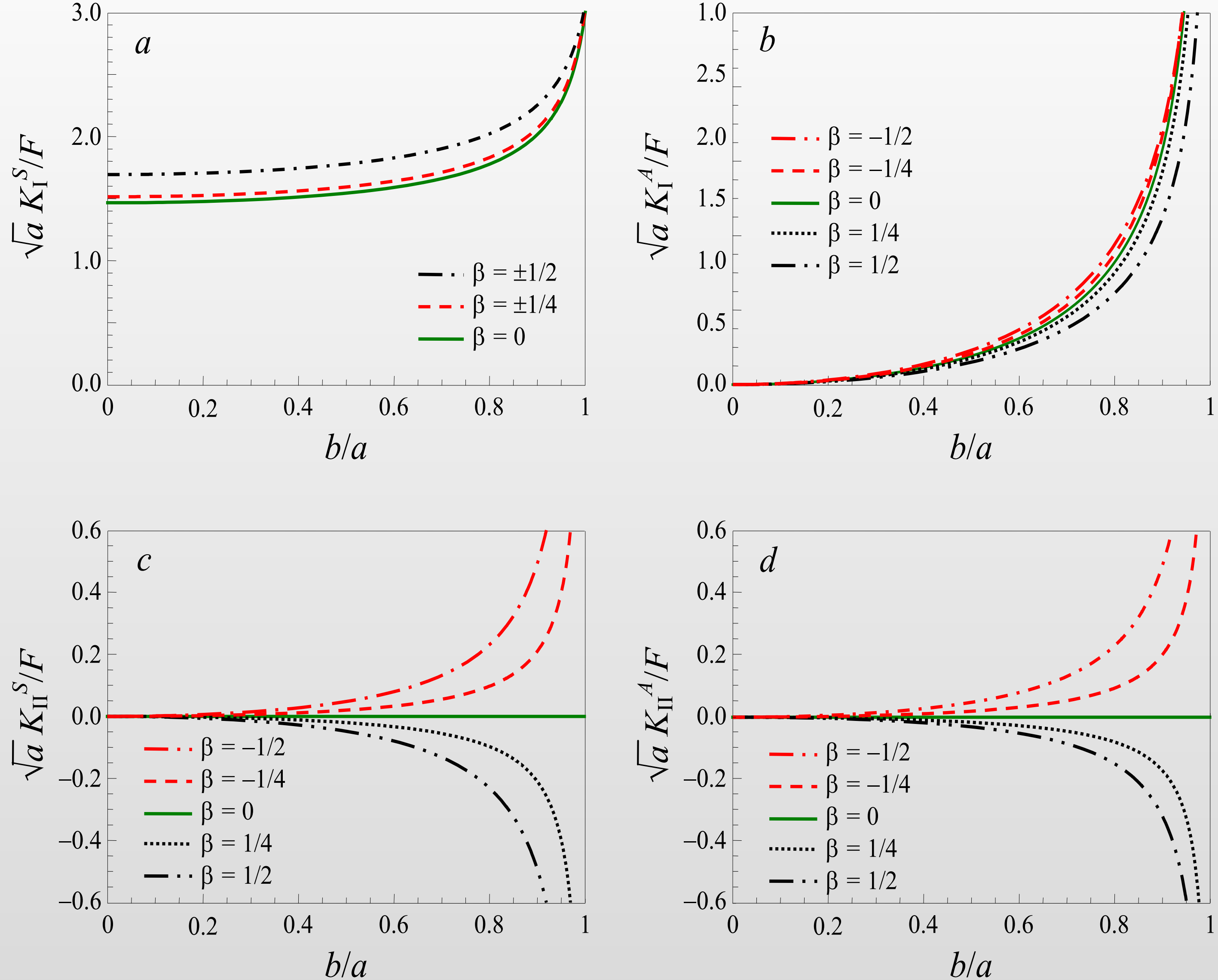
Plane strain: SIF



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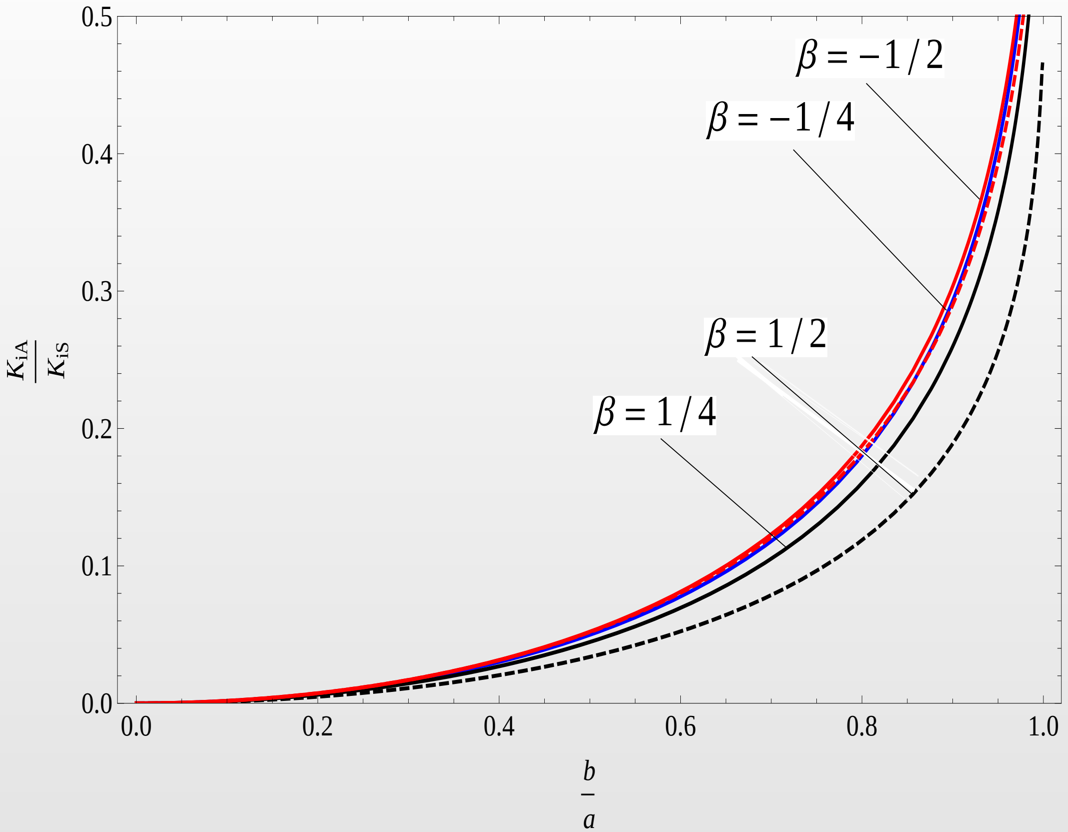
Symmetric vs skew-symmetric SIF



- As $b/a \to 1$ increase, $\boxed{K_I^A \approx 40\% - 50\% \quad \text{of} \quad K_I^S}$;
- Skew-symmetric part of the loading is not negligible and needs to be taken into account;

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Antiplane strain: weight functions

- Anisotropic materials with symmetry plane at $x_3 = 0$ are considered;
- Fourier transform of weight functions:

$$[\hat{U}_3](\xi) = -\frac{H_{33}}{|\xi|}\hat{\Sigma}_3(\xi); \quad \langle \hat{U}_3 \rangle(\xi) = \frac{\eta}{2}[\hat{U}_3](\xi);$$

- Inverting we obtain:

$$\boxed{[U_3](x_1) = \frac{H_{33}}{\sqrt{2\pi}} x_1^{\frac{1}{2}}; \quad \langle U_3 \rangle(x_1) = \frac{\eta}{2\sqrt{2\pi}} H_{33} x_1^{\frac{1}{2}};}$$

- $H_{33} = \left[\sqrt{\tilde{s}_{44}\tilde{s}_{55} - \tilde{s}_{45}^2}\right]^{(1)} + \left[\sqrt{\tilde{s}_{44}\tilde{s}_{55} - \tilde{s}_{45}^2}\right]^{(2)};$
  $\eta = \left(\left[\sqrt{\tilde{s}_{44}\tilde{s}_{55} - \tilde{s}_{45}^2}\right]^{(1)} - \left[\sqrt{\tilde{s}_{44}\tilde{s}_{55} - \tilde{s}_{45}^2}\right]^{(2)}\right) / H_{33}$
- Homogeneous material $\Rightarrow \eta = 0$, Mode III is symmetric;

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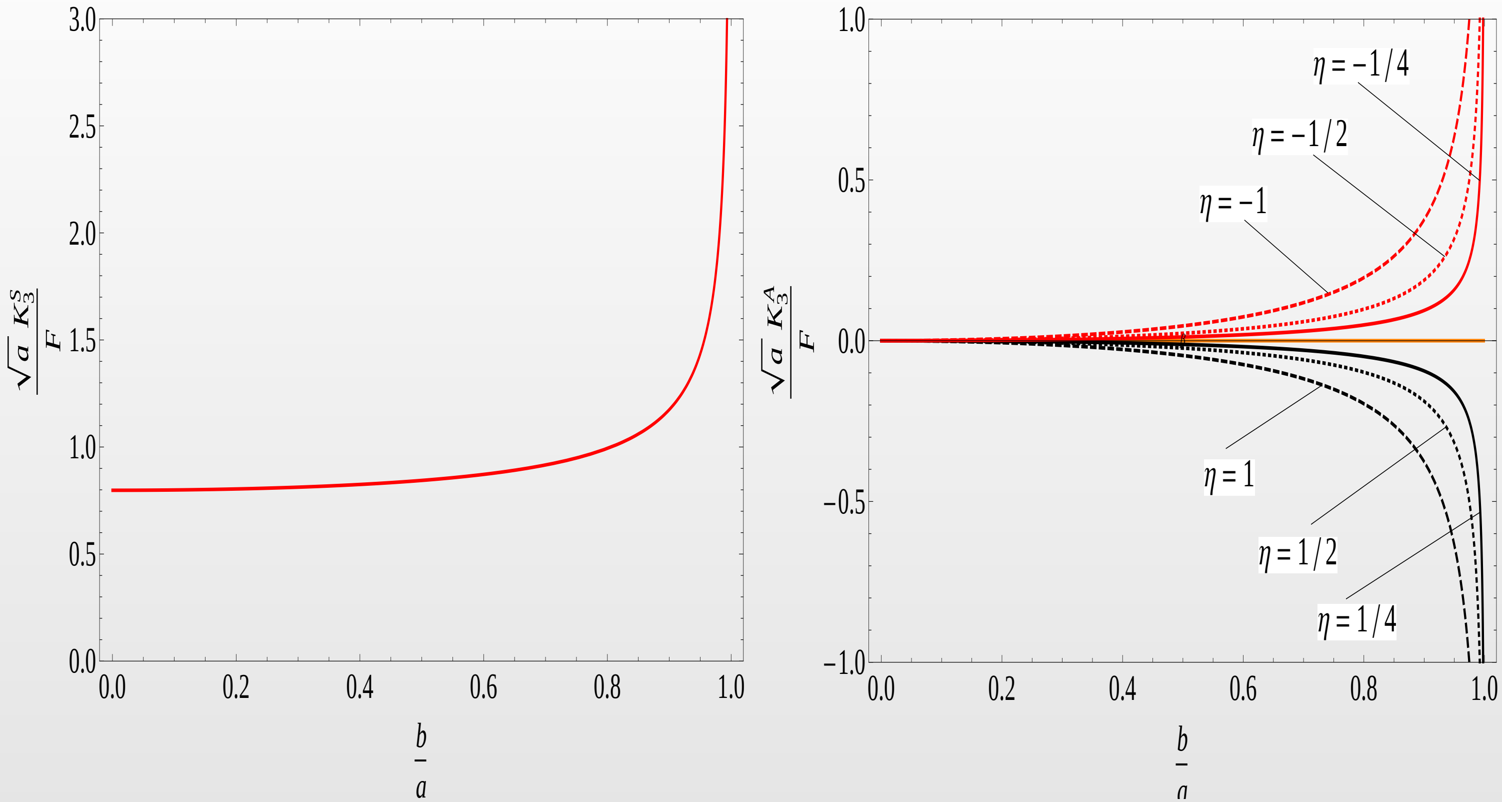

- Same loading configuration directed along x_3:

$$\langle p_3 \rangle (x_1) = -\frac{F}{2}\delta(x_1 + a) - \frac{F}{4}\delta(x_1 + a + b) - \frac{F}{4}\delta(x_1 + a - b)$$

$$[p_3](x_1) = -F\delta(x_1 + a) + \frac{F}{2}\delta(x_1 + a + b) + \frac{F}{2}\delta(x_1 + a - b)$$

- Homogeneous material $\Rightarrow \eta = 0$, K_3 is symmetric;

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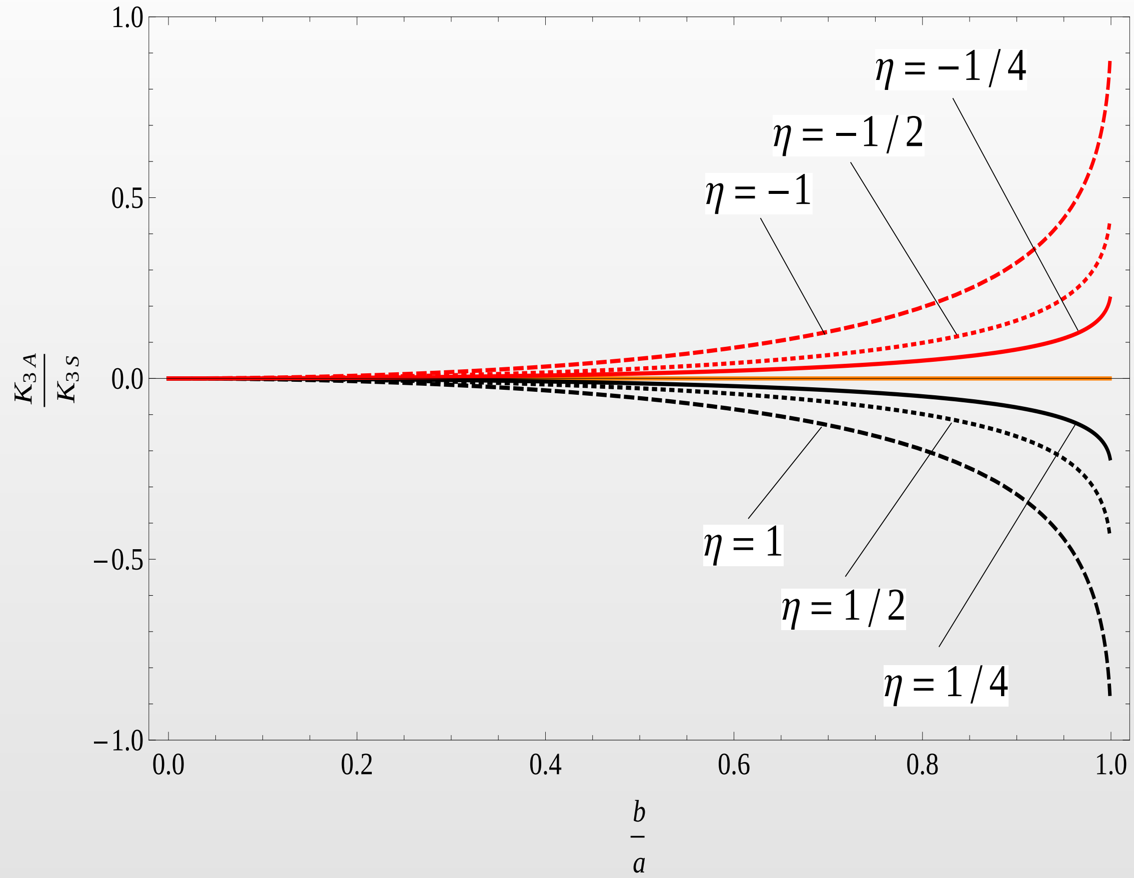

- As for plane strain, K_3^A increase with b/a, expecially for $|\eta| > 1/2$;
- For $b/a > 0.5$, skew-symmetric part of the loading is not negligible;

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Conclusions

- A new general approach for deriving the weight functions for 2D interfacial cracks in anisotropic bimaterials has been developed;
- For perfect interface conditions, the new method avoid the use of Wiener-Hopf technique and the challenging factorization problem connected;
- Both symmetric and skew-symmetric weight functions can be derived by means of the new approach;
- Weight functions can be used for deriving singular integral formulation of interfacial cracks in anisotropic media;
- The proposed method can be applied for studying interfacial cracks problems in many materials:monoclinic, orthotropic, cubic, piezoelectrics, poroelastics, quasicrystals;

Furter developments:
-Applications to steady state moving cracks and wavy cracks;
-Analysis of inclusions effects of interface cracks propagation;
-Extension to 3D case;

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