On Reverse Super Vertex-Magic Labeling

U.Masthan Raju
Research Scholar, Department of Mathematics
Rayalaseema University
Kurnool, AP, India
e-mail:masthan.u@gmail.com

Shaik Sharief Basha
Department of Mathematics
Vellore Institute of Technology
Vellore, TN, India
e-mail: shariefbasha.s@vit.ac.in

Abstract—For a graph *G*(*V, E*) an injective mapping *f* from $V \cup E$ to the set $\{1,2,3,...,v+\varepsilon\}$ is a reverse vertex-magic labeling if there is a constant *h* so that for every vertex $v \in V$, $f(v) - \sum f(uv) = h$ where the difference runs over all vertices *u* adjacent to *v* . A vertex-magic labeling *f* is called super vertex-magic labeling if $f(E) = \{1,2,3,...,\varepsilon\}$ and $f(V) = \{\varepsilon+1, \varepsilon+2,...,\varepsilon+v\}$. A graph *G* is called a reverse super vertex-magic if there exists a reverse super vertex-magic labeling of *G*. In this paper, we established some properties of reverse super vertex magic trees and exhibit reverse super vertex-magic labeling of a kite graph..

Keywords-*reverse Vertex-magic labeling, reverse super vertex-magic labeling, kite graph.*

*****

I. Introduction

In this paper, we consider only undirected finite simple graph. The graph *G* has vertex set *V* = *V*(*G*) and edge set *E* = *E*(*G*) and we take $v = |V(G)|$ and $\varepsilon = |E(G)|$.
MacDougall, Miller, Slamin and Wallis [1] introduced the notion of a vertex magic total labeling in 1999. For a graph *G*(*V, E*) an injective mapping *f* from $V V \cup E[\ E$ to the set $\{1,2,3,...,v+\varepsilon\}$ is a vertex-magic total labeling if there is a constant *h* so that for every vertex $v \in V$, $f(v) + \sum f(uv) = k$ *h* where the sum runs over all vertices *u* adjacent to *v*. A vertex-magic labeling *f* is called super vertex-magic [2] labeling if $f(E) = \{1,2,3,...,\varepsilon\}$ and $f(V) = \{\varepsilon+1, \varepsilon+2, \varepsilon+3,..., \varepsilon+v\}$. A graph *G* is called a super vertex-magic if there exists a super vertex-magic labeling of *G*.
In [5], S.VenkataRamanaetalintroduced the concept of reverse super vertexmagic labeling of a graph. A reverse vertex-magic labeling f is a bijection f from $V \cup E$ onto the integers {1,2,3,……, *v* + *ε*} such that for all vertex u, f[N(u)] - f(u)is a constant.
A reverse vertex-magic labeling f is called reverse super vertex-magic labeling if f(E) ={ 1,2,3,……, *ε*} and f(V) = { *ε*+1, *ε*+2, *ε*+3,……, *ε*+ *v* } . A graph G is called reverse super vertex-magic if there exists a reverse super vertex-magic labeling of G.

II. Main Results

Theorem 1. *No reverse super vertex-magic graph has two or more isolated vertices or an isolated edge.*

Proof. If *f* is a reverse super vertex-magic labeling of a graph *G* with constant *k* then any isolated vertex *x* has a label *f* (*x*) = k. So, there cannot be two such vertices.

Suppose there is an isolated edge *xy*. Then *f* (*xy*) – *f(x)* = *f* (*xy*)- *f* (*y*) = k. Hence *f* (*x*) = *f* (*y*) which is a contradiction. Hence there is no isolated edge.

Theorem 2. Let T be a tree with n internal vertices and tn leaves. Then T does not admit a reverse super vertex-magic labeling if $t > \frac{(3n+1)}{n}$.

Proof:- If *T* has *n* internal vertices and *tn* leaves then $v = (t+1)n$ and $\varepsilon = tn + n - 1$. So the labels used for the edges are $\{1,2,3,...,tn+n-1\}$ and for the vertices are $\{tn+n, tn+n+1,...,2tn+2n-1\}$. The maximum possible sum of weights on the leaves is

$$[(tn+2n-1+1)+(tn+2n-1+2)+...+(tn+2n-1+tn)]$$
$$-[(n-1+1)+(n-1+2)+...+(n-1+tn)]$$
$$= \left[tn(tn+2n-1) + \frac{tn(tn+1)}{2}\right] - \left[(n-1)tn + \frac{tn(tn+1)}{2}\right]$$

$$= tn(tn+n)$$

Since there are *tn* leaves, we get

$$tnk \le tn(tn+n)$$

$$k \le tn+n \qquad \rightarrow (1)$$

On the other hand, the minimum possible sum of weights on the internal vertices occurs when the smallest labels $\{1,2,3,...,n-1\}$ are assigned to internal edges (because they will be added twice), the remaining edges are assigned to the labels $\{n, n+1, n+2,..., \varepsilon\}$ and the remaining vertices are assigned to the labels $\{\varepsilon+1, \varepsilon+2,..., \varepsilon+n\}$. Hence the minimum possible sum of weights on the internals is

$$= \left[2(1+2+...+n-1) + (n+n+1+...+\varepsilon\right]$$
$$-\left[\varepsilon+1+\varepsilon+2+...+\varepsilon+n\right]$$

$$= \frac{n(n-1)}{2} + \frac{(\varepsilon+n)(\varepsilon+n+1)}{2}$$
$$-2\left[n\varepsilon + \frac{n(n+1)}{2}\right]$$
$$= \frac{n(n-1)}{2} + \frac{(tn+2n-1)(tn+2n)}{2}$$
$$-2\left[n(tn+n-1) + \frac{n(n+1)}{2}\right]$$
$$= \frac{n}{2}\left[t^2n+(4n-)t+5n-3\right]-\left[2n(tn+n-1)+n(n+1)\right]$$

$$= \frac{n}{2}\left[t^2n+(4n-)t+5n-3-4tn-4n+4-2n-2\right]$$
$$= \frac{n}{2}\left[t^2n-t-n-1\right]$$

Since there are *n* internal vertices,

$$nk \geq \frac{n}{2}\left[t^2n-t-n-1\right]$$
$$k \geq \frac{1}{2}\left[t^2n-t-n-1\right] \qquad \rightarrow (2)$$

Therefore no labeling will be possible when

$$\frac{1}{2}\left[t^2n-t-n-1\right] > tn+n$$

That is, when $t^2n-(2n+1)t-(3n+1)>0$

$$t > \frac{(2n+1)+\sqrt{(2n+1)^2+4n(3n+1)}}{2n}$$
$$= \frac{2n+1+4n+1}{2n} = \frac{3n+1}{n}$$

Theorem 3. If ¢ is the largest degree of any vertex in a tree T with º vertices and " edges then T does not admit a super vertex-magic labeling wherever $\Delta > \dfrac{-1+\sqrt{1+16\nu}}{2}$.

Proof. Let c be the vertex of maximum degree ¢. The minimum possible weight of c is $\varepsilon+1-(1+2+3+...+\Delta)$. Therefore,

$$k \geq \frac{\Delta(\Delta+1)}{2} - (\varepsilon+1)$$
$$k \geq \frac{\Delta(\Delta+1)}{2} - \nu \qquad \rightarrow (3)$$

Since there is an internal vertex of degree Δ there are at least ε leaves in T. So the maximum possible sum of weights on the leaves is at most the sum of the Δ largest labels from f (E) and the Δ largest labels from f (V) . Hence,

$$\Delta k \leq [(\varepsilon+\nu-\Delta+1)+(\varepsilon+\nu-\Delta+2)+...+(\varepsilon+\nu-\Delta+\Delta)]$$
$$-\left[(\varepsilon-\Delta+1)+(\varepsilon-\Delta+2)+...+(\varepsilon-\Delta+\Delta)\right]$$
$$= \left[(\varepsilon+\nu-\Delta)\Delta + \frac{\Delta(\Delta+1)}{2}\right] - \left[\Delta(\varepsilon-\Delta) + \frac{\Delta(\Delta+1)}{2}\right]$$
$$= \Delta\nu$$
$$k \leq \nu$$

So labeling will be impossible whenever

$$\nu < \frac{\Delta(\Delta+1)}{2} - \nu$$

.

That is, when $\Delta^2+\Delta-4\nu>0$

$$\Delta > \frac{-1+\sqrt{1+16\nu}}{2}$$. ¤

Remark. The following table shows the maximum degree permitted by the restriction given in Theorem 3 for some small values of ν .

ν	3	5	7	9	11
Δ	2	3	3	4	5

Theorems 2 and 3 do not provide sufficient condition for a graph to be a reverse super vertex-magic, since we can prove that there is a tree with 7 vertices and Δ = 3 shown in Figure 1, which does not admit any reverse super vertex magic labeling.

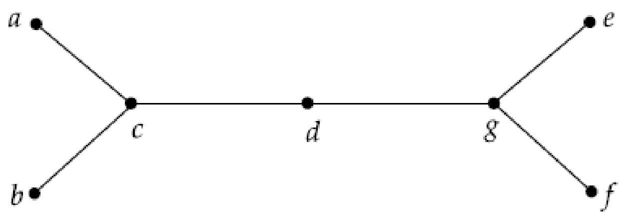

Figure 1

The reason is as follows: The vertex sum varies from 3 to 9. Since the minimum vertex sum itself is 3, the labels 1 and 2 can be assigned only to the internal edges. Therefore, the vertex sum of *d* is 3. The remaining labels 3, 4, 5, 6 are assigned to the edges *ac*, *be*, *ge*, *g f* . Hence one of the leaves must have a vertex sum 3, which contradicts the fact that vertex sums are consecutive integers.

Theorem 4. Let G be a graph obtained by joining a pendant vertex with a vertex of degree 2 of a comb graph. Then G admits reverse super vertex-magic labeling.

Proof. Let the vertex $V=\{a_1,a_2,a_3,...,a_t\}\cup\{a_{11},a_{12},a_{21},a_{31},...,a_{t1}\}$ and the edge set $E=\{a_1a_{11},a_1a_{12},a_2a_{21},...,a_ta_{t1}\}\cup\{a_ia_{i+1}:1\leq i\leq t-1\}$.

Here $\nu=2t+1$ and ε = 2t . Define $f:E\rightarrow\{1,2,3,...,\varepsilon\}$ as follows

$$f(a_ia_{i+1}) = t-i \ \ if \ 1\leq i\leq t-1$$

 ISSN: 2321-8169
30 - 33

$f(a_1 a_{12}) = t$,

$f(a_i a_{i1}) = t + i \text{ if } 1 \le i \le t$

The vertex labelings are as follows:

$f(a_{i1}) = 2t + 1 + i \quad \text{if } 1 \le i \le t$

$f(a_{12}) = 2t + 1$

$f(a_i) = 4t + 2 - i \quad \text{if } 1 \le i \le t$

It can be easily verified that f is a reverse super vertex-magic labeling with a reverse vertex-magic constant $k = t + 1$.

Example. Example of a reverse super vertex-magic labeling of a graph G with h = 26 is given in Figure 2.

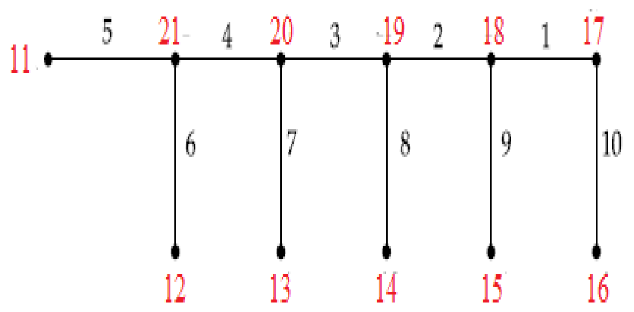

Figure 2

Definition. An (n, t) -kite graph consists of a cycle of length n with a t-edge path (the tail) attached to one vertex of a cycle

Theorem 5. An (n, t) -kite graph admits a reverse super vertex-magic labeling iff n + t is odd.

Proof. Let G be an (n, t) -kite graph. Let the vertex set $V = \{v_1, v_2, v_3, \ldots, v_n\} \cup \{u_1, u_2, u_3, \ldots, u_t\}$ and the edge set

$$E = \{e_i = v_i v_{i+1}, e_n = v_n v_1 : 1 \le i \le n-1\}$$
$$\cup \{x_i = u_i u_{i+1}, x_t = u_t v_1 : 1 \le i \le t-1\}.$$

Hence $v = \varepsilon = n + t$.

Suppose G admits a reverse super vertex-magic labeling f with a reverse super vertex-magic constant k. Then k=   , as kis an integer $v = n + t$, must be odd.

Conversely assume that v is odd. Hence either *n* or *t* is odd .We consider two cases

Case (i) *n* is odd and *t* is even.

Define $f : V \cup E \to \{1, 2, 3, \ldots, 2n + 2t\}$ as follows: For $1 \le i \le n$,

$$f(e_i) = \begin{cases} \dfrac{t+i+1}{2} & \text{if } i \text{ is odd} \\ t + \dfrac{n+1+i}{2} & \text{if } i \text{ is even} \end{cases}$$

For $1 \le i \le t$,

$$f(x_i) = \begin{cases} \dfrac{t+n+1}{2} + \dfrac{i+1}{2} & \text{if } i \text{ is odd} \\ \dfrac{i}{2} & \text{if } i \text{ is even} \end{cases}$$

The vertex labelings are as follows:

$f(v_i) = n + 2t + i \qquad \text{if } 1 \le i \le n,$

$f(u_i) = n + t + 2 - i \qquad \text{if } 1 \le i \le t.$

It can be easily verified that *f* is a super vertex-magic labeling of *G* with $k = \dfrac{n+t-1}{2}$.

.

Case (ii) *n* is even and *t* is odd. We consider two sub cases.

Subcase (i) *t* > *n*. For $1 \le i \le n$

$$f(e_i) = \begin{cases} n - \dfrac{t-1}{2} & \text{if } i \text{ is odd} \\ \dfrac{3n+t+1}{2} - \dfrac{i}{2} & \text{if } i \text{ is even} \end{cases}$$

and for $1 \le i \le t$,

$$f(x_i) = \begin{cases} \dfrac{3n+t+i}{2} & \text{if } i \text{ is odd} \\ \dfrac{i-(t-n)}{2} & \text{if } i = t-n+2, t-n+4, \ldots, t \\ n + \dfrac{i}{2} & \text{if } i \text{ is even} \end{cases}$$

The vertex labelings are as follows:

$f(u_1) = t + 2n$,

$$f(u_i) = \begin{cases} 2t + n + i & 2 \le i \le t - n + 1 \\ 2n - 1 + i & t - n + 2 \le i \le t \end{cases}$$

$f(v_i) = 2t + n - i, \quad \text{for } 1 \le i \le n.$

It can be easily verified that *f* is a super vertex-magic labeling of *G* with $k = \dfrac{n+t-1}{2}$.

Subcase (ii) $t \le n$. For $1 \le i \le n$

$$f(e_i) = \begin{cases} \dfrac{t-i}{2} & \text{if } i = 1, 3, \ldots, t-2 \\ \dfrac{3t+2n+1}{2} - \dfrac{i+1}{2} & \text{if } i = t, t+2, \ldots, n \\ t + \dfrac{n}{2} - \dfrac{i}{2} & \text{if } i = 2, 4, \ldots, n \end{cases}$$

$$f(x_i)=\begin{cases} t-\dfrac{i}{2} & if\ \ i=2,4,\ldots,t-1,\ 1\le i\le t \\ \dfrac{3t+n+1}{2}-\dfrac{i+1}{2} & if\ \ i=1,3,\ldots,t,\ 1\le i\le t \end{cases}$$

The vertex labelings are as follows:

$$f(v_1)=2n+t+1,$$

$$f(v_i)=\begin{cases} n+2t-i & if\ \ 2\le i\le t-1 \\ 3n+2t-i & if\ \ t\le i\le n \end{cases}$$

$$f(u_1)=n+t+4$$

$$f(u_i)=2n+t+2-i,\ \ if\ 2\le i\le t.$$

It can be easily verified that *f* is a super vertex-magic labeling with $k=\dfrac{n+t-1}{2}$.

Example. Example of a super vertex-magic labeling of a kite graph with *n* = 5 and *t* = 8 is given in Figure 3.

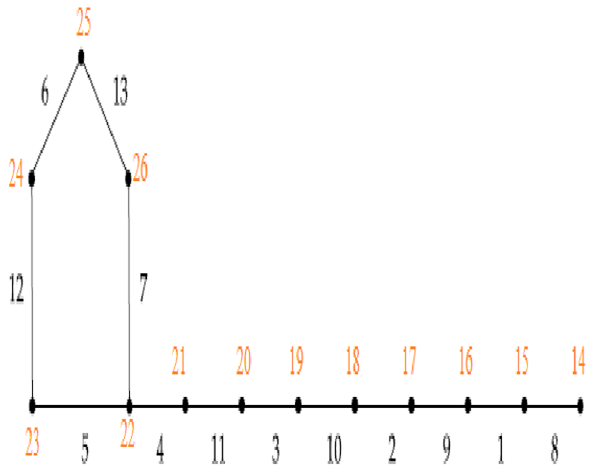

Figure 3

Example. Examples of a super vertex-magic labeling of a kite graph with n = 4, t = 9 ($t > n$) , and n = 8, t = 5 ($t < n$) are given in Figure 4.

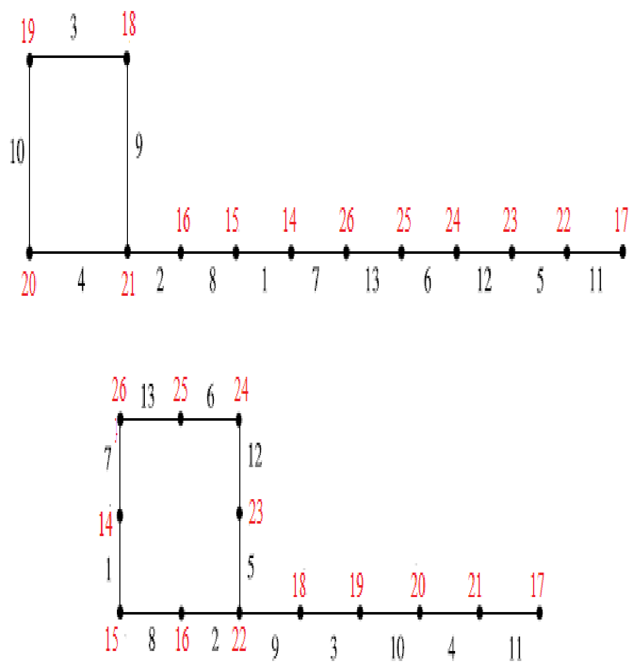

Figure 4

III. CONCLUSION

According to result and discussion we found the reverse super vertex *magic valuation* of the (n,t)-kite *graph*is $k=\dfrac{n+t-1}{2}$, for *n* is odd,*t* is even and *n* is even, *t* is oddfor n odd where $t\le n$ and t >*n*.

REFERENCES

[1]. J. A. MacDougall, M. Millar, Slamin and W. D. Wallis, Vertex-magic total labelings of Graphs, Utilitas Math., Vol. 61 (2002), pp. 3–21.

[2]. M. Millar, J. A. MacDougall, Slarnin and W. D. Walls, Problems in magic total labeling, In Proceedings of AWOCA'99, (1999), pp. 19–25.

[3]. I. D. Gray, J. MacDougall, J. P. McSorley and W. D. Wallis, Vertex magic labeling of trees and forests, Discrete Mathematics and Combinatorics Commons Published in Discrete Mathematics, 26(1-3), pp.285-298.

[4]. S. ShariefBasha , K. Madhusudhan Reddy & MD.Shakeel,Algorithm To Construct Reverse Super Vertex Magic Labeling Of Complete Graphs, Global Journal of Pure and Applied Mathematics, Volume 11, Number 1 (2015), pp. 1-7.

[5]. V. Swaminathan and P. Jeyanthi, Super vertex-magic labeling, Indian J. Pure Appl. Math., Vol. 34 (6) (June 2003), pp. 935–939.

[6]. S.VenkataRamana ,S.Shareefhbasha, Reverse super vertex-magic labeling of a graph, Ph.D. Thesis.

[7]. S. Venkata Ramana and S.Sharief Basha, Reverse Super Vertex-Magic Strength of a Graph, Bulletin of pure & Applied Sciences (Mathematics), New Delhi, Volume:2 2006, pp.317-323.