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Performance Analysis of MUSIC and Smooth MUSIC Algorithm for DOA Estimation

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Abstract— Smart Antenna Systems is one amongst speedily developing areas of wireless communication. With effective direction of arrival (DOA) and Beam forming techniques Smart Antenna Systems persuade is most effective in terms of quality of signals in wireless communication. This paper analyzed and compares the performance of MUSIC and Smooth MUSIC DOA estimation algorithm on the uniform linear array (ULA) which are used in design of smart antenna system. MUSC algorithm is high resolution subspace based method which is used for DOA estimation of uncorrelated signals while smoothing of MUSIC is introduced for DOA estimation of completely correlated signal. The angular resolution of DOA estimation techniques improves as number of elements in array, snapshots and values of SNR increases.

Keywords- *Direction of arrival (DOA), Smart Antenna system, MUSIC, Smooth MUSIC, Uniform Linear Array (ULA)*

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I. INTRODUCTION

The demand for wireless communication has been dramatically increases from few years as wireless communication systems are evolving from the second generation systems to the third and fourth generation systems, which will provide high data rate multimedia services [1]. Adaptive Antenna Array also known as Smart Antenna has been used widely for the estimation of Direction-of-Arrival (DOA) of incident source signals is capable of adjusting its radiation pattern according to the surroundings. This is typically done by combining the incident signals from array elements to form a steering beam proportional to the target signal information. This can increase the performance characteristics such as capacity of a wireless system which can effectively reducing multipath and co-channel interference. A smart antenna takes advantage of diversity effect at the transmitter and receiver or both.

Smart Antennas used to identify the Direction of arrival (DOA) of the signal that is required to trace and find the intended mobile set. Direction of arrival (DOA) estimation is the process of estimating the direction of an incident signal from mobile devices to the Base Transceiver Station. In this way, Signal-to-Interference-and-Noise Ratio (SINR) improves by producing nulls along the direction of interference. Hence Direction of arrival for incident signals is an important processing step in many sensors systems, i.e., radar, sonar, Measure Electronic Surveillance (MSE), submarine acoustics, geodesic location, optical interferometry, etc. [2]

There are many types of DOA algorithms that have been proposed such as conventional spectral-based, subspace spectral-based and statistical methods are used for DOA estimation. Conventional method for DOA estimation is based on the concepts of beamforming and null-steering. This technique electronically steer beams in all directions and look for the peaks in the output power. The sum and delay or Bartlett [3] and Minimum Variance Distortionless Response (MVDR) or Capon [4] etc. are conventional spectral-based method. These methods are highly dependent on physical size of an array antenna [4], require low computational complexity [4] but these methods have low resolution and accuracy that leads to the introduction of sub-spaced based method. DOA estimation of these algorithm is based on Eigen values and steering vectors. High resolutions Multiple Signal Classification (MUSIC) [5], Smooth MUSIC [9], Root- MUSIC [6], and Estimation of Signal Parameter via Rotational Invariance Technique (ESPRIT) [7] are sub-spaced based method having superior accuracy and resolution performance over the conventional methods.

Performance MUSIC and Smooth MUSIC algorithm depend on the source number estimation, number of snapshots, number of array elements and high computational complexity. Multiple Signal Classification (MUSIC) algorithm is high resolution subspace based method which is used to estimate DOA of incoherent signals. Under the impulse noise condition, the performance of the MUSIC algorithm will degenerate. Smoothing of MUSIC is introduced to estimate DOA of coherent and completely correlated signal. For coherent signals improved MUSIC remove the correlation between incident signals [9].

MUSIC and Smooth MUSIC algorithms were developed and simulated in MATLAB software. The Paper is organized as follows- Firstly, a signal model is developed for DOA estimation in Section II. The MUSIC algorithm is described in details in Section III. The Smooth MUSIC for coherent signal is described in details in Section IV. In

Section V, results of MUSIC and Improved MUSIC algorithm are presented and compared. The conclusion is given in the further section.

II. SIGNAL MODEL

Fig. 1 shows a uniform linear array (ULA) M with equispaced sensors.

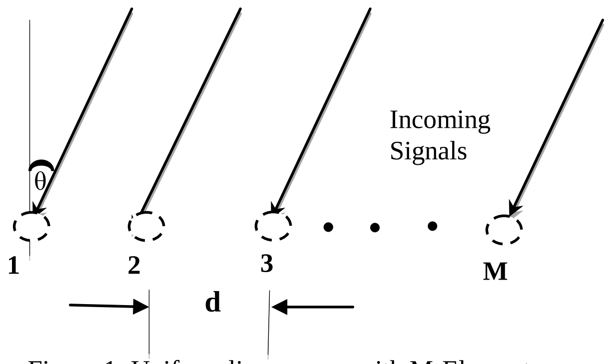

Figure 1: Uniform linear array with M-Element

- Number of elements = M
- Inter-element spacing = d
- Number of incident signals = D
- Number of Data samples = k

Consider, D signals which are incident on ULA and the received input data vector at M-elements that are separated by a distance d can be expressed as a linear combination of N incident waveforms and noise. The signal vector u (k) can be defined as

$$u(k) = \sum_{l=1}^{D} a(\theta_l)\, s_l(k) + n(k) \qquad (1)$$

$$u(k) = [a(\theta_1) .. a(\theta_D)] \begin{bmatrix} s_1(k) \\ . \\ . \\ . \\ s_D(k) \end{bmatrix} + n(k) \qquad (2)$$

$$u(k) = A\, s(k) + n(k) \qquad (3)$$

Where $A = [\, a(\theta_1)\, a(\theta_2) \dots \dots \dots a(\theta_D)]$ is the matrix of steering vectors, $s^T(t) = [s_1(k)\, s_2(k) \dots\dots s_D(k)]$ is the signal vector and $n(t) = [n_1(k)\, n_2(k) \dots\dots n_D(k)]$ is a noise vector with components of variance σ_n^2.
The input covariance matrix is given by

$$R_{uu} = A\, E[ss^H]A^H + E[nn^H] \qquad (4)$$

$$R_{uu} = AR_{ss}A^H + \sigma_n^2 \qquad (5)$$

Where R_{ss} is the signal correlation matrix.

III. MUSIC ALGORITHM

The MUSIC algorithm was proposed by Schmidt in 1979 [15]. MUSIC algorithm is a high resolution Multiples Signal Classification subspace DOA estimation algorithm which based on exploiting the Eigen structure of the input covariance matrix. [7]

Figure 2: Shows the flowchart of MUSIC algorithm. Direction of arrival estimation is the process of estimating the direction of an incomming signal from mobile devices to the Base Transceiver Station.

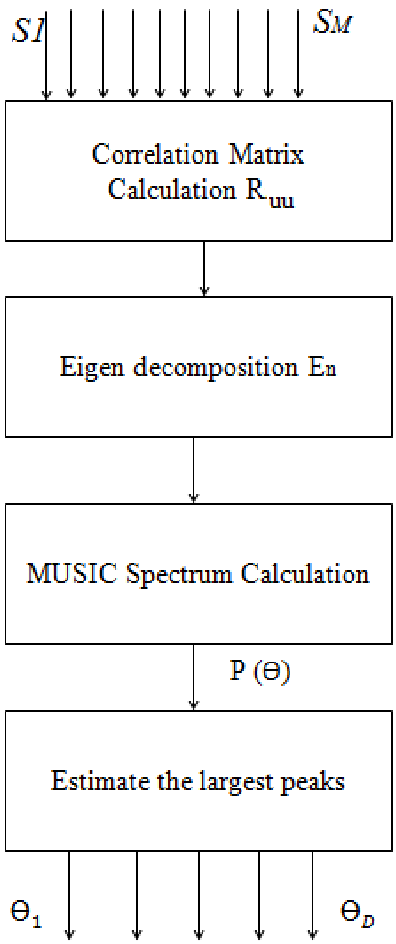

Figure 2: MUSIC implementation flow chart.

MUSIC deals with the decomposition of correlation matrix into two orthogonal matrices, signal-subspace and noise subspace [4]. Estimation of direction is performed from one of these subspaces, assuming that noise in each channel is highly uncorrelated [4]. This makes the correlation matrix diagonal [4]. The correlation matrix from equation (5) is

$$R_{uu} = AR_{ss}A^H + \sigma_n^2$$

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Where H – "Hermitian" means conjugate transpose. The eigenvectors of the covariance matrix R_{uu} belong to either of the two orthogonal subspaces, signal subspace and the noise subspace. The array correlation matrix has M Eigen values $(\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_M)$ along with M Eigenvectors $(E_1, E_2, E_3, \dots, E_M)$.

If the Eigen values are arranged in largest to smallest order, the next step is dividing the matrix E into two subspaces $[E_N\ E_S]$. E_N is the noise subspace composed of M – D Eigenvectors associated with the noise and dimension of noise subspace is $M \times (M - D)$, while E_S is the signal subspace composed of M Eigenvectors associated with the incoming signal. Due to the orthogonality of noise subspace and the array steering vector corresponding to signal components at the angles of arrival $(\theta_1, \theta_2, \theta_3, \dots, \theta_M)$, the matrix product $a^H(\theta) E_N E_N{}^H a(\theta)$ = 0 for θ corresponding to the DOA of a multipath component. Then the DOAs of the multiple incoming signals can be finding out by locating the peak of a MUSIC spatial spectrum given by

$$P_{MUSIC}(\theta) = \frac{1}{a^H(\theta) E_n E_n{}^H a(\theta)} \qquad (6)$$

Orthogonality between noise subspace E_N and array steering vectors a (θ) will minimize the denominator and hence will create sharp peaks at the angle of arrival.

IV. SMOOTH MUSIC ALGORITHM

The idea behind smoothing technique of MUSIC algorithm was proposed by Evans in 1982 [14]. Spatial smoothing is an effective preprocessing technique for DOA estimation of coherent sources [8]. MUSIC algorithm considers that incident signals are incoherent (uncorrelated). The covariance matrix R_{uu} is a non-singular matrix i.e. full-rank matrix as long as the incoming signals on the array are not coherent. The covariance matrix R_{uu} will lose its non-singularity property in case of the correlated signals. Thus the performance of MUSIC algorithm degenerate severely in coherent or highly correlated signal environment [14] as encountered in multipath propagation where multiple version of same signal arrive within the symbol duration. Smooth MUSIC involves modification of covariance matrix through a preprocessing scheme. Smoothing of correlation matrix is required to remove correlation between the signals. Smooth MUSIC algorithm is proposed by conjugate reconstruction of the data matrix of the MUSIC algorithm

Uniform linear array having M identical elements array are divided into overlapping sub arrays of size p and introducing phase shift between these as shown in Figure 3. Such that the array elements (0... p - 1) form the first subarray an (1... p) form the second subarray. The number of subarrays is L = M - p +1,

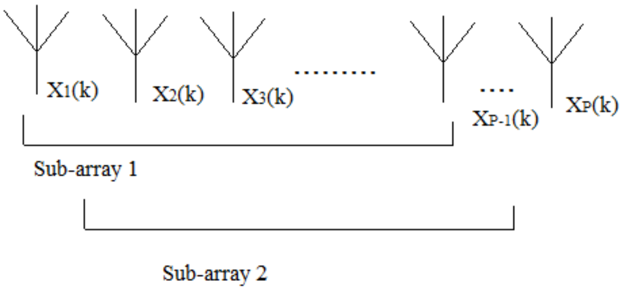

Figure 3: Uniform linear array is divided into several sub-arrays

Make a transformation matrix J, J is an Mth-order anti-matrix, known as the transition matrix, i.e.

$$J = \begin{bmatrix} 0 & 0 \dots\dots & 1 \\ 0 & 0 \dots\dots & 0 \\ \dots & \dots & \dots \\ 1 & 0 & 0 \end{bmatrix} \qquad (7)$$

similarly, the smoothed covariance matrix can be expressed as

$$R_s = R_{uu} + J\, R_{uu}^H\, J \qquad (8)$$

The spatial correlation matrix R of the sensor array is then defined as the sample mean of the covariance matrices of the forward sub-arrays:

$$R = \frac{1}{L} \sum_{1}^{L} R_s \qquad (9)$$

Here L is the number of overlapping sub-arrays. By applying spatial smooth MUSIC algorithm, M element sensor array can detect up to M/2 correlated signals.

V. RESULTS

The MUSIC and Smooth MUSIC algorithm of DOA estimation are simulated using MATLAB. In these simulations, it is considered a uniform linear array antenna formed by elements with the equally spaced distance of $\lambda/2$. The noise is Gaussian white noise, SNR=20dB and number of snapshots is 100. The simulation has been run for three independent narrow band signals; angle of arrival is -20^0 and 40^0. The performance has been analyzed for different array elements and SNR.

A. Simulation Results of MUSIC Algorithm

Case.1: MUSIC spectrum for varying number of array elements

As shown in Figure4, it is clear that as number of array elements increases (M=500), Peaks of spectrum become sharper and MUSIC can accurately estimate the direction of arrival of the incoming signals. And if number

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of array element (M=50) decreases, then angular resolution of Music algorithm decreases.

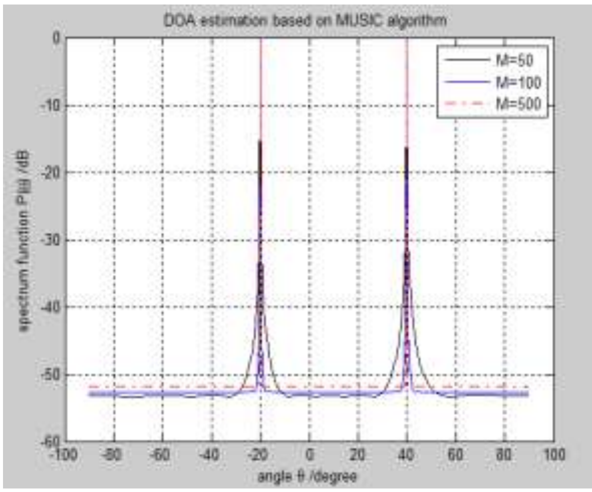

Figure4. MUSIC spectrum for varying number of array elements

Case.2: MUSIC spectrum for varying number of snapshots.

As shown in Figure5, as number of snapshots (N=1000) increases, Precise detection of incoming signals and Resolution capacity increases this is because increase the number of snapshots, will make narrower beam width around incoming signals directions. If number of snapshot (N=5) then resolution of MUSIC algorithm decreases.

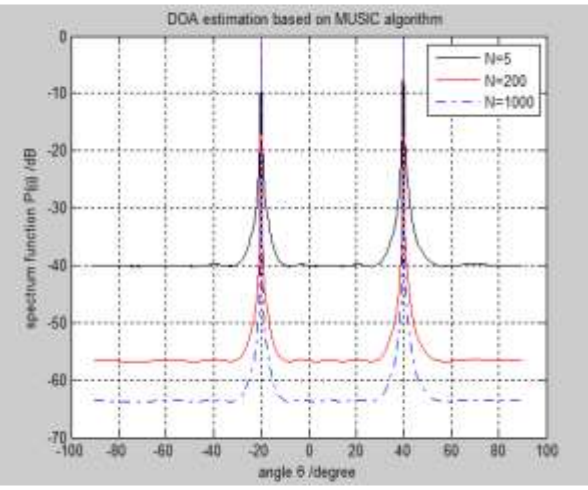

Figure5. MUSIC spectrum for varying number of array elements

Case.3: MUSIC spectrum for varying signal to noise ratio

The effect of varying the signal to noise ratio with two different values SNR_1= -20, SNR_2= 0 and SNR_3= 20 and other condition remains unchanged are as shown in Figure6. It is clear that as the value of SNR increases, precise detection of incoming signal and angular resolution capacity increases also the spectral beam width becomes narrower. The value of SNR can affect the performance of the MUSIC algorithm.

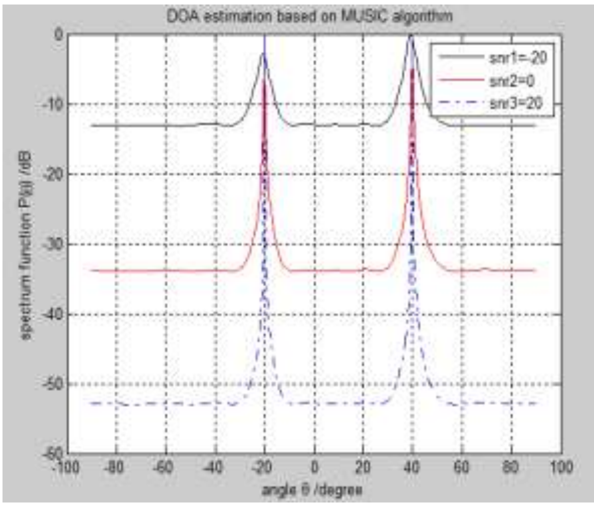

Figure6. MUSIC spectrum for varying SNR

B. Simulation Results of Smooth MUSIC Algorithm

When the signals are coherent (correlated), let the incident angle is same (-40°, 20°) the SNR is 20dB, the element spacing is half of the input signal wavelength, array element number is 10, and the number of snapshots is 200. The simulation results for varying number of array elements are shown in Figure 7.

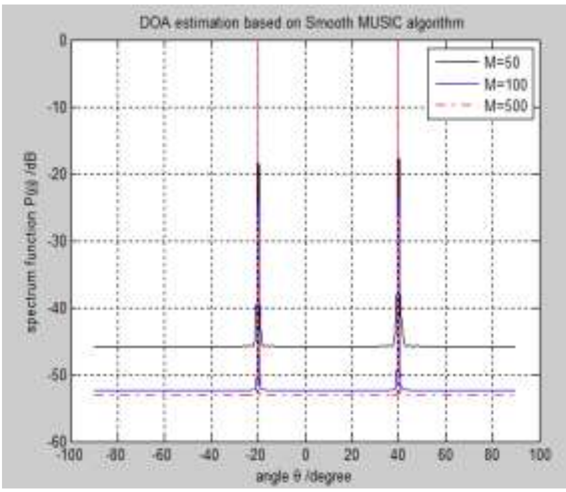

Figure7. Smooth MUSIC spectrum for varying number of array elements

C. Simulation Results for Comparison of MUSIC and Smooth MUSIC Algorithm

In these simulations, it is considered a linear array antenna formed by 10 elements that are equally spaced with

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the distance of λ/2. The SNR is 20dB and number of snapshots is 100. The simulation has been run for two independent narrow band signals, angle of arrival is -40^{0}and 20^0.

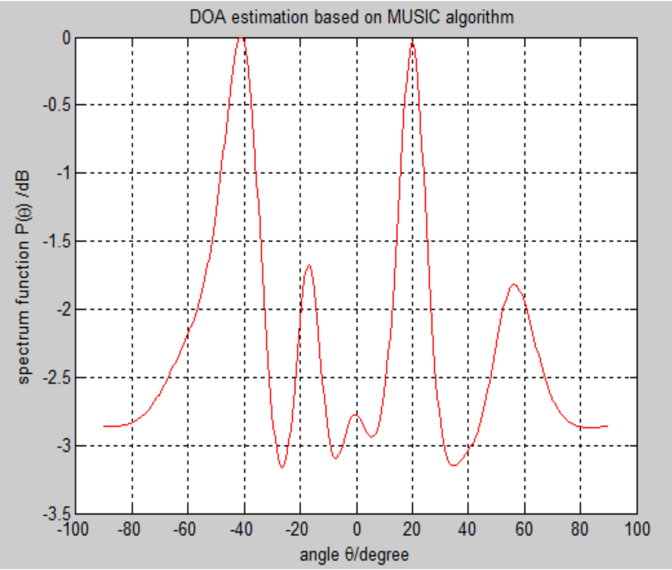

Figure8. M= 10, N= 100

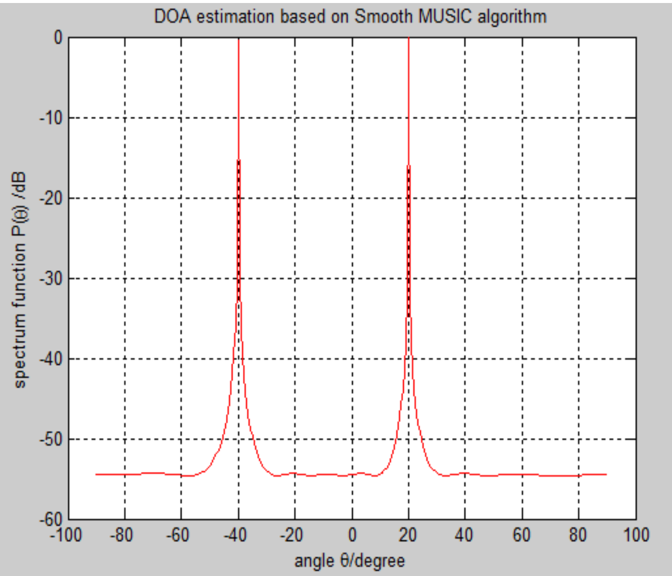

Figure9. M= 10, N= 100

Figure 8 and 9: MUSIC and Smooth MUSIC spectrum for same specification

VI. CONCLUSION

In this paper, a subspace based estimation using MUSIC and Smooth MUSIC algorithm are investigated. The simulation result of Music algorithm shows that angular resolution of Music algorithm improves by increasing number of elements in the array, snapshot of signals and increasing value of SNR. MUSIC can estimate incoherent (uncorrelated) signal very well but it fails to detect correlated signals. Smooth MUSIC estimates accurate DOA of signal under coherent condition while MUSIC algorithm shows interference.

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