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Single Server Interdependent Queueing Model using Baileys Bulk Service rule

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Abstract— In this paper, we consider the single server queueing system having Baileys bulk service rule with phase wise. In this model various system characteristics like probability that the system emptiness, variability of the system size and the coefficient of variation are obtained.

Keywords- *Queuing system, service process, arrival process, bulk service, interdependence, joint probability, marginal probabilities.*

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I. Introduction

In the bulk service queueing models, Bailey (1954) and Jaiswal (1960) considered units arrive at random form a single queue in order of arrival and are served in batches, the size of each batch being either a fixed number of customers or the whole queue length whichever is smaller. Jaiswal (1961) extended this model to the case, where at a service epoch if m $(0 \le m \le s)$ persons are already present with the server then $(s-m)$ persons or the whole queue length whichever is smaller will be taken into service. This service rule is termed as Bailey's Bulk Service. However, in these models the arrival and service processes are independent. But in some situations, like at an elevator or at a bus stop. etc., the service processes depends on the arrival processes in order to have optimal operating policies. So, for this kind of situations, we develop and analyze the interdependent queueing model with Bailey's Bulk service rule.

In this paper we considered the arrival processes is Poissonian and the service processes is Erlangian with interdependent arrival and service processes.

In this model the system behavior is analyzed by obtaining the difference-differential equations of the model and solving them through generating function techniques. The system characteristics like, mean queue length, variability of the system size and coefficient of variation are derived and analyzed in the light of the dependence parameter.

II. $M/E_k^{[x]}/1$ INTERDEPENDENT MODEL WITH PHASE WISE SERVICE

In this section, we consider the single server queueing system with interdependent arrival and service process having the Bailey's bulk service rule with phase wise. Here, we assume that the server serves only at instants $t_1, t_2, \cdots, t_n, \cdots$ (i.e., the service is available at time instants $t_1, t_2, \cdots, t_n, \cdots$). If m $(0 \le m \le s)$ persons are present in the waiting line at time t_n then the server takes a batch of *X* persons {*i.e.*, the server takes $(s-m)$ persons or whole queue length whichever is smaller}, here *X* is a random variable and S is the service capacity. Along with, we assume that the number of arrival of the customers and the number of service completions in each phase are correlated and follows a bivariate Poisson distribution of the form

$$P[X_1 = x_1, X_2 = x_2 / t] = e^{-(\lambda+\mu-\epsilon)t} \sum_{j=0}^{\min(x_1,x_2)} \frac{(\epsilon t)^j [(\lambda-\epsilon)t]^{x_1-j} [(\mu-\epsilon)t]^{x_2-j}}{j!(x_1-j)!(x_2-j)!} \quad \ldots(1)$$

here $x_1, x_2 = 0,1,2,\cdots$; $0 < \lambda, \mu$ and $\epsilon < \min(\lambda, \mu)$

$P[X_1 = x_1, X_2 = x_2 / t]$ is the joint probability of x_1 arrivals and x_2 services during time t.

The marginal distribution of arrival and services are Poisson with parameters λ and μ respectively. Thus inter arrival times and service times follow negative exponentials distributions of the form $\lambda e^{-\lambda t}$ and $\mu e^{-\mu t}$ respectively where λ is the mean arrival rate and μ is the mean service rate (Feller 1969). ∈ is the covariance between the number of arrivals and services at time t. This dependence structure turns out to be independent structure if ∈ = 0 (Teicher1954).

Let b_m be the probability that there are m customers present with the server in the system at a service epoch. Then the server takes X customers {i.e., $(s-m)$ customers or the whole queue length whichever is smaller}. S is the maximum size of the batch that is to be taken into service.

We have $b_m = 0$ if $m > s$ and $\sum_{m=1}^{S} b_m = 1$ with this dependence structure we develop $M/E_k^{[x]}/1$ Interdependent model with Bailey's bulk service.

III. POSTULATES OF THE MODEL

The postulates of the model with this dependence structure are

1. The occurrence of the events in non-overlapping time intervals is statistically independent.
2. The probability that no arrivals and no service completions occur in an infinitesimal interval of time Δt is $1 - [(\lambda + \mu - \epsilon)\, t] + O(\Delta t)$
3. The probability that no arrival and one service completion occurs in Δt is $(\mu - \epsilon)\, t + O(\Delta t)$
4. The probability that one arrival and no service completion occurs in Δt is $(\lambda - \epsilon)\, t + O(\Delta t)$
5. The probability that one arrival and one service completion accrues in Δt is $\epsilon t + O(\Delta t)$. This postulate is due to the dependence structure between the arrivals and service completions.
6. The probability that the occurrence of an event other than the above events during Δt is $O(\Delta t)$

There is an equivalence between the postulates and the process. Further for given values of λ, μ the covariance $\epsilon = r$, where r is the correlation coefficient between arrivals and service. Since ϵ is a function of r and is treated as dependence parameter (ϵ). This is the structure given by Rao K.S (1986).

Let $P_n(t)$ be the probability that there are n customers waiting in the queue at time t and service is in the r^{th} phase.

Using the phase-type technique analogue of MORSE (1957), we can have the differential equation of the model as

$$P'_{n,i}(t) = -(\lambda+\mu-2\epsilon)P_{n,i}(t) + (\mu-\epsilon)P_{n,i+1}(t)$$

$$+(\lambda-\epsilon)P_{n-1,i}(t) + (\mu-\epsilon)\sum_{m=0}^{S} B_m P_{S-m,1}(t)$$

$$n>0,\ i \le k$$

$$P'_{0,i}(t) = -(\lambda+\mu-2\epsilon)P_{0,i}(t) + (\mu-\epsilon)P_{0,i+1}(t)$$

$$+(\mu-\epsilon)\sum_{m=0}^{S} B_m P_{S-m,1}(t)$$

$$1 \le r,\ i \le k \qquad ..(3)$$

here $B_m = \sum_{q=0}^{m} b_q$ .....(4)

Assuming the system is in steady – state, the state transition equations of the model are

$$-(\lambda+\mu-2\epsilon)P_{n,i} + (\mu-\epsilon)P_{n,i+1}$$

$$+(\lambda-\epsilon)P_{n-1,i} + (\mu-\epsilon)\sum_{m=0}^{S} B_m P_{S-m,1} = 0$$

$$-(\lambda+\mu-2\epsilon)P_{0,i} + (\mu-\epsilon)P_{0,i+1}$$

$$+(\mu-\epsilon)\sum_{m=0}^{S} B_m P_{S-m,1} = 0.$$

To solve these steady-state equations, we adopt the generating function approach.

Let $P(z) = \sum_{n=0}^{\infty} z^n P_n$

be the generating faction of P_n

Following the heuristic argument of JAISWAL(1961). We get the probability generating faction of

$$P_n(y) = \frac{(\mu-)^k \left[\sum_{q=0}^{s-1} \{y^s \varphi_{s-q}(1) - y^q \varphi_{s-q}(y)\} p_q\right]}{y^s \left(\mu+\lambda-2\in - (\lambda -)y\right)^k - (\mu-)^k \sum_{m=0}^{s} b_m y^m}$$

Applying Roche's Theorem, for the denominator, we get,

$$[(\mu+\lambda-2\epsilon)-(\lambda-\epsilon)y]^k y^S - (\mu-\epsilon)^k \sum_{m=0}^{S} b_m y^m = 0.$$

It can be shown to have $(s-1)$ zero' s inside the unit circle and one at $y=1$ and the remainder outside the unit circle $|y| = 1$. However this requires the condition,

$$k(\lambda-\epsilon) < (\mu-\epsilon)\left(S - \sum_{m=0}^{S} m b_m\right)$$

The condition is necessary for statistical equilibrium. Thus $P(y)$ can be written as

$$P(y) = \frac{C}{(y_0 - y)}$$

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Where y_i 's are the roots of modules greater than one of the equation

$$[(\mu+\lambda-2\in)-(\lambda-\in)y]^k y^s - (\mu-\in)^k \sum_{m=0}^{s} b_m y^m = 0 \quad .(13)$$

Using the boundary condition *P(1)=1*, we obtain

$$C = \pi (1-y_i) \quad .(14)$$

Thus

$$P(y) = \frac{[\mu+\lambda-2\in-(\lambda-\in)y]^k-(\mu-\in)^k}{(\mu-\in)^{k-1}(\lambda-\in)(1-y)}\,\pi\left(\frac{1-y_i}{y-y_i}\right)$$

Where y_i 's are given in equation (12)

Using the probability generating function, we can analyze the system behavior of this model. Expanding equation (14) and collecting the coefficient of y^n, will give us the probability that there are n customers in the system.

IV. MESURES OF EFFECTIVENESS

The probability that the system is empty can be obtained as

$$P(0) = P_0 = \frac{[\mu+\lambda-2\in]^k-(\mu-\in)^k}{k(\mu-\in)^{k-1}(\lambda-\in)}$$

Where y_i 's are as given in equation (12)

Using the equation (15) the values of P_0 are computed for various values of s and for given values of λ, μ and k and are given in table (1). From table (1) it is observed that the values of P_0 increases as $\in$ increases for fixed values of λ, μ and k. It is also noticed that the values of P_0 increases as the batch size s increases for fixed values λ, μ and k.

The Average Number of customers in the system can be obtained by differentiating $P(y)$ with respect to *y* and substituting $y=1$.

From equation (14) and using *L-Hospital's* rule, we have

$$L = \sum\left[\frac{1}{y_i-1}\right] + \left[\frac{(k-1)(\lambda-\in)}{2(\mu-\in)}\right] \quad .(16)$$

Where y_i 's given in equation (12)

Using equation (16), we have computed the values of *L* for various values of $\in$ and s are presented in table (2)

From table (2) it is observed that the values of *L* decreases as $\in$ increases for fixed of λ, μ and k. And it is also noticed from table (2) that the values of *L* decreases as the batch size s increases .

The variability of the system size is obtained by using the formula

$$V = P''(1) + P'(1) - [P'(1)]^2 \quad .(17)$$

Differentiating $p(z)$ and putting $z=1$, we get,

$$V = \left[\sum\frac{1}{(y_i-1)^2} + \sum\frac{1}{(y_i-1)} + \left(\sum\left[\frac{1}{y_i-1}\right]\right)^2\right] + \frac{1}{3}(k-1)(k-2)\left(\frac{\lambda-\in}{\mu-\in}\right) - (k-1)\left(\frac{\lambda-\in}{\mu-\in}\right)\sum\frac{1}{y_i-1} + \frac{(k-1)}{2}\left[\frac{\lambda-\in}{\mu-\in}\right] - \left[\sum\frac{1}{y_i-1} + \frac{(k-1)}{2}\left[\frac{\lambda-\in}{\mu-\in}\right]\right]^2 \quad .(18)$$

Using equation (17) we have computed the values of *V* for various values of k an $\in$ and for fixed values of λ, μ and $\in$, also for various values of s and $\in$ for fixed values of λ, μ and k

The coefficient of variation of the system is

$$CV = \frac{\sqrt{V}}{L} \quad .(19)$$

Where *V* and *L* are as given in equation (18) and (16) respectively.

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The values of V and $C.V$ are computed for various values of $\in$, s and for fixed values of λ, μ and k are presented in table (3) and table (4).

It is observed that as s increases the variability of the system size decreases and the coefficient of variation increases as the dependence parameter increases the variability of the system size deceases and the coefficient of variation increases for fixed values of k, μ and λ. It is also noticed that as k increases the system variability increases for fixed values of λ, μ, $\in$ and s, the coefficient of variation decreases for fixed values of λ, μ, $\in$ and s. As the dependence parameter increases the variability of the system size decreases and the coefficient of variation increases for fixed values of λ, μ, k and s

Table 1. VALUES OF P_0 $k = 2$, $\lambda = 1$, $\mu = 6$

s/$\in$	0	0.2	0.4	0.6	0.8
1.	0.3611	0.4792	0.7716	0.7294	0.8781
2.	0.6270	0.6868	0.7533	0.8252	0.9069
3.	0.6938	0.7403	0.7925	0.8519	0.9202
4.	0.7228	0.7640	0.8107	0.8643	0.9266

Table 2. Values of L $k = 2$, $\lambda = 1$ $\mu = 6$

s/$\in$	0	0.2	0.4	0.6	0.8
1.	1.7499	1.0767	0.2923	0.3689	0.1408
2.	0.5857	0.4502	0.3259	0.2104	0.1023
3.	0.4335	0.3457	0.2588	0.1725	0.0864
4.	0.3762	0.3030	0.2306	0.1556	0.0789

Table 3. Values of V $k = 2$, $\lambda = 1$ $\mu = 6$

S/$\in$	0	0.2	0.6	0.8
1.	4.7704	2.2149	.5008	0.1948
2.	0.9091	0.6405	0.2515	0.1120
3.	0.6047	0.4542	0.1993	0.0931
4.	0.5019	0.3860	0.1770	0.0843

Table 4. Values of C. V $k = 2$, $\lambda = 1$ $\mu = 6$

S/$\in$	0	0.2	0.6	0.8
1.	1.2481	1.3822	1.9183	2.8825
2.	1.6279	1.7775	2.3839	3.2709
3.	1.7937	1.9494	2.5881	3.5220
4.	1.8834	2.0434	2.7031	3.6823

CONCLUSION

The interdependent waiting line systems are extended to interdependent queueing systems with Bailey's Bulk Service. In this paper the units arrive at random form a single queue in order of arrival and are served in batches, the size of each batch being either a fixed number of customers or the whole queue length whichever is smaller.

If m $(0 \le m \le s)$ persons are present in the waiting line at time when the server turns for picking the batch, then the server takes a batch of $(s-m)$ persons or the whole queue length whichever is smaller, where s is the service capacity.

The behavior of the system is analyzed through system characteristics with the dependence parameter.

It is observed that the probability that the system emptiness is increasing when the mean dependence rate is increasing. This is useful for utilizing the service facility on secondary jobs. These models are also including the models given by Jaiswal (1961) and Bailey (1954).

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