

# Game Theoretic Approach for Cognitive Radio Networks

K. J. Kulkarni

Dept. of Electronics & Telecommunication  
Maharashtra Institute of Technology  
Pune, India  
kalyanikul@gmail.com

B. S. Chaudhari

Dept. of Electronics & Telecommunication  
Maharashtra Institute of Technology  
Pune, India

**Abstract**— Cognitive radio is a promising approach to make efficient use of spectrum by rapidly adapting the changes in wireless environment. In such networks, nodes are expected to be aware of their environment and must be able to learn from the outcomes of past decisions. Game theory which comprise of analytical tools designed to study interactive decision-making processes can be applied effectively to solve some of such complex problems. This paper discusses the significance of using game theory for cognitive radio networks and resource allocation, and various game theoretic approaches used for it.

**Keywords**-Cognitive Radio, Game Theory, Non-cooperative Game Models

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## I. INTRODUCTION

Researchers have demonstrated that a large amount of licensed bands are under-utilized leading to creation of spectrum hole. The spectrum hole is nothing but a band of frequencies assigned to a primary user but at particular time and specific geographic location, the band is not being utilized by the user. Large numbers of spectral bands are unused either in number of time slots or at number of locations. The efficiency of spectrum utilization can be improved significantly by allowing a secondary user to access the spectrum hole that is unoccupied by the primary user [1]. To deal with requirements and constraints of users, there is need of dynamic systems to allocate frequency band and transmit signal. Cognitive radio can sense the spectrum utilization of the primary user and opportunistically access the spectrum holes. Some major problems in cognitive radio are opportunistic spectrum access, coordination among secondary users and cognitive medium access control [2]. Game theory which comprise of analytical tools designed to study interactive decision-making processes can be applied effectively to solve some of such complex problems.

Game theory is a mathematical platform for analysis of processes in which outcome is function of inputs from various decision-makers who have conflicting interests with reference to outcome of the process. Every game has the components such as a set of players, a set of actions, a set of priorities or preferences, method for determining outcomes according to the actions selected by players and rules that govern the game.

## II. GAME THEORY FOR COGNITIVE RADIO

For modeling a cognitive radio network, let  $N$  is a number of cognitive radios under consideration;  $i, j$ : Particular cognitive radios in  $N$ , and  $A_j$  is set of actions available to cognitive radio  $j$ . Action space may include number of changes or adaptations that Cognitive Radio may subject. It may adapt power levels, bandwidths, channel coding techniques, source coding techniques, medium access control (MAC) algorithms and modulations. In general, we can say that  $A_j$  is multidimensional set. Players in the game are the nodes in a network. Action set of each player consists of various adaptations or variable parameters available to the radio,  $A_j$ . Action space for the game is formed from the Cartesian product of the radios' available adaptations. Utility function for each player is provided by cognitive radio's goal and the

arguments. Valuations for utility functions are taken from the outputs of cognitive radios observation and orientation steps. Figure 1 shows an illustration of how different components in the cognitive radio cycle can be mapped in to game.

Preference relationship is represented by a utility function. Utility function assigns a numerical value to each possible outcome. If the utilities are higher, function is more desirable. In wireless scenario, players may prefer outcomes that give lower bit error rates, higher signal to noise ratio (SNR), and lower power consumption. But in real environment, these goals will be in conflict. One of the most challenging aspects of game theory is to model these preferences optimally. Game theory is more appropriate in the scenario where we can reasonably expect decisions of one player to impact decisions of other players.

Nodes in these networks are autonomous agents which make decisions about delay parameters required, modulation transmit power etc. While working in this scenario, some nodes may behave selfishly i. e. only for their own user's interests. Some of the nodes may look at the overall performance or Quality of Service (QoS) of the network as a whole. In the worst case nodes may behave maliciously seeking to ruin the performance of other users.

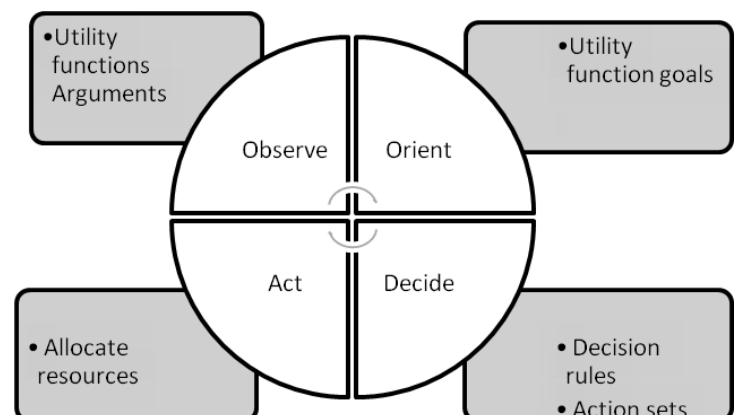


Fig. 1 Components of cognitive radio cycle

Game theory analyzes the situations in which players objectives are in conflict. In the direct application of game theory, user's actual preferences are modeled by the player's utility functions.

This theory has been widely considered by a number of authors to solve the problem of uplink power control in CDMA cellular systems. In such case, the number of cellular phones can be considered as players. Each player's action is the power level it selects. The power may be limited to a level between 0 and maximum power  $P_{max}$ . This can be modeled as a game by assigning own transmit power level and SNR as players pay-off functions. SNR is a function of its own transmit power and transmit powers of other players in the cell. When the player increases his power level, SNR may be improved but at the same time it may decrease SNR of other players. Thus, there exists a conflict. For a fixed power level, players prefer higher SNR to lower SNR. That is, players seek for the best possible channel conditions. Similarly for fixed

SNR, players will prefer lower power levels to higher ones in order to conserve power. Game theoretic solution of this conflict may lead to inefficient allocation or selfish game may lead to excessive power allocation. This scenario can be modeled as a repeated game where users punish each other for the excessive use of spectrum. Since there are multiple nodes making individual power control decisions that affect performance of every node in the network, this interactive decision problem can be modeled by a game theory. The various game-theoretic techniques applicable to the design, analysis and optimization of wireless networks can be summarized as shown in Fig. 2.

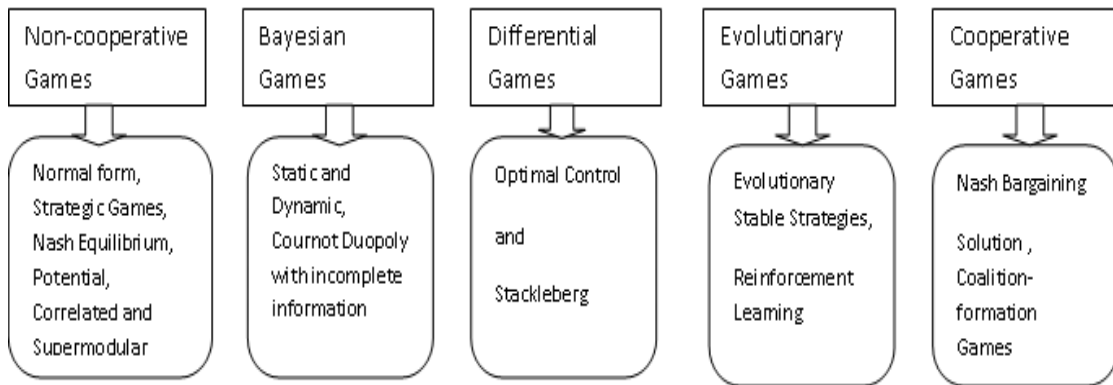


Fig. 2 Summary of Game-theoretic Techniques

### III. NON-COOPERATIVE GAME MODELS FOR COGNITIVE RADIO

The various variation of non-cooperative games which can be applied to cognitive radio systems are as discussed below.

#### A. Normal Form Game Model

Many interactive decision processes can be well modeled by *normal form game*. This game has following rules along with game elements:

- Synchronous single-shot play: All players make their decisions simultaneously and make only a single decision.
- Complete information: The players know their own utility functions and utility functions for all the players in the game. All the players follow the decisions.

Normal form game is defined by

$$\Gamma = \langle N, A, \{u_j\}_{j \in N} \rangle$$

where  $N$  is set of players,  $A$  is the action space, and  $u_j$  is the utility function for player  $j$ .

#### B. Strategic Form Games for Spectrum Allocation

A game consists of a principal and a finite set of players,  $n = \{1, 2, \dots, N\}$ . The principal sets the basic rules of games with each of the players,  $i \in n$  selects a strategy  $s_i \in S_i$  with the objective of maximizing its utility,  $u_i$ . Primary user is the principal who sets the bandwidth required for various levels of network and users are the players who decide how much

spectrum should be accessed to utilize the service. This situation can be modeled as non-cooperative game where each player selects his strategies without coordination of others. Strategy profile is the vector containing the strategies of all the players.

$$S = (s_i)_{i \in N} = (S_1, S_2, \dots, S_N) \tag{1}$$

$S_{-i}$  is the collective strategy of all the players except player  $i$ . Utility function characterizes each player's sensitivity to everyone's actions. Nash Equilibrium (NE) can be used to solve this problem by using a joint strategy where no player or node increases his utility by unilaterally deviating. NE is a mutual best response from each player to other players' strategy.

Pricing of network resources can be modeled as a game between network service provider, the primary user and a finite set of users or traffic flows, the players. Different users may have different QoS requirements. User is free to choose level of service to request among all service classes supported by the network. This choice is the user's strategy  $S_i$  and it will differ as per the priority of the parameters. For example, application may require smaller bandwidth. NE is also used in dynamic pricing where price charges per unit bandwidth depends on the total amount of bandwidth currently reserved by other users.

Strategic form game has also been employed in the flow control; where each user determines traffic load he will offer to the network in order to satisfy some performance objective.

### C. Repeated Games

A repeated game is a sequence of stage games in which each stage game is the sequence of normal form games. Based on the knowledge of past outcomes, players choose strategies. These strategies may be fixed or adaptive.

Players participate in repeated interactions and hence consider the effects of their chosen strategy in any round will have on their opponent's strategies.

Each player will try to maximize pay-off in multiple rounds. More efficient equilibrium can be obtained. These types of games are prominently used for sequential actions. These are also refereed as extensive form representation and can be designed to punish players who deviate from the agreed rules. Repeated games are represented by inserting time or state,

$$\Gamma = \langle N, A, \{u_j\}_{j \in N}, \{T_j\}_{j \in N} \rangle \quad (2)$$

where  $T_j$  represents the times at which player  $j$  can change its decisions. In repeated games, convergence is one of the parameters to be focused. Finite improvement path (FIP) property and the weak finite improvement path property are the two significant properties of the normal game that lead to convergence. FIP property implies that there is at least one NE and this will occur if the radios implement rational decision rules and network has either random or round-robin decision timings. Weak FIP implies that there exists at least one NE but further improvement can be possible.

### D. Markov Games

Markov games are a class of Stochastic Games, in which history at each stage of the game can be summarized by the state. Movement from one state to another state follows a Markov Process. The state at each next round of the state depends on current state and current action profile. When the game has finite number of states and actions, Markov Perfect Equilibrium exists. Markov game theory is also appealing in power control in CDMA systems. Increase in power to improve SNR of one user may eventually increase power requirement by another user to maintain his SNR. Game can be modeled by using clear trade-offs that can be expressed in utility function and clear independencies in user decisions.

### E. Potential Games

Potential games are a class of games in which there exists a mechanism called as potential function that reflects the change in value accrued by a unilaterally deviating player. Further depending on potential functions, games can be classified as exact potential game, weighted potential game, ordinal potential game, generalized ordinal potential game, and generalized  $\mathcal{E}$ -potential game. Power control is a set of real-time algorithms in order to maximize performance metric. Potential game approach is used in [3], for the power allocation in cognitive radio networks. Primary user is mobile satellite system and wireless terrestrial OFDM users are cognitive users. Utility function is defined such that at a time at most one player can change its strategy. It is demonstrated that heuristic approach needs huge amount of information is to be forwarded to the network whereas game theory is more fair

towards those users who experience lower rates. Non-cooperative game approach is widely used [4], for radio resource management in wireless ad-hoc networks. It is shown that Nash Equilibrium is attained for the non-cooperative game.

### F. Supermodular Games

Supermodular games are referred to games, in which there are increasing differences, an increase in  $a_i$  results in a corresponding increase in  $a_j$ . Supermodular games are useful for cognitive radios that implement decision rules that perform local optimizations or referred to game theory yields best response decision rules. Non-cooperative power control game developed by D. Goodman is utilized in [5] for power control in cognitive radio. New sigmoid efficiency function is defined with signal to interference ratio as a utility function in the super modular game. It is shown that non-pricing game will provide higher utilities and users will stably operate on comparatively low power.

Correlated equilibrium has conceptual and computational advantages over Nash Equilibrium. New and more fair pay-offs can be achieved. Each player in a game chooses its action according to own observation and strategy assigns an action for every possible observation. If no player could deviate from strategy then it is called as correlated equilibrium. Correlated equilibrium concept is used to solve transmission control problem [6]. In [7], a game theoretical approach with new solution concept, the correlated equilibrium is demonstrated and proved to be better compared with the non-cooperative Nash equilibrium in terms of spectrum utilization efficiency and fairness among the distributive users. An adaptive algorithm based on no-regret learning is constructed that guarantees the convergence. Bandwidth allocation problem in next generations is addressed in [8], for the 4G heterogeneous Oligopoly market conditions. Market competition is modeled by the Cournot game.

### G. Stackleberg Games

In a variety of non-cooperative games, there exists a hierarchy among the players. Leader imposes his own strategy upon the others and the followers react to this strategy. Followers might have many possible reactions to a given strategy of the leader. Stackleberg equilibrium is used in defining equilibrium point in these types of hierarchical games.

Non-cooperative games models the scenario of multiple selfish cognitive users and they are engaged in a non-cooperative competition. Nash Equilibrium [9] can be defined to get the optimal solution. Using the repeated and dynamic game models, the behavior of the users can be studied and incomplete information can be determined.

## IV. CONCLUSION

Dynamic Spectrum Access has been the key concept behind the cognitive radio which has been proposed as a solution to improve utilization of the limited radio resource. Some of the major challenges in cognitive radio are opportunistic spectrum access, coordination among the secondary users and the cognitive medium access control.

Game theory can be an effective tool which studies the mathematical models of conflict and cooperation between the cognitive users. The paper discussed various non-cooperative game models, which can be considered for the spectrum sensing and resource allocation issues.

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