Effect of the geometry on the nonlinear vibrations of functionally graded cylindrical shells

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Abstract

In this paper, the effect of the geometry on the nonlinear vibrations of functionally graded (FGM) cylindrical shells is analyzed. The Sanders-Koiter theory is applied to model nonlinear dynamics of the system in the case of finite amplitude of vibration. Shell deformation is described in terms of longitudinal, circumferential and radial displacement fields; the theory considers geometric nonlinearities due to the large amplitude of vibration. Simply supported boundary conditions are considered. The displacement fields are expanded by means of a double mixed series based on harmonic functions for the circumferential variable and Chebyshev polynomials for the longitudinal variable [**1**]. Both driven and companion modes are considered, allowing for the travelling-wave response of the shell. The functionally graded material is made of a uniform distribution of stainless steel and nickel, the material properties are graded in the thickness direction, according to a volume fraction power-law distribution.

The first step of the procedure is the linear analysis, i.e. after spatial discretization mass and stiff matrices are computed and natural frequencies and mode shapes of the shell are obtained, the latter are represented by analytical continuous functions defined over all the shell domain.

In the nonlinear model, the shell is subjected to an external harmonic radial excitation, close to the resonance of a shell mode, it induces nonlinear behaviors due to large amplitude of vibration. The three displacement fields are re-expanded by using approximate eigenfunctions, which were obtained by the linear analysis; specific modes are selected. An energy approach based on the Lagrange equations is considered, in order to reduce the nonlinear partial differential equations to a set of ordinary differential equations.

Numerical analyses are carried out in order characterize the nonlinear response, considering different geometries and material distribution. A convergence analysis is carried out in order to determine the correct number of the modes to be used; the role of the axisymmetric and asymmetric modes carefully analyzed [**2**]. The analysis is focused on determining the nonlinear character of the response as the geometry (thickness, radius, length) and material properties (power-law exponent N and configurations of the constituent materials) vary; in particular, the effect of the constituent volume fractions and the configurations of the constituent materials on the natural frequencies and nonlinear response are studied.

Results are validated using data available in literature [**3**], i.e. linear natural frequencies.

References

[**1**] F. Pellicano, Vibrations of circular cylindrical shells: Theory and experiments, *Journal of Sound and Vibration* 303 (2007) 154-170.

[**2**] C. Loy, K. Lam, J. Reddy, Vibration of functionally graded cylindrical shells, *International Journal of Mechanical Sciences* 41 (1999) 309-324.

[**3**] S. Pradhan, C. Loy, K. Lam, J. Reddy, Vibration characteristics of functionally graded cylindrical shells under various boundary conditions, *Applied Acoustics* 61 (2000) 111-129.