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Performance Comparison of Two Inner Coding Structures in Concatenated Codes for Frequency-Hopping Spread Spectrum Multiple-Access Communications

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***Abstract*—**In this paper, we compare the performance of two inner coding structures for concatenated codes in slow frequency-hopping spread spectrum multiple-access communication systems. It is assumed that two outer code symbols are transmitted during a hop. The first structure consists of one inner codeword per one outer code symbol, while the second structure consists of one inner codeword per two outer code symbols. We analyze the overall block error probability in asymptotic region and show that the performance of the second scheme is superior to the first one.

Keywords- *Concatenated code; inner code; slow hopping; frequency hopping; spread spectrum; multiple access*

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I. INTRODUCTION

We consider a frequency-hopped (FH) spread-spectrum multiple-access (SSMA) communication system utilizing concatenated codes. The basic features of the FHSS communication system are discussed in [1], and the key characteristics of the concatenated codes are discussed in [2]. In FH SSMA systems, many users are desired to share a given bandwidth with a given error probability and errors occur primarily due to multiple access interference (MAI). To mitigate the MAI, a concatenated coding system is employed in this paper. We assume throughout this paper that two outer code symbols are transmitted during a hop.

Concatenated codes form a class of error-correcting codes that are derived by combining an inner code and an outer code. They were conceived as a solution to the problem of finding a code that has both exponentially decreasing error probability with increasing block length and polynomial-time decoding complexity. Concatenated codes became widely used in space communications in the 1970s and recently adopted in Digital Television Terrestrial Broadcasting (DTTB) [3].

The most natural choice for outer codes is Reed-Solomon (RS) codes in concatenated codes. Because the RS codes, being maximum-distance-separable codes, make highly efficient use of redundancy, and well suited to burst error correction [4]. We will use RS codes as outer codes throughout this work. The inner code we consider in this paper is error detecting or correcting binary block code. The inner code corrects e_c errors and detects e_d errors provided $2e_c + e_d < d_{min}$, where d_{min} is the minimum distance of the inner code. When an error is detected, every symbol of the inner code is erased. There are, however, errors that are not detected nor corrected by the inner code, which results in errors at the output of the inner decoder. The purpose of the outer code is to correct the errors and erasures of the inner code.

Since we assume that two outer code symbols are transmitted during a hop, we can consider two kind of inner coding structures in the concatenated code. The first structure consists of one inner codeword per one outer code symbol; we will call this $(N, K)(N, K)$ scheme. And the second structure consists of one inner codeword per two outer code symbols; we will call this $(2N, 2K)$ scheme. In this paper, we analyse the overall block error probability of the concatenated codes with $(N, K)(N, K)$ and $(2N, 2K)$ schemes in asymptotic region and the performance of two inner coding schemes.

This paper is organized as follows. The system and the channel model are introduced in Section II. The $(N, K)(N, K)$ and $(2N, 2K)$ schemes are presented Section III and Section IV, respectively. The asymptotic analysis and numerical results are provided in Section V. Finally, a conclusion is made in Section VI.

II. SYSTEM AND CHANNEL MODEL

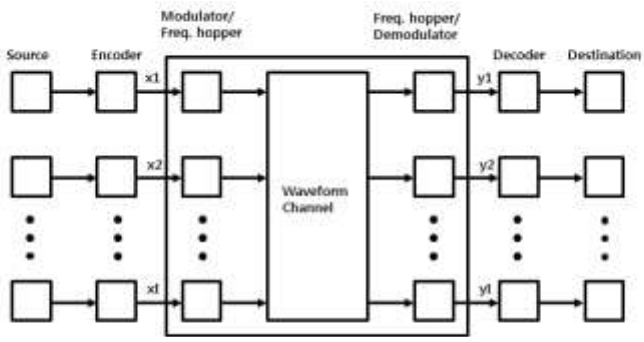

Figure 1. A FH SSMA communication system with I simultaneous users

We consider a FH/SS packet radio network in which I users wish to communicate simultaneously over a common channel as shown in Figure 1. Each source generates messages, which are independent of other users. There are I separate encoders, one for each source. The j^{th} encoder receives only the message from the j^{th} source and produces a codeword $(x_{j1}, x_{j2},...,x_{jn})$, $x_{ji} \in \boldsymbol{X}$, where $\boldsymbol{X}$ is a common input alphabet. At the j^{th} receiver, the j^{th} frequency dehopper, which has knowledge of the j^{th} hopping pattern dehops the received signal. And the dehopped signal is demodulated to produce the output vector $(y_{j1}, y_{j2},...,y_{jn})$, $y_{ji} \in$

Y, where ***Y*** is a common output alphabet. Decoding is done independently at each of the I receiver and thus there is no cooperation between users on either the transmitting and the receiving side. Then individual channels can be characterized by $P(y_i \mid x_j)$, $j \in \{1, 2, \ldots, I\!-\!1\}$, which is identical for all users [1][5].

We assume that all FH transmitters adjust their timings of frequency changes (synchronous frequency hopping), and transmit two outer code symbols during a hop (slow frequency hopping). Thus, the multi-user interference level during a hop will remain constant throughout the hop. We assume that the hopping pattern is essentially random, which makes the interference during a hop independent of that of the other hop intervals. When I users transmit their packets simultaneously, it is probable that i+1, $i \in \{0, 1, 2, \ldots, I\text{-}1\}$, users occupy a particular frequency slot simultaneously. If a frequency slot is occupied by i+1 users, the slot can be modeled by a binary symmetric channel (BSC) Δ_i with a channel crossover probability p_i given by

$$p_i = \frac{2^i - 1}{2^{i+1}} \tag{1}$$

On the other hand, the probability of the channel Δ_I being chosen, i.e. the probability of i+1 users occupying the same frequency slot, $P_{h,I}(i)$, is given by

$$P_{h,I}(i) = \binom{I-1}{i}\left(\frac{1}{q}\right)^{i}\left(1-\frac{1}{q}\right)^{I-1-i}, \quad i \in \{0, 1, \cdots, I-1\} . \tag{2}$$

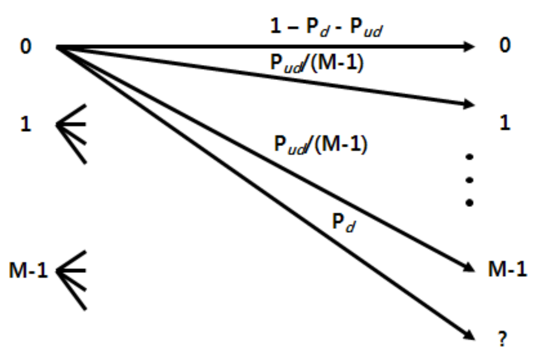

Figure 2. *M*-ary erasures and errors channel model

At the receiver side, the demodulated data is first decoded with the inner decoder, and then the symbols from the inner decoder are decoded with the outer decoder. When an inner code is used for detecting errors, a *super channel* created by the inner code can be modeled by *M*-ary erasures and errors channel as shown in Figure 2. The purpose of the outer code is to correct the errors and erasures of the inner code. From the minimum distance property [4], the (n, k) Reed-Solomon code with bounded distance decoding can correct up to $e = n - k$ erasures or up to $t = (n\text{–}k)/2$ errors. More generally, it can correct any combination of l erasures and m errors provided that $2m + l$ does not exceed $n - k$. Thus the probability of overall block (an outer codeword) error, P_E, for the memoryless channel is given by

$$P_E = \sum_{2t+e>n-k} \binom{n}{t,e} P_d^e P_{ud}^t (1-P_d-P_{ud})^{n-t-e} . \tag{3}$$

where

$$\binom{n}{t,e} = \frac{n!}{t!\,e!\,(n-t-e)!} . \tag{4}$$

III. (*N*, *K*)(*N*, *K*) INNER CODING SCHEME

TABLE I. THE KINDS OF EVENTS, JOINT PROBABILITIES AND NUMBER OF OCCURRENCES IN THE (*N*, *K*)(*N*, *K*) SCHEME

Kinds of Events	Joint Probability	No. of Occurrences
$E_{d_1}E_{d_2}$	$P_{d_1 d_2}$	n_1
$E_{ud_1}E_{ud_2}$	$P_{ud_1 ud_2}$	n_2
$E_{c_1}E_{c_2}$	$P_{c_1 c_2}$	n_3
$E_{d_1}E_{ud_2}$	$P_{d_1 ud_2}$	n_4
$E_{ud_1}E_{d_2}$	$P_{ud_1 d_2}$	n_5
$E_{d_1}E_{c_2}$	$P_{d_1 c_2}$	n_6
$E_{c_1}E_{d_2}$	$P_{c_1 d_2}$	n_7
$E_{ud_1}E_{c_2}$	$P_{ud_1 c_2}$	n_8
$E_{c_1}E_{ud_2}$	$P_{c_1 ud_2}$	n_9

In the (N, K)(N, K) scheme considered in this section, the two inner decoder operate independently. Then the two outer symbols which have passed through the super channel, have joint probabilities and the number of occurrences are given Table I. In Table I, E_i represents that the event of an outer symbol is i, $i \in \{d, ud, c\}$ where d, ud, c represent detected error, undetected error, and corrected error, respectively. $E_{i1}E_{j2}$ denotes that the first outer symbol is E_i and the second one is E_j, $i, j \in \{d, ud, c\}$. n_k, $k \in \{1, 2, \ldots, 9\}$, express the number of events occurring. Hence the condition $\sum_{k=1}^{9} n_k = n/2$ should be satisfied, since there are $n/2$ transmissions to send an outer codeword. The distribution of $n/2$ outer symbol pairs, passed though the *super channel* created by the inner code, is given by *multinomial*. Then the overall block error probability, $P_E^{'}$ is given by

$$P_E^{'} = \sum_{2t+e>n-k} \frac{\left(\frac{n}{2}\right)!}{\prod_{k=1}^{9}(n_k)!} P_{d_1d_2}{}^{n1} P_{ud_1ud_2}{}^{n2} P_{c_1c_2}{}^{n3} \times P_{d_1ud_2}{}^{n4+n5} P_{d_1c_2}{}^{n6+n7} P_{ud_1c_2}{}^{n8+n9} \tag{5}$$

Where

$$e \,\square\, 2n1 + n4 + n5 + n6 + n7$$
$$t \,\square\, 2n2 + n4 + n5 + n8 + n9 \tag{6}$$

In (5), the probabilities $P_{i_1 j_2}$, $i, j \in \{d, ud, c\}$, are derived in **Appendix A**.

IV. (2*N*, 2*K*) INNER CODING SCHEME

If we employ one inner codeword to limit two outer symbols, the super channel model created by the inner code is *M*-ary erasures and errors channel, which is the same as Figure

2. When errors are detected or corrected by the inner code, the inner codeword is decoded to corresponding two outer erasures or corrected symbols respectively. But if errors are undetected by the inner code, it causes two cases. One is that the inner codeword is decoded to two undetected outer symbols. The other is that it is decoded to an undetected (or corrected) and a corrected (or undetected) outer symbol respectively. We ignore the latter case, since the probability of the latter case occurring is much lower than that of the former. Thus we can show that the overall block error probability is given by

$$P_E^{''} = \sum_{4t+2e>n-k} \frac{\left(\frac{n}{2}\right)!}{t!\,e!\left(\frac{n}{2}-t-e\right)!} P_d^e P_{ud}^t \left(1-P_d-P_{ud}\right)^{\frac{n}{2}-t-e}. \quad (7)$$

In (7), P_d , P_{ud} and P_c can be derived from *total probability theorem* [6].

V. NUMERICAL RESULTS AND DISCUSSIONS

It is very difficult to compare equation (5) with equation (7) in a finite region, since we must find all combination of n_k, $k=1,2,\ldots,9$, to calculate the overall block error probability $P_E^{'}$ in equation (5). It is a very time consuming job in computer computation. Hence we compare these two inner coding schemes in asymptotic region.

From *Weak Law of Large Numbers* [6], it can be shown that (see **Appendix B**)

$$\lim_{n,k\to\infty} P_E^{'} = \begin{cases} 0, & r<1-S^{'} \\ \frac{1}{2}, & r=1-S^{'} \\ 1, & r>1-S^{'} \end{cases} \quad (8)$$

where

$$S^{'} = P_{d_1 c_2} + 2P_{ud_1 c_2} + P_{d_1 d_2} + 3P_{d_1 ud_2} + 2P_{ud_1 ud_2} \quad (9)$$

and

$$\lim_{n,k\to\infty} P_E^{''} = \begin{cases} 0, & r<1-S^{''} \\ \frac{1}{2}, & r=1-S^{''} \\ 1, & r>1-S^{''} \end{cases} \quad (10)$$

where

$$S^{''} = P_d + 2P_{ud}. \quad (11)$$

Then we compare $1-S^{'}$ with $1-S^{''}$, which are the maximum achievable outer code rates for *error free* communication in asymptotic region. We have plotted $1-S^{'}$ and $1-S^{''}$ as a function of simultaneously active users I for N = 15, 20, 30 in Figure 3, 4, 5, respectively. We assume outer block length n is 2048, which is large enough to simulate asymptotic region. Inner message length, K, is, therefore, 11.

We can see several facts from Figure 3, Figure 4, and Figure 5. First, we can see that the (2N, 2K) scheme gives better performance than the (N, K)(N, K) scheme when N =15. Particularly, as the channel traffic (number of users simultaneously accessing a given frequency slot) gets larger, the performance differences between (2N, 2K) and (N, K)(N, K) scheme becomes more significant. But we can also note that there is no performance differences when N = 20 and N = 30. This means that the shorter the redundancy of the inner code is, the better the performance of (2N, 2K) scheme is than (N, K)(N, K) scheme.

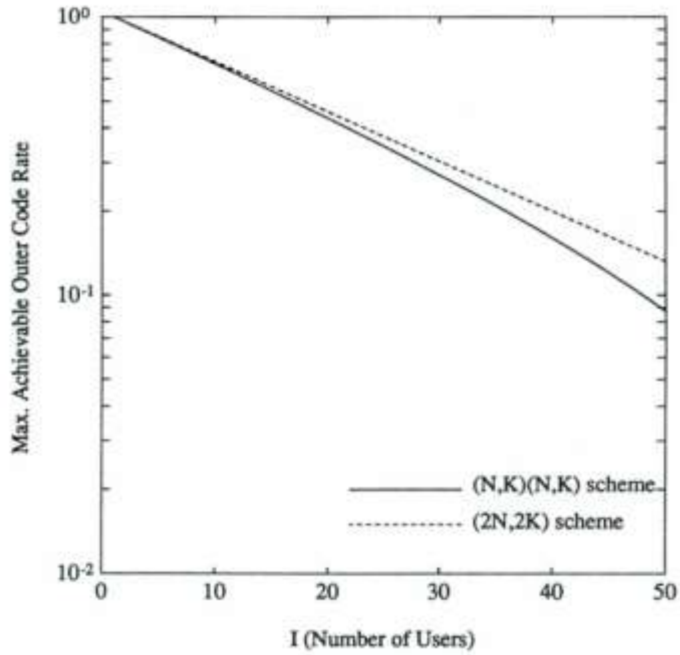

Figure 3. Maximum achievable outer code rate by varying I. (q = 25, $n = 2^K$ = 2048, K = 11, N = 15)

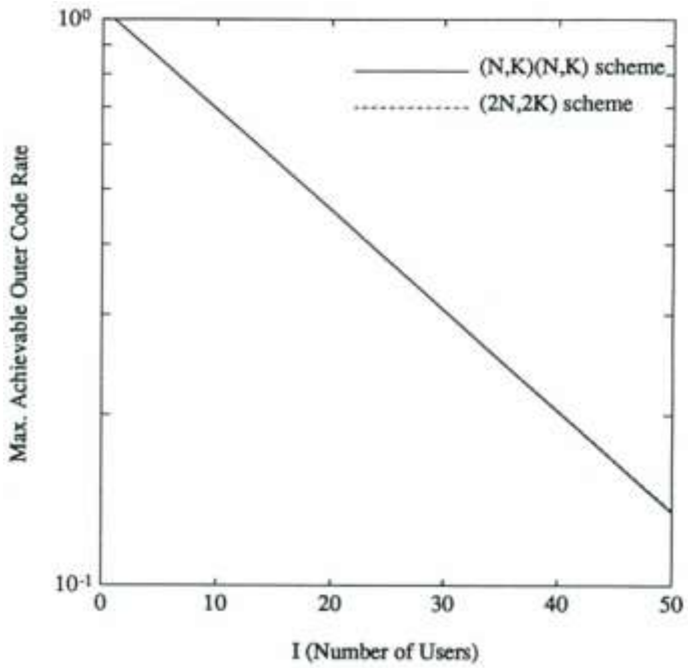

Figure 4. Maximum achievable outer code rate by varying I. (q = 25, $n = 2^K$ = 2048, K = 11, N = 12)

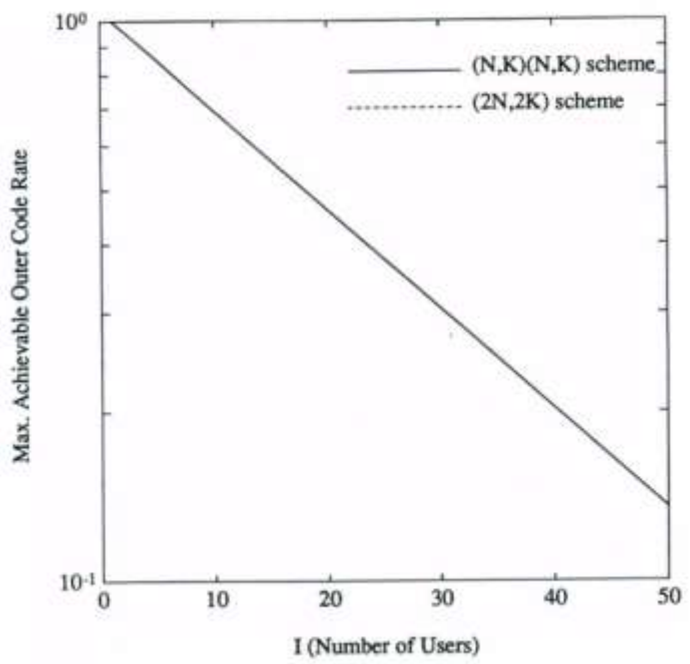

Figure 5. Maximum achievable outer code rate by varying I. (q = 25, $n = 2^K$ = 2048, K = 11, N = 30)

VI. Conclusions

In this paper, we considered two kinds of inner coding structures of concatenated codes for slow FH/SS systems where particularly two outer code symbols are transmitted during a hop. The first structure consists of one inner codeword per one outer code symbol, and second structure consists of one inner codeword per two outer code symbols. We investigated the overall block error probability of two schemes in asymptotic region and showed that the performance of the second scheme is superior to the first one.

Appendix A

In this appendix, we derive the probabilities $P_{i_1 j_2}$, $i, j \in \{d, ud, c\}$, in (5). Let us define

$$\begin{aligned} P_{i_1 i_2|\Delta_k} &\triangleq P(E_{i_1} E_{j_2} | \Delta_k \text{ is selected}) \\ P_{i_j} &\triangleq P(j^{th} \text{ outer symbols is } E_i) \end{aligned} \tag{12}$$

and

$$P_i \triangleq P_{i_1} = P_{i_2} \text{ for } i \in \{d, ud, c\},\ j \in \{1, 2\}. \tag{13}$$

Then,

$$\begin{aligned} P_{i_1 j_2} &= \sum_{k=1}^{I-1} P_{i_1 j_2|\Delta_k} P\ (\Delta_k \text{ is selected}) \\ &= \sum_{k=1}^{I-1} P_{i_1|\Delta_k} P_{i_1|\Delta_k} P\ (\Delta_k \text{ is selected}) \\ &= \begin{cases} \sum_{k=1}^{I-1} P^2_{i|\Delta_k} P(\Delta_k \text{is selected}), & i = j \\ \sum_{k=1}^{I-1} P_{i|\Delta_k} P_{j|\Delta_k} P(\Delta_k \text{is selected}), & i \neq j \end{cases} \end{aligned} \tag{14}$$

where we assume that two received inner codewords are independent. This assumption can be proved as follows.

Based on the *memoryless channel*, we have input vector $\mathbf{X} = (X_1 X_2) = (x_1 x_1, \cdots, x_{2N})$ and output vector $\mathbf{Y} = (Y_1 Y_2) = (y_1, y_2, \cdots, y_{2N})$. From the definition,

$$P(\mathbf{Y}|\mathbf{X}) = \prod_{i=1}^{2N} P(y_i | x_i), \tag{15}$$

which implies the channel crossover probability, p, is time invariant and independent from bit to bit. Then (15) is written by

$$\begin{aligned} P(\mathbf{Y}|\mathbf{X}) &= \left\{\prod_{i=1}^{N} P(y_1|x_1)\right\}\left\{\prod_{i=N}^{2N} P(y_1|x_1)\right\} \\ &= P(Y_1|X_1) P(Y_2|X_2). \end{aligned} \tag{16}$$

Therefore, each output symbols depends only on corresponding each input symbols, indicating that we can assume two input symbols X_1, X_2 are independent.

Appendix B

In this appendix, we first provide the proof of (8). Let random variables X_i, $Y_{n/2}$ and $Z_{n/2}$ be defined as

$$X_i = \begin{cases} 0, & \text{if the two received symbols are both corrected} \\ 1, & \text{if one received symbol is corrected and the other is erased} \\ 2, & \text{if the two received symbols are both erased or one is corrected and the other is in error} \\ 3, & \text{if one received symbol is erased and the other is in error} \\ 4, & \text{if the two received symbols are both in error} \end{cases}, \tag{17}$$

$$Y_{n/2} \triangleq \sum_{i=1}^{n/2} X_i\ , \tag{18}$$

and

$$Z_{n/2} \triangleq \frac{Y_{n/2} - E[Y_{n/2}]}{\sqrt{Var[Y_{n/2}]}}, \tag{19}$$

respectively. In (19), $E[Y_{n/2}]$ and Var $[Y_{n/2}]$ are the mean and the variance of $Y_{n/2}$. Then $Y_{n/2}$ is the total number of erasures and twice the number of errors in the received outer codeword. Therefore the overall block error probability, $P_E^{'}$, is given by

$$P_E^{'} = P\left(Y_{n/2} > n - k\right) = P\left(\frac{Y_{n/2}}{n/2} > 2(1-r)\right). \tag{20}$$

From the *weak law of large number* [6],

$$\lim_{n\to\infty} P\left(\left|\frac{Y_{n/2}}{n/2} - E\left[X_i\right]\right| > \varepsilon\right) = 0 \qquad (21)$$

For any ε, this implies

$$\lim_{n\to\infty} P\left(\frac{Y_{n/2}}{n/2} > 2(1-r)\right) = \begin{cases} 1, & 2(1-r) > E\left[X_i\right]+\varepsilon \\ & \left(\leftrightarrow r < 1-\frac{E[X_i]}{2}\right) \\ 0, & 2(1-r) < E\left[X_i\right]+\varepsilon \\ & \left(\leftrightarrow r < 1-\frac{E[X_i]}{2}\right) \end{cases} . \qquad (22)$$

Therefore, we get

$$P_E^{'} = \begin{cases} 1, & r < 1-\frac{E[X_i]}{2} \\ 0, & r > 1-\frac{E[X_i]}{2} \end{cases} . \qquad (23)$$

Here we assume $X_i, i = 1, 2, \ldots, n/2$, are the independent and identically distributed random variable. Then, from *central limit theorem* [6]

$$\lim_{n/2\to\infty} P(Z_{n/2} \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du \quad . \qquad (24)$$

Also it can be also shown that

$$\lim_{n\to\infty} P_E^{'} = 0.5 \ \text{ if } \ r = 1-\frac{E[X_i]}{2} \quad . \qquad (25)$$

We now have

$$E[X_i] = 2P_{d_1c_2} + 4P_{ud_1c_2} + 2P_{d_1d_2} + 6P_{d_1ud_2} + 4P_{ud_1ud_2} \qquad (26)$$

Combining (23), (25), and (26) yields

$$\lim_{n\to\infty} P_E^{'} = \begin{cases} 0, & r < 1-P_{d_1c_2} - 2P_{ud_1c_2} - P_{d_1d_2} - 3P_{d_1ud_2} - 2P_{ud_1ud_2} \\ 0.5, & r = 1-P_{d_1c_2} - 2P_{ud_1c_2} - P_{d_1d_2} - 3P_{d_1ud_2} - 2P_{ud_1ud_2} \\ 1, & r > 1-P_{d_1c_2} - 2P_{ud_1c_2} - P_{d_1d_2} - 3P_{d_1ud_2} - 2P_{ud_1ud_2} \end{cases} \qquad (27)$$

Following again the above procedure, the proof of (10) is straightforward.

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REFERENCES

[1] E. Geraniotis and M. B. Pursley, "Error probabilities for slow-frequency-hopped spread-spectrum multiple-access communications over fading channels," IEEE Trans. Communions, vol. COM-31, pp. 996-1009, May 1982.

[2] G. D. Forney, Concatenated Codes, MIT Press, Cambridge, MA, 1966.

[3] Z. Yang, M. Han, C. Pan, A. Men, and L. Yang, "A novel scheme of coding and modulation for digital television terrestrial broadcasting," Proc. of IEEE PIMRC, pp. 376-379, 2003.

[4] T. Blahut, Theory and Practice of Error Control Codes, Addison Wesley Pub. Co, 1983.

[5] H. Sato, "Two-user communication channels," IEEE Trans. on Inform. Theory, vol. IT-23, pp. 295-304, May 1977.

[6] A. Papoulis, Probability, Random Variables, and Stochastic Processes, McGrawHill Inc, Kogakusha, Ltd., 1965.