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Application of Graph Theory in Computer Science

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Abstract: The field of mathematics have important roll in various fields. One of the important area in mathematics is Graph Theory. Which used in structural modeling in many area's. The structural arrangements of various objects or technologies lead to new inventions and modification in the existing environment for enhancement in those field. The field of graph theory started from problem of Konigsberg bridge in 1735. This paper given an overview of the application of graph theory in heterogeneous field to some extent but mainly focuses on computer science application but uses graph theoretical concepts.

Keywords: *Edges, Vertex, Graph, Bipartite graph Spanning Graph, Networks.*

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I. Introduction:

Graph theoretical ideas are highly utilized by computer science applications. Especially in research areas of computer science such data mining, image segmentation, clustering, image capturing, networking etc., For example a data structure can be designed in the form of tree which in turn utilized vertices and edges. Similarly modeling of network topologies can be done using graph concepts, in the same way the most important concept of graph coloring is utilized in resource allocation, scheduling. Also, path walks and circuits in graph theory are used tremendous applications say travelling salesman problem, database design concepts, resource networking. This leads to the development of new algorithms and new theorems that can be used in tremendous applications.

II. History of Graph theory:

The origin of graph theory started with the problem of Koinsber bridge, in 1735. This problem lead to the concept of Eulerian Graph. Euler studied the problem of Koinsberg bridge and constructed a structure to solve the problem called Eulerian graph. In 1840, A.F. Mobius gave the idea of complete graph and bipartite graph and Kuratowski proved that they are planar by means of recreational problems. The concept of tree, (a connected graph without cycles) was implemented by Gustav Kirchhoff in 1845, and he employed graph theoretical ideas in the calculation of currents in electrical networks or circuits. In 1852, Thomas Gutherie found the famous four color problem. Then in 1856, Thomas. P. Kirkman and William R.Hamilton studid cycles on polyhydra and invented the concept called Hamiltonian graph by studying trips that visited certain sites exactly once. In 1913, H.Dudency mentioned a puzzle problem. Eventhough the four color problem was invented it was solved only after a century by Kenneth Appei and Wolfgang Haken. This time is considered as the birth of Graph Theory.

Calcy studied particular analytical foums from differential calculus to study the trees. This had many implications in theoretical chemistry. This lead to the invention of enumerative graph theory. Any how the terms "Graph" was introduced by Sylvester in 1878 where he drew an analogy between "Quantie invariants" and covariants of algebra and molecular diagrams. In 1941, Ramsey worked on colorations which lead to the identification of another branch of graph theory called extremely graph theory, in 1969, the four color problem was solved using computers by Heinrich. The study of asymptotic graph connectivity gave rise to random graph theory.

III. Graphical representation of algorithm:

An algorithm will be defined in the formof a facility graph whose nodes represent the facilities required to execute the algorithm and whose edges represent the links required among these facilities. An algorithm A is executable by a computing system S if A is isomorphic to a sub graph of S. This means that there is a 1-1 mapping from the nodes of A into the nodes of S that preserves nodes of A into the nodes of S that preserves node labels and adjacencies between nodes. This implies that S contains all the facilities and connections between facilities required by A, so, A can be embedded in S. If G_1 and G_2 represents a system and an algorithm respectively, the G_2 is executable by G_1. The representation of G_2 is as follows.

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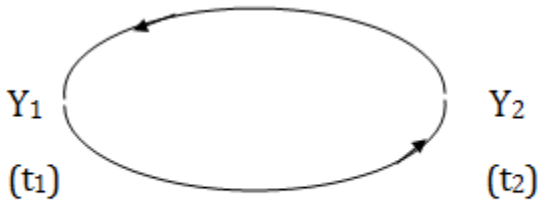


Fig. (G_2)

The isomorphism are as follows:

(y_1, y_2) (x_1,x_2)

(y_1,y_2) (x_3,x_2)

A k-fault F in a system S is the removal of any k nodes {x_1, x_2, x_3,m x_4, …… x_5} from S. All edges connected to these nodes are also removed. The resultant graph will be denoted by SF. It can be written F = {{x_1,x_2,x_3,x_4 ……. X_5}

The basic concepts relating to fault tolerance system is given by the authors.

1. A system S is fault tolerant with respect to algorithm A and fault F, if A is executable by SF.
2. S is fault tolerant with respect to a set of algorithms {A_1,A_2,A_3, ……..A_p} and a set of faults {F_1,F_2, ……. F_q}, if Ai is executable by S^F_j for all i and j where 1<i<p.

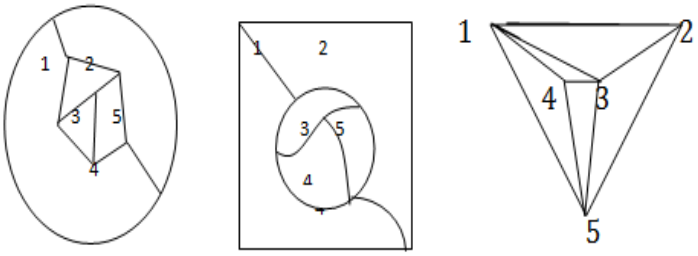
If S is k-fault tolerance with respect to A, then S is j-FT (Fault Tolerance) with respect to A, for all j where 0<j<k. G_1 is fault tolerance with respect to G_2. It is not 2 FT because if the fault F = {x_2, x_4} is present, then G_2 cannot be executed by G_2^F. The Optimal k-FT single loop system is given in this paper

1. A single loop system C_n is a graph consisting of n>3 nodes each of degree 2.
2. Let C be any optimal k-FT realization of the single loop system C_n the degree of every node in C is at least k+2.
3. If m is the number of edges in any k-FT single loop system, then $m \geq \left\lceil \frac{(k+2)(n+k)}{2} \right\rceil$

   In this paper we have used Hamiltonian circuits and Hamiltonian path concepts to design optimal k-FT systems. The central idea focused here is to stress the importance of the graph model called facility graph to represent computing system and the algorithms they execute. This model gives a precise definition of the basic concept of a k-fault tolerant system.

IV. Graph theory in symbol recognition:

Here, the authors have discussed the paper "Symbol Recognition by Error tolerant sub graph matching between region adjacency graphs"



(Segmented graph) (Region Adjacency) (Dual graph)

The distorted image subgraph is matched with the model graph. That is the image region and the model region are modeled as the image subgraph and model graph. The cost of adding a neighbor to the graphs consisting of the matched regions and the neighbor region candidates. This method is applicable to any region adjacency graph representation.

Automatic channel allocation for small wireless local are networks using graph coloring algorithm approach:

In this paper the authors focuses channel allocation issue in wireless LAN by means of modeling the network in the form of a graph and solving it using graph coloring methodology.

The graph model is constructed and called as interference graph since the access points are interfering with some other access points in the same region. The graph is called as interference graph, which is constructed y the access points as nodes. An undirected edge is connecting these nodes if the nodes interfere with each other when using the same channel. Now, the channel allocation problem is converted into graph coloring problem. i.e vertex coloring problem. A vertex coloring function f:v(G)-C where C is the set of colors corresponds to the channels on the access points. These channels are preferably non overlapping edges. A coloring algorithm is developed by the authors called DSATUR (Degree of Saturation) for coloring purpose. The algorithm is a heuristic search. i.e It finds vertices with large number of differently colored neighbors. If this subset contains only one vertex it is chosen for coloring. If the subset contains more than one vertex then the coloring is done based on the order of decreasing number of uncolored neighbors. If more than one candidate vertex is available then the final selection is replaced by a deterministic selection function to select the vertex. The protocol operating is done by identifying the neighbors by means of listening the massages generate by the access

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points. After finishing this, the interference graph is constructed and the coloring algorithm is applied. The correspondence between the channels and the graph is that as the channels listen the massages in regular intervals as the same way the coloring algorithm should be kept running at regular intervals.

A floor plan and the corresponding interference graph is given below.

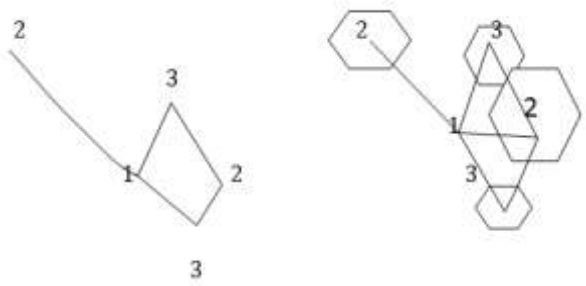


Fig : Interference Graph

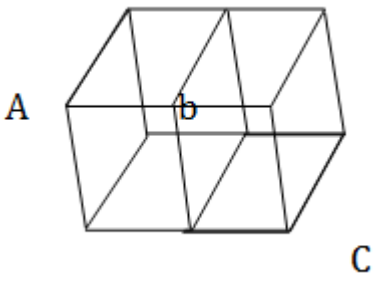
Finally we will conclude that based on the refinement strategy the interference graph can be reconstructed by adding an edge in case of additional edges.

V. Clustering of web documents using graph model:

We will discussed here the enhanced representation of web documents by means of clustering them. They used graphs instead of vectors. Here they have used the classical k-means clustering algorithm which uses the maximum common subgraph distance measure instead of usual distance measure and the concept of median graphs instead of centroid calculations.

(k-means clustering is a method of cluster analysis which aims to partition n observations into k clusters in which each observation belongs to the cluster with the nears mean.)

(A median graph is an undirected graph in which any three vertices a,b and c have a unique median: a vertex m(a,b,c) that belongs to shortest paths between any two of a, b and c



Since, traditional clustering methods are working purely on numeric feature vectors, the original data needs to be converted to a vector of numeric values by discarding possibly useful structural information. Otherwise, new customized algorithms have to be developed for specific representation.

Modeling the web documents as graphs has two significant benefits.

1. It keeps the inherent structure of the original documents, rather to arrive numeric feature vectors that contain term frequencies.
2. There is no necessity to build new algorithm completely from scratch. But the extension of classical algorithms can be developed to deal with graphs that use numerical vectors.

The graph model of the web documents has constructed by the following method.

1. Each word that appears in the web documents except stop words is a vertex in the graph representing that document. This is excuted by a node labeling function which gives labels to each node. Even though a word is repeated more than once, it is represented as only one vertex. Therefore each vertex in the graph represents a unique word and is labeled with a unique term which is not used to label any other node.
2. If any word say b follows another word say a then there is a directed edge between these two words a and b. if these two words are in a section say s, then the edge between a and b will be labeled as s.
3. Some punctuation marks are not taken into account for edge between a and b will be labeled as s.
4. Three sections are defined here. They are sections for title tag. Section link and Section text.
5. The nodes and their corresponding incident edges to the words such as the, end, of, a, for, to, etc., are removed because these words don't play much role.
6. A stemming check is performed for plural forms.
7. The most infrequent words are removed from each page by leaving maximum nodes say m for each and every graph where m is the user defined parameter.

the example of the graph representation of the web document is given below. The edges are labeled based on the titles, links and texts.

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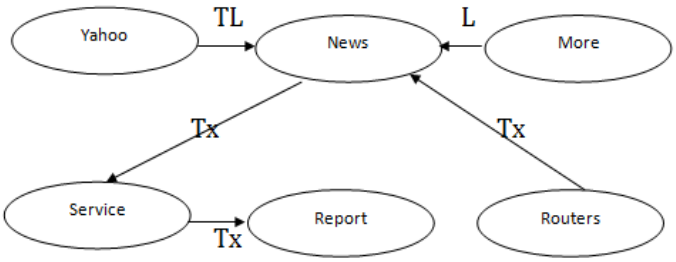


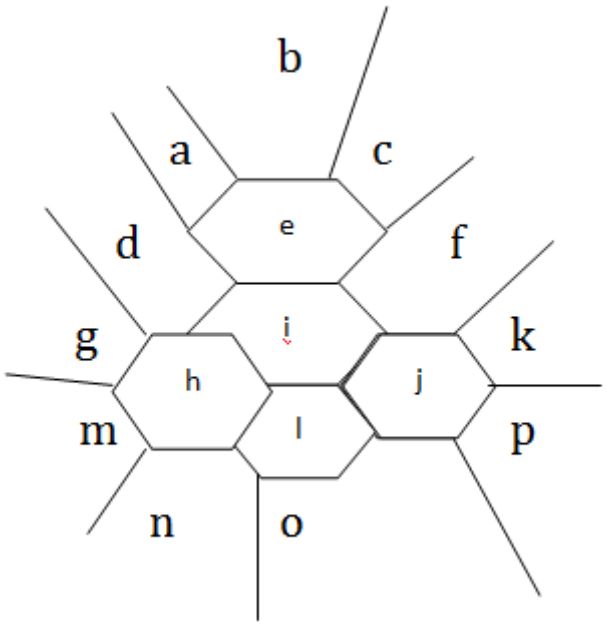
The authors have compared the results obtained by the graphical method with the vector methods and showed that the flustering web documents as graph model have give much improvement. [1]

Modeling sensor networks as graph:

The sensor networks have got variety of applications. Tracking of mobile objects, collection of environmental data, defence applications, health care etc.,

In this paper the authors have covered a message running tree tree with minimum cost to track the moving objects in wireless sensor networks. The sensor network is modeled as a graph to analyze the communication efficiency. The authors have taken voronoi graph to model the sensor networks. (A Voronoi diagram is a special kind of decomposition of a metric space determined by distances to a specified discrete set of objects in the space)

Because voronoi graph is constructed in a plane in the form of polygons with nodes as the sensors and the polygon boundaries can be considered as the sensing range of each sensor. Consider the plane as the sensing field and S be the of sensors. The sensing field is partitioned into a voronoi graph as shown.



Any object located in a polygon of the voronai graph is closest to the sensor in the corresponding polygon. The polygon can be considered as the sensing range of these sensors. Among these sensors one sensor will be the cluster head for reporting function. Two sensors are considered as neighbors if their sensing range share a common boundary in the voronai graph. In the diagram a, b are neighbours. Similarly, e,f,e,d; e,l; are also neighbours. When the objects cross the boundary of one sensor e.e the sensing range of one sensor, and enter into the sensing range of another sensor it should be reported to the neighbouring sensor properly by the previous sensor. The event rate between two neighbouring sensors implies the strength of the detection. Since, it is assumed that the sensor's transmission range is large enough such that any two neighbours can directly communicate with eachother, the network is represented as an undirected weighted graph G(VG,EG,WG) where V belongs VG, edge (u,v) belongs Eg. V implies the sensors, u,v implies the neighbours, WG(u,v) the weighted edge of (u,v) belongs EG. The authors have used the concept of covering. (K-cover, is defined as a set of sensors M such that each point in the sensor network is "covered" by at least K different sensors in M, and the communication graph induced by M is connected.)

Graph based and structural methods for fingerprint classification:

Fingerprint classification is mainly used in criminal investigation. This classification of fingerprints uses databases for storage of fingerprints. But the database will become large and the storage capacity will be more if it is stored as it is. In this paper the author gives an overview regarding graph based classification of fingerprints. Various approaches are available for the classification namely structural approaches, statistical approaches and graph based approaches. Here it is discussed only the graph based approaches. Previously, the work was done on the basis of segmenting the fingerprint images into regions containing ridges having homogeneous orientations. But this type of structural information is not useful in identifying the fingerprints based on other classes, since they have also the same structural arrangements. The graph based approach uses simple relational graphs.

The following diagram shows relational graph for fingerprint orientation field segmentations.

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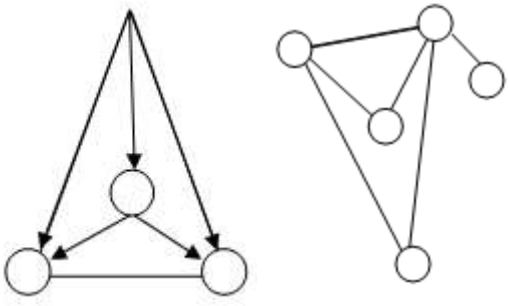

This method serves as a guide for segmentation. Relational graphs appear to be more appropriate since the nodes naturally correspond to the regions extracted by the segmentation algorithm. Each graph node can be associated to a segmentation region and the edges join two nodes according to the adjacency relationship of the computers a measure of the dissimilarity between the graph representing the input pattern to be classified and a certain graph prototype. This similarity measure is called edit distance. This is done by deformation model which can be done by substituting inserting or deleting nodes or edges.

The main idea stressed here is the graph based representation of the fingerprints I much useful for classification propose than the other structural methods.

VI. Conclusion:

The main aim of this paper is to present the importance of graph theoretical ideas in various areas of compute applications for researches that they can use graph theoretical concepts for the research. An overview is presented especially to project the idea of graph theory. So, the graph theory section of each paper is given importance than to the other sections. Researches may get some information related to graph theory and its application in computer field and can get some ideas related to their field of research.

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