Applying Digital MIMO Dynamic Sliding Surface Control to Idle Speed Control of Spark Ignition Engine

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Abstract—The good idle speed control of spark ignition engine makes the customers comfortable and drives smooth. To derive its controller, a control-oriented object based model of spark ignition engine is discussed in this paper. To achieve accuracy tracking and reduce the control effort, a digital MIMO dynamic sliding surface control rule is proposed in this paper. The sliding mode controller will drive the system to tracking the desired response. The existence condition of the discrete sliding mode is expanded to two conditions. The first one is used to keep the sliding motion toward to the sliding surface. The next one is to keep the sliding motion convergent. The dynamic sliding surface will decrease the approaching time. At the same time, the output magnitude of the switching part will be reduced efficiently. Finally, the simulation results will demonstrate the performance of the proposed control rule. The different working conditions will show the robustness of the proposed controller for parameter variation and the external load.

Keywords- *dynamic sliding surface control, idle speed, ignition engine.*

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I. INTRODUCTION

The engine is wide applied in industrial application [1-11]. The different kinds of engine, such as diesel engine, spark ignition engine and etc., have been developed in recent researches [1-6]. The spark ignition engine is especially applied to the personal cars. The stability of idle speed of spark ignition engine will affect the user's comfort. Therefore, the major objective is the idle speed control of a spark ignition engine in this paper. A linear model of the Fiat Dedra engine [7] is used to study the idle speed control problem. The engine dynamics based on the intake manifold process, combustion and rotational dynamics compose a multi-input multi-output (MIMO) system. A time delay caused by the transports of air and fuel in combustion process is also present and affects this system. At the same time, the parameters of the linear model measured on different operating conditions are so different that there exist the uncertainty problems in design of such engine systems. Therefore, the controller of the idle speed must be able to overcome the parameter deviations and against the external disturbances.

The variable structure control (VSC) [12] has been applied to many industrial fields. The advantages of the order-reducing and against to external disturbance cause the proper robustness of the VSC. However, in order to lock on the sliding surface, the hard switching method causes the chattering phenomena. The high frequency switching does not make the VSC to be practical [13-14].To smooth the chattering, some soft switching function is taken to replace the sign function. At the same time, the additional integral controller is used to eliminate the steady state error. Mostly variable structure controller is divided into the equivalent controller and the switching controller. The first controller is working under the ideal condition. The second controller is used to force the sliding motion toward the sliding surface and make the stability. Hence the integral gain and the switching gains are selected to satisfy the existence condition, that is

$$\sigma(t)\dot{\sigma}(t) < 0 \tag{1}$$

Change the differential equation to difference equation in order to apply to the discrete system. When applying the condition to discrete systems, the existence condition becomes to

$$\sigma(k)[\sigma(k+1) - \sigma(k)] < 0 \tag{2}$$

However, referring to the reference [15], the system is controlled by the controller only in each sampling time. The controller cannot modify the response during the sampling interval. It may happen that the condition (2) is not only satisfied but also the sliding motion is divergent, shown in fig 1.

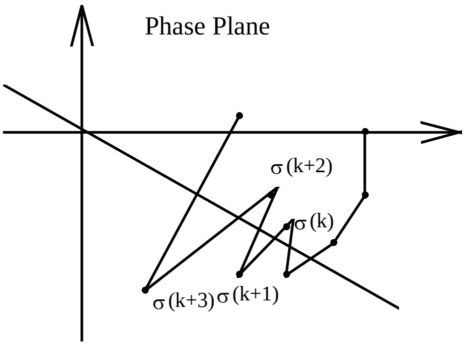

Figure 1. Discrete sliding mode

The condition (2) only makes the sliding motion toward to the sliding surface. However, it cannot guarantee the sliding mode convergent to this surface. The condition (2) is only the necessary condition not the sufficient condition in discrete systems. To make up the drawback, the reference [15] introduces one additional restriction, that is

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$$|\sigma(k+1)| < |\sigma(k)| \tag{3}$$

Combining equation (2) and (3) can make sure the sliding motion convergent. However, the sliding surface is changed to sliding region shown in fig 2.

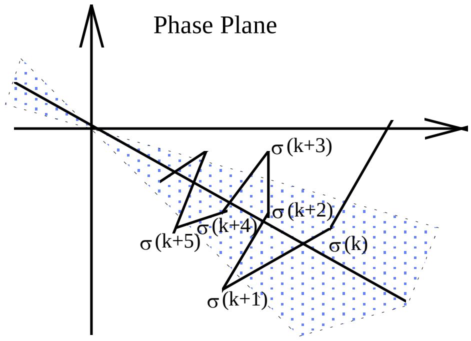

Figure 2. Discrete sliding mode with sliding region

The choice of the switching gain becomes three states. This change causes some difficulty in implementation of hardware. To maintain the binary choice, one restriction of different viewpoint is introduction in reference [16], that is

$$|\sigma(k+1) - \sigma(k)| < \frac{\xi}{2} \tag{4}$$

where ξ is a small positive constant. The varying of each step of sliding motion is restricted. Then, condition (2) makes the sliding mode toward to the surface. The condition (4) makes the sliding motion oscillated on this surface within a small range ξ shown in fig 3.

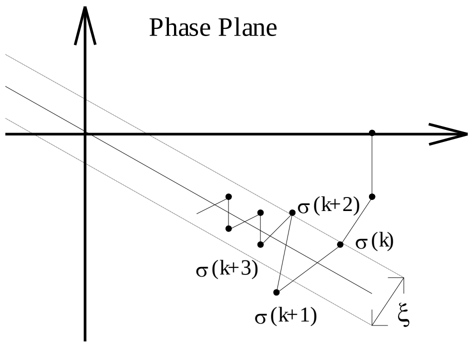

Figure 3. New discrete sliding mode

To reduce the time of out of control when the motion is not fixed on the sliding surface, the high switching gain is usually chosen to speed up the reaching time. However, in discrete systems the controller only modifies the control signal at each sampling time. High switching gains can speed up the reaching time, but the chattering often be enlarged. To eliminate the chattering and decrease the reaching time, this paper introduces the dynamic sliding surface control (DVSSC) rule shown in fig 4.

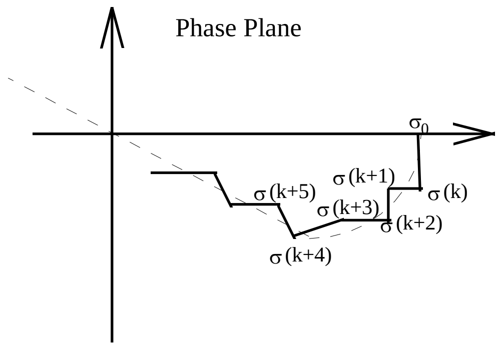

Figure 4. Discrete sliding mode with dynamic sliding surface

This method uses the first order of difference equation of sliding variable, that is

$$\sigma(k+1) = -p\sigma(k), \quad \sigma_0 = \sigma(x_0) \tag{5}$$

The initial of system's states sets the initial value of the difference equation. Obviously, at the beginning the system's dynamic is positioned on the dynamic sliding surface. Hence, the reaching can be reduced efficiently. Only small switching gain in needed to eliminate the parameter variation and external disturbance. Therefore, the chattering phenomena can be eliminated.

The novel digital MIMO dynamic sliding surface Control is proposed in this manuscript to apply to the idle speed control of spark ignition engine. To make sure the existence of sliding motion, the continuous condition is expanded to two conditions [13-14]. The existing conditions of the digital sliding mode are $[\sigma_i(k+1) - \sigma_i(k)]\sigma_i(k) < 0$ and $|\sigma_i(k+1) - \sigma_i(k)| < \frac{\xi_i}{2}$, where $\sigma_i(k)$ means each sliding surface. The dynamic sliding surfaces are defined as $\mathbf{S}_{m\times 1}(k+1) = -\mathbf{P}_{m\times m}\mathbf{S}_{m\times 1}(k)$. The detailed design procedures will be discussed in this manuscript. Finally, the simulation results will demonstrate the potential of the proposed approach even the external disturbances existed.

II. THE LINEAR MODEL OF THE SPARK IGNITION ENGINE

The block diagram of the linear model of the Fiat Dedra engine is shown in fig 5. It is consisted of three main physical phenomena inner the engine. They are intake manifold dynamics, combustion and rotational dynamics.

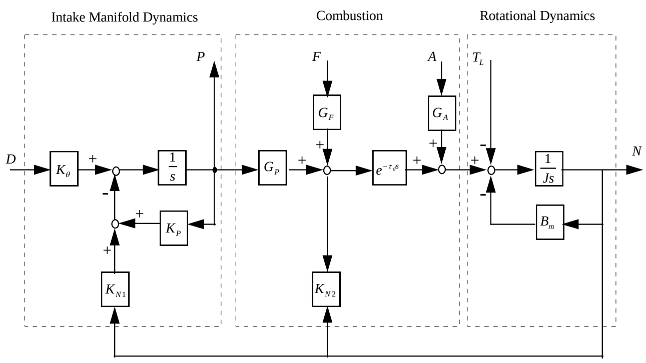

Figure 5. Block diagram of the linear model of the engine

In the intake manifold dynamics, one can assume that the temperature is approximately constant. Therefore, one can treat the modeling development of the manifold as the air pump. The relationship between the rate of change of the manifold pressure $P(t)$ and the duty cycle of the throttle valve $D(t)$ is

$$\dot{P}(t)=K_{\theta}D(t)+K_{N1}N(t)-K_{p}P(t) \tag{6}$$

where K_{θ} is the constant gain, K_{N1} is the pumping feedback gain and K_{p} is the gas gain.

The combustion process generating the required torque is depended on the air, fuel $F(t)$, engine speed $N(t)$ and spark advance position $A(t)$. Those elements don't have immediately influence on the combustion process except the spark advance position. A time delay called induction-to-power (IP) stroke lag, τ_{d}, is produced by the transport delay. The amplitude of the time delay depends on the number of the cylinders and the valve in applied engine [1]. The relationship of the combustion process can be described as

$$T_{e}(t)=\left[G_{p}P(t)+G_{F}F(t)+K_{N2}N(t)\right]delay(\tau_{d})+G_{A}A(t) \tag{7}$$

where G_{p}, G_{F}, K_{N2} and G_{A} are the relative gains. Finally, the rotational dynamic is given by the Newton's second law as

$$J\dot{N}(t)=T_{e}(t)-T_{L}(t)-B_{m}N(t) \tag{8}$$

where J is the engine inertia, $T_{L}(t)$ is the external torque load and B_{m} is viscous friction constant.

III. The Dynamic Digital MIMO Sliding Surface Control

If the MIMO system is defined as n states and p outputs, that is

$$\dot{\mathbf{X}}_{n\times1}=\mathbf{A}_{n\times n}\mathbf{X}_{n\times1}+\mathbf{B}_{n\times m}\mathbf{U}_{m\times1}+\mathbf{F}_{n\times1} \tag{9α}$$

$$\mathbf{Y}_{p\times1}=\mathbf{C}_{p\times n}\mathbf{X}_{n\times1} \tag{9β}$$

where

$$\mathbf{X}_{n\times1}=\begin{bmatrix}x_{1}\\x_{2}\\\vdots\\x_{n}\end{bmatrix},\quad \mathbf{A}_{n\times n}=\begin{bmatrix}-a_{11}&-a_{12}&\cdots&-a_{1n}\\-a_{21}&-a_{22}&\cdots&-a_{2n}\\\vdots&&\ddots&\\-a_{n1}&-a_{n2}&\cdots&-a_{nn}\end{bmatrix},$$

$$\mathbf{B}_{n\times m}=\begin{bmatrix}b_{11}&b_{12}&\cdots&b_{1m}\\b_{21}&b_{22}&\cdots&b_{2m}\\\vdots&&\ddots&\\b_{n1}&b_{n2}&\cdots&b_{nm}\end{bmatrix},\quad \mathbf{U}_{m\times1}=\begin{bmatrix}u_{1}\\u_{2}\\\vdots\\u_{m}\end{bmatrix},\quad \mathbf{F}_{n\times1}=\begin{bmatrix}f_{1}\\f_{2}\\\vdots\\f_{n}\end{bmatrix},$$

$$\mathbf{Y}_{p\times1}=\begin{bmatrix}y_{1}\\y_{2}\\\vdots\\y_{p}\end{bmatrix},\text{ and }\mathbf{A}_{n\times n}=\begin{bmatrix}-a_{11}&-a_{12}&\cdots&-a_{1n}\\-a_{21}&-a_{22}&\cdots&-a_{2n}\\\vdots&&\ddots&\\-a_{n1}&-a_{n2}&\cdots&-a_{nn}\end{bmatrix}$$

Let the output error integral controller is included in this system, that is

$$\dot{\mathbf{Z}}_{p\times1}=\mathbf{R}_{p\times1}-\mathbf{Y}_{p\times1} \tag{10}$$

where

$$\mathbf{Z}_{p\times1}=\begin{bmatrix}z_{1}\\z_{2}\\\vdots\\z_{p}\end{bmatrix},\text{ and }\mathbf{R}_{p\times1}=\begin{bmatrix}r_{1}\\r_{2}\\\vdots\\r_{p}\end{bmatrix}.$$

Then, by forward difference to replace the analog differential operation, the system model will be re-written as

$$\begin{bmatrix}\mathbf{Z}_{p\times1}(k+1)\\\mathbf{X}_{n\times1}(k+1)\end{bmatrix}=\begin{bmatrix}\mathbf{I}_{p\times p}&-T\mathbf{C}_{p\times n}\\\mathbf{0}_{n\times p}&\mathbf{I}_{n\times n}-T\mathbf{A}_{n\times n}\end{bmatrix}\begin{bmatrix}\mathbf{Z}_{p\times1}(k)\\\mathbf{X}_{n\times1}(k)\end{bmatrix}+\begin{bmatrix}\mathbf{0}_{p\times m}\\T\mathbf{B}_{n\times m}\end{bmatrix}\mathbf{U}_{m\times1}(k)$$
$$+\begin{bmatrix}T\mathbf{I}_{p\times p}\\\mathbf{0}_{n\times p}\end{bmatrix}\mathbf{R}_{p\times1}(k)+\begin{bmatrix}\mathbf{0}_{p\times n}\\T\mathbf{I}_{n\times n}\end{bmatrix}\mathbf{F}_{n\times1}(k)$$

$$\mathbf{Y}_{p\times1}(k)=\mathbf{C}_{p\times n}\mathbf{X}_{n\times1}(k)$$

Let the sliding surface is chosen to be

$$\mathbf{S}_{m\times1}(k)=\Omega^{0}_{m\times p}\left[\mathbf{Y}_{p\times1}(k)-\mathbf{K}_{p\times p}\mathbf{Z}_{p\times1}(k)\right]+\Omega_{m\times n}\mathbf{X}_{n\times1}(k)$$
$$=\begin{bmatrix}-\Omega^{0}_{m\times p}\mathbf{K}_{p\times p}&\Omega^{0}_{m\times p}\mathbf{C}_{p\times n}+\Omega_{m\times n}\end{bmatrix}\begin{bmatrix}\mathbf{Z}_{p\times1}(k)\\\mathbf{X}_{n\times1}(k)\end{bmatrix}$$

where $\Omega^{0}_{m\times p}$ and $\Omega_{m\times n}$ are switching gain matrix and $\mathbf{K}_{p\times p}$ is the integral gain matrix. They are defined as

$$\mathbf{S}_{m\times1}=\begin{bmatrix}\sigma_{1}\\\sigma_{2}\\\vdots\\\sigma_{n}\end{bmatrix},\quad \Omega^{0}_{m\times p}=\begin{bmatrix}\sigma^{0}_{11}&\sigma^{0}_{12}&\cdots&\sigma^{0}_{1p}\\\sigma^{0}_{21}&\sigma^{0}_{22}&\cdots&\sigma^{0}_{2p}\\\vdots&&\ddots&\\\sigma^{0}_{m1}&\sigma^{0}_{m2}&\cdots&\sigma^{0}_{mp}\end{bmatrix},$$

$$\Omega_{m\times n}=\begin{bmatrix}\sigma_{11}&\sigma_{12}&\cdots&\sigma_{1n}\\\sigma_{21}&\sigma_{22}&\cdots&\sigma_{2n}\\\vdots&&\ddots&\\\sigma_{m1}&\sigma_{m2}&\cdots&\sigma_{mn}\end{bmatrix},\quad \mathbf{K}_{p\times p}=\mathrm{diag}\{K_{1},K_{1},\cdots,K_{p}\},$$

and $\mathbf{P}_{m\times m}=\mathrm{diag}\{p_{1},p_{1},\cdots,p_{m}\}$

At the same time, let the dynamic sliding surface be an one ordered differential function as

$$\mathbf{S}_{m\times1}(k+1)=-\mathbf{P}_{m\times m}\mathbf{S}_{m\times1}(k) \tag{11}$$

If the sliding motion is locked on the dynamic sliding surface, then substitute eqs. (9~10) into eq. (11), one has

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$$
\begin{aligned}
&\mathbf{S}_{m\times1}(k+1)+\mathbf{P}_{m\times m}\mathbf{S}_{m\times1}(k)=\\
&+\left\{\begin{array}{l} T\left[\Omega^0_{m\times p}\mathbf{C}_{p\times n}+\Omega_{m\times n}\right]\mathbf{A}_{n\times n}\\ +\left[\mathbf{I}_{m\times m}+\mathbf{P}_{m\times m}\right]\left[\Omega^0_{m\times p}\mathbf{C}_{p\times n}+\Omega_{m\times n}\right]\\ +T\Omega^0_{m\times p}\mathbf{K}_{p\times p}\mathbf{C}_{p\times n}\end{array}\right\}\mathbf{X}_{n\times1}(k)\\
&+T\left[\Omega^0_{m\times p}\mathbf{C}_{p\times n}+\Omega_{m\times n}\right]\mathbf{B}_{n\times m}\mathbf{U}_{m\times1}(k)\\
&+T\left[\Omega^0_{m\times p}\mathbf{C}_{p\times n}+\Omega_{m\times n}\right]\mathbf{F}_{n\times1}(k)\\
&-\left[\mathbf{I}_{m\times m}+\mathbf{P}_{m\times m}\right]\left[\Omega^0_{m\times p}\mathbf{K}_{p\times p}\right]\mathbf{Z}_{p\times1}(k)\\
&-T\left[\Omega^0_{m\times p}\mathbf{K}_{p\times p}\right]\mathbf{R}_{p\times1}(k)\\
&=0
\end{aligned}
\tag{12}
$$

If there exists one diagnostic matrix such that

$$\tilde{\mathbf{U}}_{m\times1}(k)=\mathbf{D}^{-1}_{m\times m}\left[\Omega^0_{m\times p}\mathbf{C}_{p\times n}+\Omega_{m\times n}\right]\mathbf{B}_{n\times m}\mathbf{U}_{m\times1}(k)$$

where

$$\mathbf{D}_{m\times m}=\mathrm{diag}\{d_1,d_2,\cdots,d_m\}$$

Then one can solve the coupling effect of $\mathbf{U}_{m\times1}(k)$ in $\mathbf{S}_{m\times1}(k+1)-\mathbf{S}_{m\times1}(k+1)$.

Let the system's parameters exist the nominal values and the derivate values, that is $\mathbf{A}_{n\times n}=\mathbf{A}^0_{n\times n}+\Delta\mathbf{A}_{n\times n}$, $\mathbf{B}_{n\times m}=\mathbf{B}^0_{n\times m}+\Delta\mathbf{B}_{n\times m}$, $\mathbf{D}_{m\times m}=\mathbf{D}^0_{m\times m}+\Delta\mathbf{D}_{m\times m}$. Let the control function be transferred to diagonal form, that is

$$\left[\Omega^0_{m\times p}\mathbf{C}_{p\times n}+\Omega_{m\times n}\right]\left[\mathbf{B}^0_{n\times m}+\Delta\mathbf{B}_{n\times m}\right]\mathbf{U}_{m\times1}(k)=\left[\mathbf{D}^0_{m\times m}+\Delta\mathbf{D}_{m\times m}\right]\tilde{\mathbf{U}}_{m\times1}(k)$$

where

$$\Delta\mathbf{A}_{n\times n}=\begin{bmatrix}-\Delta a_{11} & -\Delta a_{12} & \cdots & -\Delta a_{1n}\\ -\Delta a_{21} & -\Delta a_{22} & \cdots & -\Delta a_{2n}\\ \vdots & & \ddots & \\ -\Delta a_{n1} & -\Delta a_{n2} & \cdots & -\Delta a_{nn}\end{bmatrix},$$

$$\Delta\mathbf{B}_{n\times m}=\begin{bmatrix}\Delta b_{11} & \Delta b_{12} & \cdots & \Delta b_{1m}\\ \Delta b_{21} & \Delta b_{22} & \cdots & \Delta b_{2m}\\ \vdots & & \ddots & \\ \Delta b_{n1} & \Delta b_{n2} & \cdots & \Delta b_{nm}\end{bmatrix}, \text{ and}$$

$\Delta\mathbf{D}_{m\times m}=\mathrm{diag}\{\Delta d_1,\Delta d_2,\cdots,\Delta d_m\}$.

Let the control function can be divided into the equivalent controller and the switching controller, that is $\tilde{\mathbf{U}}_{m\times1}(k)=\tilde{\mathbf{U}}^{eq}_{m\times1}(k)+\tilde{\mathbf{U}}^{s}_{m\times1}(k)$. The equivalent controller is used to guide the sliding motion along the sliding surface without the parameter variation and external disturbance. The switching controller is used to force the motion to move toward the sliding surface. According to eq. (12), the equivalent control function can be found as

$$
\begin{aligned}
\tilde{\mathbf{U}}^{eq}_{m\times1}(k)=&-\left[\mathbf{D}^0_{m\times m}\right]^{-1}\left\{T\left[\Omega^0_{m\times p}\mathbf{C}_{p\times n}+\Omega_{m\times n}\right]\mathbf{A}_{n\times n}+\left[\mathbf{I}_{m\times m}+\mathbf{P}_{m\times m}\right]\left[\Omega^0_{m\times p}\mathbf{C}_{p\times n}+\Omega_{m\times n}\right]\right.\\
&\left.+T\Omega^0_{m\times p}\mathbf{K}_{p\times p}\mathbf{C}_{p\times n}\right\}\mathbf{X}_{n\times1}(k)\\
&+\left[\mathbf{D}^0_{m\times m}\right]^{-1}\left[\mathbf{I}_{m\times m}+\mathbf{P}_{m\times m}\right]\left[\Omega^0_{m\times p}\mathbf{K}_{p\times p}\right]\mathbf{Z}_{p\times1}(k)+T\left[\mathbf{D}^0_{m\times m}\right]^{-1}\left[\Omega^0_{m\times p}\mathbf{K}_{p\times p}\right]\mathbf{R}_{p\times1}(k)
\end{aligned}
\tag{13}
$$

Let the switching control is defined as eq (9), that is

$$\tilde{\mathbf{U}}^{s}_{m\times1}(k)=\Phi^0_{m\times p}(k)\left[\mathbf{Y}_{p\times1}(k)-\mathbf{K}_{p\times p}\mathbf{Z}_{p\times1}(k)\right]+\Phi_{m\times n}(k)\mathbf{X}_{n\times1}(k)+\Psi_{m\times1}(k) \tag{14}$$

where the switching gain matrix is

$$\Phi^0_{m\times p}(k)=\begin{bmatrix}\phi^0_{11}(k) & \phi^0_{12}(k) & \cdots & \phi^0_{1p}(k)\\ \phi^0_{21}(k) & \phi^0_{22}(k) & \cdots & \phi^0_{2p}(k)\\ \vdots & & \ddots & \\ \phi^0_{m1}(k) & \phi^0_{m2}(k) & & \phi^0_{mp}(k)\end{bmatrix},$$

$$\Phi_{m\times n}(k)=\begin{bmatrix}\phi_{11}(k) & \phi_{12}(k) & \cdots & \phi_{1n}(k)\\ \phi_{21}(k) & \phi_{22}(k) & \cdots & \phi_{2n}(k)\\ \vdots & & \ddots & \\ \phi_{m1}(k) & \phi_{m2}(k) & & \phi_{mn}(k)\end{bmatrix},\quad \Psi_{m\times1}(k)=\begin{bmatrix}\psi_1(k)\\ \psi_2(k)\\ \vdots\\ \psi_m(k)\end{bmatrix}.$$

Then, the difference of the sliding surface will be

$$
\begin{aligned}
&\mathbf{S}_{m\times1}(k+1)-\mathbf{S}_{m\times1}(k)=\\
&\left\{\begin{array}{l}-\left[\mathbf{I}_{m\times m}+\Delta\mathbf{D}_{m\times m}\left[\mathbf{D}^0_{m\times m}\right]^{-1}\right]\left[\mathbf{I}_{m\times m}+\mathbf{P}_{m\times m}\right]\mathbf{S}^0_{m\times p}\\ +T\Phi^0_{m\times p}(k)\end{array}\right\}\left[\mathbf{Y}_{p\times1}(k)-\mathbf{K}_{p\times p}\mathbf{Z}_{p\times1}(k)\right]\\
&+\left\{\begin{array}{l}T\left[\mathbf{S}^0_{m\times p}\mathbf{C}_{p\times n}+\mathbf{S}_{m\times n}\right]\Delta\mathbf{A}_{n\times n}-\left[\mathbf{I}_{m\times m}+\Delta\mathbf{D}_{m\times m}\left[\mathbf{D}^0_{m\times m}\right]^{-1}\right]\left[\mathbf{I}_{m\times m}+\mathbf{P}_{m\times m}\right]\mathbf{S}_{m\times n}\\ -T\Delta\mathbf{D}_{m\times m}\left[\mathbf{D}^0_{m\times m}\right]^{-1}\left[\mathbf{S}^0_{m\times p}\mathbf{C}_{p\times n}+\mathbf{S}_{m\times n}+\mathbf{S}^0_{m\times p}\mathbf{K}_{p\times p}\mathbf{C}_{p\times n}\right]\\ +T\Phi_{m\times n}(k)\end{array}\right\}\mathbf{X}_{n\times1}(k)\\
&+\left\{\begin{array}{l}T\Delta\mathbf{D}_{m\times m}\left[\mathbf{D}^0_{m\times m}\right]\mathbf{S}^0_{m\times p}\mathbf{K}_{p\times p}\mathbf{R}_{p\times1}(k)\\ +T\left[\mathbf{S}^0_{m\times p}\mathbf{C}_{p\times n}+\mathbf{S}_{m\times n}\right]\mathbf{F}_{n\times1}(k)+T\Psi_{m\times1}(k)\end{array}\right\}
\end{aligned}
$$

To satisfy $\left[\sigma_i(k+1)-\sigma_i(k)\right]\sigma_i(k)<0$ for $i=1,\cdots,m$, one has

$$\phi^0_{ij}(k)=\begin{cases}\phi^{0+}_{ij}(k)<\alpha^0_{ij}=\sup\left\{\left[\left(1+\dfrac{\Delta d_i}{d^0_i}\right)(1+p_i)\sigma^0_{ij}\right]\Big/(Td_i)\right\}\\ \quad \text{if } \left[y_j(k)-k_jz_j(k)\right]\sigma_i(k)>0\\ \phi^{0-}_{ij}(k)>\beta^0_{ij}=\inf\left\{\left[\left(1+\dfrac{\Delta d_i}{d^0_i}\right)(1+p_i)\sigma^0_{ij}\right]\Big/(Td_i)\right\}\\ \quad \text{if } \left[y_j(k)-k_jz_j(k)\right]\sigma_i(k)<0\end{cases}$$

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$$\phi_{ij}(k)=\begin{cases} \phi_{ij}^{+}(k)<\alpha_{ij}=\sup\left\{\left[\left[-T\sum_{q=1}^{n}\left[\sum_{l=1}^{p}\left[\sigma_{il}^{0}c_{lq}\right]+\sigma_{iq}\right]\Delta a_{qj}+\left(1+\frac{\Delta d_i}{d_i^0}\right)(1+p_i)\sigma_{ij}\right.\right.\right. \\ \qquad \left.\left.\left.+T\frac{\Delta d_i}{d_i^0}\left[\sum_{l=1}^{p}\left[\sigma_{il}^{0}c_{lj}\right]+\sigma_{ij}+\sum_{l=1}^{p}\left[\sigma_{il}^{0}k_l c_{lj}\right]\right]\right]\right]\Big/(Td_i)\right\} \\ \quad \text{if } x_j(k)\sigma_i(k)>0 \\ \phi_{ij}^{0-}(k)>\beta_{ij}=\inf\left\{\left[\left[-T\sum_{q=1}^{n}\left[\sum_{l=1}^{p}\left[\sigma_{il}^{0}c_{lq}\right]+\sigma_{iq}\right]\Delta a_{qj}+\left(1+\frac{\Delta d_i}{d_i^0}\right)(1+p_i)\sigma_{ij}\right.\right.\right. \\ \qquad \left.\left.\left.+T\frac{\Delta d_i}{d_i^0}\left[\sum_{l=1}^{p}\left[\sigma_{il}^{0}c_{lj}\right]+\sigma_{ij}+\sum_{l=1}^{p}\left[\sigma_{il}^{0}k_l c_{lj}\right]\right]\right]\right]\Big/(Td_i)\right\} \\ \quad \text{if } x_j(k)\sigma_i(k)<0 \end{cases}$$

$$\psi_i(k)=\begin{cases} \psi_i^{+}(k)<\alpha_i^{\psi}=\sup\left\{\left[\left[-T\frac{\Delta d_i}{d_i^0}\sum_{j=1}^{p}\left[\sigma_{ij}^{0}k_j r_j(k)\right]-T\sum_{j=1}^{n}\left[\sum_{l=1}^{p}\left[\sigma_{il}^{0}c_{lj}\right]+\sigma_{ij}\right]f_j(k)\right]\right]\Big/(Td_i)\right\} \\ \quad \text{if } \sigma_i(k)>0 \\ \psi_i^{-}(k)>\beta_i^{\psi}=\inf\left\{\left[\left[-T\frac{\Delta d_i}{d_i^0}\sum_{j=1}^{p}\left[\sigma_{ij}^{0}k_j r_j(k)\right]-T\sum_{j=1}^{n}\left[\sum_{l=1}^{p}\left[\sigma_{il}^{0}c_{lj}\right]+\sigma_{ij}\right]f_j(k)\right]\right]\Big/(Td_i)\right\} \\ \quad \text{if } \sigma_i(k)<0 \end{cases}$$

To satisfy $\left|\sigma_i(k+1)-\sigma_i(k)\right|<\frac{\xi_i}{2}$ for $i=1,\cdots,m$, one can let $\sum_{j=1}^{p}\varsigma_{ij}^{0}+\sum_{j=1}^{n}\varsigma_{ij}+\varsigma_i^{\psi}=\frac{\xi_i}{2}$ for $i=1,\cdots,m$. Then

$$\begin{aligned}\left|\sigma_i(k+1)-\sigma_i(k)\right| &= \left|\sum_{j=1}^{p}\left\{\left[-\left(1+\frac{\Delta d_i}{d_i^0}\right)(1+p_i)\sigma_{ij}^{0}+Td_i\phi_{ij}^{0}(k)\right]\left[y_j(k)-k_j z_j(k)\right]\right\}\right. \\ &+\sum_{j=1}^{n}\left\{\left[T\sum_{q=1}^{n}\left[\sum_{l=1}^{p}\left[\sigma_{il}^{0}c_{lq}\right]+\sigma_{iq}\right]\Delta a_{qj}-\left(1+\frac{\Delta d_i}{d_i^0}\right)(1+p_i)\sigma_{ij}\right.\right. \\ &\left.\left.-T\frac{\Delta d_i}{d_i^0}\left[\sum_{l=1}^{p}\left[\sigma_{il}^{0}c_{lj}\right]+\sigma_{ij}+\sum_{l=1}^{p}\left[\sigma_{il}^{0}k_l c_{lj}\right]\right]+Td_i\phi_{ij}(k)\right]x_j(k)\right\} \\ &\left.+T\frac{\Delta d_i}{d_i^0}\sum_{j=1}^{p}\left[\sigma_{ij}^{0}k_j r_j(k)\right]+T\sum_{j=1}^{n}\left[\sum_{l=1}^{p}\left[\sigma_{il}^{0}c_{lj}\right]+\sigma_{ij}\right]f_j(k)+Td_i\psi_i(k)\right| \\ &\le \sum_{j=1}^{p}\left|\left\{\left[-\left(1+\frac{\Delta d_i}{d_i^0}\right)(1+p_i)\sigma_{ij}^{0}+Td_i\phi_{ij}^{0}(k)\right]\left[y_j(k)-k_j z_j(k)\right]\right\}\right| \\ &+\sum_{j=1}^{n}\left|\left\{\left[T\sum_{q=1}^{n}\left[\sum_{l=1}^{p}\left[\sigma_{il}^{0}c_{lq}\right]+\sigma_{iq}\right]\Delta a_{qj}-\left(1+\frac{\Delta d_i}{d_i^0}\right)(1+p_i)\sigma_{ij}\right.\right.\right. \\ &\left.\left.\left.-T\frac{\Delta d_i}{d_i^0}\left[\sum_{l=1}^{p}\left[\sigma_{il}^{0}c_{lj}\right]+\sigma_{ij}+\sum_{l=1}^{p}\left[\sigma_{il}^{0}k_l c_{lj}\right]\right]+Td_i\phi_{ij}(k)\right]x_j(k)\right\}\right| \\ &+\left|T\frac{\Delta d_i}{d_i^0}\sum_{j=1}^{p}\left[\sigma_{ij}^{0}k_j r_j(k)\right]+T\sum_{j=1}^{n}\left[\sum_{l=1}^{p}\left[\sigma_{il}^{0}c_{lj}\right]+\sigma_{ij}\right]f_j(k)+Td_i\psi_i(k)\right| \\ &<\frac{\xi_i}{2}=\sum_{j=1}^{p}\varsigma_{ij}^{0}+\sum_{j=1}^{n}\varsigma_{ij}+\varsigma_i^{\psi}\end{aligned}$$

$i=1,\cdots,m,\quad j=1,\cdots,n$

To solve the above equation, one can has the limits of the switching gains as $L_{ij}^{0}<\phi_{ij}^{0}<M_{ij}^{0}$, $i=1,\cdots,m,\quad j=1,\cdots,n$ and $L_{ij}<\phi_{ij}<M_{ij}$, $i=1,\cdots,m,\quad j=1,\cdots,n$ and $L_i^{\psi}<\psi_i^{\psi}<M_i^{\psi}$, $i=1,\cdots,m$, where $i=1,\cdots,m$

$$L_{ij}^{0}(k)=\sup\left\{\left[\left(1+\frac{\Delta d_i}{d_i^0}\right)(1+p_i)\sigma_{ij}^{0}\right]\Big/(Td_i)\right\}-\sup\left\{\zeta_{ij}^{0}\Big/\left[Td_i\left[y_j(k)-k_j z_j(k)\right]\right]\right\}$$

$$M_{ij}^{0}(k)=\inf\left\{\left[\left(1+\frac{\Delta d_i}{d_i^0}\right)(1+p_i)\sigma_{ij}^{0}\right]\Big/(Td_i)\right\}+\sup\left\{\zeta_{ij}^{0}\Big/\left[Td_i\left[y_j(k)-k_j z_j(k)\right]\right]\right\}$$

$$\begin{aligned}L_{ij}=\sup&\left\{\left[-T\sum_{q=1}^{n}\left[\sum_{l=1}^{p}\left[\sigma_{il}^{0}c_{lq}\right]+\sigma_{iq}\right]\Delta a_{qj}+\left(1+\frac{\Delta d_i}{d_i^0}\right)(1+p_i)\sigma_{ij}\right.\right. \\ &\left.\left.+T\frac{\Delta d_i}{d_i^0}\left[\sum_{l=1}^{p}\left[\sigma_{il}^{0}c_{lj}\right]+\sigma_{ij}+\sum_{l=1}^{p}\left[\sigma_{il}^{0}k_l c_{lj}\right]\right]\right]\Big/(Td_i)\right\} \\ &-\sup\left\{\zeta_{ij}\Big/\left[Td_i x_j(k)\right]\right\}\end{aligned}$$

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$$M_{ij} = \inf\left\{\left[-T\sum_{q=1}^{n}\left[\sum_{l=1}^{p}\left[\sigma_{il}^{0}c_{lq}\right]+\sigma_{iq}\right]\Delta a_{qj} + \left(1+\frac{\Delta d_i}{d_i^0}\right)(1+p_i)\sigma_{ij} + T\frac{\Delta d_i}{d_i^0}\left[\sum_{l=1}^{p}\left[\sigma_{il}^{0}c_{lj}\right]+\sigma_{ij}+\sum_{l=1}^{p}\left[\sigma_{il}^{0}k_l c_{lj}\right]\right]\right]\Big/(Td_i)\right\} + \sup\left\{\zeta_{ij}/\left[Td_i x_j(k)\right]\right\}$$

$$L_i^{\psi} = \sup\left\{\left[\begin{array}{l} -T\dfrac{\Delta d_i}{d_i^0}\sum_{j=1}^{p}\left[\sigma_{ij}^{0}k_j r_j(k)\right] \\ -T\sum_{j=1}^{n}\left[\sum_{l=1}^{p}\left[\sigma_{il}^{0}c_{lj}\right]+\sigma_{ij}\right]f_j(k)\end{array}\right]\Big/(Td_i)\right\} - \sup\left\{\zeta_i^{\psi}/(Td_i)\right\}$$

$$M_i^{\psi} = \inf\left\{\left[\begin{array}{l} -T\dfrac{\Delta d_i}{d_i^0}\sum_{j=1}^{p}\left[\sigma_{ij}^{0}k_j r_j(k)\right] \\ -T\sum_{j=1}^{n}\left[\sum_{l=1}^{p}\left[\sigma_{il}^{0}c_{lj}\right]+\sigma_{ij}\right]f_j(k)\end{array}\right]\Big/(Td_i)\right\} + \sup\left\{\zeta_i^{\psi}/(Td_i)\right\}$$

In order to simplify the switching gain choice, one can let $\phi_{ij}^0 = \phi_{ij}^{0+} = -\phi_{ij}^{0-}$, $\phi_{ij} = \phi_{ij}^{+} = -\phi_{ij}^{-}$, $\psi_i = \psi_i^{+} = -\psi_i^{-}$, then the bounds of the switching gains are

$$-\min\left\{\left|M_{ij}^0\right|,\left|\alpha_{ij}^0\right|\right\} < \phi_{ij}^0 < -\max\left\{\left|L_{ij}^0\right|,\left|\beta_{ij}^0\right|\right\}$$
$$-\min\left\{\left|M_{ij}\right|,\left|\alpha_{ij}\right|\right\} < \phi_{ij} < -\max\left\{\left|L_{ij}\right|,\left|\beta_{ij}\right|\right\}$$
$$-\min\left\{\left|M_i^{\psi}\right|,\left|\alpha_i^{\psi}\right|\right\} < \psi_i < -\max\left\{\left|L_i^{\psi}\right|,\left|\beta_i^{\psi}\right|\right\}$$

IV. THE SIMULATION RESULTS

Usually the engine can be assumed to work under many different operating points. The particular operating points are chosen to valid the normal operating conditions [7]. According to reference [7], the Fiat Dedra engine can be assumed to work in three standard operating points. The parameters of the linear model of the Fiat Dedra engine in three conditions are shown in table 1.

TABLE I. TABLE 1 THE PARAMETERS OF THE LINEAR MODEL OF THE ENGINE

	Cond. A	Cond. B	Cond. C
K_θ	2.1608	3.4329	2.1608
K_P	0.1027	0.1627	0.1027
K_{N1}	0.0357	0.1139	0.0357
G_P	0.5607	0.2539	0.5607
K_{N2}	2.0183	1.7993	1.2993
G_A	4.4962	2.2047	2.22962
B_m	2.0283	1.8201	1.8201
J	1.0000	1.0000	1.0000

The controller object is to maintain the idle speed while working under different condition and different external load added at 5 sec. The designed controller will against the parameter variation and the external disturbance and keeps the robust property. The fig. 6 and fig. 7 show the response of the manifold pressure and the related idle speed under condition A with different external load and the time-delay. Obviously, the idle speed can be quickly adjusted back to the setting idle speed while the different load loaded within 1.5 sec.

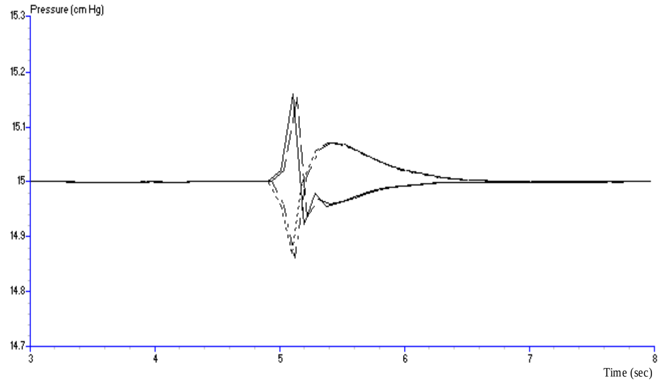

Figure 6. The time response of the manifold pressure on condition A ($P_0 = 15$ cm Hg), solid line with $T_L = 10\sin(N)$ kgw and time delay $\tau_d = 0.03$ sec, dashed line with $T_L = 10\sin(N)$ kgw and no time delay, dotted line with $T_L = 10$ kgw and time delay $\tau_d = 0.03$ sec, and dashed dotted line with $T_L = 10$ kgw and no time delay

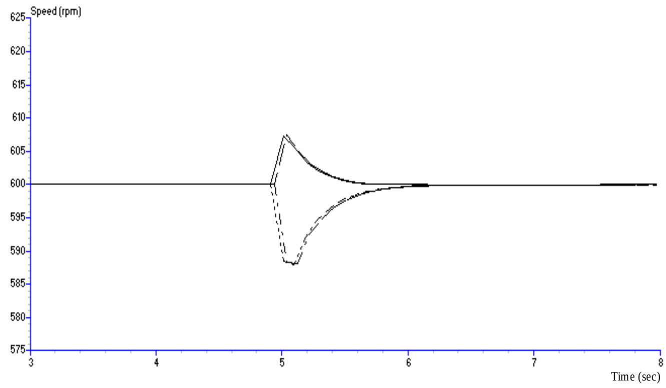

Figure 7. The time response of the idle speed on condition A ($N_0 = 600$ rpm), solid line with $T_L = 10\sin(N)$ kgw and time delay $\tau_d = 0.03$ sec, dashed line with $T_L = 10\sin(N)$ kgw and no time delay, dotted line with $T_L = 10$ kgw and time delay $\tau_d = 0.03$ sec, and dashed dotted line with $T_L = 10$ kgw and no time delay

The time response of the manifold pressure and the idle speed on condition B are shown in fig. 8~9. Even the manifold pressure seems to be changed in order to offer more power, the idle speed still maintain the constant speed in the short time.

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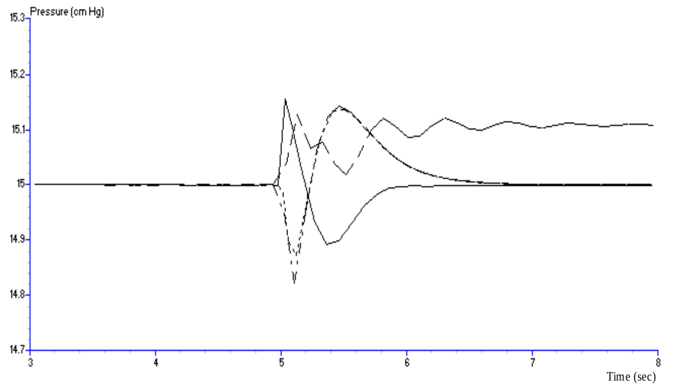

Figure 8. The time response of the manifold pressure on condition B ($P_0 = 15$cm Hg), solid line with $T_L = 10\sin(N)$ kgw and time delay $\tau_d = 0.03$ sec, dashed line with $T_L = 10\sin(N)$ kgw and no time delay, dotted line with $T_L = 10$ kgw and time delay $\tau_d = 0.03$ sec, and dashed dotted line with $T_L = 10$ kgw and no time delay

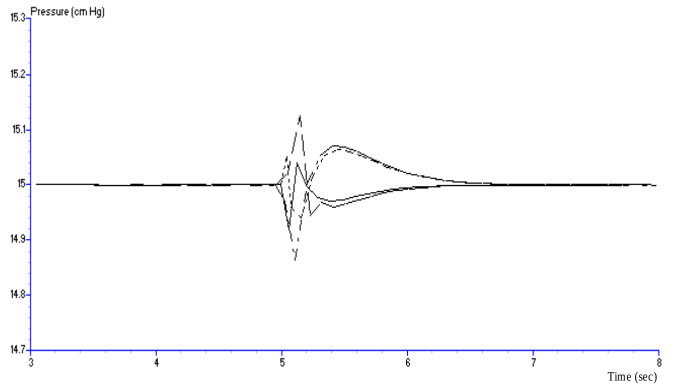

Figure 10. The time response of the manifold pressure on condition C ($P_0 = 15$cm Hg), solid line with $T_L = 10\sin(N)$ kgw and time delay $\tau_d = 0.03$ sec, dashed line with $T_L = 10\sin(N)$ kgw and no time delay, dotted line with $T_L = 10$ kgw and time delay $\tau_d = 0.03$ sec, and dashed dotted line with $T_L = 10$ kgw and no time delay

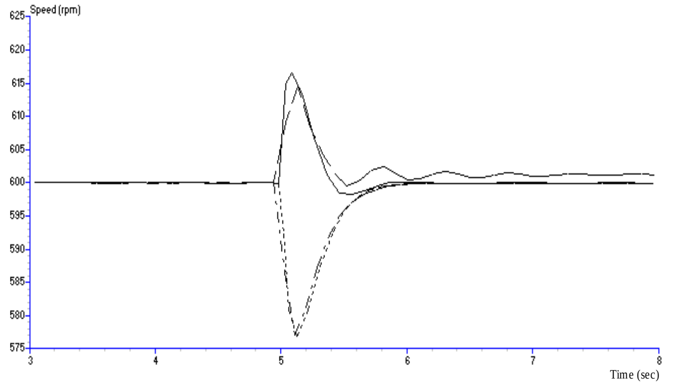

Figure 9. The time response of the idle speed on condition B ($N_0 = 600$ rpm), solid line with $T_L = 10\sin(N)$ kgw and time delay $\tau_d = 0.03$ sec, dashed line with $T_L = 10\sin(N)$ kgw and no time delay, dotted line with $T_L = 10$ kgw and time delay $\tau_d = 0.03$ sec, and dashed dotted line with $T_L = 10$ kgw and no time delay

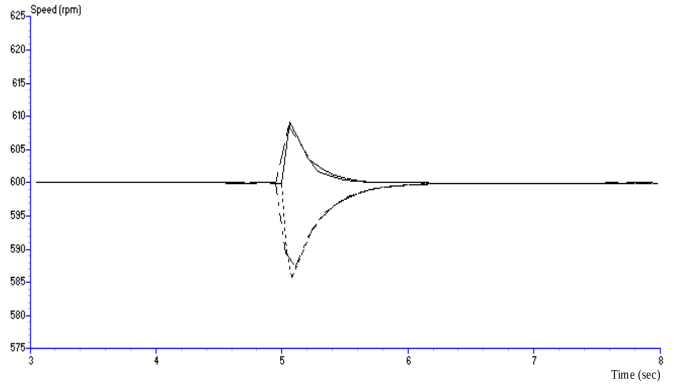

Figure 11. The time response of the idle speed on condition C ($N_0 = 600$ rpm), solid line with $T_L = 10\sin(N)$ kgw and time delay $\tau_d = 0.03$ sec, dashed line with $T_L = 10\sin(N)$ kgw and no time delay, dotted line with $T_L = 10$ kgw and time delay $\tau_d = 0.03$ sec, and dashed dotted line with $T_L = 10$ kgw and no time delay

The time response of condition C is shown in the following two figures. Obviously, the idle speed is pulled back to setting point quickly with different load added. It shows the robustness to against the external load.

V. CONCLUSION

The novel discrete dynamic sliding mode controller is proposed in this paper. The detailed design procedure is also shown, too. According to the novel existence conditions of the discrete sliding mode, the integral gain matrix and the switching gain matrix are chosen properly. The controller is not only keep the motion toward the sliding surface, but also make sure the distance from the sliding surface convergent in each sampling time. The proposed controller is applied to the idle speed control of the spark ignition engine. The engine usually works under different conditions according to the environments. The different kinds of the load are also included due to the ramp of the road. The simulation results show the performance of the proposed control scheme. Obviously, the proposed controller can against the parameter variation and external disturbance.

REFERENCES

[1] Hazell, P.A. and Flower, J.O., "Sampled-data theory applied to the modeling and control analysis of compression ignition engines, - part I," International Journal of Control, p.549-562, 1971.

[2] Hazell, P.A. and Flower, J.O., "Discrete modeling of spark-ignition engines for control purposes," International Journal of Control, p. 625-639, 1971.

[3] Powell, B.K., Cook, J.A. and Grizzle, J.W., "Modeling and analysis of an inherently multi-rate sampling fuel injection engine idle speed control loop," American Control Conference, p. 1543-1548, 1987.

[4] Powell, B.K., Cook, J.A. and Grizzle, J.W., Modeling and analysis of an inherently multi-rate sampling fuel injection engine idle speed control loop, Journal of Dynamic Systems, Measurement and Control, p. 405-411, 1987.

[5] Cook, J.A. and Powell, B.K., "Modeling of an internal combustion engine for control analysis," IEEE Control Systems Magazine, p. 20-26, 1988.

[6] Moskwa, J.J. and Hedrick, J.K., "Modeling and validation of automotive engines for control algorithm development," Journal of Dynamic Systems, Measurement and Control, p. 278-285, 1992.

[7] Abate, M.B., Barmish, R., Murillo-Sanchez C. and Tempo, R., "Application of some new tools to robust stability analysis of spark ignition engines: a case study," IEEE Transactions on Control Systems and Technology," p. 22-30, 1994.

[8] Nishimura, Y. and Ishii, K., "Engine idle stability analysis and control," SAE 860415, 1987.

[9] Moskwa, J.J., "Sliding mode control of automotive engines," American Control Conference, p. 1040-1045, 1989.

[10] Bidan, P., Boverie, S. and Chaumerliac, V., "Nonlinear control of a spark-ignition engine," IEEE Transactions on Control Systems and Technology, p. 4-13, 1995.

[11] Shiao, Y. and Moskwa, J.J., "Cylinder pressure and combustion heat release estimation for SI engine diagnostics using nonlinear sliding observers," IEEE Transactions on Control Systems and Technology, p. 70-78, 1995.

[12] Utkin, V.I., "Sliding Modes in Control and Optimization,". Springer-Verlag, 1992.

[13] Chuang, C.W., Chiang, C.L., Kao, C.C. and Fung, R.R., "Applying Modified MIMO DIVSC to Synchronism and Tension Control of Dual Motor Systems," Journal of Dynamic Systems Measurement and Control-Transactions of The ASME, vol. 131, no. 5, pp. 054501-1~054051-7, 2009.

[14] Chuang, C.W., Huang. C.L., Kao, C.C. and Fung, R.F., "Synchronization and tension control of dual motor systems via MIMO discrete pseudo model following integral variable structure control," Mechanism and Machine Theory, vol. 44, no. 2, pp. 499-510, 2009.

[15] Chern, T.L., Chuang, C.W. and Jiang, R.L., "Design of Discrete Integral Variable Structure Control Systems and Its Application to Brushless DC Motor Control," Automatica, pp 773-779, 1996.

[16] Nise, N.S., "Control systems engineering, Second Edition," New York, The Benjamin Cummings Publishing Company, 1995.