On Fault-Tree Analysis: A Possibilistic Approach

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Abstract-Top event failure probabilities are normally calculated using the exact values of the components failure probabilities in case of fault-tree analysis. There are some systems where evaluation of failure probabilities of components are very difficult based on the past occurrences because of system environments change. Moreover, failure of component which has never failed before must often be considered. Therefore, an approach based on possibility of failure i.e a fuzzy set defined in probability space has been introduced & finally possibility of failure of top event based on fuzzy fault-tree model is calculated considering the trapezoidal nature possibility of failure of components by developing a generalized computer program.

Keywords- *Possibility of failure, Probability of failure, Fuzzy set, Fault-Tree Analysis*

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I. INTRODUCTION

In this paper possibility of failure i.e fuzzy set [1-4] defined in probability space or simply fuzzy failure probability is proposed for fault-tree analysis. Using this idea one can allocate a degree of uncertainty to each value of the probability of failure & consequently different aspects of uncertainty, probability & possibility can be treated simultaneously. Say, if information that 'the probability of failure is between 0.03 and 0.13 and is perhaps around 0.09' is given, it can be represented by a fuzzy set i.e fuzzy failure probability which includes the probability of failure as a limiting case. So the present approach of fuzzy failure probability is more suitable than that of failure probability.

Fault-tree is very useful for the determination of system reliability [5] where component failure ensures the system failure. Basically fault-tree is a logic diagram which helps to determine the probability of top event if the failure probability of components is known. Normally in fault-tree analysis the components failure probability are treated as exact values based on the previous occurrences which are very uncertain & thus associated with some degree of uncertainty.

So in the present approach the fuzzy failure probability or possibility of failure concept has been introduced. In this paper possibility of failure of the top event is determined using the possibility of failure of components based on fault-tree model of a system. In this paper possibility of failures are considered as trapezoidal in nature & finally applied to a considered example system for demonstration.

II. FORMULATION OF FUZZY FAULT- TREE ANALYSIS

First part of fault-tree analysis is the construction of fault-tree & next part is the evaluation part. This paper mainly deals with the fuzzy evaluation of failure probability of the top event of a fault-tree. This method is necessary for system reliability evaluation using the component failure events. Top event is the system failure event based on the system failure criteria & for this reason minimal cut concepts are necessary for the construction of fault-tree. Actually minimal cut is a set of components failure of which ensures the system failure. First order minimal cut means a set of single components failure of which ensures system failure. Similarly second order minimal cut means a set of two components failure of which ensures system failure & so on. Basically fault-tree is a logic model that contains a combination of events which lead to the top event. The most common logic operations that uses in fault-tree are 'AND' & 'OR' operation. Figure-1 shows an example fault-tree where rectangle indicates the intermediate or top event that is the output of the logic gate & the circle indicates the failure event of components. For the fault-tree as shown in Fig.-1 the output of the 'AND' gate is

$Z_1 = Y_2.\ Y_3$ and $Z_2 = Y_4.\ Y_5.\ Y_6$ and the output of the 'OR' gate is

$TE = Y_1 + Y_2.\ Y_3 + Y_4.\ Y_5.\ Y_6$

Now applying De Morgan's theorem the failure probability of top event is

$TE = 1- [\ (1- Y_1)\ (1- Y_2.\ Y_3)\ (1- Y_2.\ Y_3)]$ which is a fuzzy set defined in [0,1]. For analysis of fault- tree following type of fuzzy set is considered in this approach.

A. Fuzzy Set Representation of the possibility of failure probability of Y_i

A fuzzy set 'A' of 'X' is characterized by a membership function $\mu_A(x)$ which is associated with a number in the interval [0,1], representing the degree of x belonging to X. Normally failure probabilities of components are considered as fixed values because they are determined based on the previous values which is not correct. Therefore, to represent this fuzzy set approach is necessary. As per the basic concept of fuzzy set theory, a degree of uncertainty can be allocated to each value of the failure probability. In this paper fuzzy failure probabilities are considered as trapezoidal in nature. By the use of these known trapezoidal fuzzy failure probabilities of components, the fuzzy probability of the top event of the fault-tree is calculated. The considered fuzzy failure probability of *ith* component is as follows-

$Y_i \equiv (\ y_1, y_2, y_3, y_4)$ & is represented by a membership function (Fig.-2) as follows where

$$\mu_{Y_i} = \begin{cases} 0, & 0 \leq Y_i \leq y_1 \\ \dfrac{Y_i - y_1}{y_2 - y_1}, & y_1 < Y_i \leq y_2 \\ 1, & y_2 < Y_i \leq y_3 \\ \dfrac{y_4 - Y_i}{y_4 - y_3}, & y_3 < Y_i \leq y_4 \\ 0, & y_4 < Y_i \leq 1 \end{cases}$$

In this possibilistic approach the product of two fuzzy failure probabilities are taken as

$Y_i\ .\ Y_j \approx (y_1 z_1,\ y_2 z_2,\ y_3 z_3,\ y_4 z_4)$

Where $Y_j \equiv (z_1, z_2, z_3, z_4)$
and the complement of fuzzy failure probability is taken as
$1 - Y_i \equiv (\ 1 - y_1,\ 1 - y_2,\ 1 - y_3,\ 1 - y_4)$

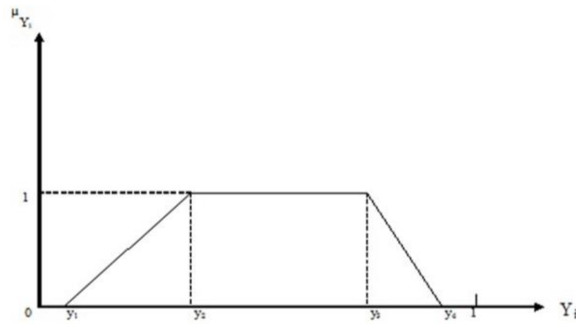

Fig. 2. Membership function of fuzzy probability of Y_i

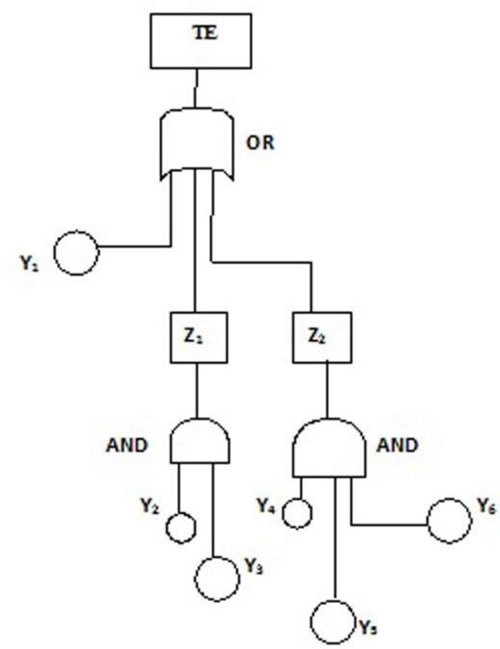

Fig.-1. Fault-Tree Example

III. ILLUSTRATION WITH EXAMPLE SYSTEM

To illustrate the proposed approach first consider the typical fault-tree example as shown in Fig.-1 & the

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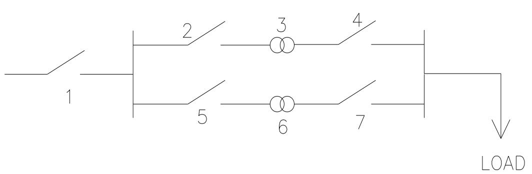
assumed fuzzy failure probabilities of components are shown in Table-1. Secondly an example power system network is considered as shown in Fig.-3 & in this case the assumed fuzzy failure probabilities of components are shown in Table-2. Table-3 contains the minimal cuts up to 3rd. order of the example power system network. These minimal cuts are determined by observation but based on the criterion that the system fails if load bus fails. However for larger system to determine these cuts software are necessary. In this approach to determine the top event fuzzy probability a generalized computer program written in fortran has been developed & the result obtained by running this program is given below.

The top event fuzzy probability for the typical fault-tree example is obtained as follows:

(0.0056, 0.0085, 0.0155, 0.0228).

The top event fuzzy probability for example power system network is obtained as follows:

(0.0091, 0.0160, 0.0524, 0.0818).



Fig–3. Example Power System Network

Table-1

Fuzzy Failure Probability data of components as shown in typical fault-tree example (Fig.-1)

Components	y_1	y_2	y_3	y_4
1	0.005	0.007	0.01	0.014
2	0.02	0.04	0.07	0.09
3	0.03	0.04	0.08	0.10
4	0.1	0.16	0.21	0.26
5	0.21	0.26	0.36	0.40
6	0.16	0.21	0.28	0.35

Table-2

Fuzzy Failure Probability data of components as shown in the example Power System Network (Fig.-3)

Components	y_1	y_2	y_3	y_4
1	0.008	0.01	0.04	0.05
2	0.01	0.03	0.04	0.07
3	0.004	0.006	0.008	0.01
4	0.02	0.04	0.06	0.09
5	0.01	0.03	0.04	0.07
6	0.004	0.006	0.008	0.01
7	0.02	0.04	0.06	0.09

Table-3

Minimal Cuts(up to 3rd. Order) of the example Power System Network

Order of Minimal Cut	No. of Minimal Cut	Components in the Minimal Cuts
1st.	1	1
2nd.	9	(2,5),(2,6),(2,7), (3,5),(3,6),(3,7) (4,5),(4,6),(4,7)
3rd.	27	(1,2,5),(1,2,6),(1,2,7), (1,3,5),(1,3,6),(1,3,7), (1,4,5),(1,4,6),(1,4,7), (2,5,6),(2,5,7),(2,6,7), (3,5,6),(3,5,7),(3,6,7), (4,5,6),(4,5,7),(4,6,7), (5,2,3),(5,2,4),(5,3,4), (6,2,3),(6,2,4),(6,3,4), (7,2,3),(7,2,4),(7,3,4),

IV. CONCLUSION

In this paper fault-tree analysis under uncertainty has been presented. Normally in conventional approach component failure probabilities are treated as crisp in nature which is not correct. So the present approach is more predictive & useful. The presents approach shows how to draw the fault-tree diagram using the system minimal cuts & finally shows how to determine the top event failure probability under uncertainty. This approach is very much useful to the reliability engineers to determine the system reliability under uncertainty. Lastly a generalized computer program has been developed to determine the top event fuzzy probability because hand calculation is not possible due to the complexity of the trees & applied to some example system.

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