# Performance Enhancement of Interference Alignment Techniques for MIMO Multi Cell Networks 

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#### Abstract

In order to increase the ability of a wireless communication network and to professionally diminish interference of the signals one of the hopeful technique is Interference alignment (IA). In this proposed work Multiple Input Multiple Output (MIMO) user of a network is considered in three folds where Gaussian Interference Broadcast Channel (GIBC) is as medium. In the first fold Transmit and Receive Beam forming vectors are designed, next to execute interference alignment using the Multiple Access Channel - Broadcast Channel Duality (MAC$B C D$ ) when improving the capacity of users in each cell and at last to maintain equality among users Rate Balancing (RB) technique is proposed.


Keywords- Inteference Alignment;Multiple input and Multiple ouput; Guassian Interference Broadcast Channel; Multi Access ChannelBroadcast Channel; Balancing Technique.

## I. Introduction

Understanding the capacity of wireless networks is the "Holy Grail" of network information theory [9]. Employing many or several antennas at the transmitter and receiver in wireless systems, popularly known as MIMO technology, has rapidly gained in popularity over the past decade due to its powerful performance enhancing capabilities. Communication in wireless channels is impaired predominantly by multi-path fading. Multi-path is the arrival of the transmitted signal at an intended receiver through differing angles and/or differing time delays and/or differing frequency (i.e., Doppler) shifts due to the scattering of electromagnetic waves in the environment. Consequently, the received signal power fluctuates in space (due to angle spread) and/or frequency (due to delay spread) and/or time (due to Doppler spread) through the random superposition of the impinging multi-path components. This random fluctuation in signal level, known as fading, can severely affect the quality and reliability of wireless communication. Additionally, the constraints posed by limited power and scarce frequency bandwidth make the task of designing high data rate, high reliability wireless communication systems extremely challenging. The benefits of MIMO technology that help achieve such significant performance gains are array gain, spatial diversity gain, spatial multiplexing gain and interference reduction [1]. The optimality of IA is to move toward to attain Shannon capacity of interference networks at high Signal to Noise Ratio (SNR). Yet, the scopes up to which interference can be aligning over a limited number of signal dimensions are unknown. Global Channel knowledge is another apprehension for interference alignment schemes [2]. The likelihood of IA in signaling vector space is stand only on beam forming for user K, MIMO interference channels. The possible issue of formative the solvability of a multivariate polynomial system is mull over widely in algebraic geometry. Bezout's theorem gives that generic polynomial systems are solvable only if the number of
equations does not surpass the number of variables. On this instinct, the signal space IA problems are classified as either proper or improper depending on the No. of equations and variables. Exact connections between improper and proper systems is done using Bernshtein's theorem, in which each transmitter use no more than one beam forming vector. In multi beam case dependencies the coefficients of a polynomial system is not generic as required by both the theorems, the connection between improper and proper systems can further be strengthened by adding standard information on theoretic external bounds in the feasibility analysis [3], [7]. The distributed MIMO perspective is applicable not only for wireless networks but also for networks of single-antenna nodes which may accomplish MIMO behavior by message sharing and collective relaying clusters of neighboring nodes [4]. An amalgamation of two broadcast channels i.e. from transmitter's view and two multiple access channels i.e. from receiver's view, these perspectives are used where two signaling schemes for such a scenario is developed, where Linear filters were employed at transmitter and receiver which molder the system into either two Non Interfering MultipleAntenna Broadcast Sub Channels or two Non Interfering Multiple Antenna Multiple Access Sub Channels. The foremost purpose in designing of the filters is to take advantage of the structure of the channel matrices to achieve the highest Multiplexing Gain (MG). It is proven that the proposed NonCooperative Signaling Schemes outperform other known NonCooperative Schemes in terms of MG. In meticulous, it is revealed that in some precise cases, the achieved MG is the similar as the MG of the system if full cooperation is provided either between the transmitter or between the receivers [5],[8]. The "Fig.1" is structure of the MIMO cellular network, where base-station equipped with ' $L$ ' antennas communicates with ' $P$ ' users in which each equipped with ' $M$ ' antennas. The downlink channel from the base-station to the users is a Broadcast Channel (BC) while uplink channel from the users
to the base-station is a Multiple-Access Channel (MAC). The set of rate tuples i.e. $\left(R_{1}, R_{2} \ldots R_{P}\right)$ can be consistently support on the downlink or uplink comprises the ability rate region of links.


Figure 1. MIMO cellular system
The main disadvantages of the existing system are a) Sum rate is less b) Inter cell interference is high and c) Inter user interference is high. To overcome these drawbacks the proposed method achieves the optimal solution where the sum rate is high, inter cell and user interference is less.

## II. INTERFERENCE ALIGNMENT OBJECTIVE

The main objective of this proposed interference alignment scheme uses the principle of MAC-BCD to maximize capacity of each cell while performing interference alignment to users in other cells. The problem in its dual form is convex and can be solved using interior point methods. Since this method performs interference alignment only to users in other cells, and inter user interference in each cell is dealt with using a capacity maximization problem, the number of antennas required at each BS is smaller than if interference alignment is performed for every users, including users within each cell for a moderate range of SNR.

## III. PROPOSED SYSTEM MODEL

The proposed interference alignment scheme uses the principle of MAC-BCD to maximize capacity of each cell while performing interference alignment to users in other cells. The problem in its dual form is convex and can be solved using interior point methods. Since our method performs interference alignment only to users in other cells, and inter user interference in each cell is dealt with using a capacity maximization problem, the number of antennas required at each BS is smaller than if interference alignment is performed for every users, including users within each cell for a moderate range of SNR. The block diagram is shown in the below "Fig. 2".


Figure 2. Block Diagram
The minimum differential feedback rate expression for timecorrelated MIMO Rayleigh block fading channels by taking into account both the channel estimation errors and the channel quantization distortion, which are assumed as independent and identically distributed (IID) complex Gaussian variables. We investigate the relationship between the ergodic capacity and the feedback interval with feedback channel capacity constraint in a periodic differential feedback system. We also prove that there exists an optimal feedback interval to achieve the maximum ergodic capacity. We present the approximate optimal feedback intervals, and verify the theoretical results in a practical differential feedback system by Lloyd's quantization algorithm.

## IV. BLOCK DIAGRAM MODULES EXPLANATION

## A. Modules of block diagram

1) System model
2) Grouping method
3) Interference alignment scheme using MAC-BCD

The MIMO-IFBC model consists of a cellular network with $L$ cells, each cell consists of $K$ users. Assume that each user is equipped with $N r$ antennas and each cell has one BS consisting of $N t$ antennas. The channel in each cell can be regarded as MIMO-IFBC. An example for the case of $L=3$ and $K=2$ is illustrated in "Fig. 1". As shown in "Fig.3", the BS 1 sends data to user 1 while introducing both inter-user interference and inter-cell interference. Similarly, BS2 and BS3 introduce interference to other users. We assume each base stations aims to convey $d_{s}$ data streams to its corresponding user, where $d_{s \leq} \min \left(N_{t}, N_{r}\right)=N_{r}$, we assumed $N_{r}$ $<\mathrm{N}_{\mathrm{t}}$. We refer to the $\mathrm{k}^{\text {th }}$ user in the $1^{\text {th }}$ cell is written as " 1 "


Figure 3. A multi-cell interference alignment scheme shown for the case of three cells and two users in each cell

$$
\begin{equation*}
X^{[k, i]}=\sum_{i=1}^{d_{s}} v_{i}^{[k, i]} s_{i}^{[k, i]}=V^{[k, l]} S^{[k, l]} \tag{1}
\end{equation*}
$$

Where $s_{i}^{[k, i]}$ enotes the $i^{\text {th }}$ transmitted symbol
for the $\mathrm{k}^{\text {th }}$ user in the $1^{\text {th }}$ cell. Satisfying an average power constraint as " 2 "

$$
\begin{equation*}
E\left[\left\|X^{[k, l]}\right\|^{2}\right] \leq P^{[k, l]} \tag{2}
\end{equation*}
$$

and " 3 "

$$
\begin{equation*}
v_{i}^{[k, l]} \in C^{N_{t} \times 1} \tag{3}
\end{equation*}
$$

is the linear transmit beam forming vector. The transmitter beam forming matrix for the user $[k, 1]$ is written as " 4 "

$$
\begin{equation*}
V^{[k, l]}=\left[v_{1}^{[k, l]} v_{2}^{[k, l]} \ldots . v_{d_{s}}^{[k, l]}\right] \in C^{\mathrm{Ntx} d_{s}} \tag{4}
\end{equation*}
$$

and its corresponding data signal vector is denoted by " 5 "

$$
\begin{equation*}
S^{[k, l]}=\left[s_{1}^{[k, l]} s_{2}^{[k, l]} \ldots \ldots s_{d_{s}}^{[k, l]}\right]^{T} \in C^{\mathrm{ds} \mathrm{\times 1}} \tag{5}
\end{equation*}
$$

Therefore the received signal of the $\mathrm{k}^{\text {th }}$ user in the $\mathrm{l}^{\text {th }}$ cell can be written as in " 6 " and " 7 "

$$
\begin{align*}
y^{[k, i]} & =\sum_{i=1}^{L} H_{i}^{[k, l]} \sum_{j=1}^{K} X^{[j, i]}+n^{[k, l]}  \tag{6}\\
& =\underbrace{H_{i}^{[k, l]} V^{[k, l]} s^{[k, l]}}_{i}+ \\
\sum_{j=1, j \neq k}^{K} H_{i}^{[k, l]} V^{[k, l]} s^{[k, l]} & +\sum^{\sum_{i=1, i \neq l}^{L} \sum_{j=1}^{K} H_{i}^{[k, \lambda]} V^{[k, l]} s^{[k, l]}}+n^{[k, i]} \tag{7}
\end{align*}
$$

Where $\mathrm{n}^{[\mathrm{k}, 1]}$ is the additive white Gaussian noise vector with variance $\sigma^{2}$ per entry at the receiver of user $[\mathrm{k}, 1]$, and $\mathrm{H}_{\mathrm{i}}{ }^{[\mathrm{k}, 1]}$ is the Nr X Nt channel matrix from the BS i to the user $[\mathrm{k}, 1]$. The signal at the receiver for the user $[k, l]$ after receiver beam former is written as in " 8 " and " 9 "

$$
\begin{gather*}
y^{[k, l]}=U^{[k, l] H} y^{[k, l]}  \tag{8}\\
=U^{[k, i] H} H_{l}^{[k, l]} V^{[k, l]} S^{[k, i]}+U^{[k, i] H} \\
+\left(\sum_{j=1, j \neq k}^{K} H_{l}^{[k, i]} V^{[j, l]} S^{[j, l]}+\sum_{i=1, i \neq l}^{L} \sum_{j=1}^{K} H_{l}^{[k, l]} V^{[j, l]} S^{[j, l]}\right)+\check{n}^{[k, l]} \text { (9) } \tag{9}
\end{gather*}
$$

Where

$$
\begin{equation*}
U^{[k, l]}=\left[u_{1}^{[k, l]} u_{2}^{[k, l]} \ldots \ldots u_{d_{s}}^{[k, l]}\right] \in C^{\mathrm{Nrx} d_{s}} \tag{10}
\end{equation*}
$$

" 10 " denotes the receiver beam forming matrix for the user [ $k$, $l]$, and $\mathbf{n}^{[k, l]}$ is the effective noise component at the output of the beam former which is distributed according to $C N(0,1)$. In order to decode the useful signal efficiently, both the ICI and IUI should be aligned into the interference space at the receiver. The desired signal should be linearly independent of the interference. Hence the signal space for the desired signal should be larger than or equal to the dimension of the data vector $d[k, l]$. Both ICI and IUI are aligned into the subspace which is orthogonal to $\mathbf{U}[k, l]$. Therefore the following condition " 11 ", " 12 " and " 13 " must be satisfied for the $k^{\text {th }}$ user in the $l^{\text {th }}$ cell

$$
\begin{gather*}
U^{[k, i] H} H_{i}^{[k, i]} V^{[j, i]}=0, \forall i \neq l, j \in\{1,2, \ldots . K\}  \tag{11}\\
U^{[k, i] H} H_{i}^{[k, l]} V^{[m, i]}=0, \forall i \neq k  \tag{12}\\
\operatorname{rank}\left\{U^{[k, i] H} H_{i}^{[k, i]} V^{[k, i]}\right\}=d^{[k, i]} \tag{13}
\end{gather*}
$$

## C. Grouping Method Explanation

To maximize the sum rate performance of the MIMO-IFBC, the transmitter and the receiver beamforming matrices are usually designed by applying an iterative optimization algorithm .The iterative scheme performs interference alignment implicitly and it normally requires a considerable number of iterations. In this section, we extend the grouping method as our multi-cell scenario. This interference alignment scheme not only mitigates both ICI and IUI simultaneously in the multi-cell multi-user MIMO-IFBC, but also it does not
require any iterative computation. To explain, we start with a simple example of $(N t, N r, K, L, d s)=(10,6,2,3,2)$. Suppose the BS $l$ wants to transmit two sets of independent symbols $\mathbf{s}[1, l]=\left[\mathrm{s}_{1}^{[1,1]}, \mathbf{s}_{2}{ }^{[1,1]}\right]^{T}$ and $\mathbf{s}[2, l]=\left[\mathrm{s}_{1}{ }^{[2,1]}, \mathrm{s}_{2}{ }^{[2,1]}\right]^{T}$ to user $[1, l]$ and user $[2, l]$ respectively. For this example, for the case of the first cell $(l=1)$, in order to transmit the symbol $\mathbf{s}[1,1]$ without causing any interference to the users, the beamformer $\mathbf{V}[1,1]$. The grouping scheme is shown in "Fig. 4".


Figure 4. The extension of the grouping scheme shown for the case of three cells and two users in each cell

1) Step 1: Grouping the users and designing the receiver beamforming matrices.
In the first step, the users are grouped to a particular interference space at different cells. Let us explain the grouping procedure in detail using the same example as in Fig. 2. The users $[1,2]$ and $[2,2]$ in the $2 n d$ cell are grouped together (Group 1) by designing the receiver beamforming matrices $\mathbf{U}[1,2]$ and $\mathbf{U}[2,2]$ such that the ICI channels from BS 1 are aligned in the same subspace. Hence from the view point of BS 1 , users in the $2 n d$ cell are placed in the same interference space. Since the effective ICI channels are aligned to each other, the BS 1 can treat the two different ICI channel vectors corresponding to user [1,2] and user [2,2] as a single ICI channel vector which spans $d s$ dimensional subspace. In other words, the interference channels for users [1, 2] and [2, 2] from BS 1 span the same subspace as follows in " 14 "
$G_{1}=\operatorname{span}\left\{H_{1}^{[1,2] H} U^{[1,2]}\right\}=\operatorname{span}\left\{H_{1}^{[2,2] H} U^{[2,2]}\right\}(14)$
Where span (.) denotes the subspace spanned by the column vectors of a matrix. We can determine the intersection
subspace satisfying the condition above equation solving the following matrix " 15 ".
$\left[\begin{array}{ccc}I_{N_{\mathrm{t}}} & -H_{1}^{[1,2] H} & 0 \\ I_{N_{\mathrm{t}}} & 0 & -H_{1}^{[2,2] H}\end{array}\right]\left[\begin{array}{c}G_{1} \\ U^{[1,2]} \\ U^{[2,2]}\end{array}\right]=F_{1} X_{1}=0$
where G1 accounts for the subspace spanned by the aligned effective interference channels from BS 1 to the user [1,2] and user [2, 2] after applying the receiver beamforming. Similar to Group 1, we can group all the users in cell 3 to a particular interference space when seen from BS 2 (Group 2), and all the users in cell 1 to a particular interference space when seen from BS 3 (Group 3).
$G_{2}=\operatorname{span}\left\{H_{2}^{[1,3] H} U^{[1,3]}\right\}=\operatorname{span}\left\{H_{2}^{[2,3] H} U^{[2,3]}\right\}$
$G_{3}=\operatorname{span}\left\{H_{3}^{[1,1] H} U^{[1,1]}\right\}=\operatorname{span}\left\{H_{3}^{[2,1] H} U^{[2,1]}\right\}$
As a result, all the users are put into three groups. From the view point of each BS, the interference to users in each group can be treated as interference to one destination. Hence in our example of $(N t, N r, K, L, d s)=(10,6,2,3,2)$, each BS only requires 10 transmit antennas to remove both IUI and ICI at the same time. However a zero forcing based transmitter precoder will require minimum of 12 antennas to cancel all ICI and IUI at each user receiver. Hence, there is a saving in terms of complexity of the BS.
2) Step 2: Designing the transmit beamforming matrices The transmit beamforming vectors for the two users in the first cell $\mathbf{V}[1,1]$ and $\mathbf{V}[2,1]$ are designed. Hence BS 1 can send the symbols $\mathbf{s}[1,1]$ and $\mathbf{s}[2,1]$ to the users [1, 1] and [2, 1] respectively without introducing any interference to users in the $2 n d$ and $3 r d$ cells. The "Equations 19 to 25 " are shown below.

## D. Interference Alignment Scheme Using MAC-BCD

The interference alignment scheme using the grouping method in pre can ensure zero IUI and ICI at the receiver of each user, hence this method is appropriate at high SNR region as the intention was to fully utilize the available degrees of freedom for transmission. However, for a moderate range of SNR, due to the perfect interference alignment, the interference alignment scheme using the grouping method proposed in previous section may result in a lower network capacity. Also, due to the perfect interference alignment for the intra cell users, the number of antennas required at the BS could still be comparably high. Therefore, to use interference alignment only for the inter-cell users to ensure no ICI, but interference alignment is not performed for the intra-cell users. Instead, the IUI among users within each cell is treated by designing the beam formers to maximize capacity of each cell using the principle of MAC-BC duality. For multiple-cells with multiple MIMO users, the basic approach of the proposed method is to write the beam former matrix for the $k^{\text {th }}$ user in the $l^{\text {th }}$ cell as in " 18 ".

$$
\begin{equation*}
\bar{V}^{[k, l]}=V^{[k, l]} W_{k, l} \tag{18}
\end{equation*}
$$

$$
\begin{align*}
& V^{[1,1]} \subset \operatorname{null}\left(\left[G_{1}\left(U^{[1,3] H} H_{1}^{[1,3]}\right)^{H}\left(U^{[2,3] H} H_{1}^{[2,3]}\right)^{H}\left(U^{[2,1] H} H_{1}^{[2,1]}\right)^{H}\right]^{H}\right)  \tag{19}\\
& V^{[2,1]} \subset \operatorname{null}\left(\left[G_{1}\left(U^{[1,3] H} H_{1}^{[1,3]}\right)^{H}\left(U^{[2,3] H} H_{1}^{[2,3]}\right)^{H}\left(U^{[1,1] H} H_{1}^{[1,1]}\right)^{H}\right]^{H}\right)  \tag{20}\\
& V^{[1,2]} \subset \operatorname{null}\left(\left[G_{2}\left(U^{[1,1] H} H_{2}^{[1,1]}\right)^{H}\left(U^{[2,1] H} H_{2}^{[2,1]}\right)^{H}\left(U^{[2,2] H} H_{2}^{[2,2]}\right)^{H}\right]^{H}\right)  \tag{21}\\
& V^{[2,2]} \subset \operatorname{null}\left(\left[G_{2}\left(U^{[1,1] H} H_{2}^{[1,1]}\right)^{H}\left(U^{[2,1] H} H_{2}^{[2,1]}\right)^{H}\left(U^{[1,2] H} H_{2}^{[1,2]}\right)^{H}\right]^{H}\right)  \tag{22}\\
& V^{[1,3]} \subset \operatorname{null}\left(\left[G_{3}\left(U^{[1,2] H} H_{3}^{[1,2]}\right)^{H}\left(U^{[2,2] H} H_{3}^{[2,2]}\right)^{H}\left(U^{[2,3] H} H_{3}^{[2,3]}\right)^{H}\right]^{H}\right)  \tag{23}\\
& V^{[2,3]} \subset \operatorname{null}\left(\left[G_{3}\left(U^{[1,2] H} H_{3}^{[1,2]}\right)^{H}\left(U^{[2,2] H} H_{3}^{[2,2]}\right)^{H}\left(U^{[1,3] H} H_{3}^{[1,3]}\right)^{H}\right]^{H}\right) \tag{24}
\end{align*}
$$

Generally

$$
\begin{equation*}
V^{[k, i]} \subset n u l l\left(\left[G_{i}\left(U^{[t(t=1 \ldots K), s(s \neq i, i+1)]^{H}} H_{i}^{[t(t=1 \ldots K), s(s \neq i, i+1)]}\right)^{H}\left(U^{[t(t \neq k), i]^{H}} H_{i}^{[t(t \neq k), i]}\right)^{H}\right]^{H}\right) \tag{25}
\end{equation*}
$$

## V. SIMULAION RESULTS AND CONCLUSION

## A. Result Analysis

The performance of the proposed hybrid interference alignment and MAC-BC duality based scheme with the extension of the grouping method in terms of the sum rate. We focus on two system configurations $(N t, N r, K, L, d s)=(5,3,2$, 3, 1) and $(N t, N r, K, L, d s)=(10,6,2,3,2)$. For these configurations, the proposed scheme is able to achieve 6 degrees of freedom and 12 degrees of freedom, respectively.


Figure 5. The achievable rates for the proposed hybrid interference alignment scheme and comparison to the extension of the grouping method ( $\mathrm{DoF}=6$ ).

To be specific, to achieve a degrees of freedom of 6, the extension of the grouping scheme requires $N t=5$ antennas for the example of $(N t, N r, K, L, d s)=(5,3,2,3,1)$, but the
conventional zero-forcing beamforming scheme requires $N t=$ 6 antennas for the case of $(N t, N r, K, L, d s)=(6,3,2,3,1)$.

The "Fig. 5 to Fig 9" are the simulated results of the proposed work. Where in "Fig. 5"and "Fig. 6" shows the comparisons of availability rates with $\mathrm{DoF}=6$ and 12 ". Rate Balancing with DoF $=6$ and $\mathrm{DoF}=12$ is shown in "Fig. 7 and 8 ". Probability function of MAC-BC duality is shown in "Fig. 9".


Figure 6. The achievable rates for the proposed hybrid interference alignment scheme with different number of antennas setting ( $\mathrm{DoF}=12$ ).

## B. Conclusion

An interference alignment scheme for a network with multiple cells and MIMO users under a Gaussian interference broadcast channel scenario. We first extended the grouping method in
the multi-cell scenario to jointly design the transmitter and receiver beamforming vectors using a closed form expression without a need for iterative computation. The grouping method can ensure no ICI and IUI at each user's complexity at the basestation as compared to the conventional zero-forcing beamforming scheme. Then we proposed a hybrid interference alignment scheme based on the principle of MACBC duality. This proposed scheme removes the ICI using interference alignment while maximizes the total capacity of the corresponding cell using MAC-BC duality. Since, interference alignment is not perform explicitly to all users in the network, but the users within each cell are dealt with using capacity maximization, the number of transmit antennas required is generally lower than the existing grouping method. Finally, a hybrid rate balancing and interference alignment technique was introduced to maintain fairness among users. Hence the proposed technique is able to maximize the data rate while balancing the rate achieved by each user. The simulation results demonstrated the performance of the algorithms for various SNR values


Figure 7. Rate Balancing of interference alignment algorithm using the MAC-BCD
( $\mathrm{DoF}=6$ ).


Figure 8. Rate balancing of the proposed hybrid interference alignment algorithm using the MAC-BCD
( $\mathrm{DoF}=12$ ).


Figure 9. Probability density function using the MAC-BC duality

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