Wavelet Thresholding for Image Noise Removal

Different Techniques of Thresholding Used for Wavelet Image Denoising

Debalina Jana
Assistant Professor, Dept. of AEIE
College of Engineering and Management, Kolaghat
Mecheda, India
debalina@cemk.ac.in

Kaushik Sinha
Assistant Professor, Dept. of IT
College of Engineering and Management, Kolaghat
Mecheda, India
kaushik-sinha@cemk.ac.in

Abstract— This work addresses to a study on the different techniques of thresholding used for noise removal from an image using Discrete Wavelet Transform (DWT). The wavelet transform technique has already proven its capability for image denoising. But thresholding is the heart of noise removal from an image. Only the application of proper thresholding can remove noise from an image. The most popular thresholding are VisuShrink, SureShrink, BayesShrink etc. In this paper, the use of such popular thresholding techniques are discussed.

Keywords*-Discrete Wavelet Transform, additive white Gaussian noise, Thresholding, Image Denoise, VisuShrink, SureShrink, BayesShrink, NormalShrink, NeighShrink*

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I. INTRODUCTION

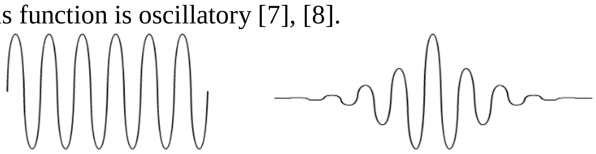
Wavelet Transform has the capability to represent signals with a high degree of scarcity. Wavelet thresholding is a technique of signal estimation that exploits the capabilities of wavelet transform for noise removal from signals. A digital image can be thought of 2 dimensional signal and it can be expressed mathematically as a function of *f(x,y)*, where x and y represent the spatial coordinate of the image, and *f(x,y)* carries the corresponding intensity value.

Noise removal from images corrupted by additive white Gaussian noise (AWGN) are classical problem in image processing. Images can be corrupted by noise due to some common reason such as its acquisition, processing, compression, transmission, and reproduction. In the past few years, several successful researches focused on removing the noise from the image. Especially the case of AWGN, a number of techniques using wavelet-based thresholding. Donoho and Johnstone [2], [6] proposed hard and soft thresholding [1] methods for denoising. Wavelet shrinkage method proposed by Donoho [1], [3], and [4] is the pioneer work for noise removal from signal using the wavelet transform.

The aim of this paper is to study various thresholding techniques [25] such as VisuShrink, SureShrink, BayesShrink, NormalShrink, NeighShrink and determining the best one for image denoising.

II. WAVELET TRANSFORM

The term wavelet means a small wave. The smallness refers to the condition that this (window) function is of finite length (compactly supported). The wave refers to the condition that this function is oscillatory [7], [8].



(a) (b)

Figure 1. Representation of a (a) wave, and a (b) wavelet

The wavelet transform (WT) is a powerful tool of signal processing for its multiresolutional possibilities [8]. Unlike the Fourier transform, the WT is suitable for handling the non-stationary signals – variable frequency with respect to time.

A. Continuous Wavelet Transform (CoWT)

For a prototype function ψ (t) ∈ L2(ℜ) called the mother wavelet, the family of functions can be obtained by shifting and scaling this ψ (t) as [7], [8]

$$\Psi_{a,b}(t) = \frac{1}{\sqrt{a}}\Psi\left(\frac{t-b}{a}\right) \tag{1}$$

Where, a, b ∈ℜ, (a > 0). The CoWT of a function f (t) ∈ ℜ is then defined as

$$CoWT_f\,(a,b) = \int_{-\infty}^{\infty} \Psi^*_{a,b}\,(t) f(t)dt = \langle \Psi_{a,b}(t) f(t) \rangle \tag{2}$$

Since, the CoWT behaves like orthonormal basis decomposition, it is isometric and it preserves energy [8]. Hence the function f(t) can be recovered from its transform by the following reconstruction formula

$$f(t) = \frac{1}{C_\Psi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} CoWT_f\,(a,b)\,\Psi_{a,b}(t)\frac{dadb}{a^2} \tag{3}$$

B. Discrete Wavelet Transform (DWT)

The discrete wavelet transform (DWT) is a linear transformation that operates on a data vector whose length is an integer power of two, transforming it into a numerically different vector of the same length [8].

It separates data into different frequency components, and then matches each component with resolution to its scale. DWT is computed with a cascade of filters followed by a factor 2 subsampling (Fig. 2).

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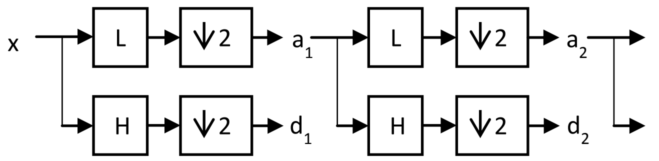

Figure 2. Discrete Wavelet Transform Tree

H and L denotes high and low-pass filters respectively, ↓2 denotes subsampling. Outputs of these filters are given by equations (12) and (13).

$$a_{j+1}[p] = \sum_{n=-\infty}^{+\infty} l[n-2p]a_j(n) \quad (4)$$

$$d_{j+1}[p] = \sum_{n=-\infty}^{+\infty} h[n-2p]a_j(n) \quad (5)$$

Elements a_j are used for next step (scale) of the transform and elements d_j, called wavelet coefficients, determine output of the transform. l[n] and h[n] are coefficients of low and high-pas filters respectively. Assume that on scale j+1 there is only half from number of a and d elements on scale j.

DWT algorithm for two-dimensional pictures is similar. The DWT is performed firstly for all image rows and then for all columns (Fig. 5).

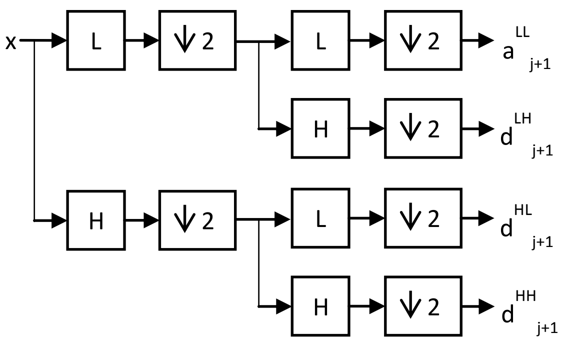

Figure 3. Wavelet Decomposition for 2D Signals

A vector contains energies of wavelet coefficients calculated in sub-bands at successive scales. As a result of this transform there are 4 subband images at each scale (Fig 4).

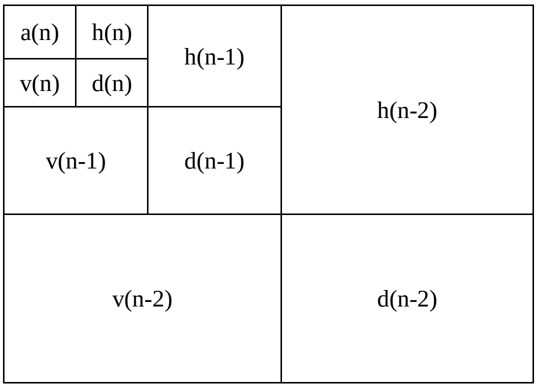

Figure 4. Sub band Images for Wavelet Decomposition

Sub band image ‘a’ is used only for DWT calculation at the next scale.

III. Noise Removal from Image using Thresholding

Two main limitations in image accuracy are categorized as blur and noise. Blur is intrinsic to image acquisition systems, as digital images have a finite number of samples and must satisfy the Shannon–Nyquist sampling conditions. The second main image perturbation is noise. There are different types of noises that can affect an image. Some of them are salt and pepper noise, Poisson noise, Gaussian white noise etc. [10], [21]. Gaussian white noise is given by equation (6)

Y = X + sqrt(variance) × random(s) + mean; (6)

Where, X is the input image, Y is the output image, s is the size of X. The value of mean and variance is taken as input. In this paper the image denoising is done based on the image which is corrupted by Gaussian white noise.

The image and noise model is given as:

$$x = s + \sigma.g \quad (7)$$

Where, s is an original image and x is a noisy image corrupted by additive white Gaussian noise g of standard deviation σ. Both images s and x are of size N by M (mostly M = N and always power of 2) [11-13].

The standard thresholding of wavelet coefficients is governed mainly by either ‘hard’ or ‘soft’ thresholding function [2] as shown in figure 5. The first function in Fig. 5(a) is a ‘hard’ function, and Fig. 5(b) is a ‘soft’ function.

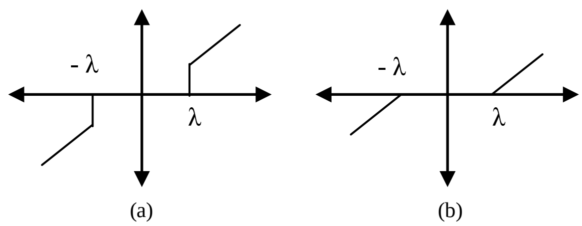

Figure 5. Thresholding functions; (a) hard, (b) soft

The hard thresholding function is given as

$$z = hard(w) = \begin{pmatrix} w, & for\ |w| > \lambda \\ 0, & for\ |w| \le \lambda \end{pmatrix} \quad (8)$$

Similarly, soft thresholding function is given as [14]

$$z = soft(w) = \begin{pmatrix} signum(w) \times max(|w| - \lambda, 0), & for\ |w| > \lambda \\ w, & for\ |w| \le \lambda \end{pmatrix} \quad (9)$$

Where, w and z are the input and output wavelet coefficients respectively, λ is a selected threshold value for both (8) and (9). In this paper, the soft thresholding technique is used for all the shrinkage methods.

Any digital image may have some amount of noise due to many reasons. Noise removal algorithm tries to remove this noise from the image in such a way that, the resulting denoised image should not have any noise. The ability of capturing the energy from a signal can be done very easily by wavelet techniques for which a signal corrupted by Gaussian white noise. Thus noise removal of natural images becomes very effective by this technique. The basic experimental setup of the wavelet transform based image de-noising is showed in Fig. 6.

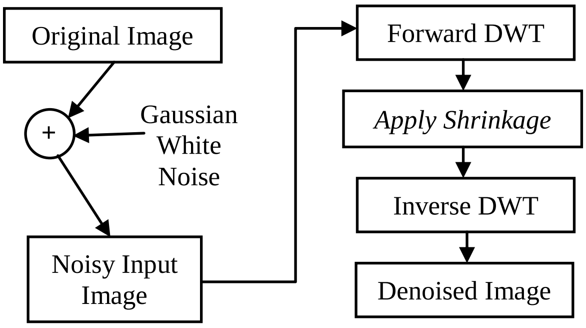

Figure 6. Basic Steps for Image Denoising

The performance of various denoising algorithms is quantitatively compared using MSE (Mean Square Error) [16] and PSNR [17] (Peak Signal to Noise Ratio) as

$$MSE = \frac{1}{NM}\sum_{n=1}^{N}\sum_{m=1}^{M}|s(n,m) - y(n,m)|^2 \quad (10)$$

$$PSNR = 10\log_{10}\left(\frac{255^2}{MSE}\right) \quad (11)$$

Where, s is an original image and y(n,m) is a recovered image from a noisy image s(n,m).

The problem of image denoising is mainly dependent upon the selection or determination of optimum threshold. It has been observed that if the value of the threshold is too low, then maximum noise elements will retain in the image and there will be very less difference between input and output image. On the other hand, if the threshold value is sufficiently or much higher, then the result will be the loss of noisy as well as original values of the image, resulting in too smooth output image [15]. The following are the methods of threshold selection for image denoising based on wavelet transform.

A. *VisuShrink*

The VisuShrink is a thresholding technique which is created by applying the Universal threshold proposed by Donoho and Johnstone [1-4]. This threshold is given by

$$T_{UNIV} = \sigma\sqrt{2logM} \quad (12)$$

Where, σ is the noise variance and M denotes the total number of pixels present in the image. It is shown in [2], [23] that the maximum of any M values iid (independent and identically distributed) as $N(0,\sigma^2)$ will be smaller than the universal threshold with high probability, with the probability approaching 1 as M increases. Thus, with high probability, a pure noise signal is estimated as being identically zero. However, for noise removal from image, VisuShrink is found to be very productive for generating smooth image. This is because the universal threshold or UT is derived with high probability constraints, the estimate should be at least as smooth as the original signal. So the UT tends to be high for large values of M, removing many signal coefficients along with the noise. So, this threshold does not adapt well to discontinuities in the signal.

B. *SureShrink*

The subband adaptive threshold [22-24] is applied for calculating the SureShrink threshold. A separate threshold value is calculated for each detail subband based upon SURE (Stein's unbiased estimator for risk), a method for estimating the unbiased loss $||\hat{\mu} - \mu||^2$. In our case let wavelet coefficients in the ith subband be $\{X_i: i = 1,\ldots, d\}$, $\hat{\mu}$ is the soft threshold estimator $\hat{X}_i = \eta_t(X_i)$, Stein's result [2] is applied to get an unbiased estimate of the risk $E||\hat{\mu}^{(t)}(x) - \mu||$:

$$SURE(t,X) = d - 2\#\{i: X_i \le t\} + \sum_{i=1}^{d}\min(|X_i|,t)^2 \quad (13)$$

For an observed vector x (set of noisy wavelet coefficients in a subband), we could find the threshold as

$$T_{SURE} = \arg\min SURE(t,X) \quad (14)$$

As the SureShrink gives better result than VisuShrink in terms of PSNR as it is subband adaptive technique.

C. *BayesShrink*

BayesShrink [9], [18], [22-23], [26] is an adaptive threshold for image denoising via wavelet soft-thresholding. This method is useful for images corrupted by Gaussian white noise. The threshold is generated by generalized Gaussian distribution (GGD) for the wavelet coefficients in each subband and try to find the threshold T which minimizes the Bayesian Risk. The reconstruction using BayesShrink is smoother and more visually appealing than one obtained using SureShrink. The basic model is expressed as follows:

$$Y = X + V \quad (15)$$

Here Y is the wavelet transform of the degraded image, X is the wavelet transform of the original image, and V denotes the wavelet transform of the noise components following the Gaussian distribution $N(0, \sigma_v^2)$. Here, since X and V are mutually independent, the variances σ_y^2, σ_x^2 and σ_v^2 of y, x and v are given by

$$\sigma_y^2 = \sigma_x^2 + \sigma_v^2 \quad (16)$$

The noise variance σ_v^2 can be estimated from the first decomposition level diagonal subband a(n) (Fig. 4) by the robust and accurate median estimator [18].

$$\sigma_v^2 = \left[\frac{median(|a(n)|)}{0.6745}\right]^2 \quad (17)$$

The variance of the sub-band of degraded image can be estimated as:

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$$\sigma_y^2 = \frac{1}{M}\sum_{m=1}^{M} A_m^2 \quad (18)$$

where A_m are the wavelet coefficients of sub-band under consideration, M is the total number of wavelet coefficient in that sub-band.

The bayes shrink thresholding technique performs soft thresholding, with adaptive data driven, sub-band and level dependent near optimal threshold given by [18]:

$$T_{BAYES} = \begin{cases} \frac{\sigma_v^2}{\sigma_x}, & if\ \sigma_v^2 < \sigma_y^2 \\ \max\{|A_m|\}, & otherwise \end{cases} \quad (19)$$

Where, $\sigma_x = \sqrt{\max(\sigma_y^2 - \sigma_v^2, 0)}$

In the case, where $\sigma_v^2 > \sigma_y^2$, σ_x is taken to be zero, this means $T_{BAYES} \rightarrow \infty$, or, in practice, $T_{BAYES} = \max(|A_m|)$, and all coefficients are set to zero.

D. NornShrink

The optimum threshold value for Normal Shrink or NormShrink is given by [19], [24]:

$$T_{NORM} = \frac{\lambda \sigma_v^2}{\sigma_y^2} \quad (20)$$

Where, the parameter λ is given by the following equation:

$$\lambda = \sqrt{log\left(\frac{L_k}{J}\right)} \quad (21)$$

L_k is the length of the sub-band at kth scale. And, J is the total number of decomposition. σ_v is the estimated noise variance, calculated by equation (17) and σ_y is the standard deviation of the subband of noisy image, calculated by using equation (18). Normal Shrink also performs soft thresholding with the data driven subband dependent threshold T_{NORM}, which is calculated by the equation (20).

E. NeighShrink

Let, $g = \{g_{ij}\}$ denotes the matrix representation of the noisy input image. Then, w = Wg denotes the matrix of wavelet coefficients of the signal under consideration. For every value of w_{ij}, let B_{ij} is a neighbouring window around w_{ij}, w_{ij} denotes the wavelet coefficient to be shrinked. The neighbouring window size can be represented as L×L, where L is a positive odd number. A 3×3 neighbouring window centered at the wavelet coefficient to be shrinked is shown in Fig 7.

As per the concept of this technique, corresponding terms in the summation is omitted when the previous summation (equation 22) has pixel indexes out of the wavelet subband range. The shrinked wavelet coefficient according to the NeighShrink is given by this formula [20]:

$$w'_{ij} = w_{ij}\,\beta_{ij} \quad (23)$$

The shrinkage factor β_{ij} can be defined as:

$$\beta_{ij} = \left(1 - \frac{T_{UNIV}^2}{S_{ij}}\right) \quad (24)$$

Here, T_{UNIV} is universal threshold given by equation (12).

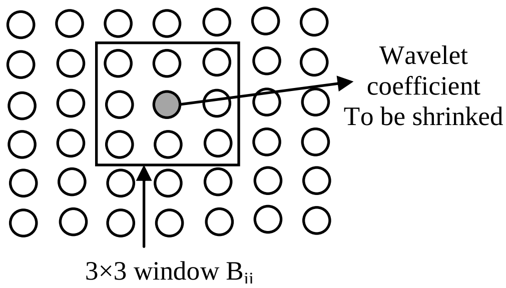

Figure 7. An illustration of how neighbour shrinkage is done using a 3×3 window centered at the wavelet coefficient to be shrinked

IV. EXPERIMENTAL RESULTS AND DISCUSSION

The experiment is done as shown in Fig 6. The experiments are conducted on natural gray scale test images like Lena and Boat of size 512×512. The kind of noise, added to original image, is Gaussian of different noise levels σ = 2 and 5. Then forward DWT is applied up to the desired level and then the corresponding threshold mechanism is applied, such as VisuShrink, SureShrink, NeighShrink etc.

After this step, inverse DWT is applied to get the denoised image. The PSNR values as given in equation (10) and (11), are obtained as shown in table I. The PSNR from various methods are compared in Table I and the data are collected from an average of fifteen runs on the image Lena, Peppers and Boat of size 512×512. It is a comparison between Visu Shrink, SURE Shrink, Bayes Shrink, Normal Shrink and Neigh Shrink

TABLE I
PSNR VALUES FOR IMAGES OF SIZE 512×512

Image Name	**Shrinkage Technique**				
	Visu Shrink	**SURE Shrink**	**Bayes Shrink**	**Norm Shrink**	**Neigh Shrink**
lena					
σ = 2	26.1803	33.7220	35.9144	35.0045	35.9928
σ = 5	19.1898	19.5900	35.0903	33.7347	35.8830
peppers					
σ = 2	26.3049	33.7821	36.8849	36.1934	36.9494
σ = 5	19.2599	19.6490	35.7767	35.3154	35.6305
boat					
σ = 2	25.9841	33.6372	35.3604	35.0121	35.8012
σ = 5	19.0265	19.4223	34.3730	34.0192	34.5619

From the PSNR values shown in table I, it is very much clear that, as we increase the value of noise level (σ), PSNR value gradually decreases.

After applying different shrinkage techniques, the denoised images are obtained. Some of them are shown in the Fig 8-11.

(a) (b)

Figure 8. Experimental Image of lena (512×512): (a) Original, (b) After adding Gaussian White Noise of σ=2, variance=30

Figure 9. From top left clockwise, noisy image, denoised images using VisuShrink, BayesShrink and SureShrink techniques respectively

(a) (b) (c)

Figure 10. Pepper Image is taken for experiment; (a)noisy image, (b)denoised image using VisuShrink, (c) denoised image using NeighShrink

The window size for neigh shrink is taken in this experiment is 5×5, and the output obtained is shown in Fig. 10(c).

Different images have given different values of PSNR, but it can be seen that each shrinkage technique is producing consistent result. That is, for a single image, all shrinkages are maintaining their property.

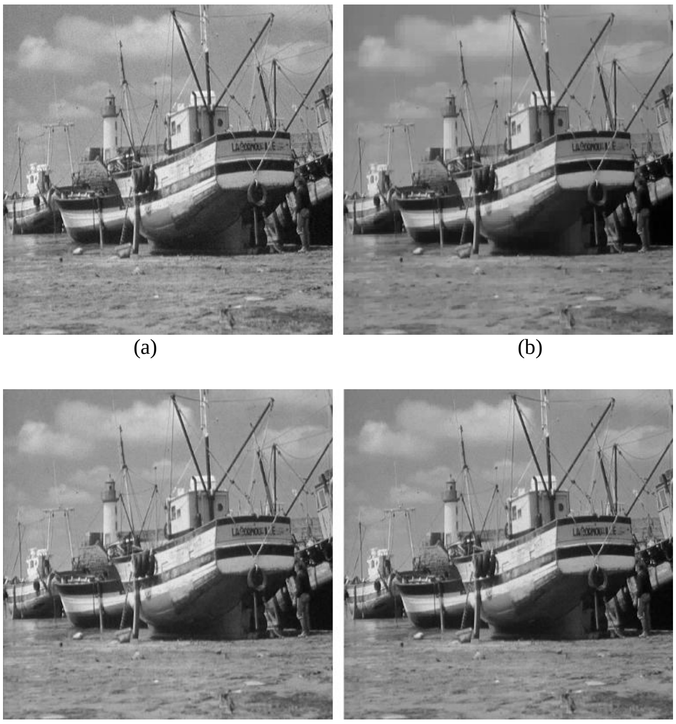

Figure 11. Boat Image; (a) noisy input image, (b) denoised image using VisuShrink, (c) using Norm Shrink, (d) using Neigh Shrink

V. CONCLUSION

In this paper, the advantages and applications of popular standard DWT and its extensions are realized for image denoising. The experiments were conducted for the study and understanding of different thresholding techniques which are the most popular.

It was seen that wavelet thresholding is an effective method of denoising noisy signals. We first tested hard and soft on noisy versions of the standard 1-D signals and found the best threshold. We then investigated many soft thresholding schemes such as VisuShrink, SureShrink, BayesShrink, NormShrink and NeighShrink for denoising images.

Individual software codes are developed for simulation of selected applications such as denoising and DWTs. The performance is statistically validated and compared to determine the advantages and limitations of all type of shrinkage techniques. Promising results are obtained using individual implementation of existing algorithms incorporating novel ideas into well-established frameworks.

The results show that NeighShrink gives somewhat better result than other shrinkage techniques in terms of PSNR. However, in terms of processing time, universal thresholding and normal shrinkage are much faster than other methods.

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