Analysis of Vertical Handover using Analytical Hierarchcy Process & Analytical Network Process

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Abstract—In this technologically driven era we are surrounded by many wireless network, in such a heterogeneous wireless environment we need a very smart and intelligent handoff algorithm. An algorithm which can provide global connectivity, for anyone, at anyplace, at anytime, to provide high-speed, high quality and reliable communication channel for mobile devices. Traditional handoff decision algorithm was based on single criteria such as received single strength which are not efficient and intelligent enough to minimize the number of unnecessary handoffs, decision and call-dropping and blocking probabilities. This research presents a novel approach for the design and implementation of a multi-criteria vertical handoff algorithm for heterogeneous wireless networks.

Keywords-component; Throughput, vertical handover, AHP, ANP

I. INTRODUCTION

In today's world a major concern of every nation is to improve safety and security conditions of transportation system [1], therefore Intelligent Transportation System (ITS) has merged remote sensing and communication technologies for the betterment of transportation system.

Applications of ITS is divided into two section: safety and non safety applications [2][3]. The major requirement of Intelligent Transportation System (ITS) is to have "always best connected" (ABC) network to improve safety conditions of vehicle, which is also one of the important criteria of 4G due to various heterogeneous network.

Present wireless communication system consist of various heterogeneous network which are characterized by different parameters. Hence there lies the decision problem of figuring out and choosing best available network. To satisfy the need of the user vertical handover becomes a necessity.

The need of vertical handover in heterogeneous network is not for the connectivity purpose but it is for the satisfaction of the user requirements. In vertical handover seamlessness and automation are two important issues, which is referred in ABC concept as always connected to a best available network in presence of various network [4].

The scope of our work is to compare vertical handover decision problem based on ANP and AHP considering mobile communication scenario and choosing best vertical handover algorithm between them. Authors are involved for multi channel solutions of ITS challenges. In IT'S for safety application remote sensing is used. The effectiveness of Digital radar to avoid collision is presented by the authors in [19][20]. For safety and non safety applications seamless communication is a major requirement. To provide ubiquitous communication in a heterogeneous radio access network scenario, authors have taken an initiative to design a very efficient vertical handover algorithm [21][24]. Both remote sensing and communication are amalgamated in the work carried out by R.Bera et.al[25]. This work is an extension of the work presented in [20][25]. Here, a novel context vertical handover model is developed and simulated to provide uninterrupted connection for V2V (Vehicle to Vehicle) and V2I(Vehicle to Infrastructure) communication.

II. LITERATURE SURVEY

To implement "Continuous Air-interface for Long and Medium range (CALM)" and to achieve a vision of 4G; vertical handover is a mandatory requirement [5]. The concept of always being best connected in any environment is discussed by Gustaffsson and Jonsson [4]. A robust vertical handover decision algorithm with the ability to decide best network at best time in presence of various heterogeneous networks based on static and dynamic factors is presented in [6]. Decision algorithm considering velocity and movement pattern of mobile station is described by W. Lee, E. Kim et.al[7]. On the other hand, L. Ma and D. Jia [8] discuss the comparative and cooperative relationship of WiMAX, WLAN and WCDMA which are important ITS technologies. Network selection algorithm using Analytical Hierarchy Process (AHP) is presented by Song and A. Jamilipour [9], here two Radio Access Network (RAN), UMTS and WLAN are considered. Comparison of different vertical handover algorithms using (AHP) is explained by Navarro.E, S.Wong [10].Vertical handover decision criteria among WLAN, UMTS and GPRS based on AHP is presented in the work carried out by Isakson Lennart and Fiedler Markus[11]. K. Nitiwong et.al[12] describe network selection decision when a new cell arrives in a mobile condition. Problems regarding horizontal and vertical handover for mobile IPv6 are discussed by C.W. Lee et.al in their work [13]. Network selection strategy in integrated CDMA-WLAN and initiation of handover is discussed and analyses of QoS at various velocity of mobile terminal is shown in the work presented by Kim,J et.al [14]. A vertical handover algorithm which maximizes the life time of a battery is proposed in the work carried out by SuKyoung,L et.al [15].

In the work presented by Q. Wei et.al[16] and T. Ahmed [17] context repository and adaptability manager is proposed for network selection process. Vertical handover algorithm based on multi criteria decision tool, Analytical Network Process (ANP) is proposed in the work carried by Shubhajit Datta et.al [18].

From the thorough review of literature, we can conclude with the following criticisms:

- 1. The traditional vertical handover algorithms are provided with certain constraint, which does not take under consideration of seamless connection in mobile terminal.
- 2. Many VHO algorithms include very complex mathematics and very long execution time.
- 3. The traditional VHO algorithm has not considered mobility of receiver as one of its criteria.

The main objective of this work is to compare the results of VHO decision algorithm using AHP and ANP, considering the same attributes in both the cases.

III. PROBLEM DEFINITION

The motive of our work is to design and simulate an application specific vertical handover among Wimax, UMTS and WLAN for ITS using Analytical Hierarchy Process(AHP) and Analytical Network Process (ANP) and comparing their result.

For this case we have considered a scenario where user moves from point A to E as shown in "Fig 1".

At position A, the vehicle is in sub-urban area with 10 km/hr speed. At this position all three networks are present. He downloads some files at a position referred as A.

Before his downloading is finished he enters position B which has coverage of UMTS and WiMAX. The speed of the car at position B is 60Km/hr. At position C it is covered only by UMTS network; at this point he makes a call. He reaches point D before terminating the call and the speed of vehicle is reduced to 15 Km/hr.

At position E he does video chatting and speed of the vehicle is 40Km/hr. In all this cases, handover is required at a right position in a right time to satisfy the user need.

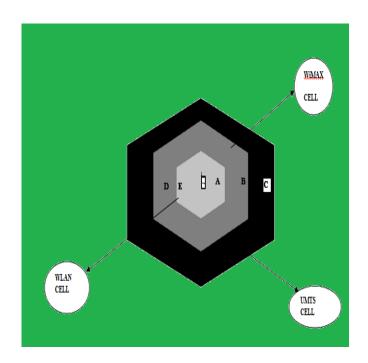


Figure 1. A Typical Radio Network Availability Scenario for ITS

IV. PROPOSED SOLUTION

A. Analytical Hierarchy Process(AHP)

The Analytical Hierarchy Process is a mathematical tool developed by Satty[26-30] to solve complex decision making problem. There are three principles of AHP method.

- 1. Structure of the model.
- 2. Comparative judgement of the alternatives and the criteria.
- 3. Synthesis of the priorities.

The steps of AHP are as follows:

- 1. At first a complex decision problem is modeled into hierarchy as depicted in "Fig2", with the objective at the top followed by different criteria in the middle and then at the bottom the different alternatives.
- 2. In the second step pair-wise comparison of alternatives and criteria is done. After the problem has been dissolved to its hierarchy, prioritization starts in

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order to determine the relative importance of the criteria within each level. From the second level onwards pair-wise comparison of each alternative with respect to criteria starts. In AHP, multiple pair-wise comparison is done based on a standardized comparison scale of nine table presented in [22].

- 3. Two factors are identified objective factors and subjective factors. Objective factors are measured directly whereas subjective factors are quantified from human knowledge. In a subjective factor consistency ratio has to be evaluated before final decision.
- 4. Develop a normalized matrix. The sum of every column of a normalized matrix is always one.
- 5. Now the average of each row of the normalized matrix is formed and this is known as priority vector. The sum of column of a priority vector is also one.
- 6. Consistency ratio is determined. According to Satty[26] consistency ratio should not be grater than 10%.
- 7. Final decision of choosing one of the alternatives is determined after multiplying priority vector of criteria with the priority vector of alternative with respect to each criterion taking under consideration the main objective. The alternative having highest weight is chosen.
- B. Design of Analytical Hierarchy Process(AHP)

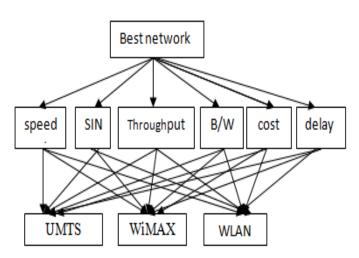


Figure 2: AHP model for best network selection

"Fig2" shows the AHP model for selection of best network among UMTS, WiMAX and WLAN under consideration of six criteria which are speed, SINR, throughput, delay, bandwidth and cost. The following tables i.e. Table1, Table2, Table3, Table4 and Table5 show the pair-wise comparison of alternatives with the given criteria, with their respective consistency ratio (CR).

Table 1: First level AHP Matrix for deciding relative priority of influencing factors. (**CR=0.0138**)

	SP EE D	SI N R	TH RO UG HPU T	BA ND WI DT H	C OS T	DE LA Y	PRIORITY VECTOR
SPEED	1	2	2	4	8	7	0.382962
SINR	1/2	1	1⁄2	3	5	3	0.18968
THRUO GHPUT	1/2	2	1	2	6	3	0.22945
BAND WIDTH	1/4	1/ 3	1⁄2	1	3	2	0.09906
COST	1/8	1/ 5	1/6	1/3	1	1/2	0.036708
DELAY	1/7	1/ 3	1/3	1/2	2	1	0.061951

Table 2: Second level AHP Matrix for deciding relative priority of networks for SINR only (CR=0.0176)

SINR	WiMAX	UMTS	WLAN	PRIORITY VECTOR
WiMAX	1	8	2	0.594688
UMTS	1/8	1	1/8	0.340374
WLAN	1/2	6	1/2	0.064938

Table 3: Second level AHP Matrix for deciding relative priority of networks for bandwidth only(CR=0.0019)

Bandwidth	WiMAX	WiMAX UMTS		PRIORITY VECTOR
WiMAX	1	7	1	0.458263
UMTS	1/7	1	1/8	0.062616
WLAN	1	8	1	0.479121

Table 4: Second level AHP Matrix for deciding relative priority of networks for delay only. (CR=0.0015)

Delay	WiMAX	UMTS	WLAN	PRIORITY VECTOR
WiMAX	1	3	1/3	0.23634
UMTS	1/3	1	1/8	0.081935
WLAN	3	8	1	0.681735

Table 5: Second level AHP Matrix for deciding relative priority of networks for cost only(CR=0.0036)

Cost	WiMAX	UMTS	WLAN	PRIORITY VECTOR
WiMAX	1	1/2	1/5	0.12202
UMTS	2	1	1/3	0.229651
WLAN	5	2	1	0.648329

Speed of the vehicle is converted into 9-point scale first as it is objective factor before formation of AHP matrix [22].

C. Analytical Network Process

From the literature review it is clear that the AHP is a multicriteria decision technique used for solving complicated decision problem; where the lower cluster of the hierarchy is dependent on its upper cluster. But all the problems cannot be framed in hierarchy as in some case both upper and lower cluster are dependent on each other [31]. In such a case where interdependence is required, Analytical Network Process (ANP) is used.

Satty [31] has suggested to solve the problems of dependence of alternatives on criteria by AHP, and use of ANP when both criteria and alternatives depend on each other.

The steps of ANP are as follows:

- 1. Problem is well defined and then it is decomposed into a network.
- 2. Like AHP, in ANP pair-wise comparison of alternative with respect to criteria is done and in addition to it, pair-wise comparison of criteria with respect to alternative is also done.
- 3. Supermatrix is formed: supermatrix concept is same as MARKOV chain process to obtain the rate of influence of one factor on the other.
- 4. Selection of the best alternative is done by seeing the highest weight alternative.

"Fig3" shows the ANP model for selection of best network among UMTS, WiMAX and WLAN under consideration of five criteria which are speed, SINR, throughput, delay, bandwidth and cost. It is clear from the figure that both alternatives and criteria depend on each other.

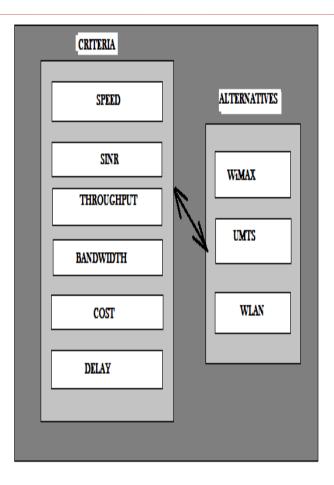


Figure 3: ANP model for best network selection

In ANP method same steps of AHP are followed, apart from it two more steps are included.

In this process influence of criteria with respect to alternative is also processed.

Finally Supermatrix is formed(Table6 & 7), showing influence of criteria and alternatives on each other.

Table 7. Limited SUPERMATRIX.

Table		UMTS	WIMAX	WLAN	B/W	COST	DELAY	SINR	SPEED	THROUG HPUT
	UMTS	0.1192	0.1192	0.1192	0.1192	0.1192	0.1192	0.1192	0.1192	0.1192
	WiMAX	0.20858	0.20858	0.20858	0.20858	0.20858	0.20858	0.20858	0.20858	0.20858
	WLAN	0.17450	0.17450	0.17450	0.17450	0.17450	0.17450	0.17450	0.17450	0.17450
	B/W	0.10649	0.10649	0.10649	0.10649	0.10649	0.10649	0.10649	0.10649	0.10649
	COST	0.02150	0.02150	0.02150	0.02150	0.02150	0.02150	0.02150	0.02150	0.02150
	DELAY	0.05501	0.05501	0.05501	0.05501	0.05501	0.05501	0.05501	0.05501	0.05501
	SINR	0.06165	0.06165	0.06165	0.06165	0.06165	0.06165	0.06165	0.06165	0.06165
	SPEED	0.15528	0.15528	0.15528	0.15528	0.15528	0.15528	0.15528	0.15528	0.15528
	THROUGHPUT	0.10008	0.10008	0.10008	0.10008	0.10008	0.10008	0.10008	0.10008	0.10008

6.Unweighted SUPERMATRIX

	UMTS	WIMAX	WLAN	B/W	COST	DELAY	SINR	SPEED	THROUG HPUT
UMTS	0.000	0.000	0.000	0.06262	0.22965	0.08193	0.06494	0.58155	0.06494
WiMAX	0.000	0.000	0.000	0.45826	0.12202	0.23633	0.59469	0.30900	0.59469
WLAN	0.000	0.000	0.000	0.47912	0.64833	0.64833	0.34037	0.10945	0.34037
B/W	0.05683	0.15476	0.038718	0.000	0.000	0.000	0.000	0.000	0.000
COST	0.03103	0.03646	0.05881	0.000	0.000	0.000	0.000	0.000	0.000
DELAY	0.09953	0.10714	0.12057	0.000	0.000	0.000	0.000	0.000	0.000
SINR	0.17756	0.07462	0.14512	0.000	0.000	0.000	0.000	0.000	0.000
SPEED	0.37635	0.49561	0.15286	0.000	0.000	0.000	0.000	0.000	0.000
THROUG HPUT	0.258669	0.22142	0.13552	0.000	0.000	0.000	0.000	0.000	0.000

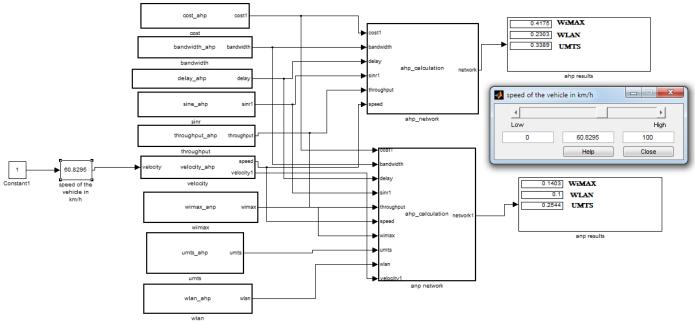


Figure 4. SIMULINK model for ANP and AHP based Vertical Handover.

V. IMPLEMENTATION DETAILS

"Fig4" shows simulation model using AHP and ANP for network selection in Vehicular Communication. Here the weight of different network (WiMAX, WLAN and UMTS) is calculated using AHP and ANP and then these weights are provided to the decision maker block as its input. Network availability detector shows the presence of the network.

Since UMTS has highest coverage area, so it's available everywhere. Network availability detector has to detect only the availability of WiMAX and WLAN. Result of this block is fed as the input to the other blocks. Network availability will be shown if Network availability detector receives received signal strength (RSS) from that particular network.

In "fig4" the speed of the vehicle is taken as dynamic attribute and the results of AHP and ANP is displayed on the display unit.

Here only conversational application is taken under consideration. Decision maker follows all the steps of AHP and ANP before making final decision. Finally the weights of each network are displayed on AHP and ANP display unit.

VI. RESULTS AND DISCUSSIONS

Simulation results related to network selection among WiMAX, UMTS and WLAN is presented in this section. Table8 shows AHP and ANP weights of the network WiMAX,UMTS and WLAN at different speed of the vehicle. Here one assumption made is that the user is present in a particular network at least for a minute to reduce frequent handover.

SPEED		AHP		ANP				
	WiMAX	WLAN	UMTS	WiMAX	WLAN	UMTS		
10	0.444	0.3034	0.2389	0.1616	0.2429	0.2622		
15	0.4553	0.3097	0.2217	0.1634	0.2379	0.2654		
20	0.4691	0.3176	0.2	0.1657	0.2316	0.2694		
25	0.4871	0.3279	0.1717	0.1687	0.2233	0.2747		
30	0.5077	0.3386	0.1404	0.1718	0.2143	0.2806		
40	0.4718	0.2955	0.2194	0.1593	0.1	0.2702		
50	0.4422	0.26	0.2844	0.149	0.1	0.2616		
60	0.4185	0.2316	0.3366	0.1407	0.1	0.2547		
70	0.3964	0.205	0.3852	0.133	0.1	0.2483		
80	0.3791	0.1842	0.4234	0.1269	0.1	0.2433		
90	0.3637	0.1672	0.4558	0.122	0.1	0.2388		
100	0.3465	0.1819	0.4583	0.1263	0.1	0.2338		

Table 8: Results of AHP and ANP at different speed.

In the literature review, we have seen that previously designed VHO algorithm were based on QoS analysis . These algorithms had limitation of considering multiple constraint resources. Comparing network selection by AHP and ANP method is not shown in any of the work discussed under literature survey except few. Analyzing the results of network selection by AHP and ANP method shows that ANP is a good option for solving decision making problems as it has the following advantages:

- 1. It overcomes the limitation of QoS based VHO algorithm.
- 2. It does not have complex mathematical formulas and method and therefore it does not take long time period for execution.

- 3. It takes equal consideration of alternatives on the attributes before making final decision which AHP does not.
- 4. It provides optimum solution for any kind of situation.

The following RAN (UMTS, WiMAX and UMTS) is taken as it is supported by the CALM. Platform used for the design of this VHO algorithm are Embedded MATLLB and SIMULINK. The vision of the author is to develop SDR based reconfigurable hardware to be able to perform VHO in heterogeneous network environment.

VII. CONCLUSION

Thus, the comparison of the two vertical handover decision problems based on ANP and AHP considering mobile communication scenario was implemented successfully. This comparison helps in choosing the best vertical handover algorithm based on users criteria.

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