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Analysis of ECG signal by Polynomial Approximation

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Abstract— ECG (Electrocardiogram) originated due to depolarization and repolarization of heart generates massive volume of digital data. One hour of ECG signal requires 1 GB of memory. So, ECG signals needs to be compressed for efficient transmission and storage purpose. This paper proposes a technique to denoise and then approximate ECG signal. The proposed method achieves 0.037 MSRE and 42.59 dB SNR in denoising the ECG. Polynomial of order 31 can be used to successfully approximate ECG segments which provide minimum error with relevant compression ratio.

Keywords*-ECG, SNR, moving average,CR, PRD, MSE, etc.*

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I. INTRODUCTION

The ECG represents the electrical activity of the heart and already provides a lot of essential information to physicians for diagnosis of heart diseases. Fig.1 shows a simple ECG model characterized by a number of waves P, QRS, T, U related to the heart activity. These waves are the result of contraction and expansion of the heart muscles. The P wave is due to the depolarization of atria whereas QRS complex reflects the rapid depolarization of right and left ventricles [2]. The T-wave represents the repolarisation (or recovery) of the ventricles. The QRS complex is a major wave in each ECG beat since the duration, the amplitude, and the morphology are used for cardiac arrhythmias, conduction abnormalities, ventricular hypertrophy, myocardial infarction, etc [1] [2].

While recording ECG in a clinical environment it is usually contaminated by power line interference, EMG (electromyogram) signal caused due to high frequency signal related to muscle activity. These recordings are critically often contaminated by cardiac artifact [3]. Baseline wander elimination (a low frequency signal caused mainly by the breathing action) is considered as a classical problem. It is considered as an artifact which produces atrifactual data when measuring the ECG parameters, especially the ST segment measures are strongly affected by this wandering. In most of the ECG recordings the respiration, electrode impedance change due to perspiration and increased body movements are the main causes of the baseline wandering, the electrode motion is usually represented by a sharp variation of the baseline. Wherever Times is specified, Times Roman or Times New Roman may be used. If neither is available on your word processor, please use the font closest in appearance to Times. Avoid using bit-mapped fonts if possible. True-Type 1 or Open Type fonts are This corrupted noise prevents considerably the accurate analysis of the ECG signal and useful information extraction [4].

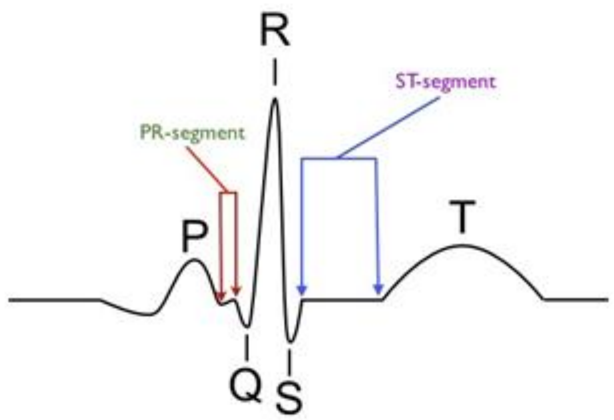

Fig1: ECG signal (Google source).

Many researchers have worked on development of methods for reduction of baseline wander noise. Zahoor-uddin, presented Baseline Wandering Removal from Human ECG signal using Projection Pursuit Gradient Ascent Algorithm & showed the comparative study of the results of different algorithms like Kalman filter, cubic spline [5]. Manpreet Kaur et al in 2011 has compared the performance of different filtering methods for baseline removal from ECG signal. [6].Mahesh S. Chavan et al in 2008 compared the results of Butterwoth filter and Elliptic filter for the suppression of Baseline and Powerline interferences [7]. Ch. Renumadhavi et al evaluated SNR which is used as a performance indicator for the comparison of filters [8]. V.S. Chouhan and S.P. Mehta in [9] developed an algorithm for total removal of Baseline drift from ECG signal & deploy least square error correction & median based correction.

Signal reconstruction and compression is an issue of vital importance in signal storage and transformation. Many techniques have been proposed in this field, such as Fourier transforms (FT), discrete cosines transform (DCT) and Wavelet transforms. The main difference among these methods is the basis functions used. The most common basis function is the polynomial basis. The Polynomials are just about the simplest mathematical functions that exist, requiring only multiplications and additions for their evaluation. Yet they also have the flexibility to represent very general nonlinear relationships. Polynomials are frequently used in signal processing; e.g., for filtering noisy signals [15], for interpolation of data [16], and for data compression [17]. All these applications use low-order polynomials (e.g., degree 2-5) to approximate a signal on a small interval.

ECG interpolation including splines functions & polynomials of maximum degree 3, have been proposed in [26] and [27]. The representation of ECG signals using second degree polynomials is studied by Nygaard *et al* in [28]. When using cubic splines or quadratic polynomials for ECG compression, the signal should be pre-processed in order to extract some particular points such as extrema, zero crossing and inflexion points that could be used as the interpolation nodes. Legendre polynomials with high degree were used for ECG data compression in [29] Generalized Jacobi polynomials are tested for ECG compression in [30]. On the contrary, this paper is concerned with the approximation of a signal by a high-degree polynomial on a large interval. Similar techniques use expansions in terms of sine and cosine (Fourier descriptors [18] and discrete cosine transform [19]), sampled square waves [20], Hermite functions [21] and principal components [22][23].

II. METHODOLOGY

A. *Moving average*

One of the most common tools for smoothing data is the MOVING AVERAGE filter, often used to try to capture important trends in repeated statistical surveys. The Finite Impulse Response (FIR) filter is applied to a set of data points by creating an average of different subsets of the full data set[10]-[12]. The moving average filter is optimal for a common task: reducing random noise while retaining a sharp step response. This makes it the premier filter for time domain encoded signals. A subset of fixed size obtained from sample values in a given signal by taking their average, the moving average is obtained. Then this subset is shifted forward for an element in the given signal. First element in subset is deleted whereas adding a next element to the end of the subset of the given signal. The new subset has the same size as previous one and it is averaged again [12] [13]. Same process is repeated over the entire signal. In this paper moving average is defined for samples as follows:

Suppose $s_m = \{x_m, x_{m+1}, \ldots, x_{N+(m-1)}\}$ is a subset of sample values in an ECG signal $S = \{x_1, x_2, x_3, \ldots, x_m, \ldots\}$ and the fixed size of the subset *is* N . Then, a new series of $\{A_1, A_2, \ldots Am, ..\}$ is called the moving average of S which is obtained by the following calculation[14]:

$$A_m = \frac{x_m + x_{m+1} + \ldots + x_{N+(m+1)}}{N} \qquad (1)$$

Algorithm

Step 1. Choose a proper size N of the subset $s_i = \{x_m, x_{m+1}, \ldots, x_{N+(m-1)}$ in a given signal S .

Step 2. Calculate the average of every N sample values to obtain the moving average.

Step 3**.** Maintain the same length, create series of $\{A_1, A_2, \ldots Am, ..\}$ of signal S.

The testing criteria for denoising performance consist of Signal to Noise Ratio (SNR) and Mean Square Relative Error (MSRE).

Basically signal to noise ratio (SNR) is an engineering term for the power ratio between a signal and noise.

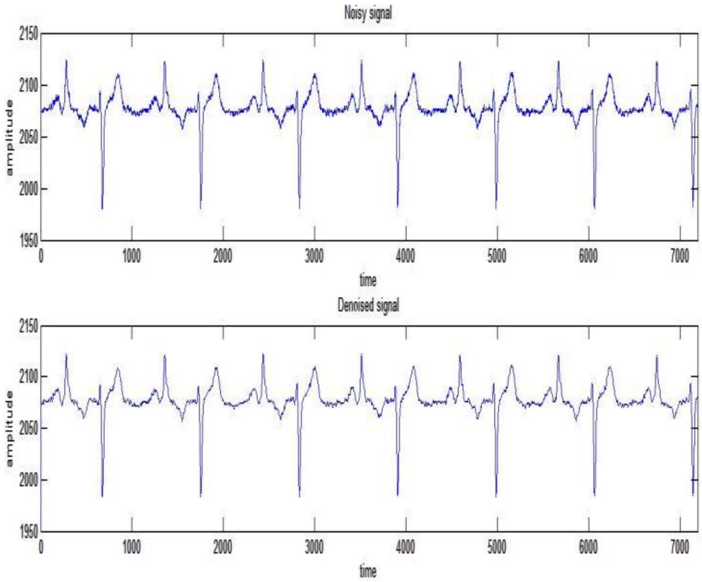

Fig 2: Noisy signal from MIT -BIH ANSI/ AAMI EC13 and denoised ECG signal.

B. *Polynomial Approximation*

Given a set of $n + 1$ data points (x_i, yi) where no two x_i are the same, for a polynomial p of degree at most n with the property.

$$p(x_i) = y_i, i = 0, \ldots, n. \qquad (4)$$

Suppose that the interpolation polynomial is in the form

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0 \qquad (5)$$

The statement that p interpolates the data points means that

$$p(x_i) = y_i \qquad \text{for all } i \in \{0,1,\ldots,n\}. \qquad (6)$$

If we substitute equation (1) in here, we get a system of linear equations in the coefficients a_k . The system in matrix-vector form reads

$$\begin{bmatrix} x_0^n & x_0^{n-1} & x_0^{n-2} & \cdots x_0 & 1 \\ x_1^n & x_1^{n-1} & x_1^{n-2} & \cdots x_1 & 1 \\ \vdots & \vdots & \vdots & \cdots \vdots & \vdots \\ x_n^n & x_n^{n-1} & x_n^{n-2} & \cdots x_n & 1 \end{bmatrix} \begin{bmatrix} a_n \\ a_{n-1} \\ \vdots \\ a_0 \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_n \end{bmatrix}.$$

We have to solve this system for a_k to construct the interpolate *p(x)*. The matrix on the left is commonly referred to as a Vander monde matrix.

If f is $n + 1$ times continuously differentiable on a closed interval I and $p_n(x)$ be a polynomial of degree at most n that interpolates f at $n + 1$ distinct points $\{x_i\}$ (i=0,1,...,n) in that interval. Then for each x in the interval there exists ξ in that interval such that

$$f(x) - p_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}\prod_{i=0}^{n}(x - x_i) \qquad (10)$$

The trigonometric form of the Chebyshev polynomials of first kind is given by

$$c_x(x) = \cos(n\cos^{-1}(x)) \quad \text{for } n \geq 0 \qquad (11)$$

the Chebyshev polynomials satisfy

$$c_0(x) = 1,$$
$$c_1(x) = x,$$
$$c_2(x) = 2x^2 - 1,$$
$$\ldots$$
$$c_{n+1}(x) = 2x\,c_n(x) - c_{n-1}(x)\ , n \geq 1.$$

$c_n(x)$ changes sign (and has a zero) n times in the interval of interest. The zeros of the nth order Chebyshev polynomial occur at

$$x_j = \cos\left(\frac{j - 0.5}{n}\pi\right) \qquad 1 \leq j \leq n \qquad (12)$$

The Chebyshev polynomials also have the property of bounded variation. The local maxima and minima of Chebyshev polynomials on [−1, 1] are exactly equal to 1 and −1, respectively, regardless of the order of the polynomial. It is this property which makes them valuable for minimax approximation. In fact, an excellent approximation to the nth order minimax polynomial on an interval can be obtained by finding the polynomial that satisfies $c_n(x)$ = f (x) at the zeros of the $(n+1)^{th}$ order Chebyshev polynomial[28] [29].

Algorithms:

Step 1. Construct the function of p(x) for N samples of segment of ECG signal.

Step 2. Decide order of n for approximation.

Step 3.Find Chebyshev points x_j.

Step 4.Calculate coefficient and create polynomial p(x).

Step 5.Calculate minimax error

Reconstructed segments of ECG signal are tested on the basis of CR, PRD and MSE.

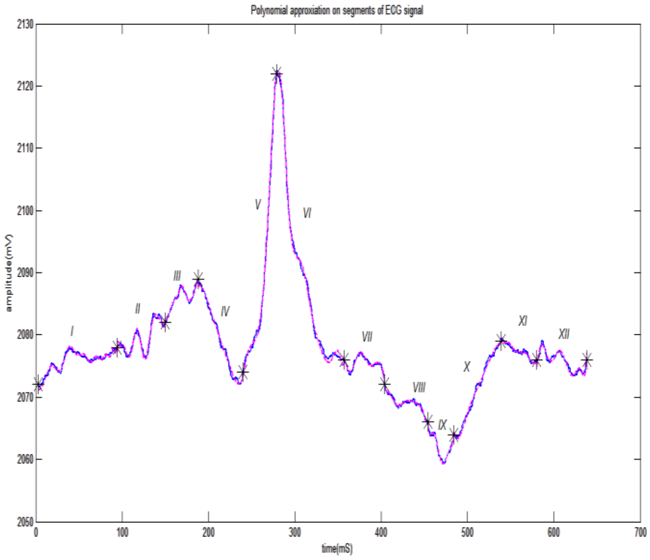

Fig 3: Polynomial Approximation on segments of one cardiac cycle of ECG signal.

III. RESULT AND DISCUSSION

An ECG signal is not linear, rather more curvaceous consisting of waves of various shapes. Many author uses SNR as an objective method to analyze the performance of ECG denoising methods. The IIR and FIR zero phase filtering gives 12.708 and 11.679 SNR value [6]. Similarly wavelet and polynomial fitting is used for noise removal purpose provide 11.689 and 11.16 SNR value which is greater than moving average 10.989 SNR value [6]. Hamming Low Pass filter and LMS adaptive filter gives better SNR value (i.e. 21.6521, 22.5268) than Moving average gives 12.4004 SNR value [8].

As shown in fig 2 noisy signal is rough in nature. After denoising signal from moving average method produce smooth ECG signal. Signal to Noise ratio and MRSE being calculated against a subset of the MIT-BIH ANSI/AAMI Database. The optimal numerical experimental results for the subset of this standard database are summarized in Table 1.

TABLE I. SNR AND MSRE CALCULATION OF DENOISED SIGNAL.

Database	SNR(dB)	MSRE
aami3am	42.5410	0.0375
aami3bm	42.5495	0.0375
aami3cm	42.4939	0.0375
aami3dm	42.5085	0.0375
aami4a_dm	42.5622	0.0374
aami4a_hm	42.5916	0.0373
aami4am	42.5717	0.0373
aami4b_dm	42.5623	0.0374
aami4b_dm	42.5921	0.0373
aam4bm	42.5720	0.0373

The moving average filter is implemented over 10 signals taken from MIT-BIH database.SNR to all the signals is almost 42dB which is expected for a good denoising method. MSRE is also under expected limit. The denoised signal is then segmented into various sections depending upon their shapes. The segmentation was done manually. The segmented sections are then approximated by polynomials of different orders.

As shown in table II that PRD and MSE for each segment is low with relevant CR.As increasing the degree of polynomial minimax error will reduce additionally.

TABLE II. APPROXIMATION FOR SEGMENTS OF ECG SIGNAL (AAMI3AM.MAT)

Segment	Degree of polynomial	CR	PRD	MSE	Minimax error
I.	31	2.8750	0.0159	0.1086	0.5245
II.	12	4.3846	0.0204	0.1802	0.6006
III.	21	1.7727	0.0144	0.0907	0.4328
IV.	5	8.8333	0.0227	0.2235	0.8028
V.	5	6.6667	0.0268	0.3144	0.8049
VI.	10	7.1818	0.0299	0.3908	0.9435
VII.	7	6.0000	0.0113	0.0553	0.3808
VIII.	8	5.6667	0.0131	0.0736	0.4392
IX.	7	3.8750	0.0149	0.0947	0.4583
X.	6	8.0000	0.0236	0.2383	0.7702
XI.	7	5.2500	0.0139	0.0839	0.4145
XII.	8	6.5556	0.0198	0.1685	0.6566

TABLE III. APPROXIMATION FOR SEGMENTS OF ECG SIGNAL (AAMI3BM.MAT)

Segment	Degree of polynomial	CR	PRD	MSE	Minimax error
I.	39	2.3000	0.0173	0.1290	0.5147
II.	12	4.3846	0.0089	0.0344	0.2759
III.	15	2.4375	0.0185	0.1479	0.5371
IV.	9	5.3000	0.0215	0.1985	0.6517
V.	8	4.4444	0.0230	0.2157	0.6672
VI.	7	9.8750	0.0564	1.3592	1.7283
VII.	7	6.0000	0.0185	0.1482	0.5485
VIII.	7	6.3750	0.0301	0.4084	0.9021
IX.	7	3.8750	0.0121	0.0665	0.3782
X.	7	7.0000	0.0228	0.2288	0.7309
XI.	7	5.2500	0.0187	0.1511	0.6153
XII.	7	7.3750	0.0323	0.4505	1.0332

TABLE IV. APPROXIMATION FOR SEGMENTS OF ECG SIGNAL (AAMI3DM.MAT)

Segment	Degree of polynomial	CR	PRD	MSE	Minimax error
I.	43	2.0909	0.0135	0.0782	0.5245
II.	12	4.3846	0.0311	0.4187	0.6006
III.	15	2.4375	0.0139	0.0822	0.4328
IV.	10	4.8182	0.0131	0.0733	0.8028
V.	9	4.0000	0.0162	0.1116	0.8049
VI.	9	7.9000	0.0282	0.3425	0.9435
VII.	7	6.0000	0.0191	0.1682	0.3808
VIII.	7	6.3750	0.0146	0.0923	0.4392
IX.	7	3.8750	0.0124	0.0673	0.4583
X.	7	7.0000	0.0235	0.2495	0.7702
XI.	7	5.2500	0.0299	0.3831	0.4145
XII.	7	7.3750	0.0107	0.6477	0.6566

The original ECG signal (aami3am) and its approximated signal is shown in fig 3.

IV. CONCLUSION

In this paper, moving averaging-based filtration and polynomial approximation is presented. The novel method to provide better filtration gives better SNR and MRSE value. And polynomial approximation is done which provide low PRD and MSE for segments of ECG signal. According to the various papers studied during research, it can be concluded that filtering through moving average permits relatively fewer calculations and better SNR, in comparison with most existing method. For polynomial approximation, increasing the degree of polynomial will reduce the error further and decreases the compression ratio. In this research it is found the higher order polynomial is required to model QRS complex as it is more curvaceous in nature. Automatic segmentation of ECG signals may provide better and fast results.

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