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On The Existence of Discrete Solitons in a Hexagonal Lattice

Arvind Sharma and A.K. Nagar
Department of Physics, Govt. Dungar College,
Bikaner, Rajasthan 334001, India

Abstract— The discrete nonlinear Schrodinger equation on a non-square hexagonal geometry lattice due to the topological charge shows the possibility of a closed phase portrait and limit cycle in case of focusing nonlinearity. A linear stability analysis of discrete solitons is presented. The method of numerical continuation is applied to solve the dynamical equations. The phase plane analysis shows existence of Limit cycle in the phase plane indicating stability of solitons.

Keywords: *Discrete nonlinear Schrodinger equation, hexagonal lattice, limit cycle*

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INTRODUCTION

The discrete lattice systems have become popular due to experimental implementations of such systems in various branches of physics such as carbon nanotubes and AlGaAs waveguide arrays, in which, the interplay of inherent discreteness and nonlinearity led to the emergence of numerous interesting phenomena such as diffraction and diffraction management, gap solitons[1,2]. Recently, the setting of optically induced photonic lattices in photorefractive crystals include the formation of patterns such as dipole, quadrupole and necklace solitons, impurity modes, discrete vortices, rotary solitons [3], higher order Bloch modes [4] and gap vortices, the observation of two-dimensional (2D) Bloch oscillations and Landau-Zener tunneling, the observation of localization and diffraction in honeycomb, hexagonal [5] and quasi-crystalline lattices. The study of Anderson localization in disordered photonic lattices [6] and the atomic physics of Bose-Einstein condensates (BECs), when trapped in periodic potentials have also been discussed in the recent reviews [7-10].

Matcont is a numerical continuation package for dynamical analysis of continuous and discrete differential equations based on Matlab[11]. The method of phase plane analysis gives better understanding of results.

THEORETICAL ANALYSIS

The number of nearest neighbors in a hexagonal lattice is six in two dimensional case. Hence, considering the discrete geometry, the discrete nonlinear Schrodinger equation is written as, following [3].

$$i\dot{\varphi}_{l,m} = -\varepsilon\left(\sum_{l',m'}\varphi_{l',m'} - 6\varphi_{l,m}\right) - \left|\varphi_{l,m}\right|^2 \varphi_{l,m} \tag{1}$$

where the summation is to be carried over the six nearest neighbors (denoted by l',m') of the site (l,m). In case of simple hexagonal contour with a central inter site, a configuration with topological charge K is developed [3]. Here, ε represents the coupling strength between nearest neighbor nodes in the anti-continuum limit ($\varepsilon \to 0$) where the discrete wave function is

$$\varphi_j = \sqrt{\gamma_j}\exp(i\theta_j)\exp(i\gamma_j z_m) \tag{2}$$

with $\theta_j = 2\pi K/6$; $j = 1,\ldots,6$ for the six sites constituting relevant contours.

In the anti-continuum limit for the two dimensional case, on substituting the above ansatz, Eq.(2) in Eq.(1) and following [3], keeping $\gamma = 1$, we obtain the reduced eigenvalue problem for the system (1) in the form of following difference equation:

$$2z_m - z_{m+1} - z_{m-1} = \gamma_j z_m \tag{3}$$

Using the discrete Fourier transforms for the eigenvectors $z_m \sim \exp(i\pi jm/3)$, we obtain the following eigenvalues over the lattice:

$$\lambda_j = \pm\sqrt{8\varepsilon\cos\left(\frac{\pi K}{3}\right)\sin^2\left(\frac{\pi j}{6}\right)} \tag{4}$$

On applying the method of phase plane analysis, solving Eq.(3) using Matcont [11], we arrive at the phase plane contours. The salient results are discussed in the next section.

RESULTS AND DISCUSSION

Figure 1 shows the closed loop phase portrait between amplitude $|\varphi_o|^2$ and the coupling constant ε for a hexagonal lattice. The solitons have remarkable property of shape preserving, by virtue of which, they emerge unaltered after mutual interaction. From the phase plane, it may be seen that the topological charge shows the possibility of a limit cycle in case of focusing nonlinearity. The method of numerical continuation is useful in indicating the stability of discrete lattice solitons.

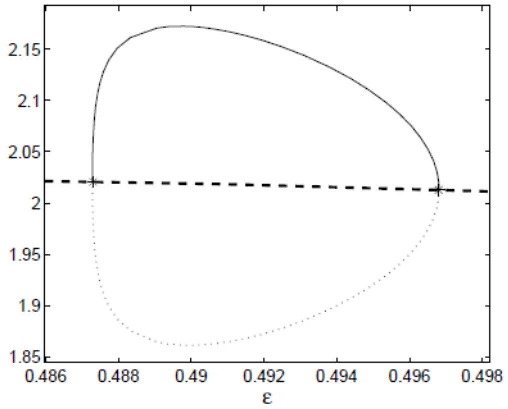

FIGURE 1. Closed loop phase portrait showing existence of Limit Cycle.

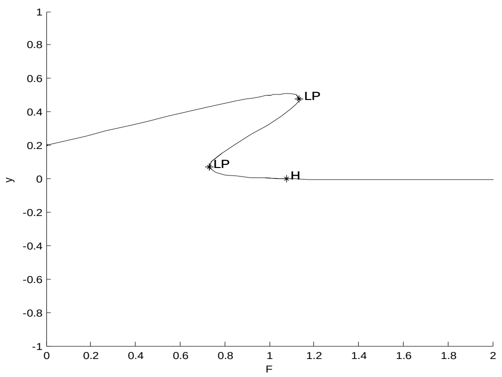

FIGURE 2. Hopf points connected by Limit cycle

Figure 2 illustrates one of the Limit cycles joining two Hopf points. This shows the possibility of stability of solitons as a result of competition between discrete lattice site nonlinearity and dispersive effects, showing a limiting scenario in which both the effects counterbalance each other giving rise to discrete self-trapped soliton structure. The Hopf point connected through Limit cycle give further information about formation of stable wave structure at the surface for ε=0.1 The results add to the findings of [13].

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