# Heuristic Method for Optimal Placement of Phasor Measurement Units (PMUs)

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Abstract—The phasor measurement units (PMUs) are very important tool for monitoring and control the power system. PMUs give real time, synchronized measurements of voltages at the buses and also current phasors which are incident to those buses where these PMUs are located. It is unnecessary and impossible to place PMU at each bus to estimate the states because high cost of PMUs and also the cost of communication facilities. It is necessary to find out the minimum number of PMUs to entire power system observable. The optimal placement of PMUs (OPP) problem solved by various techniques such as mathematical programming, metaheuristic techniques. The recently some heuristic optimization technique proposed to determine the minimum number of PMUs for various systems should be completely observable. This optimal PMUs placement (OPP) problem is pure binary optimization problem. A topological observability based three stages optimal PMU placement technique is proposed for solving this problem. For topological observability a set of minimum PMUs is required to make the system completely observable. It is assumed that there are strategic buses in every system using that the PMU placement becomes an easy task. The proposed method tested on standard IEEE bus systems and compared the results of the proposed method to the previously methods.

Keywords-Phasor measurement unit; Optimal PMUs Placement (OPP) problem; Heuristic Search Method; Optimal PMU Placement Technique.

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#### I. INTRODUCTION

The rapidly growth of the generation, transmission, power demand for electricity makes the power grid congested and more complex. It is necessary the continuous monitoring of the system operating condition to make the power system reliable. Traditionally it is done by state estimator which has access to the measurements information received from numerous substations in the monitoring system and resides in the control center computer. Until, recently used measurement technology does not measure the phase angle due to the technical difficulties related with the synchronization measurements at the remote locations. Now day's the phasor measurement units (PMUs) which are synchronized with Global positioning satellite (GPS) signal used to solve these difficulties. Conventional measurement systems replaced by synchronized measurement technology (SMT) to online monitoring, protection and control the power network [1], [2]. Phasor measurement unit (PMU) is most accurate and advanced measurement device of SMT which synchronized via signals from global positioning system (GPS) satellite. The PMUs can provide the real time, synchronized measurements of voltage phasors values at the buses where it is placed as well as current phasors of adjacent power branches which is connected to those buses. It is not needed to place these PMUs on all of the buses for complete observability of the system. It is also impossible to install these units on all of the system buses because of their high cost and the cost of communication services, which may be higher than that of the PMUs. Thus to determine the minimum number of PMUs and its location for complete observability of the system is very important. This thesis has devoted the techniques that ensure monitoring and controlling of the power system under the any contingency.

## II. PROPOSED METHOD TO OPTIMAL PLACEMENT OF PMUS

The main objective is to place the minimum number of PMUs on the strategic buses that completely observe the power network. This observability is definedby equation,

$$F(X) = AX \ge \widehat{1} \tag{1}$$

and strategic bus locations can be obtained from binary connectivity matrix [A]which gives the connectivity information of the network.

B.K.Saha Roy suggested proposed method in [9].It consists the three stages. Initially consider PMUs on all the buses. If zero injection bus is used as pseudo measurements, then the PMUs are not considered on those buses. PMUs are eliminated one by one from less valuable bus locations and placed on the most important bus locations.

The definitions of some terms used in the proposed method as follows.

*Valency*: Valency of the bus is defined as the total number of buses connected to that bus which bus valency should be calculated (Include that bus also).

 ${\it Minimum\ Valency\ (Vmin): Minimum\ valency\ of\ the\ bus.}$ 

Maximum Valency (Vmax): Maximum valency of the bus.

Radial Bus (RB): The bus which is connected to the only one bus to the network means the valency of the bus is minimum (i.e. 2) that bus is radial bus.

Equal Valency (EV) Buses: The set of buses which consists equal or same valency that stored in EV.

Allocated PMU Bus (APB): PMUs are located at important bus location while eliminating it from different bus locations, those locations of PMUs are defined as Allocated PMU Bus (APB).

SB: The set of EV buses which are connected to APB stored in SB.

Greater Valency (GV) bus: The set of buses which has valency greater than the EV.

Higher Valency (HV) bus: The group of EV buses (At least two EV buses) connected to the GV buses. Those set of GV buses stored in HV.

Observable Buses (OBs): Buses which are connected to any APBs.

Candidate bus (CB): CB consist candidate's bus locations from where PMUs are to be eliminated. At starting CB consists all buses except the ZIBs.

Zero Injection Bus (ZIB): The buses which not have any generation and load those buses are the ZIB. At ZIB no current is injected into the system. This is used as pseudo information to make system observable with less number of PMUs compared to the case when information of ZIBs is not considered.

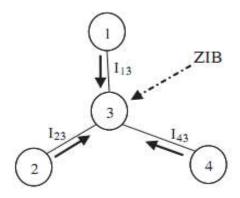


Figure 1. Modelling of ZIB

#### Modeling of ZIB:

Let us consider a typical 4-bus example as in Fig.5.1 for ZIB modeling. In the 4-bus example, buses are numbered as 1, 2, 3 and 4. Bus 3 is a ZIB. Application of KCL at bus 3 provides

$$I_{13} + I_{23} + I_{43} = 0$$
 (4.4)

Among the four buses if any three bus voltages (V) are known the 4th bus voltage can be calculated using (8). When 'K' number of connected ZIBs forms a super node with all known adjacent bus voltages, the ZIBs can be solved using (9). M is the number of bus connected to ith bus and Y\_ij is the transfer admittance between ith bus and jth bus.

$$\sum_{j=1}^{M} Y_{ij} V_j = 0$$
 for i=1,....,K (4.5).

### III. HEURISTIC USED IN THE PROPOSED METHOD

For observability optimally located minimum number of PMU is required. In the results of the exhaustive approach and other exiting methods, it is found that most of the bus locations are common in different optimal solution for any particular power system. These are the strategically important bus locations for that system. In every power system there is a pattern of strategically important bus locations. Once this pattern is identified, the PMU placement becomes an easy job. In the proposed approach the pattern is being investigated and obtained using the following heuristics.

The three stages of the proposed algorithm are listed below. *Stage-I:* In that different valency of the buses finds out and from that PMUs retained on the important bus locations.

Stage-II: Some of the lower valency buses have higher connectivity to OBs and also buses uncheck in stage-I that buses considered in stage-II and ordered properly. The buses connected to ZIBs are considered firstly. PMUs are eliminated or retained one by one from these buses subject to satisfying system observability. On testing all lower valency buses, untested higher valency buses are tested up to maximum valency.

Stage-III: This is a pruning stage. Find a non PMU bus which is connected to more than one PMU bus. Pruning checks whether placement of one PMU at the non PMU bus can make two or more PMU buses redundant in satisfying observability constraints. Pruning checks the possible ways for further reduction of PMUs from the set of PMU bus locations obtained in stage-I and stage-II.

PMU elimination starts from minimum valency buses i.e. from the radial buses. To observe a radial bus PMU is essential at radial buses or the bus to which a radial bus is connected. If PMU is placed at radial bus it makes two buses observable but if PMU is placed at bus where radial bus is connected it makes more than two buses observable. Hence PMUs are eliminated from radial buses and retained at buses where radial buses are connected. PMU elimination process continues for the higher valency buses up to maximum valency. In the process of elimination PMUs are retained at important bus locations such as (1) a higher valency buse connected to more than one running valency buses of SV, and (2) a bus from where elimination of PMU leads to the system unobservability. The tested buses (PMU eliminated or retained) are removed from the CB i.e. update CB at each iteration.

Elimination and retention of PMU obey the following rule.

**Rule-1:** Eliminate PMU from SB buses one by one subject to satisfaction of (4.2).

Rule-2: Retain PMU at any highest valency bus among HV buses

A. Procedure to find out optimal location of PMUs Stage-I

Step-1. Form a binary bus connectivity matrix (A) from the network data.

Step-2. Find out valency of the all candidate buses (CB). Also find minimum valency and maximum valency bus. Set V=Vmin

*Step-3*. Consider the PMUs in all the buses except ZIBs when ZIBs are considered as pseudo measurement.

Step-4. Find out isolated or radial buses (RB) for the system. Also find the bus connected to the radial bus (BCRB).Retain PMU at BCRB and eliminated from RB. Retained PMU buses stored in APB. Remove tested buses from CB.

Step-5. Find EV buses of valency V=V+1 from CB and also find SB buses from EV buses.

*Step-6*. Eliminate PMUs as per rule 1.If observability violate at the bus retain PMU to that bus. Update EV, APB and CB. Calculate n: n=number of buses in SV.

Step-7. Check n=0 or not. If n is not equal to zero, Find GV from CB and also find HV from GV. Calculate nh; nh=number of buses in HV.

*Step-8*.Check nh=0 or not. If nh is not equal to zero, Apply rule 2.Update APB, CB. Reset EV and HV.

Step-9. After step 9 or if n and nh is equal to zero check PMU retained condition. If PMU retained set V=2 otherwise increment V upto Vmax. If V=Vmax turn to stage-II.

#### Stage-II

*Step-1.* Set V=2, Find EV buses of valency V=V+1 from updated CB excluding ZIBs. Find OBs buses.

*Step-2.* Order the EV buses connected to maximum OBs buses and connected to the ZIBs 1st followed by the rest.

*Step-3.* Eliminate one PMU from ordered EV buses. Retain PMU when system observability violated. Update EV,CB, and APB.

Step-4. Check EV nil or not, if EV nil increase V upto Vmax at each iteration otherwise go to step-3.If V=Vmax ,go to step-1 i.e. find EV buses of valency V=V+1 otherwise check pruning condition.

### Stage-III

Do pruning, check system observability if yes again check pruning complete or not otherwise don't prune. if pruning completed optimal solution obtained so stop otherwise do pruning again.

Flow charts of three stages of the proposed algorithm are detailed in Figs. 5.2–5.4

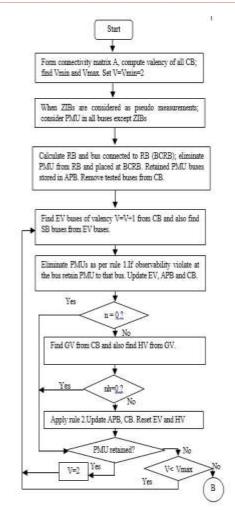


Figure 2. Flow chart of Stage I.

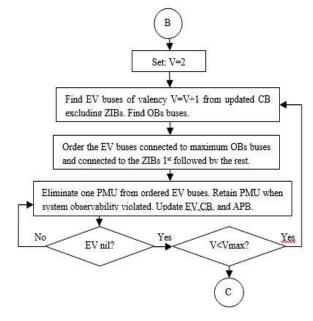


Figure 3. Flow chart of Stage-II.

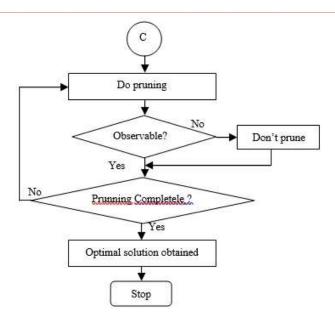


Figure 4. Flow chart of Stage-III.

## B. Test cases for PMU placement problem

IEEE 14-bus system is shown in Figure 5.1. The Information of the system, Radial Buses zero injections are given in the Table 5.1.

TABLE I. SYSTEM INFORMATION OF IEEE 14 BUS SYSTEM

System	Number of	Number of	Zero Injection	Radial
	branches	Zero Injection Buses	Bus	bus
IEEE 14 Bus	20	1	Bus-7	Bus-8
D6 <b>=</b>	812	810	B3	88

Figure 5. IEEE 14 bus system

Simulations are carried out on the IEEE 14-bus, 24-bus, 30-bus, New England 39-bus, IEEE 57-bus and 118-bus test system in MATLAB to validate the effectiveness of the proposed method. The simulation results of the test systems are explained in Table 2. In Table 3, a comparison between the results presented in Table 2 and other studies earlier done is provided. As shown here, the number of PMUs obtained using the proposed method required for full observability of the IEEE-14,24, 30, 39, 57 and 118-bus systems are providing

minimum number of PMUs for complete observability of the system.

TABLE II. SIMULATION RESULTS FOR PMU PLACEMENT PROBLEM

Test Systems	Number of PMUs	Locations of PMUs	
IEEE 14-Bus	3	2,6,9	
IEEE 24-Bus	6	1,2,8,16,21,23	
IEEE 30-Bus	7	2,3,10,12,18,24,30	
New England 39-Bus	8	3,8,12,16,20,23,25,29	
IEEE 57-Bus	11	1,6,13,19,25,29,32,38,51,54,56	
IEEE 118- Bus	28	1,6,8,12,15,17,21,25,29,34,40,45,49, 53,56,62,72,75,77,80,85, 86,90,94,101,105,110,114	

TABLE III. COMPARISON OF THE NUMBER OF PMUS LISTED IN TABLE 5.3 WITH THE NUMBERS PRESENTED IN PRIOR STUDIES.

IEEE System	14-Bus	24-Bus	30-Bus	39-Bus	57-Bus	118- Bus
Propose d	3	6	7	8	11	28
Ref. [3]	3	-	7	-	11	-
Ref. [4]	3		7	-	11	28
Ref. [5]	3	-	-	10	13	-
Ref. [6]	3		8	-	-	-
Ref. [7]	3	6	7	8	-	-
Ref. [8]	3	-	7	-	13	29
Ref. [10]	3	-	8	-	12	29
Ref. [11]	3	-	-	-	14	29

# IV. CONCLUSIONS

The optimal placement problem of Phasor Measurement Unit (PMU) is solved by method called as Heuristic search method. A code is developed in MATLAB for Heuristic search method. This code is tested for several IEEE test cases like IEEE 14-bus, 24-bus, 30-bus, New England 39-bus, IEEE 57-bus and 118-bus system. Finally, results are compared with the other previously method.

The new iterative method makes the test systems topologically observable by placing a set of minimum PMUs. The three stage algorithm is simple, fast and easy to implement. The present method obtains optimal solution using simple network connectivity information. The overall optimal solution obtained is sufficient to take care of system observability under normal operating condition. Simulation results for different networks show the effectiveness of the proposed method in obtaining the minimum number of PMUs required for complete observability of power systems.

Future scope is solving problem of PMU placement for making system observable with not only PMU data but also including other traditional data like flow measurements and injection measurements. Also taking account the one or two PMU loss while solving the PMU placement problem.

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