International Journal on Recent and Innovation Trends in Computing and Communication ISSN: 2321-8169
Volume: 4 Issue: 11 295 – 298

Fundamentals and Literature Review of Discrete Fourier Transform in Digital Signal Processing

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Abstract: Today's world is digital world. While designing and analyzing the digital phenomena the most desired factor are high performance, time and cost(economy).DFT and FFT are the most efficient mathematical technique to meet these challenges .To compute DFT using FFT is genius method ,in this method FFT decomposes DFT with N sample points, into N DFT each with single point. This paper focus on DFT, FFT and its approach to Digital signal processing.

Keywords: *Discrete Fourier Transform (DFT), Fast Fourier Transform (FFT), windowing, MATLAB.*

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1. INTRODUCTION:

Now a days science and technology introduce many DFT/FFT algorithms to compute DFT. DFT and FFT are techniques which can convert time domain sampled data into its equivalent frequency domain sampled data and vice-versa. In digital world, frequency analysis is the most common and convenient method. Here signals are sampled in time domain, so we have DFT [1] [2]. Thus the signal when converted into frequency domain will have various components in frequency and can be used to remove certain undesired frequency components.

FFT is the genius method ie most effective method to compute DFT and IDFT. Computing the N-points DFT takes O (N^2) arithmetic operations, whereas FFT can takes only O ($N\log_2 N$) operations.[2]. Actually FFT is an algorithm, which implement DFT. L.G.Johnson (1992) presented conflict free memory addressing for dedicated FFT hardware. He and Torkelson (1998) proposed a FFT-Algorithm and application, it covers, FFTs, frequency domain representation, filtering, and applications to video and audio signal processing. Its provides a good reference for any engineer planning to work in this field.

DFT plays very important role in design of digital filters, calculating Impulse response from frequency response and vice- versa.DFT and FFT are most popular efficient and important of all well-known transforms because they provides the most adequate solution in frequency domain. DFT/FFT are widely used in linear filtering, spectral analysis, digital communication, remote sensing, image/signal processing, wireless communication etc. In this paper the analysis of DFT using FFT and its approach to digital signal processing is discussed.

2. TWIDDLE FACTOR, DFT AND ITS MATRIX REPRESENTATION:

The need for computing the DFT has become increasingly important over the years, partly because the cost of computers has been steadily declining and partly because the difficulty and sophistication of our measurement has been steadily increasing. The continuous Fourier Transform is very useful in theoretical work, but not suited for computation with digital computers techniques of sampled data.

The N-point DFT of a given sequence is expressed as
$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N}$, $k = 0,1,2, \dots \dots .. N-1$ (1)and the corresponding IDFT is expressed as

$$x(n) = \frac{1}{N}\sum_{n=0}^{N-1} X(k)\, e^{j2\pi nk/N},$$
$$n = 0,1,2, \dots \dots . N-1 \quad (2)$$

Here $x(n)$ represent the time domain as n represents the discrete time domain index and X(k) represent the frequency domain components where k is the normalized frequency domain index.

2.1 TWIDDLE FACTOR AND ITS PROPERTIES:

It is defined as $W \ or \ W_N = e^{-j2\pi/N}$ .... ….(3)

Thus $e^{-j2\pi nk/N} = \left(e^{-j2\pi/N}\right)^{nk} = {W_N}^{nk}$. …...(4)

Remember to think of W_N or W as a single complex exponential of frequency $\frac{2\pi}{N}$. Hence ${W_N}^{nk}$ is a matrix of several exponentials with both $n \ and \ k$ ranging $0 \ to \ N-1$ [1].

Therefore the DFT and IDFT can be represented in terms of Twiddle factor as

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W^{nk}, \qquad k = 0,1,2, \dots \dots N-1 (5)$$

$$x(n) = \frac{1}{N}\sum_{n=0}^{N-1} X(k)\,\mathrm{W}^{-\mathrm{nk}}, n = 0,1,..N-1 \ (6)$$

If we represent $x(n)$ as a vector of N –samples as

$$x(n) = \begin{bmatrix} x(0) \\ X(1) \\ . \\ . \\ x(N-1) \end{bmatrix}$$

And X(k) is expressed as a vector as X(k) of N –samples as

$$\mathrm{X(k)} = \begin{bmatrix} \mathrm{X(0)} \\ \mathrm{X(1)} \\ . \\ . \\ \mathrm{X(N-1)} \end{bmatrix}$$

The matrix ${W_N}^{nk}$ is of $N \times N$ size as

$${W_N}^{nk} = \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & . & . & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & . & . & W_N^N \\ W_N^0 & W_N^2 & W_N^4 & . & . & W_N^{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ W_N^0 & W_N^N & W_N^{2N} & . & . & W_N^{2(N-1)} \end{bmatrix} \quad ...(7)$$

Thus $\mathrm{X(k)} = [\mathrm{W_N}]\,\mathrm{x(n)}$ and

$$x(n) = \frac{1}{N}[W_N^*]\,X(k) \ldots\ldots\ldots\ldots\ldots\ldots\ldots. (8)$$

The Twiddle factors are simply the sine and cosine basis functions written in the polar form.

2.2 APPLYING THE PROPERTIES OF SYMMETRY AND PERIODICITY TO W_N^{nk} for N=8:

1. Symmetric Property: $W_N^{k+\frac{N}{2}} = -W_N^k$
2. Periodicity Property: $W_N^{k+N} = W_N^k$

$$W_8^4 = W_8^{0+4} = -W_8^0 = -1$$
$$W_8^5 = W_8^{1+4} = -W_8^1$$
$$W_8^6 = W_8^{2+4} = -W_8^2$$
$$W_8^7 = W_8^{3+4} = -W_8^3$$
$$W_8^8 = W_8^{0+8} = W_8^0 = 1$$
$$W_8^9 = W_8^{1+8} = W_8^1$$
$$W_8^{10} = W_8^{2+8} = W_8^2$$
$$W_8^{11} = W_8^{3+8} = W_8^3$$

.. .. ..
.. .. ..

Fig.1 Properties of Twiddle factor

DFT is operates on a sampled periodic time domain signal. The signal must be periodic in order to be decomposed into the summation of the sinusoids [3] Finite number of samples (N) are available for inputting into DFT. This concept is overcome by placing an infinite numbers of groups of the same (N) samples "end-to-end", thereby forcing mathematical periodicity .Equation (5) can be written

$$X(k) = \frac{1}{N}\sum_{n=0}^{\infty}\left[\cos\left(\frac{2n\pi k}{N}\right) - \sin\left(\frac{2n\pi k}{N}\right)\right] \ldots\ldots..(9)$$

Where values of k and n are varies from0 to N-1.

3. FAST FOURIER TRANSFORM (FFT):

3.1 FFT is simply an algorithm for efficient computation of DFT. In 1948, Cooley and Tukey came up with computational breakthrough called FFT algorithm .It allowed the computation of N –DFT as a function only 2N instead of N^2, so a 256 point DFT would only requires 512 calculations a huge improvement from 65792 calculations doing it the laborious way. The algorithm was quickly and widely adopted and is the basis of all modern signal processing. In MATLAB you can do FFT of any size. The main thing one needs to know about FFT is that it works only with samples numbers that are powers of 2 such as 2,4,8,16,32,…….etc. The FFT is the DFT with constraints on the number of input samples. The other thing about FFT process to know is that it allows zero padding. Let's say we have 28 samples and we wish to do the DFT via FFT then we can insert four zeros at the end so we have 32 points (2^5) . [4].The zero padding provides us better resolution but does not provide extra information. The frequency detected is still a function of the original N-samples and not the zero padded length, although FFT does look a lot better and looks do not count.

To illustrate the saving of FFT, consider a count of complex multiplication and addition .Evaluating the DFT sum directly involves N^2 complex multiplication and N(N-1) complex additions. The well known radix -2 Cooley Tukey algorithm for N power of 2can compute the same result with only $\frac{N}{2}$ $(\log_2 N)$ complex multiplications and $N \log_2 N$ complex addition. FFT makes the use of symmetry and periodicity properties of twiddle factor W_N^{nk} to effectively reduce DFT computation time. [5].

Comparison between the calculation of DFT and FFT:

Table 1: comparison of efficiency between DFT and FFT

Sr. No.	N	Computation		FFT Efficiency
		DFT N^2	FFT $\frac{N}{2}(\log_2 N)$	
1	8	64	12	5.34:1
2	16	256	32	8:1
3	256	65336	1024	61:1
4	512	262144	2304	114:1
5	1024	16777216	5120	205:1

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3.2PROPERTIES OF DFT OR FFT:

Table 2: Properties of DFT

Sr. No.	Property	Time domain	Frequency domain
(1)	Periodicity	$x[n]=x[n+N]$	$X[k]=X[k+n]$
(2)	Linearity	$a_1x_1[n]+a_2x_2[n]$	$a_1X_1[k]+a_2X_2[k]$
(3)	Time Reversal	$x[N-n]$	$X[N-k]$
(4)	Circular time shift	$x[(n-l)]_N$	$X[k]e^{-j2\pi kl/N}$
(5)	Circular frequency shift	$x[n]e^{j\pi 2l\,n/N}$	$X((k-l))_N$
(6)	Circular convolution	$x_1[n] \circledast x_2[n]$	$X_1[k]X_2[k]$
(7)	Circular correlation	$x[n] \circledast y^*[-n]$	$X[k]Y^*[k]$
(8)	Multiplication of two sequences	$x_1[n]x_2[n]$	$\frac{1}{N}X_1[k] \circledast X_2[k]$
(9)	Complex conjugate	$x^*[n]$	$X^*[N-k]$
(10)	Parseval's theorem	$\sum_{n=0}^{N-1} x[n]y^*[n]$	$\frac{1}{N}\sum_{n=0}^{N-1} X[k]Y^*[k]$

3.3 CALCULATION OF IDFT USING FFT ALGORITHM:

The inverse DFT of an N-point signal
$X(K)$, k=0, 1, 2, 3………N-1 is defined as

$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)W_N^{-nk}$ ……… ……….(10)

If we take complex conjugate and multiplication by N, we get

$N\,x^*(n) = \sum_{k=0}^{N-1} X^*(k)W_N^{nk}$ ……………….. (11)

The RHS of equation (11) is the DFT of signal $X^*(k)$ and may be computed using any FFT algorithm.The desired output sequence / signal $x(n)$ can be obtained by taking complex conjugate of the DFT of the equation (11) and dividing by N.

Therefore $x(n) = \frac{1}{N}[\sum_{k=0}^{N-1} X(k)W_N^{-nk}]^*$… (12) Or in short $F^{-1}(x) = F(ix^*)^*$

In other words, we can calculate the IDFT directly from the FFT. Thus the IDFT algorithmsare essentially the same as FFT algorithm, all one must do its flips the numbers around at the beginning of the calculation.

Since the IFFT inherits all of the speed benefits of the FFT, it also quite practical to use it in real time in the laboratory, such as DSP. In general the idea of DSP is to use configurable digital electronics to clean up, transform, or amplify a signal by first taking FFT of the signal, removing, shifting or damping the unwanted frequency components, and transforming the signal back using IDFT on the filtered signal.

4. WINDOW BASED ANALYSIS:

4.1 Mathematical Function:It is the mathematical function that has zero value outside of some chosen interval. This technique is used to shape the time portion of the measurement data and to minimize edge effect that may result in spectral leakage otherwise in FFT spectrum. The window function when used correctly increases the spectrum resolution of the frequency domain result.[6]. For FFT both time domain and the frequency domain appears to be circular topologies, so to the end points of the time wave form are interpreted as though were connected together .[6]. The spectral leakage effect can be minimized by applying the window technique to the measured signal in the domain. Windowing reduces the amplitude of the discontinuities at the boundaries of each finite signals acquired by the digitizer [7], However appropriate window function must be applied for a specific application.

The various windows that are widely used are rectangular window, Triangular window, Hanning window, Hamming window and Blackman window.

4.2VARIOUS WINDOW FUNCTIONS AND THEIR SHAPES:

The following table gives the idea of different window functions

Table 3: Various window functions.

Sr no	Name of window function	Time domain signal $w(n)$,$0 \leq n \leq M-1$
1.	Rectangular	1
2.	Triangular (Barlelt)	$1-\frac{2\left\|n-\frac{M-1}{2}\right\|}{M-1}$
3.	Hamming	$0.54\text{-}0.46\frac{2\pi n}{M-1}$
4.	Hanning	$\frac{1}{2}\left[1-\cos\frac{2\pi n}{M-1}\right]$
5	Blackman	$0.42-0.5\cos\left(\frac{2\pi n}{N}\right)+0.08\cos\left(\frac{4\pi n}{N}\right)$

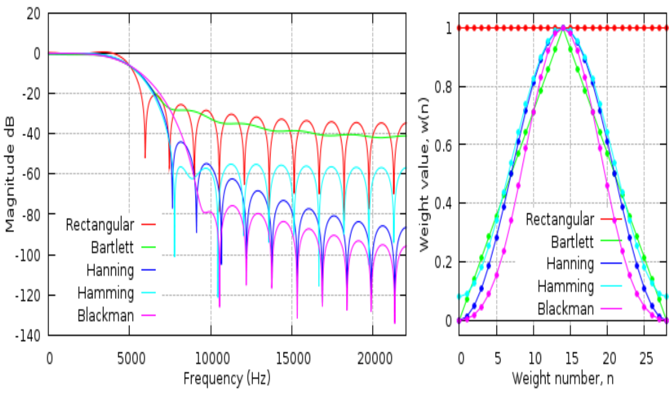

Fig.2Shape of window function in both Time and Frequency domain

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4.3 FREQUENCY DOMAIN CHARACTERISTICS OF WINDOW FUNCTIONS

Table 3: Characteristics of window functions:-

Sr. no.	Window type	Normalized transient width of main lobe	Mins top-band attenuation (dB)	Pass-band ripple (dB)	Peak amplifier of side lobe (dB)
1	Rectangular	0.9/M	-21	0,741	-13
2	Triangular	3/M	-25	0.1428	-25
3	Hanning	3.1/M	-44	0.0546	-31
4	Hamming	3.3/M	-53	0.0194	-41
5	Blackman	5.5/M	-74	0.0017	-57

The desirable characteristics of the window function are –

a) The central lobe of the frequency response of window should be contain most of the energy and should be narrow.

b) The highest side level of the frequency response should be small.

c) The side lobes of the frequency response should be decrease in energy rapidly as w tents to π [8].

5. CONCLUSION:

The DFT and FFT is useful many applications ranging from experimental instruments to rigorous mathematical analysis techniques. Thanks to modern development in digital electronics, coupled with numerical algorithms such as FFT, IDFT, windowing etc. The focus of this paper is on DFT and the efficient computation of the DFT .We demonstrated that by taking advantage of the symmetric property and periodicity property of the twiddle factor W_N^{nk} , we can reduce the number of complex multiplication and addition. In addition to the FFT algorithm described in this paper there are other efficient algorithms for computing DFT. Also various window techniques are described. Each window has its own unique advantages and disadvantages. According to the kind of signal being analyzed a specific window function should be selected. MATLAB has been used to simulate the different window techniques. There are numerous areas in which a greater degree of development in future can be expected. The numerical solution of differential equations, multiple time series analysis, filtering and image processing are some of the problems.

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