International Journal on Recent and Innovation Trends in Computing and Communication ISSN: 2321-8169
Volume: 4 Issue: 8 160 - 163

Convolution Structure of Fractional Quaternion Mellin Transform

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Abstract— The Mellin transform is a basic tool for analyzing the behavior of many important functions in mathematics and mathematical physics. A quaternion is a 4 dimensional number, a generalization of a complex number. Basically it used in image registration.

This paper is focus on the new convolution structure of Fractional Quaternion Mellin transform. Also the main properties i.e. associative, distributive, linear, shifting for the convolution of Fractional Quaternion Mellin Transform are proved.

Keywords- *Mellin transform, Fractional Mellin transform, quaternion fractional Mellin transform, quaternion..*

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I. INTRODUCTION

Mellin transform is a natural analytical tool to study the distribution of product and quotients of independent random variables. Mellin transforms has application to derive different properties in statistics and probability densities of single continuous random variable and also used in deriving densities for algebraic combination of random variable, it is also applied to fractional differential equations with a right-sided derivative and variable potential . Mellin transform also use to establish the means, variances, skewness, and kurtosis of fuzzy numbers and applied them to the random coefficient autoregressive (RCA) time series models [2,3,4].

Convolution is a powerful way of characterizing the input-output relationship of time invariant linear system.

Quaternion

A quaternion is a four-element vector that can be used to encode any rotation in a 3D coordinate system. Technically, a quaternion is composed of one real element and three complex elements, and it can be used for much more than rotations.

On October 16th, 1843, while walking with his wife to a meeting of the Royal Society of Dublin, Hamilton discovered a 4-dimensional division algebra called the quaternions.

$i^2 = j^2 = k^2 = ijk = -1$

Here i, j, k represent 90^0 degree rotations about three mutually orthogonal axes. The other basic relationships:

ij = k = -ji
jk = i = -kj;
ki = j = -ik .

Quaternion play an vital role in animation field because it compose rotation very nicely and mainly it gives spherical interpolation.

The Quaternion Fourier Transform is well suited for describing the spectral content of colour images also can be applied to image registration, edge detection, and data compression [5].

In our previous work we have proved the operational calculus, operation transform formulae, applications and convolution theorem for two dimensional fractional Mellin transform [1,7,8,9]. In this work we have discussed the fractional Quaternion Mellin transform, also the new convolution structure for fractional Quaternion Mellin transform is presented. Moreover we have proved some basic properties of convolution.

II. DEFINITIONS

2.1 Definition of Two Dimensional Fractional Mellin Transform

The two-dimensional fractional Mellin transform with parameters θ of $f(u,v)$ denoted by 2DFRMT{ $f(u,v)$} performs a linear operation, given by the integral transform.

2DFRMT

$$2DFRMT\{f(u,v)\} = \int_0^\infty\int_0^\infty K_\theta(u,v,r,s)\,f(u,v)\,du\,dv$$

where,

$$K_\theta(u,v,r,s) = u^{\frac{2\pi ir}{\sin\theta}-1}\,v^{\frac{2\pi is}{\sin\theta}-1}\,e^{\frac{\pi i}{\tan\theta}(r^2+s^2+\log^2 u+\log^2 v)}$$

2.2 Definition of Fractional Quaternion Mellin Transform

For any two dimensional quaternion function $f(u,v)$ given by

$$f(u,v) = f_r(u,v) + i f_i(u,v) + j f_j(u,v) + k f_k(u,v)$$

where $f_r(u,v), f_i(u,v), f_j(u,v)$, $f_k(u,v)$ are real, the quaternion fractional Mellin transform of $f(u,v)$ is denoted by

$$FRM^{k,l}_{\theta_1,\theta_2}(r,s) = FRM^{k,l}_{\theta_1,\theta_2}\{f(u,v)\}$$

$$= \int_0^\infty\int_0^\infty K^k_{\theta_1}(u,r)\,f(u,v)\,K^l_{\theta_2}(v,s)\,du\,dv$$

where

$$K^k_{\theta_1}(u,r) = u^{\frac{2\pi ir}{\sin\varphi_1}-1}\,e^{\frac{\pi i}{\tan\varphi_1}(r^2+\log^2 u)}, \qquad \varphi_1 = \theta_1\frac{\pi}{2}$$

$$K^l_{\theta_2}(v,s) = v^{\frac{2\pi is}{\sin\varphi_2}-1}\,e^{\frac{\pi i}{\tan\varphi_2}(s^2+\log^2 v)}, \qquad \varphi_2 = \theta_2\frac{\pi}{2} \quad .$$

III. CONVOLUTION STRUCTURE OF FRACTIONAL QUATERNION MELLIN TRANSFORM

For any real scalar or complex signal $f(u,v)$ and convolution kernel $g(u,v)$ and

$$h(u,v) = (f * g)(u,v)$$
$$= \left(e^{\frac{-\pi i}{\tan\varphi_1}\log^2 u - \frac{-\pi i}{\tan\varphi_2}\log^2 v}\right)[f(u,v) * g(u,v)]$$

where * is the QFRMT convolution operator then

$$F_{\theta_1,\theta_2}\{h(u,v)\}(r,s) = e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}$$
$$\{F_{\theta_1,\theta_2}[f(t,q)]\}\{F_{\theta_1,\theta_2}[g(t,q)]\}$$

Proof-

$$F_{\theta_1,\theta_2}[h(u,v)](r,s)$$
$$= \int_0^\infty\int_0^\infty h(u,v) u^{\frac{2\pi i r}{\sin\varphi_1}-1} v^{\frac{2\pi i s}{\sin\varphi_2}-1} e^{\frac{\pi i}{\tan\varphi_1}[r^2+\log^2 u]}$$
$$e^{\frac{\pi i}{\tan\varphi_2}[s^2+\log^2 v]}dudv$$

From given we know that

$$= e^{\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\int_0^\infty\int_0^\infty u^{\frac{2\pi i r}{\sin\varphi_1}-1} v^{\frac{2\pi i s}{\sin\varphi_2}-1} e^{\frac{\pi i}{\tan\varphi_1}\log^2 u}$$
$$e^{\frac{\pi i}{\tan\varphi_2}\log^2 v}\ [\int_0^\infty\int_0^\infty \frac{1}{tq}\tilde{f}\ (t,q)\tilde{g}\ (\tfrac{u}{t},\tfrac{v}{q})dtdq]dudv$$

[from ref. [1]]

Putting $\frac{u}{t} = w,\quad \frac{v}{q} = z$

$\frac{u}{w} = t,\quad \frac{v}{z} = q$

Differentiate with respect to *w* and *z*

$\Rightarrow \frac{-u}{w^2}dw = dt,\quad \frac{-v}{z^2}dz = dq$

$$= e^{\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\int_0^\infty\int_0^\infty u^{\frac{2\pi i r}{\sin\varphi_1}-1} v^{\frac{2\pi i s}{\sin\varphi_2}-1}$$
$$\{\int_0^\infty\int_0^\infty \frac{1}{wz} f\left(\frac{u}{w},\frac{v}{z}\right) e^{\left[\frac{\pi i}{\tan\varphi_1}\log^2(u/w)+\frac{\pi i}{\tan\varphi_2}\log^2(v/z)\right]}$$
$$g(w,z)e^{\pi i\left[\frac{\log^2 w}{\tan\varphi_1}\right]}e^{\pi i\left[\frac{\log^2 2}{\tan\varphi_2}\right]}dwdz\}\ dudv$$

Putting $\frac{u}{w} = m,\quad \frac{v}{z} = n$

$u = mw,\quad v = nz$

$du = wdm,\quad dv = zdn$

$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\{\int_0^\infty\int_0^\infty (m)^{\frac{2\pi i r}{\sin\varphi_1}-1}$$
$$e^{\frac{\pi i}{\tan\varphi_1}[r^2+\log^2(m)]} f(m,n)(n)^{\frac{2\pi i s}{\sin\varphi_2}-1}$$
$$e^{\frac{\pi i}{\tan\varphi_2}[s^2+\log^2(n)]}dmdn\}$$
$$\{\int_0^\infty\int_0^\infty (w)^{\frac{2\pi i r}{\sin\varphi_1}-1} e^{\frac{\pi i}{\tan\varphi_1}[r^2+\log^2 w]} g(w,z)(z)^{\frac{2\pi i s}{\sin\varphi_2}-1}$$
$$e^{\frac{\pi i}{\tan\varphi_2}[s^2+\log^2 2]}dwdz\} \text{------------(3.1)}$$
$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\{[K_{\theta_1}^k(m,r)f(m,n)K_{\theta_2}^l(n,s)]$$
$$\{[K_{\theta_1}^k(w,r)f(w,z)K_{\theta_2}^l(z,s)]\}$$
$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\{F_{\theta_{1,\theta_2}},[f(m,n)]\}\{F_{\theta_{1,\theta_2}},[g(w,z)]\}\text{--}$$

(3.2)

$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\{F_{\theta_{1,\theta_2}},[f(t,q)]\}\{F_{\theta_{1,\theta_2}},[g(t,q)]\}$$

(by using change of variable property).

IV. SOME BASIC PROPERTIES OF QUATERNION CONVOLUTION

4.1 Linearity property

Prove that

(i) $(A_1 f + A_2 g) * h = A_1(f * h) + A_2(g * h)$

(ii) $h * (A_1 f + A_2 g) = A_1(h * f) + A_2(h * g)$

where $A_1, A_2 \epsilon H$

Proof-

(i) Consider

$LHS = (A_1 f + A_2 g) * h$

$= \beta * h \qquad (\because \beta = (A_1 f + A_2 g))$

By using equation (3.1) and (3.2) we get

$LHS = \ \beta * h$

$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}$$
$$F_{\theta_1,\theta_2}\{\beta(m,n)\}F_{\theta_1,\theta_2}\{h(w,z)\}$$
$$e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\int_{-\infty}^\infty\int_{-\infty}^\infty\{\beta(m,n)\} K_{\theta_1,\theta_2}(m,n,r,s)dmdn$$
$$\int_{-\infty}^\infty\int_{-\infty}^\infty\{h(w,z)\} K_{\theta_1,\theta_2}(w,z,r,s)dwdz$$
$$e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\int_{-\infty}^\infty\int_{-\infty}^\infty\{[A_1 f(m,n) +$$

A2gm,n]}Kθ1,θ2m,n,r,sdmdn

$$\int_{-\infty}^\infty\int_{-\infty}^\infty\{h(w,z)\} K_{\theta_1,\theta_2}(w,z,r,s)dwdz\text{------}$$

(1)

Now we consider

$RHS = A_1(f * h) + A_2(g * h)$

By using (A)

$$= e^{-\pi i\left[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}\right]}[A_1\{F_{\theta_1,\theta_2}(f * h)\} + A_2\{F_{\theta_1,\theta_2}(g * h)\}$$

$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\{A_1[\int_{-\infty}^\infty\int_{-\infty}^\infty f(m,n)\ K_{\theta_1,\theta_2}(m,n,r,s)$$
$$dmdn\ \int_{-\infty}^\infty\int_{-\infty}^\infty h(w,z)\ K_{\theta_1,\theta_2}(w,z,r,s)dwdz]$$
$$A_2[\int_{-\infty}^\infty\int_{-\infty}^\infty g(m,n)\ K_{\theta_1,\theta_2}(m,n,r,s)dmdn$$
$$\int_{-\infty}^\infty\int_{-\infty}^\infty h(w,z)\ K_{\theta_1,\theta_2}(w,z,r,s)dwdz]\}$$

By using change of variable

$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\{[\int_{-\infty}^\infty\int_{-\infty}^\infty h(w,z)\ K_{\theta_1,\theta_2}(w,z,r,s)dwdz]$$
$$[A_1\int_{-\infty}^\infty\int_{-\infty}^\infty f(m,n)\ K_{\theta_1,\theta_2}(m,n,r,s)dmdn$$
$$+A_2\int_{-\infty}^\infty\int_{-\infty}^\infty g(m,n)\ K_{\theta_1,\theta_2}(m,n,r,s)dmdn]$$
$$= e^{-\pi i\left[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}\right]}\{\int_{-\infty}^\infty\int_{-\infty}^\infty h(w,z)\ K_{\theta_1,\theta_2}(w,z,r,s)dwdz$$
$$\{\int_{-\infty}^\infty\int_{-\infty}^\infty[A_1 f(m,n) + A_2\ g(m,n)]$$
$$K_{\theta_1,\theta_2}(m,n,r,s)dmdn\}\text{---(2)}$$

From (1) and (2) result (i) is proved.

Similarly we can proved result (ii)

$h * (A_1 f + A_2 g) = A_1(h * f) + A_2(h * g).$

4.2. Shifting Property

Prove that (i) $(\alpha f * g) = \alpha(f * g)$

(ii) $(f * \alpha g) = \alpha(f * g)$

Proof-

Consider,

$LHS = (\alpha f * g)$

By using equation (3.1) and (3.2) we get

$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}F_{\theta_1,\theta_2}\{\alpha f(m,n)\}F_{\theta_1,\theta_2}\{g(w,z)\}$$

$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\alpha\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{f(m,n)\} K_{\theta_1,\theta_2}(m,n,r,s)dmdn\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{g(w,z)\} K_{\theta_1,\theta_2}(w,z,r,s)dwdz$$

$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\alpha F_{\theta_1,\theta_2}\{f(m,n)\}F_{\theta_1,\theta_2}\{g(w,z)\}$$

$$= \alpha(f*g)$$

Similarly we can proved result (ii)

$$(f*\alpha g) = \alpha(f*g)$$

4.3 Distributive Property

Prove that $f*(g+h) = (f*g)+(f*h)$

Proof-

Consider,

$$LHS = f*(g+h)$$

$$= f*\omega \qquad (\because \varphi = g+h)$$

By using equation (3.1) and (3.2) we get

$$LHS = f*\omega$$

$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}F_{\theta_1,\theta_2}\{f(m,n)\}F_{\theta_1,\theta_2}\{\varphi(w,z)\}$$

$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{f(m,n)\} K_{\theta_1,\theta_2}(m,n,r,s)dmdn\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{\varphi(w,z)\} K_{\theta_1,\theta_2}(w,z,r,s)dwdz$$

$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{f(m,n)\} K_{\theta_1,\theta_2}(m,n,r,s)dmdn\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{g(w,z)\} K_{\theta_1,\theta_2}(w,z,r,s)dwdz + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{h(w,z)\} K_{\theta_1,\theta_2}(w,z,r,s)dwdz\}$$

$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{f(m,n)\} K_{\theta_1,\theta_2}(m,n,r,s)dmdn\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{g(w,z)\} K_{\theta_1,\theta_2}(w,z,r,s)dwdz\} +\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{f(m,n)\} K_{\theta_1,\theta_2}(m,n,r,s)dmdn\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(w,z)\} K_{\theta_1,\theta_2}(w,z,r,s)dwdz\}$$

$$= (f*g)+(f*h)$$

3.4 Associative Property

Prove that-

$$(f*g)*h = f*(g*h)$$

Proof-

Consider

$$LHS = (f*g)*h$$

$$= \delta*h \qquad (\because \delta = f*g)$$

By using equation (3.1) and (3.2) we get

$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}F_{\theta_1,\theta_2}\{\delta(m,n)\}F_{\theta_1,\theta_2}\{h(w,z)\}$$

$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{\delta(m,n)\} K_{\theta_1,\theta_2}(m,n,r,s)dmdn\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{h(w,z)\} K_{\theta_1,\theta_2}(w,z,r,s)dwdz$$

$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{(f*g)(m,n)\} K_{\theta_1,\theta_2}(m,n,r,s)dmdn\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{h(w,z)\} K_{\theta_1,\theta_2}(w,z,r,s)dwdz$$

$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}(F_{\theta_1,\theta_2}f(m,n))(F_{\theta_1,\theta_2}g(m,n))K_{\theta_1,\theta_2}(m,n,r,s)dmdn]\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{h(w,z)\} K_{\theta_1,\theta_2}(w,z,r,s)dwdz$$

$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]} e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(m,n)K_{\theta_1,\theta_2}(m,n,r,s)dmdn\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(w,z))K_{\theta_1,\theta_2}(w,z,r,s)dwdz] K_{\theta_1,\theta_2}(m,n,r,s)dmdn\}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(w,z)\} K_{\theta_1,\theta_2}(w,z,r,s)dwdz$$

$$= e^{-\pi i\left[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}\right]} e^{-\pi i\left[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}\right]}\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(m,n)K_{\theta_1,\theta_2}(m,n,r,s)dmdn] K_{\theta_1,\theta_2}(m,n,r,s)dmdn\}[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(w,z))K_{\theta_1,\theta_2}(w,z,r,s)dwdz\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(w,z)\} K_{\theta_1,\theta_2}(w,z,r,s)dwdz]$$

$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]} e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(m,n)K_{\theta_1,\theta_2}(m,n,r,s)dmdn][\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(w,z))K_{\theta_1,\theta_2}(w,z,r,s)dwdz\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(w,z)\} K_{\theta_1,\theta_2}(w,z,r,s)dwdz] K_{\theta_1,\theta_2}(m,n,r,s)dmdn\}$$

$$= f*(g*h)$$

4.5 Conjugation Property

Prove that-$\overline{(f*g)} = \bar{g}*\bar{f}$

Proof-

By using equation (3.1) and (3.2) we get

$$f*g = e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}F_{\theta_1,\theta_2}\{f(m,n)\}F_{\theta_1,\theta_2}\{g(t,z)\}$$

$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{f(m,n)\} K_{\theta_1,\theta_2}(m,n,r,s)dmdn\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{g(w,z)\} K_{\theta_1,\theta_2}(w,z,r,s)dwdz$$

$$= e^{-\pi i[\frac{r^2}{\tan\varphi_1}+\frac{s^2}{\tan\varphi_2}]}\{\int_0^{\infty}\int_0^{\infty}(m)^{\frac{2\pi i r}{\sin\varphi_1}-1} e^{\frac{\pi i}{\tan\varphi_1}[r^2+\log^2(m)]} f(m,n)(n)^{\frac{2\pi i s}{\sin\varphi_2}-1} e^{\frac{\pi i}{\tan\varphi_2}[s^2+\log^2(n)]}dmdn\}$$

$$\{\int_0^{\infty}\int_0^{\infty}(w)^{\frac{2\pi i r}{\sin\varphi_1}-1} e^{\frac{\pi i}{\tan\varphi_1}[r^2+\log^2 w]} g(w,z)(z)^{\frac{2\pi i s}{\sin\varphi_2}-1}$$

$$e^{\frac{\pi i}{\tan\ \varphi_2}[s^2+\log\ ^2 2]} dwdz \quad \}$$

$$= \overline{g} * \overline{f}$$

Hence proved.

Basic properties of Quaternion convolution1	**Linearity Property**	(i) $(A_1 f + A_2 g) * h = A_1(f * h) + A_2(g * h)$ (ii) $h * (A_1 f + A_2 g) = A_1(h * f) + A_2(h * g)$
2	**Shifting Property**	(i) $(\alpha f \ * g) = \alpha(f * g)$ (ii) $(f * \alpha g\ ) = \alpha(f * g)$
3	**Distributive Property**	$f * (g + h) = (f * g) + (f * h)$
4	**Associative Property**	$(f * g) * h = f * (g * h)$
5	**Conjugation Property**	$\overline{(f * g)} = \overline{g} * \overline{f}$

CONCLUSION

In this paper we have developed the fractional Quaternion Mellin transform. The new convolution structure for QFRMT is obtained. Useful properties Quaternion convolution are proved. It can be useful in animation world.

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