_____

Estimation of GPS User Location Using Linearization Technique and Bancroft Algorithms

B. Hari Kumar
Professor, Department of ECE
Brindavan Institute of Technology & Science
Kurnool, AP, India
hari_kumarin@yahoo.com

Dr. G. Sasibhushana Rao
Professor, Dept. of ECE
University College of Engg., Andhra University
Vizag, AP, India
sasigps@gmail.com

Abstract— This paper describes determination of GPS receiver position using Linearization technique and Bancroft algorithms. The position is determined by obtaining pseudo ranges from eight different GPS satellites. The measured ranges do not represent the true ranges as the signal coming from a GPS satellite will be contaminated by various errors like Ephemeris error; Propagation error in the form of Ionospheric and Tropospheric delays; Satellite and Receiver clock biases with respect to GPS Time (GPST); Multipath error etc. Most of these errors can be estimated accurately and can be accounted for. In this paper some standard mitigation techniques have been applied for the estimation of GPS errors and user position is then determined using the above two technique after making appropriate corrections. GPS data of Chitrakut station available in RINEX (Receiver Independent Exchange) format have been used for this purpose

Keywords: Pseudo range, GPS, GPST, RINEX, SV

_____***** _____

I.   Introduction

A GPS user estimates an apparent or pseudo range to each SV (Satellite Vehicle) by measuring the transit time of the signal. Using the pseudo ranges, user position in 3-D (latitude, longitude and height) and the time offset between the transmitter and receiver clock can be estimated after making appropriate corrections for the observed pseudoranges. If the unknown coordinates of the user position are represented by x_u, y_u and z_u and the known positions of Satellite Vehicles are with x_j, y_j, z_j, (where j = 1,2,3,4) in ECEF coordinate system, the user position (in 3-D) and time offset 't$_u$' are obtained by simultaneously solving the nonlinear equations given below.

$$\rho_j = \sqrt{(x_j - x_u)^2 + (y_j - y_u)^2 + (z_j - z_u)^2} + ct_u \; ; \; j = 1, 2, 3, 4 \qquad (1)$$

Where 'c' is the free space velocity of electromagnetic signals in m/s.

The ranges measured from satellites are called pseudo ranges since biases in the receiver clock prevent the precise measurement of actual ranges (true ranges). To determine the receiver position accurately, all the errors have to be estimated and compensated for. In this paper, the ionospheric delay is estimated using Klobuchar model [3]. Hopfield model has been used for the estimation of tropospheric delay. Satellite clock bias and the Relativistic effects also have been estimated and corrected.

II   GPS ERRORS

A. Ephemeris errors

To compute the pseudo range of the user with respect to a satellite, the user should know the position of the satellite. This information is transmitted by the satellite as part of the navigation message. The position of the satellite is dynamic one and is influenced by gravitational field, solar pressures. The estimation of the position by a ground master control station is subject to errors due to clock drifts and processing delays in the monitoring stations.

The estimated positions of the satellites are uploaded periodically to the respective ones. The errors in the estimation of the satellites position results into the pseudo range errors and are to be corrected at `the user level. The rms ranging error attributable to ephemeries error is about 2.1m

B. Multipath effect

Multipath error results from the combination of signals from more than one propagation path, distorting the signal characteristics. Sometimes the reflected signal is as strong as the direct signal. The magnitude of multipath error depends on the strength of the reflected signal and the ability of user's antenna to resist the reflected signal. The strength of the reflected signal is dependent on a reflection co-efficient of the reflecting surface that varies not only with a difference in the ground coarseness, but also with a difference in ground plant quills. A reduction of multipath effect may also be achieved by digital filtering, wideband antennas, and radio frequency absorbent antenna ground planes. Multipath error ranges from 1.2m to 2.7m.

C. Satellite Clock Error

Although all the satellites carry atomic clocks that are very accurate, these can deviate from GPS system time (GPST), which results in pseudo range error. GPS Time (GPST) is a composite time scale derived from the times kept by clocks at GPS monitor stations and aboard the satellites and is used as the reference time. Ranging errors induced by clock errors are of the order of 3.0m. The Master Control Station determines the clock error of each satellite and transmits clock correction parameters to the satellites for rebroadcast of these in the navigation message. The receiver using these parameters implements correction.

The SV clock correction for the C/A code pseudo range observation is given by the expression:

$\delta t^s = af_0 + af_1 * (t - t_{oc}) + af_2 * (t - t_{oc})^2 + \nabla t_r$ (2)

af_0 = clock bias (sec)
af_1 = clock drift (sec/sec)
af_2 = frequency drift (sec/sec^2)
t_{oc} = clock data reference time (sec)
t = current time epoch (sec)
∇t_r = correction due to relativistic effects (sec)

118

_____

D. Relativistic Effects

Both general and special theories of relativity affect the satellite clock. Need for general relativistic corrections arise when the signal source (satellite) and signal receiver (GPS receiver) are located at different gravitational potentials. Special relativity relativistic corrections arise any time the signal source or the signal receiver is moving with respect to the chosen isotropic frame, which in the GPS system is the ECI frame. In order to compensate for both of these effects, the satellite clock frequency is adjusted to 10.22999999545 MHz prior to launch. The frequency observed by the user at sea level would be 10.23 MHz, hence the user does not have to correct for this effect.

But the user has to make a correction for another relativistic effect that arises because of the slight eccentricity of the satellite orbit. When the satellite is at perigee, the satellite velocity is higher and the gravitational potential is lower because of which the satellite clocks run slower. When the satellite is at apogee, the satellite velocity is lower and the gravitational potential is higher and the satellite clocks run faster. This effect can be compensated as follows

$$\nabla t_r = Fe \sqrt{a} \; SinE_k \qquad (3)$$

$F = -4.4442807633 \times 10^{-10}$ sec/m
e = satellite orbital eccentricity
a = semi major axis of the satellite orbit

E_k = eccentric anomaly of the satellite orbit   Due to rotation of the Earth during the time of signal transmission, a relativistic error is introduced which is called the sagnac effect. During the propagation time of the SV signal transmission, a clock on the surface of the earth will experience a finite rotation with respect to the resting reference frame at the geocenter. If the user experiences a net rotation away from SV, the propagation time will increase and vice versa.

E. Atmospheric Errors

The satellite signals propagate through atmospheric layers as they travel from the satellite to the receiver. Two layers are generally considered when dealing with GPS: the Ionosphere, which extends from a height of 70 to 1000 km above the Earth, and the Troposphere which stretches to about 16 kms. Above the equator and 9 kms. Above the poles from the surface of the earth.

As the signal propagates through the ionosphere, the carrier experiences a phase advance and the codes experience a group delay. In other words, the GPS code information is delayed resulting in the pseudo ranges being measured too long as compared to the geometric distance to the satellite. (Hofmann-Wellenhof, 1992). The extent to which the measurements are delayed depends on the Total Electron Content (TEC) along the signal path, which is a measure of the electron density. Further this is dependent on three factors: the geomagnetic latitude of the receiver, the time of day and the elevation of the satellite. Significantly larger delays occur for signals emitted from low elevation satellites (since they travel through a greater section of the ionosphere), peaking during the daytime and subsiding during the night (due to solar radiation). In regions near the geomagnetic equator or near the poles, the delays are also larger (Parkinson, 1996).

The ionospheric delay is frequency dependent and can therefore be eliminated using dual frequency GPS observations; hence the two carrier frequencies in the GPS design. Single frequency users, however, can partially model the effect of the ionosphere using the Klobuchar model (Hofmann-Wellenhof, 1992). Eight parameters of this model are transmitted along with other navigation data from the satellites. These parameters depend on the time of day and the geomagnetic latitude of the receiver. The model results in an estimate of the vertical ionospheric delay, which is then combined with an obliquity factor, dependent on satellite elevation, producing a delay for the receiver-satellite line of sight. The final value provides an estimate within 50% of the true delay and produces delays ranging from 5m (night) to 30m (day) for low elevation satellites and 3-5m for high elevation satellites at mid latitudes.

The troposphere causes a delay in both the code and carrier observations. Since it is not frequency dependent (within the GPS L band range) it cannot be canceled out by using dual frequency measurements but it can, however, be successfully modeled. The troposphere is split into two parts: the dry component, which constitutes about 90% of the total refraction, and the wet part, which constitutes the remaining 10%. Values for temperature, pressure and relative humidity are required to model the vertical delay due to the wet and dry part, along with the satellite elevation angle, which is used with an obliquity/mapping function. Models put forward by Hopfield, Black and Saastamoninen are all successful in predicting the dry part delay to approximately 1-cm and the wet part to 5 cm. Ionospheric and tropospheric delays have been described in detail in the subsequent sections.

Ionospheric Delay:
The vertical time delay for the code measurement is given by

$$\Delta T_v^{iono} = A_1 + A_2 Cos\left(\frac{2\pi(t-A_3)}{A_4}\right) \qquad (4)$$

Where A_1 = 5 nano seconds

$$A_2 = \alpha_1 + \alpha_2 \varphi_{IP}^m + \alpha_3 \varphi_{IP}^{m2} + \alpha_4 \varphi_{IP}^{m3}$$

$A_4 = 14^h$ local time

$$A_4 = \beta_1 + \beta_2 \varphi_{IP}^m + \beta_3 \varphi_{IP}^{m2} + \beta_4 \varphi_{IP}^{m3}$$

The values for A_1 and A_3 are constant, the coefficients α_i, β_i, $i = 1, 2 --- 4$ are uploaded to the satellites and broadcast to the user. The parameter 't' in Eq. (4) is the local time of the ionospheric point IP and may be derived from

$$t = \frac{\lambda_{IP}}{15} + t_{UT} \qquad (5)$$

Where λ_{IP} is the geomagnetic longitude positive to east for the ionospheric point in degrees and t_{UT} is the observation epoch in Universal Time. φ_{IP}^m is the spherical distance between the geomagnetic pole and the ionospheric point.

Tropospheric Delay:
The tropospheric path delay is defined as

$$\Delta^{Trop} = \int (n-1) ds \qquad (6)$$

In general, instead of the refractive index n the refractivity N is used.

Where the refractivity $N^{Trop} = 10^6 (n-1)$

Equation becomes $\Delta^{Trop} = 10^{-6} \int N^{Trop} ds$

$$(7)$$

The tropospheric delay can be estimated using several models which are described below:

Hopfield (1969) shows the possibility of separating N^{Trop} into dry and wet components as $N^{Trop} = N_d^{Trop} + N_w^{Trop}$

(8)

Where the dry part results from the dry atmosphere and the wet part from the water vapor. Correspondingly the relations become

$$\Delta_d^{Trop} = 10^{-6} \int N_d^{Trop} ds \text{ and } \Delta_w^{Trop} = 10^{-6} \int N_w^{Trop} ds$$

$$(9)$$

$$\Delta^{Trop} = \Delta_d^{Trop} + \Delta_w^{Trop}$$

$$\Delta_d^{Trop} = 10^{-6} \int N_d^{Trop} ds + 10^{-6} \int N_w^{Trop} ds \quad (10)$$

About 90% of the tropospheric refraction arise from the dry component and 10% from the wet component. There are a number of models which give us the dry and wet refractivity at the surface of the earth (e.g., Essen and froom 1951).

Dry component of refractivity is given by

$$N_{d,o}^{Trop} = \bar{c}_1 \frac{p}{T}, \quad \bar{c}_1 = 77.64 \text{ K mb}^{-1} \quad (11)$$

Where p is the atmospheric pressure in millibar (mb) and T is the temperature in Kelvin (K). Wet component is given by

$$N_{w,o}^{Trop} = \bar{c}_2 \frac{e}{T} + \bar{c}_3 \frac{e}{T^2}, \quad \bar{c}_2 = -12.96 \text{ K mb}^{-1} \text{ and Where } e$$

$$\bar{c}_3 = 3.718 * 10^5 \text{ K}^2 \text{ mb}^{-1} \quad (12)$$

is the partial pressure of water vapor in mb. According to Hopfield model the dry refractivity as a function of the height h above the surface is given by

$$N_d^{Trop}(h) = N_{d,o}^{Trop} \left[\frac{h_d - h}{h_d} \right]^4 \quad (13)$$

Where $h_d = 40136 + 148.72 (T - 273.16)$ m. Substituting the above equation results in

$$\Delta_d^{Trop} = 10^{-6} N_{d,o}^{Trop} \int \left[\frac{h_d - h}{h_d} \right]^4 ds \quad (14)$$

The integral can be solved if the delay is calculated along the vertical direction and if the curvature of the signal path is neglected. Thus, for an observation site on the surface of the earth (i..e., h=0), the above equation becomes

$$\Delta_d^{Trop} = 10^{-6} N_{d,o}^{Trop} \frac{1}{h_d^4} \int_{h=0}^{h=h_d} (h_d - h)^4 dh \quad (15)$$

Where the constant denominator has been extracted. After integration,

$$\Delta_d^{Trop} = 10^{-6} N_{d,o}^{Trop} \frac{1}{h_d^4} \left[-\frac{1}{5} (h_d - h)^5 \Big|_{h=0}^{h=h_d} \right] \quad (16)$$

is obtained. The evaluation of the expression between the brackets gives $h_d^5 / 5$ so that $\Delta_d^{Trop} = \frac{10^{-6}}{5} N_{d,o}^{Trop} h_d$

$$(17)$$

is the dry portion of the tropospheric zenith delay.

The wet portion is much more difficult to model because of the strong variations of the water vapor with respect to time and space. Nevertheless, due to the lack of an appropriate alternative, the Hopfield model assumes the same functional model for both the wet and dry components. Thus,

$$N_w^{Trop}(h) = N_{w,0}^{Trop} \left[\frac{h_w - h}{h_w} \right]^4 \quad ; \text{ Where the mean value}$$

$$h_w = 11,000 m \quad (18)$$

is used. Sometimes other values such as $h_w = 12,000 m$ have been proposed. Unique values for h_d and h_w cannot be given because of their dependence on location and temperature.

The integration of above equation results in

$$\Delta_w^{Trop} = \frac{10^{-6}}{5} N_{w,o}^{Trop} h_w \quad (19)$$

Therefore, the total tropospheric zenith delay is

$$\Delta^{Trop} = \frac{10^{-6}}{5} \left[N_{d,o}^{Trop} h_d + N_{w,o}^{Trop} h_w \right] \quad (20)$$

With the dimension in meters. The model in its present form does not account for an arbitrary zenith angle of the signal. Considering the line of sight, an obliquity factor must be applied which, in its simplest form, is the projection from the zenith onto the line of sight. Frequently, the transition of the zenith delay to a delay with arbitrary zenith angle is denoted as the application of a mapping function.

Introducing the mapping function the above equation becomes

$$\Delta^{Trop} = \frac{10^{-6}}{5} \left[N_{d,o}^{Trop} h_d m_d(E) + N_{w,o}^{Trop} h_w m_w(E) \right]$$

$$(21)$$

Where $m_d(E)$ and $m_w(E)$ are the mapping functions for the dry and the wet part and E (expressed in degrees) indicates the elevation at the observing site (where the line of sight is simplified as straight line). Explicitly,

$$m_d(E) = \frac{1}{\sin \sqrt{E^2 + 6.25}} \quad (22)$$

$$m_w(E) = \frac{1}{\sin \sqrt{E^2 + 2.25}} \quad (23)$$

Are the mapping functions for the Hopfield model. The above equation can be represented as

$$\Delta^{Trop}(E) = \Delta_d^{Trop}(E) + \Delta_w^{Trop}(E) \quad (24)$$

$$\Delta_d^{Trop}(E)$$

$$= \frac{10^{-6}}{5} \frac{-12.96 + 3.718(10^5)}{\sin \sqrt{E^2 + 2.25}} [\frac{e}{T^2}](11000) \quad (25)$$

Measuring p,T,e at the observation location and calculating the elevation angle E, the total tropospheric path delay is obtained in meters by the equations.

III   LINEARIZATION TECHNIQUE

To determine the user position in three dimensions (x_u, y_u, z_u) and the receiver clock offset t_u, pseudoranges are to be obtained from a minimum of four satellites.

$$\rho_j = \sqrt{(x_j - x_u)^2 + (y_j - y_u)^2 + (z_j - z_u)^2} + ct_u \; ; \quad j = 1,2,3,4. \qquad (26)$$

The resulting equations can be written as a function of user coordinates and clock offset as $\quad \rho_j = f(x_u, y_u, z_u, t_u)$

$$(27)$$

Using an approximate position location ($\hat{x}_u$, $\hat{y}_u$, $\hat{z}_u$) and time bias estimate $\hat{t}_u$, an approximate pseudorange can be calculated

$$\hat{\rho}_j = \sqrt{(\hat{x}_j - x_u)^2 + (\hat{y}_j - y_u)^2 + (\hat{z}_j - z_u)^2} + c\hat{t}_u \; ; \qquad j = 1, 2, 3, 4$$

$$= f(\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u) \qquad (28)$$

The unknown user position and receiver clock offset are considered to consist of an approximate component and an incremental component as stated below.

$$x_u = \hat{x}_u + \Delta x_u$$
$$y_u = \hat{y}_u + \Delta y_u$$
$$z_u = \hat{z}_u + \Delta z_u$$
$$t_u = \hat{t}_u + \Delta t_u$$

This allows writing Eq. (27) as
$$f(x_u, y_u, z_u, t_u) = f(\hat{x}_u + \Delta x_u, \hat{y}_u + \Delta y_u, \hat{z}_u + \Delta z_u, \hat{t}_u + \Delta t_u)$$

This latter function can be expanded about the approximate point using a Taylor series. It can be shown that

$$\hat{\rho}_j - \rho_j = \frac{x_j - \hat{x}_u}{\hat{r}_j}\Delta x_u + \frac{y_j - \hat{y}_u}{\hat{r}_j}\Delta y_u + \frac{z_j - \hat{z}_u}{\hat{r}_j}\Delta z_u - c\Delta t_u$$

Where $\quad \hat{r}_j = \sqrt{(\hat{x}_j - x_u)^2 + (\hat{y}_j - y_u)^2 + (\hat{z}_j - z_u)^2}$

$$\Delta\rho_j = \hat{\rho}_j - \rho_j$$

$$a_{xj} = \frac{x_j - \hat{x}_u}{\hat{r}_j}; a_{yj} = \frac{y_j - \hat{y}_u}{\hat{r}_j}; a_{zj} = \frac{z_j - \hat{z}_u}{\hat{r}_j}$$

Where a_{xj}, a_{yj} and a_{zj} terms denote the direction cosines of the unit vector pointing from the approximate user position to the jth satellite.

Rewriting Equation (4.5) results into

$$\Delta\rho_j = a_{xj}\Delta x_u + a_{yj}\Delta y_u + a_{zj}\Delta z_u - c\Delta t_u$$

When pseudorange measurements are made to four satellites (j = 4), Equation can be represented in matrix form as

$$\Delta\rho = \begin{bmatrix} \Delta\rho_1 \\ \Delta\rho_2 \\ \Delta\rho_3 \\ \Delta\rho_4 \end{bmatrix} \quad \Delta x = \begin{bmatrix} \Delta x_u \\ \Delta y_u \\ \Delta z_u \\ -c\Delta t_u \end{bmatrix}$$

$$H = \begin{bmatrix} a_{x1} & a_{y1} & a_{z1} & 1 \\ a_{x2} & a_{y2} & a_{z2} & 1 \\ a_{x3} & a_{y3} & a_{z3} & 1 \\ a_{x4} & a_{y4} & a_{z4} & 1 \end{bmatrix}$$

We can obtain error matrix Δx from the following equation

$$\Delta x = H^{-1}\Delta\rho \qquad (29)$$

The above procedure is to be repeated till sufficient accuracy is obtained

IV   BANCROFT ALGORITHM

The set of nonlinear equations which represent the relationship between the unknown user position u = [x_u, y_u, z_u] and the known satellite positions s_j = [x_j, y_j, z_j] can be written as

$$(\rho_j - b)^2 = (x_j - x_u)^2 + (y_j - y_u)^2 + (z_j - z_u)^2 \quad (30)$$

Where b is substituted for ct_u

Expanding the above equation leads to

$$(x_j^2 + y_j^2 + z_j^2 - \rho_j^2) - 2(x_j x_u + y_j y_u + z_j z_u - b\rho_j) + (x_u^2 + y_u^2 + z_u^2 - b^2) = 0$$

$$(31)$$

Defining the Lorentz inner product as

$$\langle \vec{u}, \vec{v} \rangle = u_1 v_1 + u_2 v_2 + u_3 v_3 - u_4 v_4 \;;$$

Equation (27) can be written as

$$\langle \vec{s}_j, \vec{s}_j \rangle - 2\langle \vec{s}_j, \vec{u} \rangle + \langle \vec{u}, \vec{u} \rangle = 0 \qquad (32)$$

Equation (28) can be represented as

$$\vec{a} - B\vec{u} + \Lambda\vec{e} = 0 \text{ and}$$
$$B\vec{u} = \vec{a} + \Lambda\vec{e} \qquad (33)$$

Where

$$\vec{a} = \frac{1}{2}\begin{bmatrix} \langle \vec{s}_1, \vec{s}_1 \rangle \\ \langle \vec{s}_2, \vec{s}_2 \rangle \\ \langle \vec{s}_3, \vec{s}_3 \rangle \\ \langle \vec{s}_4, \vec{s}_4 \rangle \\ \vdots \end{bmatrix}, \qquad \vec{e} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ \vdots \end{bmatrix}$$

$$B = \begin{bmatrix} x_1 & y_1 & z_1 & -\rho_1 \\ x_2 & y_2 & z_2 & -\rho_2 \\ x_3 & y_3 & z_3 & -\rho_3 \\ x_4 & y_4 & z_4 & -\rho_4 \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \text{ and } \Lambda = \frac{1}{2}\langle \vec{u}, \vec{u} \rangle$$

If more than 4 satellites are observed, matrix B will not be a square matrix. The solution can be obtained using the least square method as given below (Bancroft, 1985).

$$\vec{u} = B^+(\vec{a} + \Lambda\vec{e}) \qquad (34)$$

Where $B^+ = (B^T B)^{-1} B^T$

V   RESULTS AND CONCLUSIONS

RINEX data from Chitrakut station (Near IIT Kanpur) is used for this purpose. The observation data of 21st January 2006 at 0hrs.0 min. 30 sec.have been used. Eight satellites (SV PRN. Nos. 3   8   11   13   19   23   27   28) are observed at the epoch time. Algorithms have been implemented to sort out the ephemeris data into matrix format and for the determination of satellites' position at the epoch time. By using clock correction parameters which are available as part of the Navigation message, the satellite clock bias and error due to relativistic effect have been obtained. The Ionospheric delay has been estimated using Kloubuchar model. All the eight coefficients for the implementation of Kloubucharmodel are available as part of Navigation message. The Tropospheric delay has been estimated using Hopfield method. The estimated errors and the corrected ranges have been represented in Table 1. Finally the user position is estimated using the linearization technique and Bancroft methods. Results indicate that the latter method is found to be more accurate in estimating the user position.

TABLE 1
ESTIMATION OF GPS ERRORS AND CORRECTION OF PSEUDORANGES

PRN	Pseudo range in meters	Sv clock Error in meters	EL (degrees)	AZ (degrees)	Iono delay in meters	Tropo delay in meters	Corrected Range in meters
3	22366411.703470	20458.9163418	31.48530	45.00000	2.5726932	4.57499	22386863.4721243
8	23797054.919460	-16229.1889608	18.31110	315.00000	3.3829168	7.56015	23780814.7874324
11	24125708.677460	90166.4669971	13.13495	135.00000	3.7884592	10.36503	24215860.9909640
13	20151227.111490	10876.5173663	80.60147	243.43495	1.5122900	2.42835	20162099.6882207
19	20845653.236480	-6991.4685115	58.63407	94.08562	1.7034061	2.80453	20838657.2600340
23	21132518.133480	46079.5912404	50.20845	180.00000	1.8785100	3.11548	21178592.7307304
27	22298850.607480	9715.7632025	41.58418	315.00000	2.1399100	3.60477	22308560.6260035
28	23500461.379470	10425.2163174	23.16312	251.56505	3.0491071	6.05881	23510877.4878739

Using the corrected pseudoranges user position is determined and the results are given below:

User Position as per the observation data:
X_u = 918074.1038m, Y_u= 5703773.5389   and
Z_u =2693918.9285m.

User position by linearization technique:
X_u= 918044.33m, Y_u= 5703780.81m   and
Z_u = 2693913.23m.

User position by Bancroft algorithm:
X_u =   918076.00m, Y_u = 5703779.01m   and
Z_u = 2693913.41m

REFERENCES

[1]   Bancroft. S, "An algebraic solution of the GPS equations", IEEE Transactions on Aerospace and Electronic Systems 21 (1985) 56–59.

[2]   B.HofmannWellenhof, .Lichtenegger and J.Collins, "GPS Theory and Practice", Springer-Verlag   Wien, New York, 1992.

[3]   Klobuchar J, "Design and characteristics of the GPS ionospheric time – delay algorithm for single frequency users", Proceedings of PLANS'86, Las Vegas, Nevada, November 4-7, pp280-286.

[4]   Hopfield HS, "Two – quartic tropospheric refractivity profile for correcting satellite data", Journal of Geophysical research, 74(18): 4487-4499.

[5]   Strang, G. and Borre, K., "Linear Algebra, Geodesy, and GPS", Wellesley Cambridge, - Wellesley, MA, 1997.

[6]   http://home.iitk.ac.in/~ramesh/gps