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Comparison of Compressed Sensing algorithms for MIMO-OFDM Systems

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Abstract—Estimation of the channel accurately in a MIMO-OFDM system is crucial to guarantee the performance of the system. In this paper the Subspace Pursuit (SP), Orthogonal Matching Pursuit (OMP), Compressed Sampling Matching Pursuit(CoSaMP) and Distributed Compressed Sensing(DCS) algorithms combined with Minimum Mean Square Error(MMSE) and Least Mean Square (LMS) tools are used to estimate the channel coefficients for MIMO-OFDM system. These algorithms are used for the channel estimation in MIMO-OFDM system to develop the joint sparsity of the MIMO channel. Simulation results shows that SP, OMP, CoSaMP and DCS algorithms combined with MMSE and LMS tools provides significant reduction in Normalized Mean Square Error (NMSE) when compared to SP ,CoSaMP, DCS algorithms with Least Square (LS) tool and also the conventional channel estimation methods such as LS, MMSE and LMS. Moreover DCS combined with LMS tool performs better than SP and OMP techniques with LMS tool with less computational time complexity.

Keywords— *MIMO-OFDM, Compressed Sensing(CS), Compressed Sampling Matching Pursuit(CoSaMP), Subspace Pursuit (SP), Orthogonal Matching Pursuit (OMP)*

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I. INTRODUCTION

Channel estimation is crucial to the performance of coherent demodulation in multi-input–multi-output orthogonal frequency division multiplexing (MIMO-OFDM) systems. It determines the channel state information (CSI) that is used to support pre-coding and scheduling [1]–[2].Generally, CSI is obtained by embedding pilot symbols in OFDM symbols and using linear channel estimators such as Least Squares (LS), Minimum Mean Square Error (MMSE) and Least Mean Square(LMS)[4] -[5].However, the multipath channels tend to exhibit sparse structures [6]. Conventional channel estimation methods do not use multipath channel inner sparsity completely. Recently, compressive sensing (CS) techniques have been applied in the field of channel estimation[7]. CS theory demonstrates that a sparse or compressive signal can be used to efficiently reconstruct the signal from very few limited sample values [8-10]. CS algorithms are extended to MIMO-OFDM channel estimation systems in [11].

The CS algorithm SP is compared with OMP for MIMO-OFDM system in [12], where it is shown that the computational complexity of SP is less than OMP. The computational complexity of SP and OMP is O(m.N.log(K)) and O(K.m.N) respectively[13],where m is number of pilots or rows in the measurement matrix, N is number of columns in the measurement matrix and K is the sparsity level.

However, in all these papers, LS tool is used for the channel estimation. LS tool does not take channel statistics into account and suffers from high mean-square error(MSE).The MSE can be reduced using MMSE tool and a better estimate can be obtained by using LMS tool which is an adaptive estimation technique[13]. It obtains more information about the channel and the estimated channel coefficients are continuously updated at every iteration. The knowledge of noise statistics and channel statistics are not necessary to estimate the channel.

In this paper, the compressed sensing algorithms such as SP, OMP, CoSaMP and DCS are combined with LS and LMS methods in the frequency domain channel estimation of MIMO-OFDM systems. A MIMO-OFDM system model is briefly described in Section II. In section III CoSaMP and DCS algorithms are illustrated. The results are compared with traditional methods such as LS, MMSE and LMS and also with SP, OMP , CoSaMP and DCS with LS approach in section IV. Performance analysis shows that DCS combined with LMS performs better than other methods with the computational complexity O(m.N) and these conclusions are shown in Section V.

II. SYSTEM MODEL

A. MIMO OFDM System Model

The Channel Impulse Response (CIR) of a system with L number of Multipath, is given by [14]

$$h(\tau,t)=\sum_{q=1}^{L}\alpha_q(t)\delta(\tau-\tau_q(t)) \qquad (1)$$

Where $\tau_q(t)\in\mathbb{R}$ and $\alpha_q(t)\in\mathbb{C}$ are real-valued delay spread and complex-valued magnitude for path q, in turn. Assuming block-fading channel where each block has channel parameters are persistent and also neglecting the symbol synchronization, CIR in discrete form is given by

$$h(\tau,t)=\sum_{q=1}^{L}\alpha_q(t)\delta((\tau-\tau_q)T_q) \qquad (2)$$

where T_q is the sampling interval. T_q must be lower compared to the maximum DS(Delay Spread)in high data rate systems. Application of Eqn (2) results in a channel with comparatively more number of zero taps and few nonzero taps. Let P be the total number of channel taps with Q of them are nonzero (Q<<P) i.e. a Q-sparse channel.

Consider OFDM system with Np subcarriers as pilots , at position t1,t2,……tNp (1≤ t1<t2<….<tNp ≤ N).

X(t1),X(t2),…….,X(tNp)and Y(t1),Y(t2),…….Y(tNp) represent the transmitted symbols and the received symbols respectively at pilot locations. The estimated transfer function on pilot subcarriers is

$$\hat{H}(k)=\frac{Y(k)}{X(k)}, \qquad k=t1, t2, \ldots, tNp \qquad (3)$$

The discrete channel transfer function $\hat{H}$ (k) (k = 1, 2,….,N) is obtained by considering the pilot subcarriers and interpolating as shown in Eqn(3). With channel sparsity included the problem can be expressed as[14]

$$y = X.F_{N_p \times L}.h + n_0 \qquad (4)$$

where h=[h(1),h(2),h(3)….,h(L)]^{T} are the CIR, X=diag{X(t1),X(t2),…….,X(tNp)} transmitted symbol and $n_0=[n_0(1),n_0(2),\ldots\ldots n_0(Np)]^T$ is the noise vector which is Additive White Gaussian in Noise (AWGN) in nature. And y is the output symbol as defined above, and

$$F_{N_p\times L}=\frac{1}{\sqrt{N}}\begin{bmatrix} 1 & w^{t_1} & \cdots & w^{t_1(L-1)} \\ 1 & w^{t_2} & \cdots & w^{t_2(L-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & w^{t_{N_p}} & \cdots & w^{t_{N_p}(L-1)} \end{bmatrix}$$

where$w=e^{-j2\pi/N}$. In fact $F_{N_p\times L}$ is a DFT sub matrix chosen by column indices [0, 1,…,L-1] and row indices [$t_1,t_2,\ldots t_{Np}$] from a standard NxN Fourier matrix[11].

$$y = A_d.h + n_0 \qquad (5)$$

where A_d=X. $F_{N_p\times L}$.

Purpose of channel estimation is to obtain h from y and A_d. If columns of Ad is less than its rows (L<Np), equation (5) can be viewed as regular LS problem and solution to it is given [4]

$$\hat{h}_{ls} = (A_d^H A_d) - 1A_d^H.y \qquad (6)$$

When the channel coefficients more than the pilots (Np>L), it significantly helps in decreasing pilots and therefore spectral efficiency is improved. Ideally, for sparse recovery problem [5] there is a feasible solution if most components of vector h are zero (Q<<L).

B. *Least Square Channel Estimation Technique*

LS is simple estimation technique and very straight forward. The received pilot signal is multiplied with the inverse of the transmitted pilot signal as given in the Eqn (3).

Mean Square Error (MSE) for the LS is,

$$MSE_{LS}=E\{(H-H_{est})H(H-H_{LS})\} \qquad (7)$$

LS technique has low complexity and simple to implement. However, LS channel technique doesn't take channel statistics into account and suffer from high mean-square error.

C. *MMSE channel Estimation technique*

The MSE can be reduced by using MMSE technique

$$H_{est}^{MMSE} = R_{H\tilde{H}_{LS}}(R_{HH}+\frac{\sigma_w^2}{\sigma_b^2} I)-1\tilde{H}_{LS} \qquad (8)$$

where $R_{H\tilde{H}_{LS}}$ and R_{HH} is the cross correlation and the autocorrelation respectively and $\frac{\sigma_w^2}{\sigma_b^2}$ is the SNR.

D. *LMS channel estimation*

Practically, channel is time varying. In order to estimate the time varying channel the channel coefficients have to be adjusted accordingly. LMS is an adaptive estimation technique gets more information about the channel. The estimated channel coefficients are continuously updated at each iteration. The knowledge of noise and channel statistics is not necessary. The LMS solely depends on the step size.

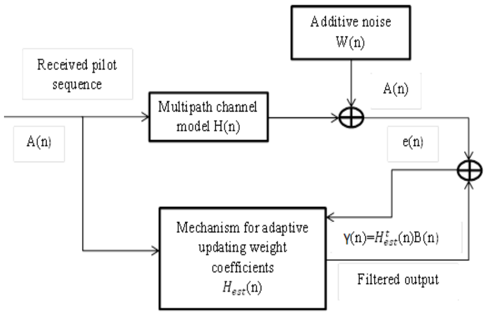

Figure 1: Illustration the LMS channel estimation [13]

Fig 1 illustrates the LMS channel estimation. B(n) is the transmitted signal, A(n)=is the received signal, and W(n) is the Additive noise. The received signal A(n) is given by

$$A(n) = H^T(n)B(n) + W(n) \qquad (9)$$

where $B=(B_0\ldots.B_{L-1})^T$,A=(A0…..AL-1)^{T,} $W=(W_0,W_1,\ldots\ldots,W_{L-1})^T$ and $H=(H_0,H_1\ldots\ldots,H_{L-1})^T$.

The output of the adaptive filter is given as in the Eqn (10)

$$Y(n) = H_{est}^T(n) + B(n) \qquad (10)$$

H_{est}(n) is the estimated channel coefficient.

The recursive method can be used to update H_{est}(n) as given in equation (11)

$$H_{est}(n+1) = H_{est}(n) + \square B(n)e*(n) \qquad (11)$$

□ is the step size for adaptive LMS algorithm. And e(n)=A(n)-Y(n) is the error vector for updating the weights..

III. COMPRESSED SENSING ALGORITHMS

Compressed sensing constructs based on the necessary fact that many signals can be categorized with only a few non-zero coefficients in appropriate basis or dictionary. The recovery process of such signals, utilizing non-zero measurements, can then be enabled by the nonlinear optimization. A Q-sparse vector $h\epsilon\mathbb{R}^L$ can be recovered from equation (5) with deliberately designed Ad $\epsilon\mathbb{R}^{Np\times L}$ and by solving l_0 norm minimization problem [14].

$$\min h\epsilon\mathbb{R}^L \|h\|_0 s.t. \|y - A_d.h\|_2 \le \sigma_n \qquad (12)$$

Where $||h||_0$ is sum of non-zero components of h and σ_n is the variance. This problem is NP hard and combinatorial. The problem can be resolved using greedy algorithms or convex optimization algorithms[5].

$$\min h\epsilon\mathbb{R}^L ||h||_1 \text{s.t.} ||y - A_d.h||_2 \leq \sigma_n \quad (13)$$

In application of time-varying channel where channel estimation is frequently carried out, it's inappropriate to choose high computational convex optimization algorithms [19]. Greedy algorithms are focused here since its complexity is less than the other one[18].

Calculation of best non-linear estimate to a signal in a complete, redundant dictionary is achieved by the Matching Pursuit[MP] which is a basic greedy algorithm. MP forms a series of sparse estimates to the signal step wise that builds a linear combination of matrix columns closest to the signal [15]. The complexity is reduced compared to traditional LS approach. However the computational time complexity of OMP is more than that of other greedy algorithms like SP and CoSaMP. This is because at each step only one atom is selected[16]. The total computational time of OMP is given by O(m.N.K).

Another greedy algorithm with less computational time and better BER performance is SP. The idea of SP is to iteratively refine S columns selection from the dictionary matrix through LS method until the stop condition is satisfied. At each step, it selects S columns rather than only one column as in MP and OMP. The sub space spanned by S columns is thus tracked down. The disadvantage of SP is that we must have prior knowledge of S before we start the algorithm. So it's required to extend SP to the instance where the sparsity is not known. The total computational time of SP is given by O(m.N.log(K)). The computational time complexity of SP is reduced compared to OMP because batch selection is done instead of one.

A. Channel Estimation Using CoSaMP Algorithm

Algorithm 1: CoSaMP

A= measurement Matrix (dictionary).
x=Sparse Approximation of b.
B=received Signal (samples obtained).
K=Sparsity required
Φ= measurement Matrix(dictionary) .
Inputs AM×N , BN×1 , K
r←B, Π←Ǿ
While stopping criterion not met do
c← AH r
Π←ΠU find Largest Indices (apply Model(|c|2),2K)
xtemp←arg minx||Asupportxsupport-y||22
Π←find Largest Indices(apply Model(|xtemp|2),K)
r ← r-A support xtemp,support
end while
x← 0, xsupport←xtemp

In CoSaMP first identifies 2K(where K is the sparsity level) atoms using a matched filter and it is combined with the support-matrix or set estimated in the earlier iteration. Candidate-set is the set of atoms which are estimated from the matched filter. The support set and its union with candidate-set of previous step can be called as union set. From, the union set, a new K-dimensional subspace is identified from the union-set using least-squares. This will reduce the reconstruction error of the sparse signal. The total computational time of CoSaMP is given by O (m.N).which is much small compared to OMP and SP.

B. Channel Estimation Using DCS Algorithm

Algorithm 2: DCS

A= measurement Matrix (dictionary).
x=Sparse Approximation of b.
B=received Signal (samples obtained).
K=Sparsity required
Φ= measurement Matrix(dictionary) .

Inputs AM×N , BN×1 , K
r←B, Π←Ǿ
While stopping criterion not met do
c← AH r
Π←ΠU find Largest Indices (apply Model(|c|2),2K)
xtemp←arg minx||Asupportxsupport-y||22
Π←find Largest Indices(apply Model(|xtemp|2),K)
r ← r-A support xtemp,support
end while
x← 0, xsupport←xtemp
b← Merge channel estimates
Find the dominant channel coefficients
c= supp (b, x)
Prune channel estimates:
y = ΦΦx
Update the estimation residual
end while

DCS is an algorithm which is based on CoSaMP. The difference between the two is that CoSaMP builds the solution by operating on the entire 2K(where K is the sparsity level) atoms that contains all the channel components in the MIMO system while DCS operates on each channel component hij between the ith receive antenna and jth transmit antenna individually. Thus the proposed DCS algorithm reduces the computational complexity with increased accuracy. The iterative DCS algorithm is stopped when the l2 norm of the residuals saturate to a constant level. In case of MIMO-OFDM channel estimation, this saturation bound is found to be at the iteration index K, the sparsity of each channel component.

IV. SIMULATION RESULTS

In this section, the OMP, SP, CoSaMP and DCS compressed sensing channel estimation methods combined with LS, MMSE and LMS methods for MIMO-OFDM system in frequency-selection Rayleigh fading channels are compared. The number of OFDM subcarriers used is 256. Out of which 12 are used as pilot subcarriers and these subcarriers are placed according to the RIP (Restricted Isometric Property) among all OFDM subcarriers [17]. The main parameters of the concerned MIMO-OFDM system are listed in Table 1.

Table 1. Parameters for the simulated MIMO-OFDM systems

Number of transmit antennas	2
Number of receive antenna	2
Channel type	Rayleigh
Input Sample period	10^{-7}
Total Number of subcarriers	N = 256
Number of pilot subcarriers	Np = 12
Number of cycles prefix	NG = 64
Delay spread	15
Doppler frequency	0.1Hz
Modulation	QAM

Figure2 gives the performance comparison of the conventional channel estimation techniques in 2x2 MIMO-OFDM systems. It shows that the performance of LMS is better than LS and MMSE. Since LMS tool is an adaptive estimation technique. Estimated channel coefficients are continuously updated at till it meets the stopping criteria

Figure3 gives the performance comparison of the LS channel estimation using CoSaMP and SP algorithms in 2x2 MIMO-OFDM systems. The plots are calculated using different sparsity levels: 20, 30 and 50.Normalised MMSE reduces as the number of non zero coefficients i.e., the sparsity levels increases.

Figure 4 and 5 gives the performance comparison of the SP, OMP and CoSaMP combined with LS and MMSE tools. CoSaMP combined with MMSE tool performs better than LS, SP and OMP combined with LS and MMSE tool.

Fig 6 gives the performance comparison of the LMS channel estimation combined with SP, OMP and CoSaMP algorithms. Since in LMS channel estimated by adaptively changing the coefficients until the error between the combined output and received signal is zero. The performance CoSaMP combined with LMS is better.

Fig 7 and 8 gives the performance comparison of the CoSaMP and DCS algorithms with LS and LMS tools respectively.DCS operates on each channel component between the receive antenna and transmit antenna individually. The DCS algorithm reduces the computational complexity and accuracy is increased.

Fig. 10(a) to (f) are the reconstructed images at the receiver for different SNRs for CoSaMp algorithm with LS tool for the original transmitted image shown in fig(9). We can observe that the quality of the image improves at higher SNR which is directly related to the BER observed.

Fig. 11(a) to (f) are the reconstructed images at the receiver for SNR's for CoSaMp algorithm with LMS tool for the original transmitted image shown in fig(9). The quality of the image improves at higher SNR and also the quality is better than CoSaMP with LS tool. Based on the results of the simulation CoSaMP with LMS tool MIMO channel estimation is superior to other CS algorithms.

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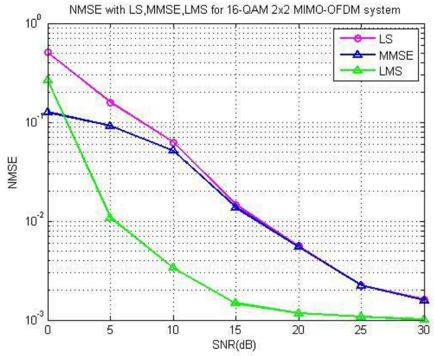

Figure2. Plot of Channel estimation in 2x2 MIMO-OFDM using conventional methods with equispased pilots.

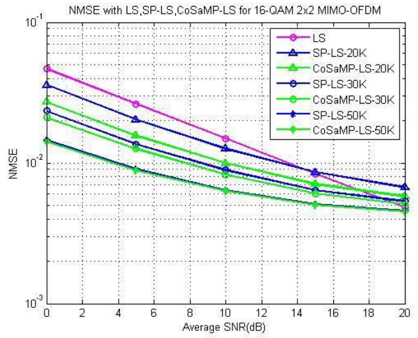

Figure 3.Plot of NMSE vs SNR for channel estimation in 2x2 MIMO-OFDM system using CoSaMP-LS and SP-LS for different sparsity levels.

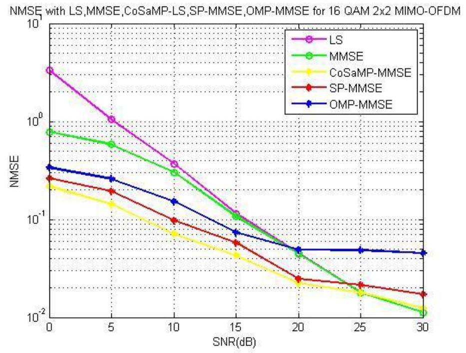

Figure4. Plot of NMSE vs SNR for channel estimation in 2x2 MIMO-OFDM system using SP-LS, SP-MMSE

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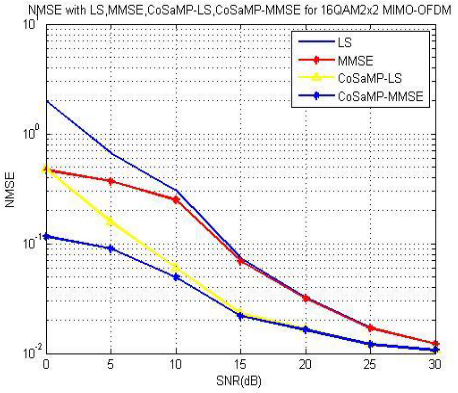


Figure 5.Plot of NMSE vs SNR for channel estimation in 2x2 MIMO-OFDM system using CoSaMP-LS,CoSaMP-MMSE.

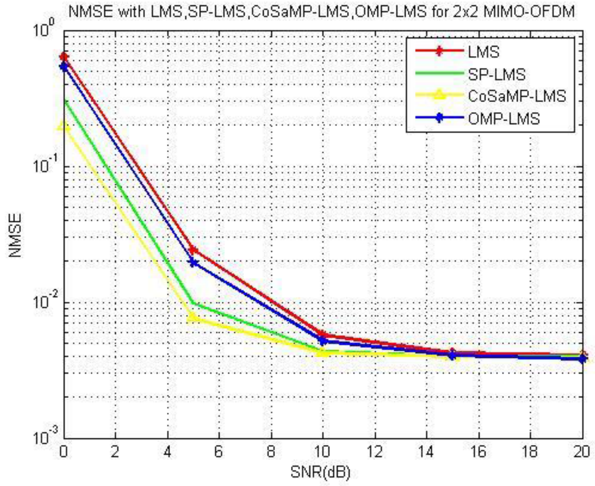


Figure6. Plot of NMSE vs SNR for channel estimation in 2x2 MIMO-OFDM system using LMS, CoSaMP-LMS,SP-LMS, OMP-LMS.

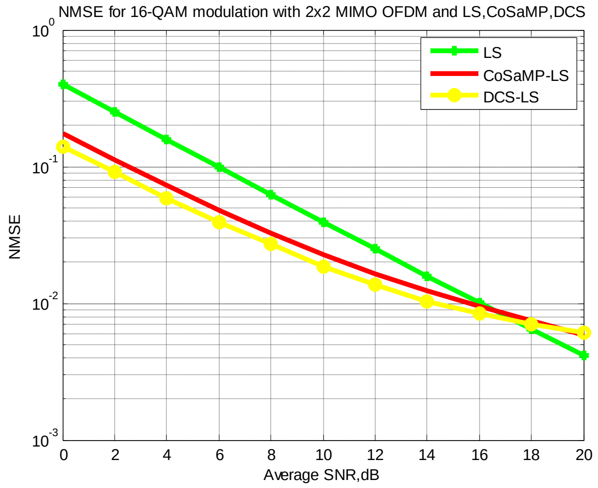


Figure7. Plot of NMSE vs SNR for channel estimation in 2x2 MIMO-OFDM system using LS, CoSaMP-LS,DCS-LS

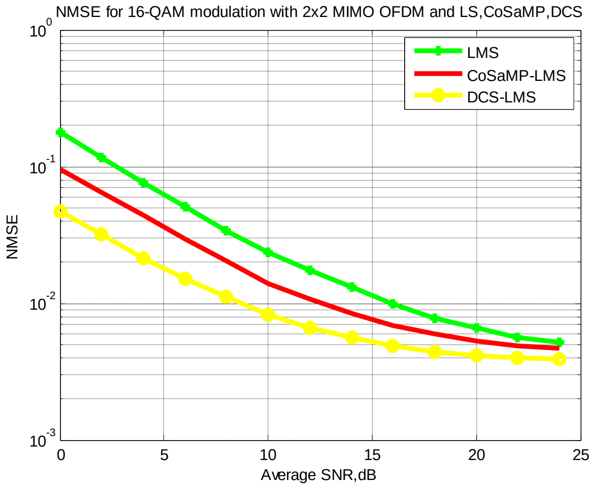
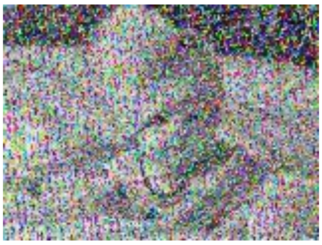


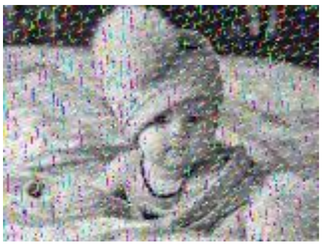
Figure 8. Plot of NMSE vs SNR for channel estimation in 2x2 MIMO-OFDM system using LMS, CoSaMP-LMS,DCS-LMS

Figure9.Original image size: 159x119

A. Simulation result of CoSaMP with LS tool for image data for different SNRs.

(a) (b)

(c) (d)


(e) (f)

Figure 10.(a)Reconstructed Output Image at SNR=5, (b) Reconstructed Output Image at SNR=10, (c). Reconstructed Output Image at SNR=15,(d). Reconstructed Output Image at SNR=20, (e). Reconstructed Output Image at SNR=25,(f). Reconstructed Output Image at SNR=30.

B. Simulation result of CoSaMP with LMS tool for image data for different SNRs.

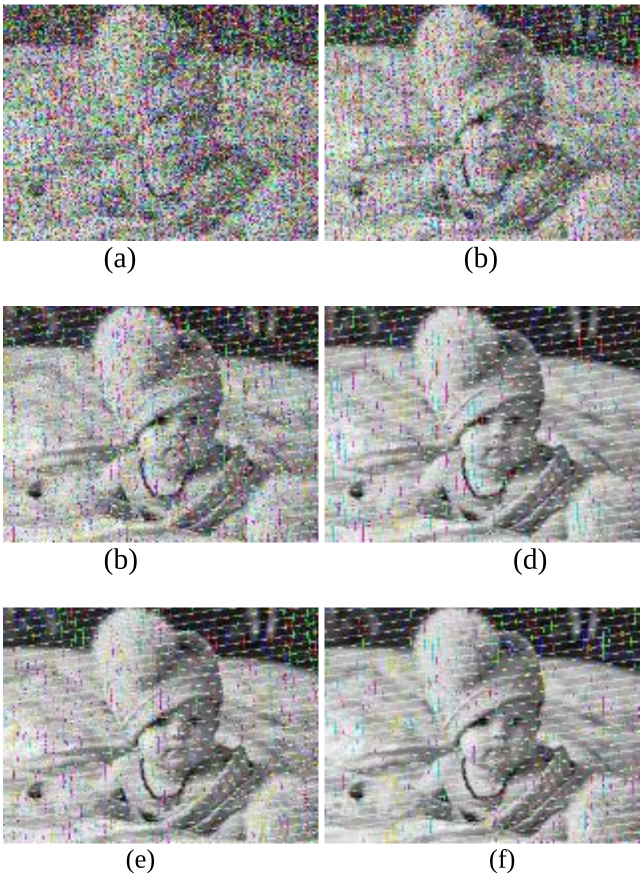


Figure 11. (a)Reconstructed Output Image at SNR=5, (b) Reconstructed Output Image at SNR=10, (c). Reconstructed Output Image at SNR=15,(d). Reconstructed Output Image at SNR=20, (e). Reconstructed Output Image at SNR=25,(f). Reconstructed Output Image at SNR=30

V. CONCLUSION

This paper presents the SP, OMP ,CoSaMP and DCS algorithms combined with LS and LMS tools for MIMO-OFDM channel estimation. The results shows that the DCS out performs the existing SP, OMP and CoSaMP algorithms combined with LMS tool with less complexity. And also as the Sparsity level increases i.e., the number of non zero coefficients are more, the NMSE reduces. Further work will continue on application of CS theory for sparse channel estimation with increase in number of antennas i.e., in highly multipath environment. Also, less complex CS algorithms are to be designed for the accurate channel estimation.

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