Integer Optimization Model of Dynamic Interactions in Social Networks

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Abstract— A social network represents a social structure containing a set of people or groups of people, with some pattern of interactions or ties between them. In most general representation, a social network can be viewed as a network of nodes (people) related to one another using edges (relationship). People interact with different numbers of individuals and with some individuals more than others and this affects behavior in fundamental ways. In reality the characteristic of the interactions is dynamic. The concept of centrality is used to measure the importance of a node's position in the network. In order to show the dynamic nature of a node the concepts of reciprocity and transitivity are used. In the optimization formulation, this paper proposes an integer linear programming model to analyze the dynamic interactions in the social networks. A feasible neighborhood search is used to solve the model. The result shows the optimal degree of dynamic interactions of a node.

Keywords- *optimization, social networks, centrality, graph, feasible neighborhood search.*

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I. INTRODUCTION

A social network can be defined as a set of people or groups of people which has some pattern of interactions or ties between them ([35], [3]). These patterns could be friendship among a group of individuals, in industry there are business relationships , and for families we have intermarriages. These are all examples of networks that have been studied in the past. From these examples we can say that social network effects can be used for understanding human behavior. People interact with different numbers of individuals and with some individuals more than others and this affects behavior in fundamental ways.

The rapid growth of online social networks has led to a resurgence of interest in research on several fields such as sociology, economics, physics, mathematics and public health. These research aim to study all kinds of phenomena. Interaction is very useful for analyzing diffusion of information. In economics, through social networks managers are able to comprehend and predict economic outcomes [16] and, in particular, to interface with both external and internal actors. The research of using social network in employment prospects ([21]; [16]), for investment ([10]) and productivity ([25]). The study of social network in behaviors phenomena such as crime [15]. In health sciences social networks have been studied as determinants of health (reviewed by ([31]), ranging from determining the patterns of infectious disease spread([17]; [28]; [19]) to the propagation of behaviors such as smoking cessation ([9]), regarding to obesity ([8]) and behavior toward suicidal ideation ([5]). Networks play great importance in evolutionary biology. In this case population structure which describe interactions can be used to facilitate the evolution of cooperative behavior ([24]; [33], [34]; [23]; [14]).

In general, a network is used to grasp information on social interactions. Each individual is represented by a node in the network, and there is an edge between two nodes if a social interaction has occurred at any point in time between the two individuals represented by these nodes. The conceptualization of social systems as graphs and networks offered the opportunity for systematic investigation and theorizing of the structure of ties among social actors beyond the pair. Whereas classical sociology tended to make a quantum leap from the individual and the pair to the triple, group, or society, graph theory offered the tools to formally describe and visualize social structure consisting of three and more actors.

Let $N = \{1, 2,\dots, n\}$ be a set of network nodes, with each node representing a social actor. The actors are often persons, but may also be groups, organizations or other social entities.

A graph can be used to represent social network in a way of specifying relationships among each node of a network. The relationship is represented by links called edges.

Using graph, this network model of social interactions has a clear understanding mathematically.. Unfortunately, from the structure point of view, this model has a major drawback is that it is essentially static in that all information about the dynamic relationship among actors is discarded. The static nature of the model can give inaccurate or inexact information about patterns in the social activities of actors.

In optimization point of view, we can use the concept of centrality to characterize the measurement whether an actor's position is the most important (or popular). The concept of centrality as applied to social communication was introduced already by [4], since then many different measures of centrality have been proposed (see, for example, [13]; [6]; [35]; [1]; [29]; [20])

. Research on the optimization of a network generally comes under the heading of discrete optimization. Therefore, in this paper, we propose a new discrete model and computational framework that enables analysis of dynamic social networks and that explicitly makes use of information about the time that social interactions occur. The model formulation is based on degree of centrality. In order to show that the actor is dynamic we use the concept of reciprocity and transitivity. We propose a feasible neighborhood search for solving the model.

II. CENTRALITY

Centrality is one of the most application tools in social network analysis. This concept is a rough indicator to measure the social power and the influence of a node based on the degree of influence of a node (actor) in the network. Previous empirical studies ([2]; [26]; [30]; [32]; [35]) provide evidence that a node (individuals or organizations) with high centrality showed an extraordinary socioeconomic position and also realized excellent levels of performance. Due to the fact that, the node can communicate directly with many other nodes. There are several types of centrality with unique characteristics, such as degree centrality, betweenness centrality, and closeness centrality. Degree centrality denotes the extent of homogeneity or heterogeneity in structural position, which is defined as the range and variability of degree [13]. Thus, degree centrality is used in this study for analyzing descriptive views of popularity of a node (actor) in the networks.

Betweenness centrality, introduced by [13] and [20], represents centrality as a mediator or an intermediary ([18]). Since betweenness centrality signifies the extent of communication to which a node lies between other pairs of nodes, it is defined as the proportion of all the shortest paths between pairs of other nodes that pass through the node. (actor) Closeness centrality, on the other hand, denotes the ability to access information or to communicate of a node (actor) through other nodes. Therefore, it focuses on how closely a node is connected to the other nodes in a network ([11]; [27]). The following Eqs. (1)–(3) show the mathematical forms of degree centrality, betweenness centrality, and closeness centrality ([11]).

Degree centrality

$$C_i = \sum_{j=1}^{n} (Z_{ij} + Z_{ji}) / i = \sum_{i=1}^{n} \sum_{j=1}^{n} Z_{ij} \tag{1}$$

where Z_{ij} = number of degree that a node i receives information from a node j and n = number of existent nodes.

Betweenness centrality (of node i)

For each pair of node (j,k), compute the shortest path between them

$$B_i = \sum_{j,k:sj \neq k \neq i} \frac{\sigma_i(j,k)}{\sigma(j,k)} \tag{2}$$

Where $\sigma_i(j,k)$ = the number of shortest paths from node j to node k that pass through node i

Closeness centrality (of node i)

$$Cl_i = \frac{n-1}{\sum_{k \in N} d(i,k)} \tag{3}$$

where n = number of nodes; N = total nodes; k = kth node in the network; and $d(i,k)$ = the length of the shortest path between node i and k.

A. Forbidden Tie

A simple directed graph from node v to node w in G is called a forbidden tie or an exception if an actor is not allowed to have a tie to actor w due to the physical constraints. Given a set X of forbidden tie, a tie $(v_1, v_2, v_3, \ldots, v_l)$ is said to avoid E if $(v_i, v_i + 1, \ldots, v_j) \notin E$ for all i, j such that $1 \leq i < j \leq l$. A network P from s to t is called a shortest E-avoiding tie if the length of P is the shortest among all E-avoiding tie from s to t. We will use the term "exception avoiding" instead of "X-avoiding" when E is equal to X, the set of all forbidden paths in G.

III. SOCIAL NETWORK MODEL

We build a model for social network based on graph formulation.

In many social settings, such as Facebook, Centrality measures address the question, "Who is the most popular person in this network?".

Based on this conjunction, the objective of the social network model is to maximize the centrality of node i. The constraints of the model consist of Density of a network's connectivity (D), Betweenness centrality (B), and Closeness centrality (Cl). In the model we impose a forbidden tie condition for a node (an actor).

The model can be formulated as a 0-1 integer programming problem.

$$\max \sum_{i \in \delta^{-1}(i);(i,j) \in E; i,j \notin x} C_i x_{ij} \tag{4}$$

Subject to

$$\sum_{(i,j) \in \delta^{+}(i)} x_{ij} \leq D_i \qquad \forall i, j \in N; i, j \notin X \tag{5}$$

$$\sum_{(i,j) \in \delta^{-}(i)} x_{ij} = \sum_{(v,j) \in \delta^{+}(i)} x_{ij} \qquad i \notin X; \forall i \in N \tag{6}$$

$$\sum_{(i,j) \in \delta^{+}(i)} \left(\tau_{ji} x_{ij} \right) \geq B_i \qquad i, j \notin X; \forall i \in E \tag{7}$$

$$\sum_{(i,j)\in\delta^{-}(i)} \left(\tau_{ij} x_{ij}\right) \le Cl_i \qquad i,j \notin X; \forall i \in E \tag{8}$$

$$x_{ij} \in \{0,1\} \qquad (i,j) \notin X, \forall (i,j) \in E \tag{9}$$

τ_{ij} consumption or prevalence factor.

IV. MODELING SOCIAL NETWORK DYNAMIC

There are several important points are necessarily to be satisfied in order we can say that a person (actor) has a dynamic interactions in the social network. These points are:

a) The number of outdegree ties,
b) Reciprocal relationship,
c) Transitivity interaction, and
d) Equilibrium.

Now we can formulate the model with the objective to maximize degree of centrality, the number of outdegree ties, and reciprocity relationship. The model can be formulated as a binary integer programming problem, which can be written mathematically as follows.

$$\max \sum_{i\in\delta^{-1}(i);(i,j)\in E;i,j\notin x} c_j x_{ij} + \sum_{(i,j)\in E} \delta_i^{+} x_{ij} + \sum_{(i,j)\in E} \rho x_{ij} \tag{10}$$

Subject to

$$\sum_{(i,j)\in\delta^{+}(i)} x_{ij} \le D_i \qquad \forall i,j \in N; i,j \notin X \tag{11}$$

$$\sum_{(i,j)\in\delta^{-}(i)} x_{ij} = \sum_{(v,j)\in\delta^{+}(i)} x_{ij} \qquad i \notin X; \forall i \in N \tag{12}$$

$$\sum_{(i,j)\in\delta^{+}(i)} \left(\tau_{ji} x_{ij}\right) \ge B_i \qquad i,j \notin X; \forall i \in E \tag{13}$$

$$\sum_{(i,j)\in\delta^{-}(i)} \left(\tau_{ij} x_{ij}\right) \le Cl_i \qquad i,j \notin X; \forall i \in E \tag{14}$$

$$x_{ij} \in \{0,1\} \qquad (i,j) \notin X, \forall (i,j) \in E \tag{15}$$

V. THE ALGORITHM

To solve the 0-1 integer programming model, we adopt the approach of examining a reduced problem in which most of the integer variables are held constant and only a small subset allowed varying in discrete steps.

The steps of the procedure can be summarized as follows.

Step 1. Solve the problem ignoring integrality requirements.

Step 2. Obtain a (sub-optimal) integer-feasible solution, using heuristic rounding of the continuous solution.

Step 3. Divide the set I of integer variables into the set I_1, at their bounds that were nonbasic at the continuous solution, and the set I_2, $I = I_1 + I_2$.

Step 4. Perform a search on the objective function, maintaining the variables in I_1 nonbasic and allowing only discrete changes in the values of the variables in I_2.

Step 5. At the solution in step 4, examine the reduced costs of the variables in I_1. If any should be released from their bounds, add them to the set I_2 and repeat from step 4, otherwise terminate.

It should be noted that the above procedure provides a framework for the development of specific strategies for particular classes of problems.

The integer results are kept in superbasic variables set. Then we conduct an integer line search to improve the integer feasible solution [20].

VI. CONCLUSIONS

This paper presents a mathematical model of dynamic social network based on graph approach. The objective of the model is to maximize centrality of an actor restricted to some constraints. We solve the model using feasible neighbourhood search.

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