Numerical Vibration Analysis of Rectangular Beams for Different End Conditions

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Abstract: In this study the vibration behaviour of rectangular beams with different end conditions i.e. for simply supported and cantilever type is determined. This behaviour is estimated for two different materials namely aluminum (Al6063L) and mild steel (304L). Initially five natural frequencies for simply supported and cantilevered conditions are found out using Rayleigh-Ritz method for both aluminum and mild steel.Vibrational behaviour of rectangular beam under different end conditions and different material properties are carried out experimentally and results are validated.

Keywords: *Rayleigh-Ritz, Numerical analysis, rectangular beams.*

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I. INTRODUCTION

In engineering field vibration behaviour of an element plays a key role without which it is incomplete. Resonance is a key aspect in dynamic analysis, which is the frequency of any system matches with the natural frequency of the system which may lead to catastrophes or system failure. Modal analysis has become a major alternative to provide a helpful contribution in understanding control of many vibration phenomena which encountered in practice [1].

Most of the works done so far have used theoretical analysis to find the natural frequencies of beams and validated it by using numerical analysis or FEM. Safa Bozkurt Coşkun, [2] have worked on Transverse Vibration Analysis of Euler-Bernoulli Beams using three different methods Adomian Decomposition Method (ADM), Variational Iteration Method (VIM) and Homotopy Perturbation Method (HPM) in order to find the analytical solution and then used FEM to validate them. Similarly J.P. Chopade and R.B. Barjibhe [3] have worked on Free Vibration Analysis of Fixed Free Beam using Euler-Bernoulli equation to find the natural frequencies theoretically and used Ansys to validate. Vibration analysis of a rectangular plate can be performed analytically for simply supported end condition. For all other combinations of boundary conditions a numerical procedure is used as for instance the Rayleigh–Ritz method or FEM, Ivo Senjanović, Marko Tomić, Nikola Vladimir, Neven Hadžić [4].

In this paper, Rayleigh–Ritz method is used to find the theoretical solution and Ansys in order to find the numerical solution. Also, an experimental validation is performed for cantilever and simply supported boundary conditions for two different materials.

II. THEORITICAL ANALYSIS

a) *Euler Bernoulli beam theory*

Euler Bernoulli beam theory also know as engineer's theory or classical beam theory is a simplification of the linear theory of elasticity which provides a means of calculation the load-carrying and deflection characteristics of beam. It covers the case for small deflections of beam that is subjected to lateral load only. It is thus a special case of Timoshenko beam theory [5].

$$EI\frac{\partial^4\omega}{\partial x^4}+\rho A\frac{\partial^2\omega}{\partial t}=f(x,y) \tag{1}$$

$$\frac{\partial^2 4}{\partial x^2}(E\frac{\partial^2\omega}{\partial x^2})+\rho A \tag{2}$$

If the beam is uniform, i.e. EI is constant; the equation of motion in above Eq. reduces to

$$c^2\frac{\partial^4\omega}{\partial x^4}+\frac{\partial^2\omega}{\partial t^2}\quad c=\sqrt{\frac{EI}{\rho A}} \tag{3}$$

Transverse vibrations of beams are an initial-boundary value problem. Hence, both initial and boundary conditions are required to obtain solution w(x, t). Since the equation involves a second order derivative with respect to time and fourth order derivative with respect to a space coordinate, two initial conditions and four boundaries are needed.

b. *Modal analysis*

The solution for problem is given by Equation. (1), it can be produced by first obtaining the natural frequency and mode shapes and then expressing the general solution as summation of modal responses. In each mode, the system will vibrate in a fixed shape ratio which lead to providing a separable displacement function into two separate time and space function w(x,t) can be defined by the following from.

$$W(x,t) = Y(x)T(t) \tag{4}$$

Consider the free vibrations problem for a uniform beam, i.e. EI is constant. The governing equation for this specific case previously was given in Equation (2). The free vibration solutions will be obtained by inserting Equation (4) into (2) and rearranging it as

$$\frac{c^2}{Y(x)}\frac{\partial^4 Y4(x)}{\partial x^4} = -\frac{\partial T(t)}{\partial t^2} \tag{5}$$

Where c is defined in (3) and (4) is a constant. Equation (5) can be rearranged as two ordinary differential equations as

$$\frac{\partial^4 Y4(x)}{\partial x^4} - \lambda^4 Y(x) = 0 \quad \frac{\partial^4 T(t)}{\partial t^2} + \omega^2 T(t) \quad \lambda^4 = \frac{\omega^2}{c^2} \tag{6}$$

General solution of equation (6) is a mode shape and consider as

$$Y(x) = C_1 \cosh \lambda x + C_2 \sinh \lambda x + C_3 \cos \lambda x + C_4 \sin \lambda x \tag{7}$$

The constant C_1, $C_{2,}$ C_3 and C_4 can be found the end conditions of the beams. Then, the natural frequency of the beam is obtained from Equation (6) as

$$\omega = \lambda^2 c \tag{8}$$

Inserting Equation (3) into Equation (11) with rearranging leads to

$$\omega = (\lambda L)\sqrt{\frac{EI}{\rho A L^4}} \tag{9}$$

Equation 9 is used to determine the natural frequencies for both cantilever and simply supported conditions. The calculations are done taking into account the material properties of two different materials. The results obtained from these calculations are tabulated as shown in table-1 for cantilever condition and table-2 for simply supported condition. Graphs are developed form these results for comparing the results for two materials.

Table 1: Comparison of natural frequency (Hz) for cantilever beam

S.no	Thickness (mm)	Steel(304L) (Hz)	Aluminium(6063L) (Hz)
1	10	840.12	842.73
2	20	1402.3	1412
3	30	1703.8	1745.7
4	40	1954.8	1942.8
5	50	2089	2064.7

Table 2: Natural frequency (Hz) for simply supported

S.no	Thickness(mm)	Steel(304L) (Hz)	Aluminum(6063L) (Hz)
1	10	972.8	940.11
2	20	1628.7	1698.4
3	30	2018.23	2052.7
4	40	2007.7	2052.7
5	50	2084.7	2064.7

III. MODAL ANALYSIS IN ANSYS:

The main aim of using Ansys is to validate the theoretical results with those of the numerical results.Beam 2node 188element is used to perform the numerical analysis using Ansys, on to which the boundary conditions for simple supported and cantilever are imposed. Modal analysis is performed in order to obtain the natural frequencies for both simply supported and cantilever conditions. This is repeated for two different materials, namely aluminum and steel.

a) *Cantilever beam*

Table 3: Natural frequency(Hz) for cantilever beam

Thickness (mm)	Steel(304L) (Hz)	Aluminium(6063L) (Hz)
10	842.63	840.2
20	1495.23	1498.5
30	1672.89	1758.4
40	2053.3	2010.2
50	2081	2095.7

b) *Simply supported*

Table4: Natural frequency(Hz) for simply supported

Thickness (mm)	Steel(304L) (Hz)	Aluminium(6063L) (Hz)
10	940.34	1012.3
20	1598.63	1418.78
30	2042.23	1983.56
40	1956.7	1983.56
50	2071.7	2154.29

IV. EXPERIMENTAL ANALYSIS

Beam are fabrication with same dimensions as that are consider in analytical analysis to compare results for both steel and aluminum materials. For both cantilever and simply supported conditions are fixtures are fabricated separately.

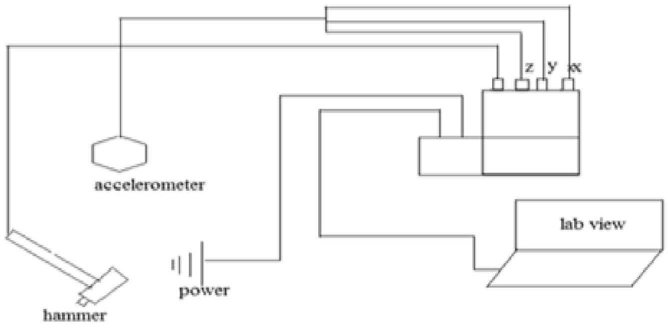

Fig1: Experimental setup line diagram

Fig2: Experimental setup for simply supported beam

Fig3: Experimental setup for cantilever beam

The experimental setup for the experimental analysis is shown in the fig5. Table 5 shows the experimental results for two materials

Table5: Natural frequency (Hz) for Cantilever and simply supported beams

Type of beam	Thickness (mm)	Steel(304L)	Aluminium(6063L)
Cantilever	10	890	901
Simply supported	10	910	990

V. RESULTS AND DISCUSSIONS:

In this project, Rayleigh Ritz method has been used in order to obtain the analytical solution for both cantilever and simply supported conditions taking into account two different material properties. Analytical solution is validated with experimental analysis using Fast Fourier Transform Analyzer. The experiment is conducted for one condition of cantilever and one condition of simply supported. Whereas theoretical and experimental solutions are obtained for 5 different thicknesses. The results obtained from all the three i.e. theoretical, numerical and experimental are compared and shown in table6. Experimental result obtained for one case is generalized

Table6: Comparison of natural frequency for cantilever and simply supported beams

Type of analysis	Cantilever		Simply supported	
	Aluminium	Steel	Aluminium	Steel
Theoretical	842.73	840.12	940.11	972.8
Numerical	840.2	842.63	1012.12	940.34
Experimental	901	890	990	910

VI. CONCLUSION

It is observed that the natural frequencies for aluminum are slightly higher than that of mild steel for both the end conditions. Later Modal analysis is performed in Ansys by assuming the material conditions of aluminum and mild steel for both the end conditions. The natural frequencies of these two materials are estimated and compared and the following is observed

i. There is an error of 6% between the frequencies of steel and aluminum.
ii. An error of 5% is observed between simply supported and cantilevered conditions of steel
iii. An error of 4% is observed between simply supported and cantilevered conditions of aluminum.

Experimental investigations are performed for similar boundary conditions of both the materials. It was observed that the experimental results are in very close agreement with both theoretical and Ansys results.

VII. REFERENCES

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