PAPR reduction techniques in optical OFDM:A Review

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Abstract—In the past few years, number of users and demand for bandwidth has increased due to increased usage of internet and real time applications like video and audio streaming and Voice over IP (VoIP). OFDM has proved to be a very reliable technique in optical communication to satisfy these needs by providing high data rates along with robustness against fiber impairments like chromatic dispersion and polarization mode dispersion. But it suffers from major problem of high peak to average power ratio. High PAPR induces nonlinearities in fibers due to Kerr effect as well as in ADC/DAC. Several techniques have been developed to combat this problem in OFDM systems. This paper reviews some of most common and recently developed PAPR reduction techniques in optical OFDM systems.

Keywords-*Orthogonal Frequency Division Multiplexing (OFDM), Peak to Average Power Ratio (PAPR), Partial Transmit Sequence (PTS), Selected Mapping (SLM), Sliding Norm Transform(SNT), Constant amplitude zero autocorrelation (CAZAC).*

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I. INTRODUCTION

The invention of the Internet has changed the information communication infrastructure. Many new applications, such as YouTube and IPTV have emerged which have increased the bandwidth demand. Optical communication systems act as a very good alternative to meet the increased demands on transmission capacity [1]. However, the performance of optical communication systems operating at high data rates is degraded significantly due to several transmission losses like fiber nonlinearities, polarization mode dispersion (PMD), and chromatic dispersion. Orthogonal frequency division multiplexing (OFDM) has acted as a reliable technique which can suppress chromatic dispersion as well as PMD along with transmission of information at high data rates [2].

Orthogonal frequency-division multiplexing (OFDM) belongs to a broader class of multicarrier modulation (MCM) in which the data information is carried over many lower rate subcarriers. The subcarriers used are orthogonal to each other. Hence it carries large information with small bandwidth demand. The orthogonality of subcarriers reduce inter-symbol interference and the guard interval present in OFDM prevents inter-carrier interference. In optical communication use of OFDM prevents PMD and chromatic dispersion which make it a desirable technique for fiber-optic communication [2].

However, in spite of all these advantages, OFDM suffers from one major problem of high peak to average power ratio (PAPR) which means that in OFDM, there is a huge difference between average signal power and peak signal power. The PAPR of OFDM signal s(t) can be evaluated as-

$$PAPR\{s(t)\} = \frac{max\,|s(t)|^2}{E\{|s(t)|^2\}} \qquad (1)$$

It happens because in OFDM, large numbers of subcarriers are present. They can be added constructively to give large peak power but due to destructive interference average power might be low which results in large differences between peak signal power and average signal power. Presence of high PAPR affects the performance of optical OFDM system because high PAPR induces fiber nonlinearities such as self-phase modulation (SPM) and cross phase modulation (XPM) in fiber transmission. It occurs because effective refractive index within fiber is dependent on input power by following relation:

$$n_{eff} = n_1 + \frac{n_1 P}{A_{eff}} \qquad (2)$$

Where P is the input power and $A_{eff}$is the effective core radius. Hence if input power exceeds a particular value, then it will induce nonlinearities in fiber due to Kerr effect. Also high PAPR requires large dynamic range of ADC/DAC and linear power amplifier for their operation in linear region, failing which they will enter non-linear region and some signal may be lost or get distorted [3].

Hence to avoid the occurrence of all these losses, PAPR of OFDM signal should be reduced. For this, several PAPR reduction techniques are used.

II. PAPR REDUCTION TECHNIQUES

Several techniques have been proposed to reduce PAPR in optical OFDM systems. Before selecting any PAPR reduction technique, several factors are needed to be considered. Some of these factors include PAPR reduction capacity, implementation complexity, power increase in transmitted signal, degradation in data rate, and increase in the bit-error rate at the receiver end. In this paper, some of most commonly used along with recently proposed PAPR reduction techniques have been reviewed. Some of these techniques include signal clipping, SLM, PTS, non-linear companding, sliding norm transform (SNT) and constant amplitude zero autocorrelation (CAZAC) etc. These techniques are discussed below:

A. Amplitude Clipping and Filtering

Amplitude clipping is the simplest approach to reduce PAPR among all PAPR reduction techniques. In this technique, a threshold value of amplitude is predefined and a soft limiter is employed to clip the peaks of signal which are above the predefined threshold value and the rest of signal is passed unclipped. If x[n] is the input OFDM signal, then clipping function is defined as:

$$F(x[n])=\begin{cases} x[n], & \text{if } |x[n]|\leq A \\ Ae^{j\angle x[n]} & \text{if } |x[n]|>A \end{cases} \quad (3)$$

where A is the predefined threshold level and $\angle x[n]$ is the angle of x[n].

Thus it reduces high PAPR of signal without any additional power and bandwidth requirement. But clipping is a non-linear process which leads to in- band distortion and out of band radiation in the signal which increases BER as well as affects the spectral efficiency [4]. Hence to remove these limitations, clipping is followed by filtering. However, filtering cannot remove in-band distortion but can reduce out of band distortion at the cost of some peak re-growth. A repeated clipping and filtering process can be implemented to achieve a desired amplitude level [5].

B. *Selected Mapping (SLM)*

In this technique, a set of data blocks is generated at the transmitter which represent the original information and then the most desirable block is chosen among them. Let us consider an OFDM system with N subcarriers. A data block is a vector $X=(x_n)_N$ which consist N complex symbols x_n , each of them representing modulation symbol transmitted over a sub–carrier. The input data blocks are multiplied by V different phase sequences. These sequences are denoted by vector $B_v= [b_{v,0}, b_{v,1}, \ldots b_{v,N-1}]$, v=0,1,…V-1 where each vector is composed of N complex symbols $b_{v,n}$. The resulting vector denoted by X_v is given by $X_v=[x_0b_{v,0}, x_1b_{v,1}, \ldots x_{N-1}b_{v,N-1}]$, v=0,1,2, ….V-1.

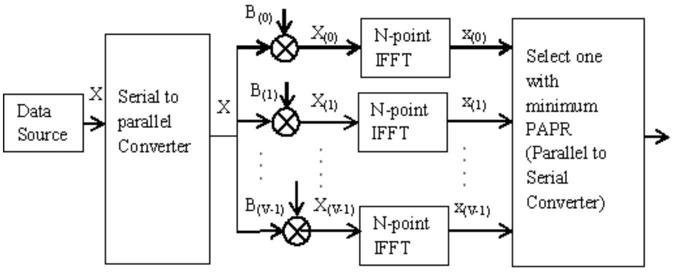

Figure 1: Block Diagram of SLM

Each of the modified sequence is further processed under IFFT operation and among those blocks X_v, v=0,1,2….V-1, the signal with lowest PAPR is selected for transmission.At the receiver, inverse operation is performed to recover original signal from modified sequence [6]. It requires side information of phase sequences to be sent with information signal.The extent to which PAPR is reduced in SLM depends on the number and design of phase sequences V [7].

C. *Partial Transmit Sequence(PTS)*

In this technique, the frequency domain input data block X is partitioned into V disjoint sub-blocks $X_m=[X_{m,0},X_{m,1},X_{m,2}\ldots X_{m,N-1}]^T$ such that $\sum_{m=0}^{V-1} X_m = X$. In general, the sub-block partitioning can be done in three ways: adjacent partition, interleaved partition and pseudorandom partition. Then these sub-blocks are transformed into time domain by taking IFFT of these sub-blocks and are weighted by a phase factor $w_m = e^{j\varphi m}$, $m = 1,2 \ldots . V-1$. Thephase factor should be chosen in such a way that PAPR can be minimized as:

$$\left[\widetilde{w_0}\ldots\widetilde{w_{V-1}}\right] = \arg\min\left(\max\left|\sum_{m=0}^{V-1} w_m X_m[n]\right|\right) \quad (4)$$

where n=0,1,2…(LN-1), L is oversampling factor.

Then the corresponding time domain signal with lowest PAPR is given by:

$$\tilde{x} = \sum_{m=0}^{V-1} \tilde{w}_m X_m \quad (5)$$

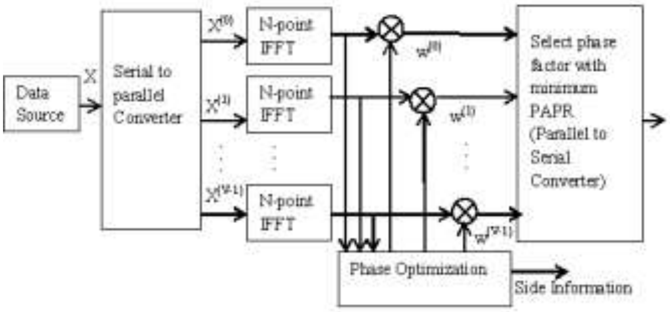

Figure 2.Block Diagram of PTS

It has proved to be a very good technique to reduce PAPR in OFDM. But it suffers from two major problems: The complexityof searching for the optimum set of phase vector as well as increase in IFFT operations when the number of sub-blocks increases and the overhead of phase factors used at transmitter side are needed to be transmitted as side information for proper decoding of the transmitted information [6][8].

D. *Tone Reservation*

This technique works on the concept of reserving some unused sub-carriers to produce a signal having reduced peak power. In this technique, some subcarriers are reserved and are not used for transmission. These sub-carriers are called peak reduction carriers (PRC).

The main objective of TR technique is to find optimal values of a time domain signal c to be added to the original signal x so that the PAPR of the transmitted OFDM signal can be minimized. Let $\{c=c_n|\ n=0,1,2..N-1\}$ denote a set of complex symbols for tone reservation at PRCs. These symbols are added to the signal during tone reservation processing and hence the modified OFDM signal is given as:

$$Y = IFFT(x + c) = X + C \quad (6)$$

The aim of achieving the proper value of c to make vector Y with reduced PAPR can be solved as a convex optimization problem that can be cast as a linear programming problem. To achieve distortion less data transmission, the reserved symbol vector C and the data symbol vector X are restricted to lie in the disjoint frequency subspaces as:

$$X_n + C_n = \begin{cases} X_n & n \in S \\ C_n & n \in S^c \end{cases} \quad (7)$$

where S denotes the index set of the data-bearing subcarriers and S^c represents the index set of the PRCs [9].

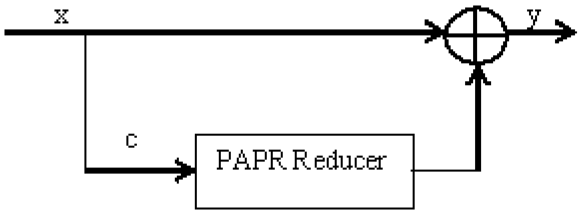

Figure 3: Tone reservation

Since there is no need to transmit the tones information separately with information signal, it reduces overhead compared to SLM and PTS. The receiver only needs to know about the positions of PRCs so that they can be ignored. Hence this technique is less complex and requires no side information to be transmitted. The only problem is that signal power transmitted increases as some of power must be used for reservation of tones [6].

E. Non-linear Companding based PAPR Reduction

In this technique, pre-distortion is applied to the OFDM signal at the transmitter side according to the power distribution of the signals. There are various non-linear companding transform techniques used for reducing PAPR but in optical communication µ-law companding has been used. Here the transmitted signal is compressed at the transmitter and expanded at the receiver. Compression is achieved by using following expression:

$$y = V\frac{\log\left(1+\mu\frac{|x|}{V}\right)}{\log(1+\mu)} sgn(x) \quad (8)$$

where V=peak amplitude of the signal, and x=instantaneous value of the input signal.

Compression involves enlarging small amplitude signals while maintaining amplitude of large signals unchanged resulting in improving quantization resolution of small amplitude signals and lowering the resolution of large signals.

At receiver, expanding is achieved by inverse of above expression as:

$$x = \frac{V}{\mu}\left[\exp\left(\frac{(|y|\log(1+\mu))}{V}\right) - 1\right] sgn(y) \quad (9)$$

In this way, signal amplitude is redistributed after transformation which results in reduced PAPR. This also introduces quantization noise, but, the effect of the quantization noise due to reduction in resolution of the peaks is relatively small as the peaks occur less frequently [10]. As µ is increased, the gain of PAPR reduction and noise is increased. Hence, companding parameters should be chosen in such a way that significant noise can be avoided [11].

F. Sliding Norm Transform

Two types of non-linear sliding norm transform based techniques have been proposed to reduce the PAPR of transmitted OFDM signal. These include L2-by-3 sliding norm transform and modified sliding norm transform (MSNT).

In discrete sliding norm transform technique, after performing IFFT, the input vector $x=[x_0,x_1,x_2..x_{N-1}]^T$ is transformed to a vector $y=[y_0,y_1,y_2....y_{N-1}]$ using a non-linear discrete transform as:

$$y_n = \begin{cases} x_0, n = 0 \\ \frac{x_n}{\sqrt{\sum_{k=0}^{n} {x_k}^2}}, n = 1,2,\ldots.N-1 \end{cases} \quad (10)$$

This discrete transformation given in equation is called L2-sliding norm transform. In order to simplify calculations, a special case of L2-DSNT called L2-by-3 is generally used. It uses three input samples x_{n-1}, x_n, x_{n+1} and a controlling parameter α in each sliding window to calculate output samples y_n as:

$$y_n = \frac{x_n}{\sqrt{\alpha + {x_{n-1}}^2 + {x_n}^2 + {x_{n+1}}^2}} \quad (11)$$

At the receiver side, the input signal x can be reconstructed using a system of equations defined by a matrix as:

$$\begin{bmatrix} \lambda_0 & 1 & 0 & 0 & 0 & 1 \\ 1 & \lambda_1 & 1 & 0 & 0 & 0 \\ 0 & 1 & \lambda_2 & 1 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & 1 & \lambda_{N-2} & 1 \\ 1 & 0 & \ldots & 0 & 1 & \lambda_{N-1} \end{bmatrix} \begin{bmatrix} {x_0}^2 \\ {x_1}^2 \\ {x_2}^2 \\ \vdots \\ {x_{N-2}}^2 \\ {x_{N-1}}^2 \end{bmatrix} = -\alpha \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

(12) where $\lambda_n = 1 - \frac{1}{{y_n}^2}$

From above equation, the original data x_n can be recovered using following equation:

$$x_n = \sqrt{{x_n}^2}\, sgn(y_n) \quad (13)$$

where "sgn(.)" is signum function.

L2-by-3 SNT technique reduces PAPR at lower computational complexity without the need of side information transmission or increase in transmitted signal power. It also improves BER of the system by reducing out-of-band distortion as observed in clipping. But it degrades the Power Spectral Density (PSD) of transmitted signal at lower value of parameter α [12].

To overcome this problem, modified Sliding Norm Transform (MSNT), which is a modification of L2-by-3 SNT, is used. In this technique, instead of using three complex value samples, only two input samples (x_{n-1} and x_n)and their absolute values are used in any sliding window, so that the computational complexity is less than that of the L2-by-3 method. Also along with parameter 'a', another parameter 'b' ($1 \le b \le 10$) is used to adjust the PSD and PAPR values in the output signal. Thus, IFFT of signal is followed by transformation of signal as:

$$y_n = \begin{cases} x_0 , n=0 \\ \dfrac{x_n}{\sqrt{a+b.|x_n|^2+|x_{n-1}|^2}} ,n=1,2,....N-1 \end{cases} \quad (14)$$

To further reduce the PAPR, the frame obtained above is reversed and then MSNT is applied again. Finally the signal is passed through DAC after S/P conversion.

Since transform applied at transmitter side is reversible in nature, at the receiver side the original signal can be reconstructed as:

$$x_n = \begin{cases} y_0, \ y_n \neq 0, n=0 \\ y_n . \sqrt{\dfrac{a+|x_{n-1}|^2}{1-b.|y_n|^2}} \\ 0, \ y_n = 0 \end{cases} \quad (15)$$

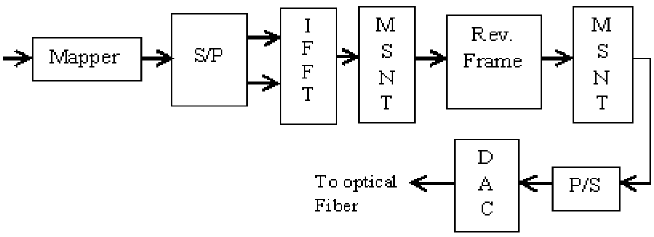

Figure 4: Block Diagram of MSNT Transmitter

MSNT algorithm simplifies transmitter and receiver side and have reduces PAPR as well as more efficient power spectral density as compared to L2-by-3 SNT algorithm [13].

G. CAZAC Matrix Transform

This technique has been recently proposed to reduce PAPR in optical OFDM. It is a very simple technique which reduces PAPR of the OFDM system with low computational complexity as well as improves receiver sensitivity and BER of the system.

In this technique, the input vector is transformed by a Constant Amplitude Zero Autocorrelation (CAZAC) transform matrix before applying IFFT. This results in the rotation of the input vector on the complex sphere in the N dimensional complex space, which results in reduced PAPR, due to lower values of side lobe of autocorrelation function of the transmitted signal.

The CAZAC sequence adopted is Zadoff-Chu sequence which is mathematically expressed as:

$$c_k = \begin{cases} exp[j\pi k(k-1)r/L]; \ L \ is \ odd \\ \exp\left[\dfrac{j\pi\,(k-1)^2 r}{L}\right]; \quad L \ is \ even \end{cases} \quad (16)$$

where k=1,2….L

This sequence is rearranged into N*N orthogonal transform matrix A with $L=N^2$ and r=1 as:

$$A = \begin{bmatrix} c_0 & c_N & \vdots & c_{N(N-1)} \\ c_1 & c_{N+1} & \cdots & c_{N(N-1)+1} \\ \vdots & \vdots & \ddots & \vdots \\ c_{N-1} & c_{2N-1} & \cdots & c_{N^2-1} \end{bmatrix} (17)$$

After normalizing by √N, the input vectorx=[x_0, x_1, ….x_{N-1}] modulated over N subcarriers is transformed by a CAZAC matrix A to obtain vector x_A=[x_{A0}, x_{A1}, …$x_{A(N-1)}$]. This transformed vector is then applied to IFFT operation.In CAZAC transform, the probability of in-phase combination of the signal peaks can be significantly reduced which leads to reduced PAPR [13].

Since matrix A is invertible in nature, at the receiver side, original information can be recovered by multiplying received signal with the inverse of matrix A after performing FFT operation as follows:

$x=y.\ A^{-1}$ (18)

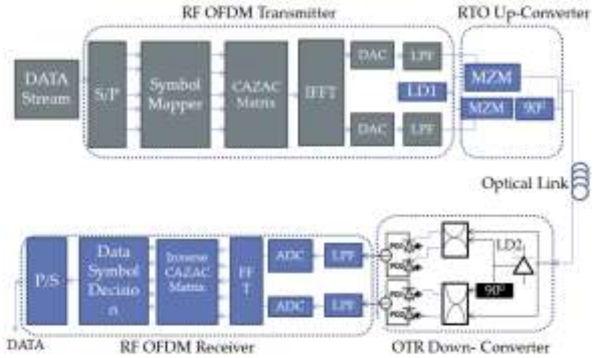

Figure 5: Block Diagram of CAZAC Matrix Transform

Hence this technique reduces PAPR of the OFDM system without the requirement of side information transmission or any data rate loss. It also improves BER of the system as well as receiver sensitivity of the system [14].

TABLE I: COMPARISON OF PAPR REDUCTION TECHNIQUES

PAPR Reduction Technique	**Power Increase**	**BER Degradation**	**Implementation Complexity**	**Data Rate Loss**
Clipping & Filtering	No	Yes	Low	No
SLM	No	No	High	Yes
PTS	No	No	High	Yes
TR	Yes	No	High	Yes
Compandin	No	No	Low	No
SNT	No	No	Low	No
CAZAC Matrix Transform	No	No	Low	No

CONCLUSION

OFDM is a very efficient and promising technique in high speed optical communication. It provides high spectral efficiency, reduction in chromatic and polarization mode dispersion as well as reduced inter-symbol interference. But it suffers from one major problem of high PAPR which results in non-linear losses along fiber as well as in ADC/DAC and high power amplifiers. In the literature,

Several techniques have been proposed which are used to reduce PAPR of OFDM systems. In this paper, seven PAPR reduction techniques are reviewed and are compared on the basis of computational complexity, BER performance, transmitted signal power and bit rate loss. All techniques can reduce PAPR significantly but different techniques have their own disadvantages also like clipping increases out-of-band and in-band distortion, SLM and PTS require transmission of side information and companding. Thus a proper PAPR reduction technique can be selected based on system requirements and available resources.

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