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Vibrations of Cracked Rotors

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Abstract—The analysis of dynamic behavior of rotors with cracks is considerably important for design aspect. In this study the effect of a single open crack on the free vibrational characteristics of a simply supported shaft are investigated using Finite element methodology. The rotor is modelled by using Timoshenko beam element. To account for the presence of the crack, the stiffness matrix of the beam element is modified. The beam element with modified stiffness matrix is then fit into the complete finite element assemblage representing a cracked rotor system. In this study the natural frequencies of cracked and un-cracked rotor are found using Ansys and then the variations in natural frequencies of the rotor with varying depths at a given location are estimated. This analysis is done for two different geometries of shaft and results are compared. Experimental investigations were carried out and frequencies of cracked and un-cracked rotor are determined and the results are compared.

Keywords- *Transverse surface cracks, Flexibility Coefficients,Frequency response.*

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I. INTRODUCTION

Development and propagation of cracks is one of the inchoate losses in Dynamic machine members. A crack may develop from some small imperfections on the surface of the body or inside of the material and mostly in the regions with higher stress concentration. Presence of crack can lead to a catastrophic failure in certain conditions. The importance of the early detection of the cracks dictates the need to study various aspects of behaviour of structures defected by cracks. One of the aspects is to analyse the vibrations of the cracked structures. Development of crack in a system changes the dynamic and vibrational behaviour of that system. With measurement and analysis of these vibrations the cracks can be identified in advance and suitable actions can be taken to prevent more damage to the system. One of a special cases of damage in mechanical structures is the development and propagation of transverse surface cracks in rotating shafts. The term transverse cracks indicate the cracks in which the crack surface is perpendicular to the axis of rotation the shaft. A significant amount of research involving the prediction of vibrationalresponse of rotor systems with a transverse crack and the detection of transverse cracks by vibration monitoring, has been completed in the past few decades but there is need for improvement on crack detection and its analysis.

II. BACKGROUND

Difficulty to inspect rotating shafts during the operation of the machines makes the detection of cracks much more difficult in these structures than in static (non-rotating) structures. Suitable symptoms are needed that can easily be measured and that are able to indicate clearly the presence of a crack in a rotating shaft. If these symptoms arise, machine may be stopped and a catastrophic failure may be avoided. Dynamic tests, based on vibration measurements are considered for identifying presence and location of cracks in complex structures.The accurate modelling of cracked shafts allows the simulation of its dynamic behaviour, as a function of different positions and depths of cracks. The comparison of calculated and measured vibrations allows the identification of not only the presence of a crack in a rotating shaft, but also of its location and depth.

This justifies the need for Reliable and accurate modelling for representing the crack and to estimate the change in the overall dynamic behaviour, caused by the local reduction of the stiffness due to presence of a crack and evaluating the response as a function of different positions and different depths of transverse crack.

In this present study shear, axial and torsional deformations in rotors are neglected and only bending vibrations of rotors in the presence of an open crack are considered. The changes in the fundamental frequencies for cracked rotors having L/D =40 and L/D=50 are presented.

III. FINITE ELEMENT MODELLING OF CRACKED ROTOR SEGMENT

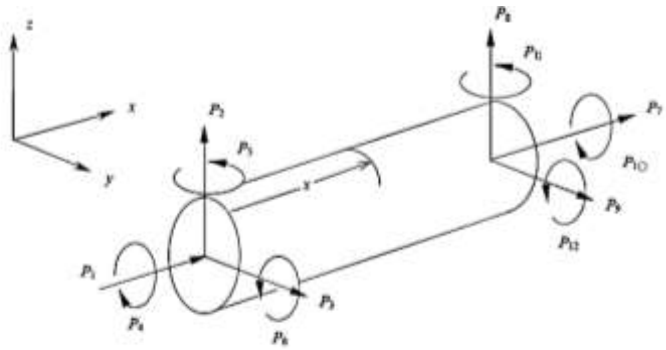

Figure 1. Shaft element showing forces acting on it [4].

Consider a shaft element containing a single transverse surface crack of depth "a" as shown in the Figure.2. To represent this segment in the finite element model of the cracked rotor, the rotor segment is represented by a beam element with six degrees of freedom per node. A small segment around the crack cross section in the rotor will be modelled as a finite beam element that will be different than the usual beam element with regard to the stiffness properties. Let the shaft element be of diameter "D" and length L. The element is loaded withbending moments P_5, P_6 and P_{11}, P_{12}, shear forces P_2, P_3 and P_8, P_9, axial forces P_1 and P_7 and torsional moments P_4, P_{10}.

A. *Flexibility of uncracked shaft*

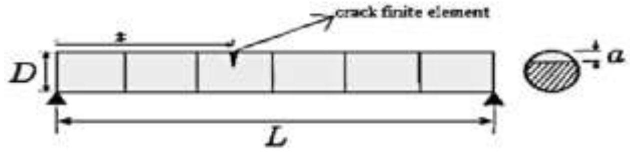

Figure 2. A simple Rotor and its finite element Model [4].

The flexibility of the uncracked shaft is found out using Castiglione's theorem from [1] i.e.

$$u_i = \frac{\partial U_o}{\partial P_i}$$

P_i is the force along i-th co-ordinate.
U_o is the total strain energy.
U_i is the displacement along the i-th co-ordinate.

$U_o = U_{axial} + U_{bending} + U_{shear} + U_{torsion}$

$$\text{Uaxial} = \int_0^x \frac{F^2}{2EA}\,dx$$

$$\text{Ushear} = \int_0^x \frac{\alpha_s V^2}{2GA}\,dx$$

$$\text{Ubending} = \int_0^x \frac{M^2}{2EI}\,dx$$

$$\text{Utorsion} = \int_0^x \frac{T^2}{2GI_o}\,dx$$

Hence

$$U_0 = \int_0^x \frac{M^2}{2EI} + \frac{\alpha_s V^2}{2GA} + \frac{F^2}{2EA} + \frac{T^2}{2GI_o}\,dx$$

Substituting all the forces and moments we get

$$U_0 = \frac{P^2 l}{AE} + \frac{\alpha_s {P_2}^2 l}{GA} + \frac{{P_2}^2 l^3}{3EI} + \frac{\alpha_s {P_3}^2 l}{GA} + \frac{{P_3}^2 l^3}{3EI} - \frac{P_3 P_5 l^2}{EI} + \frac{{P_4}^2}{GI_o} + \frac{{P_5}^2 l}{EI} + \frac{{P_6}^2 l}{EI} + \frac{P_2 P_6 l^2}{EI}$$

U_0 gives the total elastic strain energy of the uncracked shaft element, from which the individual displacements can be obtained as

$$u_1 = \frac{P_1 l}{AE}$$

$$u_2 = \left(\frac{\alpha_s\, l}{GA} + \frac{l^3}{3EI}\right)P_2 + \frac{l^2 P_6}{2EI}$$

$$u_3 = \left(\frac{\alpha_s l}{GA} + \frac{l^3}{3EI}\right)P_3 - \frac{l^2 P_5}{2EI}$$

$$u_4 = \frac{l P_4}{GI_o}$$

$$u_5 = \frac{P_5 l}{EI} - \frac{l^2 P_3}{3EI}$$

$$u_6 = \frac{P_6 l}{EI} + \frac{l^2 P_2}{2EI}$$

u_1, u_2, u_3, u_4, u_5, u_6, represents the individual nodal displacements. Thesecan be used to find out the flexibility influence coefficients.

"Flexibility Influence coefficient c_{ij} is defined as the deflection at a point i due to unitload at point j". Therefore,the flexibility coefficients obtained are:

$$C_{12} = C_{13} = C_{14} = C_{15} = C_{16} = 0 \text{ and } C_{11} = \frac{l}{AE}$$

$$C_{21} = C_{23} = C_{24} = C_{25} = 0 \text{ and } C_{26} = \frac{l^2}{2EI}, C_{22} = \frac{\alpha_s l}{GA} + \frac{l^3}{3EI}$$

$$C_{31} = C_{32} = C_{34} = C_{36} = 0 \text{ and} C_{33} = \frac{\alpha_s l}{GA} + \frac{l^3}{3EI} C_{35} = -\frac{l}{AE}$$

$$C_{41} = C_{42} = C_{43} = C_{45} = C_{46} = 0 \text{ and } C_{44} = \frac{l}{GI_O}$$

$$C_{51} = C_{52} = C_{54} = C_{56} = 0 \text{ and } C_{55} = \frac{l}{AE}, C_{53} = -\frac{l^2}{3EI}$$

$$C_{61} = C_{63} = C_{64} = C_{65} = 0 \text{ and } C_{66} = \frac{l}{EI}, C_{62} = \frac{l^2}{2EI}$$

By arranging all the flexible coefficients in the form of a matrix we get

$$C_0 = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{45} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{pmatrix}$$

Where C_0 represents the flexibility matrix of the uncracked shaft element. As only bending moments are taken into consideration (neglecting shear, torsion and axial effects) we get the flexibility matrix as

$$C_0 = \begin{pmatrix} C_{55} & C_{56} \\ C_{65} & C_{66} \end{pmatrix}$$

i.e.

$$C_0 = \begin{pmatrix} \frac{l}{EI} & 0 \\ 0 & \frac{l}{EI} \end{pmatrix} \qquad (1)$$

now the stiffness matrix of the uncracked shaft element can be obtained as

$$K_0 = \begin{pmatrix} {C_0}^{-1} & -{C_0}^{-1} \\ -{C_0}^{-1} & {C_0}^{-1} \end{pmatrix} \; (2)$$

B. *Flexibility of Cracked shafts*

According to Saint Venant's, stressfield is effected only in the region adjacent to the crack from [1]. Hence stiffness except for thecrack element is regarded unchanged. The flexibility coefficient, expressed by a stressintensity factor, is derived by means of Castiglione's theorem.

$$u_i = \frac{\partial U}{\partial P_i}$$

Where

$U = U_o + U_c$.

U is the total strain energy.
U_o is the strain energy of uncracked element.
U_c is the strain energy of cracked element.

hence we get

$$C_t = C_o + C_c$$

C_t is the total flexibility of the cracked shaft, C_o is the flexibility of uncracked element and Cc is the flexibility of cracked element. $C_o$is obtained from (1). To evaluate c_c displacement of the cracked elastic structure due to action of force given by Paris [2] is used, which is

$$u = \frac{\partial}{\partial P}\int_0^{\eta} J(\eta)d\eta \qquad (3)$$

where η is the depth of the crack and J(η) is the strain energy density function, which for a beam with a crack of depth η is,

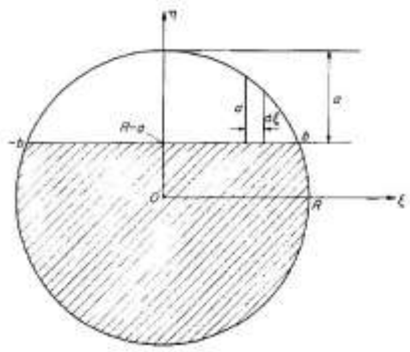

Figure 3. Geometry of cracked section of shaft [5]

$$J = \frac{(1-\upsilon^2){K_1}^2}{E}$$

For a crack with varying depth, the strain energy density is,

$$J = \int_{-b}^{b}\frac{(1-\upsilon^2){K_1}^2}{E} \qquad (4)$$

Where E is the young's modulus, υ is the Poisson ratio and 2b the width of the crack. The local flexibility due the crack is obtained by substituting (4) in (3) and partially derivating with respect to the corresponding force.

$$C = \frac{\partial u}{\partial P} = \frac{\partial^2}{\partial P^2} = \int_0^{\eta}\int_{-b}^{b}\frac{(1-\upsilon^2){K_1}^2\{\xi\}d\xi d\eta}{E} \qquad (5)$$

The solution for a strip with width value of K_I and depth $\eta = a + \sqrt{R^2 - \xi^2} - R$ is taken from taken from Paris [2]. The K_I value is

$$K_I = \frac{4M}{\pi R^4}\sqrt{R^2-\xi^2}\sqrt{\pi\eta}\ F_2(\frac{\eta}{h})$$

where $F_2(\frac{\eta}{h})$ represents the correction factor, which takes into account finite dimensions and takes form for different geometry and loading modes [5].

$$F_2\left(\frac{\eta}{h}\right) = \sqrt{\left(\frac{2h}{\pi\eta}\right)\tan\left(\frac{\pi\eta}{2h}\right)}\ 0.923 + 0.199[1-\sin\left(\frac{\pi\eta}{2h}\right)]^4/\cos\left(\frac{\pi\eta}{2h}\right)$$

where h is the local height given as $h = 2\sqrt{R^2-\xi^2}$.substituting the values of moment M in (5).

$$C_\xi = \frac{1-\upsilon^2}{E}\int_0^a\int_{-b}^{b}\frac{32(R^2-\xi^2)\ \pi\eta\ {F_2}^2(\frac{\alpha}{h})\,d\xi d\eta}{\pi^2 R^8} \qquad (6)$$

Similarly, for moment about the η axis the cracked shaft has another flexibility coefficient, which isC_η [5].

$$C_\eta = \frac{1-\upsilon^2}{E}\int_0^a\int_0^b\frac{32\xi^2\ \pi\eta\ {F_1}^2(\frac{\eta}{h})\,d\xi d\eta}{\pi^2 R^8} \qquad (7)$$

And it corresponding correction factor $F_1\left(\frac{\eta}{h}\right)$ is

$$F_1\left(\frac{\eta}{h}\right) = \sqrt{\left(\frac{2h}{\pi\eta}\right)\tan\left(\frac{\pi\eta}{2h}\right)}\ [0.752 + 2.02\left(\frac{\eta}{h}\right) + \frac{0.37\left[1-\sin\left(\frac{\pi\eta}{2h}\right)^3\right]}{\cos\left(\frac{\pi\eta}{2h}\right)}]$$

By solving (6) and (7) the values of C_ξ and C_η are obtained they are taken as C_{44} and C_{55}.Thus the flexibility matrix of the cracked shaft element C_C is obtained as

$$C_C = \begin{pmatrix} C_{44} & 0 \\ 0 & C_{55}\end{pmatrix}(8)$$

Substituting the values of Cc and C_o in the below equation i.e.

$$C_t = C_c + C_o$$

Where C_t gives the total flexibility of the cracked shaft. Hence the element stiffness matrix K_c for the cracked shaft is obtained as

$$K_c = \begin{pmatrix} {C_t}^{-1} & -{C_t}^{-1} \\ -{C_t}^{-1} & {C_t}^{-1}\end{pmatrix}(9)$$

IV. ANSYS RESULTS

The frequency of un-cracked rotor was found out using Ansys. The rotor has a diameter of 25mm and length of 1000mm between the bearing center line. The material is mild steel which has a density of 7800 kg/m^3 and young's modulus is 2*10^11N/mm^2. The natural frequency of the uncracked shaft (ω_o) was found to be 106.77Hz.

Similarly, a rotor with a crack was modelled with a crack at the mid span. The crack has a depth of 5mm in the figure shown below. The beam element considered is solid 188 and cracked rotor was meshed using tria and the frequency (ω) of the cracked shaft was found out to be 106.32Hz.

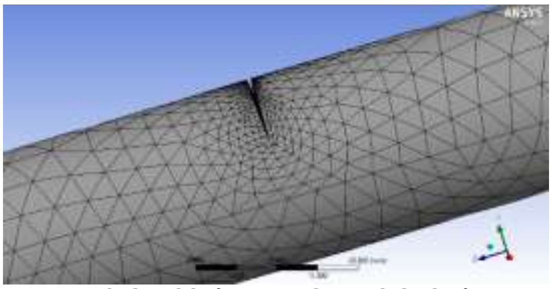

Figure 4.Meshed model of a rotor with a crack depth of 5mm.

Similarly, the frequencies were found out for different depths like 5mm,7.5mm,10mm,12.5mm. The measured value of frequency change i.e. ω/ω_owere plotted against the relative crack depth values i.e. a/d.

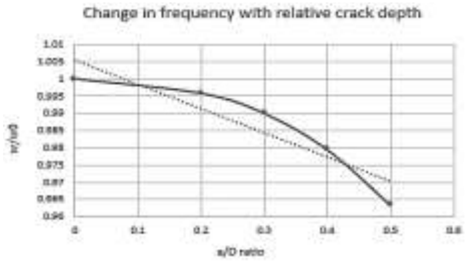

Figure 5. Frequency drop vs crack depth ratio

In the figure.5the solid line represents the change in frequency with change in relative crack depth. The straight line represents the decreasing trend in the natural frequencies. A similar procedure is adopted and the frequency of the rotor with a different shaft geometry is found out. A rotor of diameter 20mm and length 1000mm between the bearing center line (L/D=50) is modelled with similar material properties. The frequency of the un-cracked rotor in this case was found out to be 86. 282Hz.Now the rotor (with L/D=50) is modelled with different crack depths and their corresponding frequencies for are found out in this case.

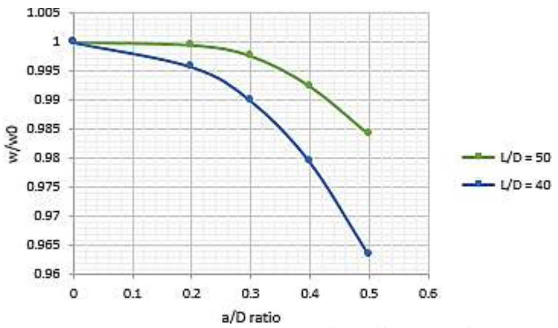

Figure 6. Frequency drop vs crack depth ratio for different shaft geometries.

The graph in figure.6 represents the changes in natural frequencies with relative changes in crack depths for different geometries. Similar trend was observed in [5] where the low slenderness ratios of rotors were considered.

V. Experimental Investigations

In this experiment steel shaft (ASI 1018 Steel) of 25mm diameter and length 1m is taken which is supported by two ball bearings at both ends. The shaft is coupled to an 0.5Hp electric motor by means of a Jaw coupling. An accelerometer is mounted on one the pillow blocks using accelerometer mounting wax i.e. a near point of the rotating shaft. The accelerometer used here is Kistler ceramic shear tri axial accelerometer which has a sensitivity of 2.5mV/g and frequency range of 1-10000hz. The accelerometer has three channels which are connected to vibrational analyzer NI9234. This analyzer NI9234 is mounted on a chassis (CDAQ 9174) which is connected to the PC by means of a usb port. PC equipped with Lab view (sound and vibrations kit) is used for processing the vibrational signals. The vibrational Signals received by the accelerometer are transformed by using an FFT relation and the output is displayed on the monitor screen. In this experiment the frequency response of un-cracked rotor is determined. The dynamic behaviour of the un-cracked shaft is determined from the amplitude vs frequency graph. The natural frequency of the un-cracked shaft was found out to be 147.5hz.

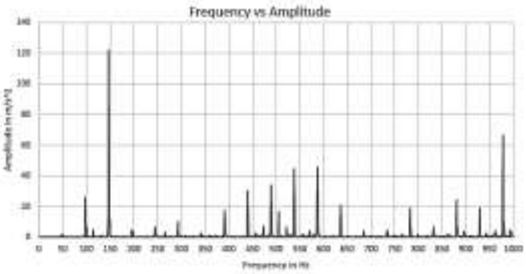

Figure 7. Frequency response of the un-cracked shaft.

Now a crack is developed on the shaft at its center i.e. at a distance of 500mm from either of the ends. Now the dynamic behavior of the cracked rotor was examined. The natural frequency of the cracked shaft was found to be 144. 5Hz

VI. Conclusions

In the present study the frequencies of the un-cracked and cracked rotor are found out. It is observed that with the introduction of a crack there is a decrease in the natural frequency of the rotor. This is due to the decrease in stiffness of the rotor. It is also observed that with the further increase in crack depth there is a decrease in their corresponding natural frequencies, this trend can be observed from figure 5. There is some difference between the theoretical and experimental results in natural frequency of the rotor with and without a crack. This is due the experimental part, as there are some parameters like accuracy of crack depth, the speed of rotation, occurrence of friction between shaft and bearings in case if low lubrication, which can cause changes in frequency values, in spite of the fixing of accelerometer to the near point of the rotating shaft. All these will cause some differences between the theoretical and experimental results, but on the other hand experimental results are in close agreement with the theoretical results. It can also be inferred from figure.6 that changes in natural frequencies with crack will be much appreciable in case of shafts with low slenderness ratio.

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