On Contra gγ-Continuous Functions

[1]K. V. Tamilselvi, [2]P. Thangaraj AND [3]O. Ravi

[1]Department of Mathematics,
Kongu Engineering College, Perundurai, Erode District,
Tamil Nadu, India.
e-mail : kvtamilselvi@gmail.com.

[2]Department of Computer Science and Engineering,
Bannari Amman Institute of Technology, Sathyamangalam,
Erode District, Tamil Nadu, India.
e-mail : ctptr@yahoo.co.in.

[3]Department of Mathematics,
P. M. Thevar College, Usilampatti, Madurai District, Tamil Nadu, India.
e-mail : siingam@yahoo.com.

Abstract. In this paper, we investigate further properties of the notion of contra gγ-continuous functions which was introduced in [4]. We obtain some separation axioms of contra gγ-continuous functions and discuss the relationships between contra gγ-continuity and other related functions.

2010 Mathematics Subject Classification: Primary: 54C08, 54C10: Secondary: 54C05

Keywords and phrases. *gγ-closed set, gγ-continuous function, contra gγ-continuous function, contra gγ-graph, gγ-normal space.*

*****

1. Introduction

In 1996, Dontchev [11] introduced a new class of functions called contra-continuous functions. He defined a function $f : X \to Y$ to be contra-continuous if the pre image of every open set of Y is closed in X. In 2007, Caldas et al. [9] introduced and investigated the notion of contra g-continuity. This notion received a proper attention and some research articles came to existence.

In this paper, we study the generalization of contra-continuity called contra gγ- continuity. It turns out that the notion of contra gγ-continuity is a weaker form of contra γ-continuity [24] and a stronger form of contra πgγ-continuity [29].

2. Preliminaries

Throughout this paper, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. The closure of A and the interior of A are denoted by cl(A) and int(A), respectively. A subset A of X is said to be regular open [34] (resp. regular closed [34]) if A = int(cl(A)) (resp. A = cl(int(A))). The finite union of regular open sets is said to be π-open [36]. The complement of a π-open set is said to be π-closed [36].

Definition 2.1. A subset A of a space X is said to be

(1) pre-open [22] if $A \subseteq int(cl(A))$; (2) pre-closed [22] if $cl(int(A)) \subseteq A$; (3) semi-open [19] if $A \subseteq cl(int(A))$; (4) β-open [1] if $A \subseteq cl(int(cl(A)))$;

(5) b-open [6] or sp-open [12] or γ-open [14] if $A \subseteq cl(int(A)) \cup int(cl(A))$;

(6) γ-closed [14] if $int(cl(A)) \cap cl(int(A)) \subseteq A$;

(7) g-closed [20] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X;

(8) gp-closed [27] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X;

(9) gγ-closed [13] if $\gamma cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X;

(10) πgp-closed [28] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open in X;

(11) $\pi g\gamma$-closed [32] if $\gamma cl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open in X.

The complements of the above closed sets are said to be open sets.

The complements of the above open sets are said to be closed sets.

The intersection of all pre-closed (resp. γ-closed) sets containing A is called pre-closure (resp. γ-closure) of A and is denoted by pcl(A) (resp. γcl(A)).

The family of all gγ-open (resp. gγ-closed, closed) sets of X containing a point $x \in X$ is denoted by $G\gamma O(X, x)$ (resp. $G\gamma C(X, x)$, $C(X, x)$). The family of all gγ-open (resp. gγ-closed, closed, semi-open, γ-open) sets of X is denoted by $G\gamma O(X)$ (resp. $G\gamma C(X)$, $C(X)$, $SO(X)$, $\gamma O(X)$).

Definition 2.2. Let A be a subset of a space (X, τ).

(1) The set $\cap\{U \in \tau : A \subseteq U\}$ is called the kernel of A [23] and is denoted by ker(A).

(2) The set $\cap\{F : F$ is gγ-closed in $X : A \subseteq F\}$ is called the gγ-closure of A [16] and is denoted by gγ-cl(A).

(3) The set $\cup\{F : F$ is gγ-open in $X : A \supseteq F\}$ is called the gγ-interior of A [16] and is denoted by gγ-int(A).

Lemma 2.3. [18] The following properties hold for subsets U and V of a space (X, τ).

(1) $x \in ker(U)$ if and only if $U \cap F \neq \emptyset$ for any closed set $F \in C(X, x)$;

(2) $U \subseteq ker(U)$ and $U = ker(U)$ if U is open in X;

(3) If $U \subseteq V$, then $ker(U) \subseteq ker(V)$.

Lemma 2.4. Let A be a subset of a space (X, τ), then

(1) $g\gamma\text{-}cl(X-A) = X - g\gamma\text{-}int(A)$;

(2) $x \in g\gamma\text{-}cl(A)$ if and only if $A \cap U \neq \emptyset$ for each $U \in G\gamma O(X, x)$ [2];

(3) If A is gγ-closed in X, then $A = g\gamma\text{-}cl(A)$.

Remark 2.5. [3] If $A = g\gamma\text{-}cl(A)$, then A need not be a gγ-closed.

Example 2.6. [3] Let $X = \{a, b, c, d, e, f\}$ and $\tau = \{\emptyset, X, \{a, b\}, \{c, d\}, \{a, b, c, d\}\}$. Take $A = \{a, b, c, d\}$. Clearly $g\gamma\text{-}cl(A) = A$ but A is not gγ-closed.

Lemma 2.7. [6] Let A be a subset of a space X. Then $\gamma cl(A) = A \cup [int(cl(A)) \cap cl(int(A))]$.

 ISSN: 2321-8169
260 - 269

Definition 2.8. [4] A function $f : X \rightarrow Y$ is called contra gγ-continuous if $f^{-1}(V)$ is gγ-closed in X for every open set V of Y.

Theorem 2.9. [2] The following are equivalent for a function $f : X \rightarrow Y$:

(1) f is contra gγ-continuous;

(2) The inverse image of every closed set of Y is gγ-open in X;

(3) For each $x \in X$ and each closed set V in Y with $f(x) \in V$, there exists a gγ-open set U in X such that $x \in U$ and $f(U) \subseteq V$;

(4) $f(g\gamma\text{-cl}(A)) \subseteq \ker(f(A))$ for every subset A of X;

(5) $g\gamma\text{-cl}(f^{-1}(B)) \subseteq f^{-1}(\ker(B))$ for every subset B of Y.

Definition 2.10. A function $f : X \rightarrow Y$ is said to be

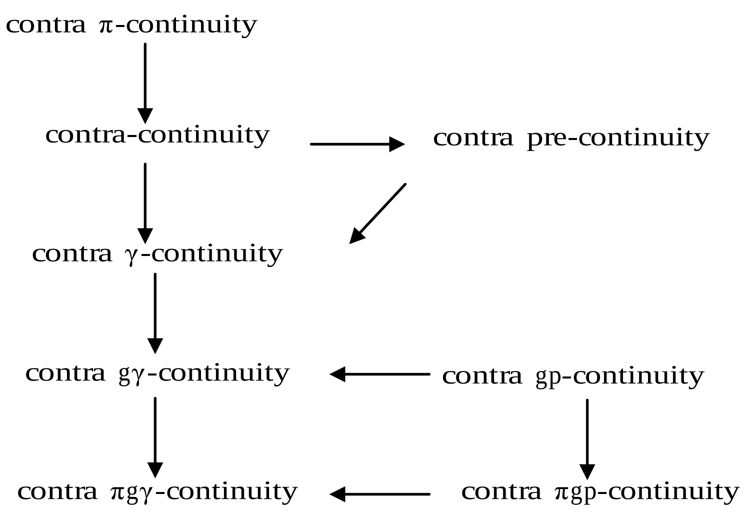
(1) completely continuous [7] (resp. perfectly continuous [26], gγ-continuous [5]) if $f^{-1}(V)$ is regular open (resp. clopen, gγ-open) in X for every open set V of Y;

(2) contra-continuous [11] (resp. contra pre-continuous [17], contra γ-continuous [24]) if $f^{-1}(V)$ is closed (resp. pre-closed, γ-closed) in X for every open set V of Y;

(3) contra g-continuous [9] (resp. contra gp-continuous [10]) if $f^{-1}(V)$ is g-closed (resp. gp-closed) in X for every open set V of Y;

(4) contra π-continuous [10] (resp. contra πgp-continuous [10], contra πgγ-continuous [29]) if $f^{-1}(V)$ is π-closed (resp. πgp-closed, πgγ-closed) in X for every open set V of Y.

Remark 2.11. [2, 10, 24, 29] For the functions defined above, we have the following implications:



None of these implications is reversible.

The following properties and characterizations of contra gγ-continuous functions are already obtained. Further properties are to be discussed in this paper.

Theorem 2.12. [2] If a function $f: X \to Y$ is contra gγ-continuous and Y is regular, then f is gγ-continuous.

Theorem 2.13. [2] Let $\{X_i : i \in \Omega\}$ be any family of topological spaces. If a function $f: X \to \prod X_i$ is contra gγ-continuous, then $Pr_i \circ f: X \to X_i$ is contra gγ-continuous for each $i \in \Omega$, where Pr_i is the projection of $\prod X_i$ onto X_i.

Definition 2.14. [2] A space (X, τ) is said to be gγ-connected if X cannot be expressed as the disjoint union of two non-empty gγ-open sets.

Remark 2.15. [2] A contra gγ-continuous image of gγ-connected space is connected.

Definition 2.16. [2] The graph G(f) of a function $f: X \to Y$ is said to be contra gγ-graph if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist a gγ-open set U in X containing x and a closed set V in Y containing y such that $(U \times V) \cap G(f) = \emptyset$.

Lemma 2.17. [2] A graph G(f) of a function $f: X \to Y$ is contra gγ-graph in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist a $U \in G\gamma O(X)$ containing x and $V \in C(Y)$ containing y such that $f(U) \cap V = \emptyset$.

Theorem 2.18. [2] If $f: X \to Y$ is contra gγ-continuous and Y is Urysohn, G(f) is contra gγ-graph in $X \times Y$.

Theorem 2.19. [2] Let $f: X \to Y$ be a function and $g: X \to X \times Y$ the graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is contra gγ-continuous, then f is contra gγ-continuous.

Definition 2.20. A topological space X is said to be

(1) gγ-normal [2] if each pair of non-empty disjoint closed sets can be separated by disjoint gγ-open sets.

(2) Ultra normal [33] if for each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets.

Theorem 2.21. [2] If $f: X \to Y$ is a contra gγ-continuous, closed injection and Y is Ultra normal, then X is gγ-normal.

3. Contra gγ-continuous functions

Further study of contra gγ-continuous functions are given in this section.

Definition 3.1. A function $f: X \to Y$ is said to be

(1) gγ-semiopen if $f(U) \in SO(Y)$ for every gγ-open set U of X;

(2) contra-I(gγ)-continuous if for each $x \in X$ and each $F \in C(Y, f(x))$,

There exists $U \in G\gamma O(X, x)$ such that $int(f(U)) \subseteq F$.

Theorem 3.2. If a function $f: X \to Y$ is contra-I(gγ)-continuous and gγ-semiopen, then f is contra gγ-continuous.

Proof. Suppose that $x \in X$ and $F \in C(Y, f(x))$. Since f is contra-I(gγ)-continuous, there exists $U \in G\gamma O(X, x)$ such that $int(f(U)) \subseteq F$. By hypothesis f is gγ-semiopen, therefore $f(U) \in SO(Y)$ and $f(U) \subseteq cl(int(f(U))) \subseteq F$. This shows that f is contra gγ-continuous.

Lemma 3.3. For a subset A of (X, τ), the following statements are equivalent.

(1) A is open and gγ-closed;

(2) A is regular open.

Proof. (1) $\Rightarrow$ (2): Let A be an open and gγ-closed subset of X. Then γcl(A) $\subseteq$ A and so int(cl(A)) $\subseteq$ A holds. Since A is open then A is pre-open and thus A $\subseteq$ int(cl(A)). Therefore, we have int(cl(A)) = A, which shows that A is regular open.

(2) $\Rightarrow$ (1): Since every regular open set is open, then, by Lemma 2.7, γcl(A) = A and γcl(A) $\subseteq$ A. Hence A is gγ-closed.

Theorem 3.4. For a function f : X $\rightarrow$ Y, the following statements are equivalent.

(1) f is contra gγ-continuous and continuous;

(2) f is completely continuous.

Proof. (1) $\Rightarrow$ (2): Let U be an open set in Y. Since f is contra gγ-continuous and continuous, f^{-1}(U) is gγ-closed and open, by Lemma 3.3, f^{-1}(U) is regular open. Then f is completely continuous.

(2) $\Rightarrow$ (1): Let U be an open set in Y. Since f is completely continuous, f^{-1}(U) is regular open, by Lemma 3.3, f^{-1}(U) is gγ-closed and open. Then f is contra gγ-continuous and continuous.

Definition 3.5.

(1) A subset A of a topological space X is said to be Q-set [21] if int(cl(A)) = cl(int(A)).

(2) Let f : X $\rightarrow$ Y be a function. Then f is called Q-continuous if f^{-1}(U) is Q-set in X for each open set U of Y.

Lemma 3.6. For a subset A of X, the following statements are equivalent:

(1) A is clopen,

(2) A is open, Q-set and gγ-closed.

Proof. (1) $\Rightarrow$ (2): Let A be a clopen subset of X. Then A is closed and open. Therefore, A is Q-set. Since every closed is gγ-closed then A is gγ-closed.

(2) $\Rightarrow$ (1): By Lemma 3.3, A is regular open. Since A is Q-set, A = int(cl(A)) = cl(int(A)). Therefore, A is regular closed. Then A is closed. Hence A is clopen.

Theorem 3.7. For a function f : X $\rightarrow$ Y, the following statements are equivalent.

(1) f is perfectly continuous;
(2) f is continuous, Q-continuous and contra gγ-continuous.

Proof. It obtained from the above Lemma.

Theorem 3.8. Let f : X $\rightarrow$ Y and g : Y $\rightarrow$ Z be a function. Then the following holds:

(1) If f is contra gγ-continuous and g is continuous, then g o f : X → Z is contra gγ-continuous;
(2) If f is gγ-continuous and g is contra-continuous, then g o f : X → Z is contra gγ-continuous;
(3) If f is contra gγ-continuous and g is contra-continuous, then g o f : X → Z is gγ-continuous.

Definition 3.9. A space (X, τ) is called T_{gs} [16] if every gγ-closed set is γ-closed.

Theorem 3.10. Let f : X → Y be a function. Suppose that X is a T_{gs} space. Then the following are equivalent.

(1) f is contra gγ-continuous;

(2) f is contra γ-continuous. **Proof.** Obvious.

Definition 3.11. For a space (X, τ), $g\tau^{\gamma} = \{U \subseteq X : g\gamma\text{-}cl(X\backslash U) = X\backslash U\}$.

Theorem 3.12. Let (X, τ) be a space. Then

(1) Every gγ-closed set is γ-closed (i.e. (X, τ) is T_{gs}) if and only if $g\tau^{\gamma} = \gamma O(X)$;

(2) Every gγ-closed set is closed if and only if $g\tau^{\gamma} = \tau$.

Proof. (1) Let $A \in g\tau^{\gamma}$. Then gγ-cl(X\A) = X\A. By hypothesis, γcl(X\A) = gγ- cl(X\A) = X\A and hence A ∈ γO(X).

Conversely, let A be a gγ-closed set. Then gγ-cl(A) = A and hence $X\backslash A \in g\tau^{\gamma} = \gamma O(X)$, i.e. A is γ-closed. (2) Similar to (1).

Theorem 3.13. If $g\tau^{\gamma} = \tau$ in X, then for a function f : X → Y the following are equivalent:

(1) f is contra gγ-continuous;
(2) f is contra g-continuous;
(3) f is contra-continuous.

Proof. Obvious.

4. Properties of Contra gγ-continuous functions

Definition 4.1. A space X is said to be gγ-T_1 if for each pair of distinct points x and y in X, there exist gγ-open sets U and V containing x and y respectively, such that y ∉ U and x ∉ V.

Definition 4.2. A space X is said to be gγ-T_2 [35] if for each pair of distinct points x and y in X, there exist U ∈ GγO(X, x) and V ∈ GγO(X, y) such that U ∩ V = ∅.

Theorem 4.3. Let X be a topological space. Suppose that for each pair of distinct points x_1 and x_2 in X, there exists a function f of X into a Urysohn space Y such that $f(x_1) \neq f(x_2)$. Moreover, let f be contra gγ-continuous at x_1 and x_2. Then X is gγ-T_2.

Proof. Let x_1 and x_2 be any distinct points in X. Then suppose that there exist an Urysohn space Y and a function f : X → Y such that $f(x_1) \neq f(x_2)$ and f is contra gγ-continuous at x_1 and x_2. Let $w = f(x_1)$ and $z = f(x_2)$. Then $w \neq z$. Since Y is Urysohn, there exist open sets U and V containing w and z, respectively such that $cl(U) \cap cl(V) = \emptyset$. Since f is contra gγ-continuous at x_1 and x_2, then there exist gγ-open sets A and B containing x_1 and x_2, respectively such that $f(A) \subseteq cl(U)$ and $f(B) \subseteq cl(V)$. So we have $A \cap B = \emptyset$ since $cl(U) \cap cl(V) = \emptyset$. Hence, X is gγ-$T_2$.

Corollary 4.4. If f is a contra gγ-continuous injection of a topological space X into a Urysohn space Y, then X is gγ-T_2 .

Proof. For each pair of distinct points x_1 and x_2 in X and f is contra gγ-continuous function of X into a Urysohn space Y such that $f(x_1) \neq f(x_2)$ because f is injective. Hence by Theorem 4.3, X is gγ-T_2.

Theorem 4.5. For a space X, the following are equivalent:

(1) X is gγ-connected;

(2) The only subsets of X which are both gγ-open and gγ-closed are the empty set ϕ and X;

(3) Each contra gγ-continuous function of X into a discrete space Y with at least two points is a constant function.

Proof. (1) ⇒(2): Suppose $S \subset X$ is a proper subset which is both gγ-open and gγ- closed. Then its complement $X - S$ is also gγ-open and gγ-closed. Then $X = S \cup (X - S)$, a disjoint union of two non-empty gγ-open sets which contradicts the fact that X is gγ-connected. Hence, $S = \emptyset$ or X.

(2) ⇒(1): Suppose $X = A \cup B$ where $A \cap B = \emptyset$, $A \neq \emptyset$, $B \neq \emptyset$ and A and B are gγ-open. Since $A = X - B$, A is gγ-closed. But by assumption $A = \emptyset$ or X, which is a contradiction. Hence (1) holds.

(2) ⇒(3): Let f : X → Y be contra gγ-continuous function where Y is a discrete space with at least two points. Then $f^{-1}(\{y\})$ is gγ-closed and gγ-open for each $y \in Y$ and $X = \cup\{f^{-1}(y) : y \in Y\}$. By hypothesis, $f^{-1}(\{y\}) = \emptyset$ or X. If $f^{-1}(\{y\}) = \emptyset$ for all $y \in Y$, then f is not a function. Also there cannot exist more than one $y \in Y$ such that $f^{-1}(\{y\}) = X$. Hence there exists only one $y \in Y$ such that $f^{-1}(\{y\}) = X$ and $f^{-1}(\{y_1\}) = \emptyset$ where $y \neq y_1 \in Y$. This shows that f is a constant function.

(3) ⇒(2): Let P be a non-empty set which is both gγ-open and gγ-closed in X. Suppose f : X → Y is a contra gγ-continuous function defined by $f(P) = \{a\}$ and $f(X \setminus P) = \{b\}$ where $a \neq b$ and $a, b \in Y$. By hypothesis, f is constant. Therefore $P = X$.

Definition 4.6. A subset A of a space (X, τ) is said to be gγ-clopen [35] if A is both gγ-open and gγ-closed.

Theorem 4.7. If f is a contra gγ-continuous function from a gγ-connected space X onto any space Y, then Y is not a discrete space.

Proof. Suppose that Y is discrete. Let A be a proper non-empty open and closed subset of Y. Then $f^{-1}(A)$ is a proper non-empty gγ-clopen subset of X which is a contradiction to the fact that X is gγ-connected.

Definition 4.8. A space (X, τ) is said to be submaximal [8] if every dense subset of X is open in X and extremally disconnected [25] if the closure of every open set is open.

Note that (X, τ) is submaximal and extremally disconnected if and only if every β-open set in X is open [15].

Note that (X, τ) is submaximal and extremally disconnected if and only if every γ-open set in X is open (We know that γ-open set is β-open) [30].

Theorem 4.9. If A and B are gγ-closed sets in submaximal and extremally disconnected space (X, τ), then A ∪ B is gγ-closed.

Proof. Let A∪ B ⊆ U and U be open in (X, τ). Since A, B ⊆ U and A and B are gγ-closed, γcl(A) ⊆ U and γcl(B) ⊆ U. Since (X, τ) is submaximal and extremally disconnected, γcl(F) = cl(F) for any set F ⊆ X. Now γcl(A ∪ B) = γcl(A) ∪ γcl(B) ⊆ U. Hence A∪ B is gγ-closed.

Lemma 4.10. Let (X, τ) be a topological space. If U, V ∈ GγO(X) and X is submaximal and extremally disconnected space, then U ∩ V ∈ GγO(X).

Proof. Let U, V ∈ GγO(X). We have X \ U, X \ V ∈ GγC(X). By Theorem 4.9, (X \ U) ∪ (X \ V) = X \ (U ∩ V) ∈ GγC(X). Thus, U ∩ V ∈ GγO(X).

Theorem 4.11. If f : X → Y and g : X → Y are contra gγ-continuous, X is submaximal and extremally disconnected and Y is Urysohn, then K = {x ∈ X : f(x) = g(x)} is gγ-closed in X.

Proof. Let x ∈ X\K. Then f(x) ≠ g(x). Since Y is Urysohn, there exist open sets U and V such that f(x) ∈ U, g(x) ∈ V and cl(U) ∩ cl(V) = ∅. Since f and g are contra gγ-continuous, f^{-1}(cl(U)) ∈ GγO(X) and g^{-1}(cl(V)) ∈ GγO(X). Let A = f^{-1}(cl(U)) and B = g^{-1}(cl(V)). Then A and B contains x. Set C = A ∩ B. C is gγ-open in X. Hence f(C) ∩ g(C) = ∅ and x ∉ gγ-cl(K). Thus K is gγ-closed in X.

Definition 4.12. A subset A of a topological space X is said to be gγ-dense in X if gγ-cl(A) = X.

Theorem 4.13. Let f : X → Y and g : X → Y be contra gγ-continuous. If Y is Urysohn and f = g on a gγ-dense set A ⊆ X, then f = g on X.

Proof. Since f and g are contra gγ-continuous and Y is Urysohn, by Theorem 4.11, K = {x ∈ X : f(x) = g(x)} is gγ-closed in X. We have f = g on gγ-dense set A ⊆ X. Since A ⊆ K and A is gγ-dense set in X, then X = gγ-cl(A) ⊆ gγ-cl(K) = K. Hence, f = g on X.

Definition 4.14. A space X is said to be weakly Hausdroff [31] if each element of X is an intersection of regular closed sets.

Theorem 4.15. If f : X → Y is a contra gγ-continuous injection and Y is weakly Hausdroff, then X is gγ-T_1.

Proof. Suppose that Y is weakly Hausdroff. For any distinct points x_1 and x_2 in X there exist regular closed sets U and V in Y such that f(x_1) ∈ U, f(x_2) ∉ U, f(x_1) ∉ V and f(x_2) ∈ V. Since f is contra gγ-continuous, f^{-1}(U) and f^{-1}(V) are gγ-open subsets of X such that x_1 ∈ f^{-1}(U), x_2 ∉ f^{-1}(U), x_1 ∉ f^{-1}(V) and x_2 ∈ f^{-1}(V). This shows that X is gγ-T_1.

 ISSN: 2321-8169
260 - 269

Theorem 4.16. Let $f : X \to Y$ have a contra gγ-graph. If f is injective, then X is gγ-T_1.

Proof. Let x_1 and x_2 be any two distinct points of X. Then, we have $(x_1, f(x_2)) \in (X \times Y)\backslash G(f)$. Then, there exist a gγ-open set U in X containing x_1 and $F \in C(Y, f(x_2))$ such that $f(U) \cap F = \emptyset$. Hence $U \cap f^{-1}(F) = \emptyset$. Therefore, we have $x_2 \notin U$. This implies that X is gγ-T_1.

Definition 4.17. A topological space X is said to be Ultra Hausdroff [33] if for each pair of distinct points x and y in X, there exist clopen sets A and B containing x and y, respectively such that $A \cap B = \emptyset$.

Theorem 4.18. Let $f : X \to Y$ be a contra gγ-continuous injection. If Y is an Ultra Hausdroff space, then X is gγ-T_2.

Proof. Let x_1 and x_2 be any distinct points in X, then $f(x_1) = f(x_2)$ and there exist clopen sets U and V containing $f(x_1)$ and $f(x_2)$ respectively, such that $U \cap V = \emptyset$. Since f is contra gγ-continuous, then $f^{-1}(U) \in G\gamma O(X)$ and $f^{-1}(V) \in G\gamma O(X)$ such that $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. Hence, X is gγ-$T_2$.

References

[1] M. E. Abd El-Monsef, S. N. El-Deeb and R. A. Mahmoud, β-open sets and β-continuous mappings, Bull. Fac. Sci. Assiut Univ., 12(1983), 77-90.

[2] M. Akdag and A. Ozkan, Some properties of contra gb-continuous functions, Journal of New Results in Science, 1(2012), 40-49.

[3] S. C. Akgun and G. Aslim, On πgb-closed sets and related topics, International Journal of Mathematical Archive, 3(5)(2012), 1873-1884.

[4] A. Al-Omari and M. S. M. Noorani, Decomposition of continuity via b-open set, Bol. Soc. Paran. Mat., 26(1-2)(2008), 53-64.

[5] A. Al-Omari and M. S. M. Noorani, Some properties of contra b-continuous and almost contra b-continuous functions, European J. of Pure and App. Math., 2(2)(2009), 213-230.

[6] D. Andrijevic, On b-open sets, Mat. Vesnik, 48(1-2)(1996), 59-64.

[7] S. P. Arya and R. Gupta, On strongly continuous mappings, Kyungpook Math. J., 14(1974), 131-143.

[8] N. Bourbaki, General topology, Part I, Reading, Ma: Addison wesley, Paris.1966.

[9] M. Caldas, S. Jafari, T. Noiri and M. Simoes, A new generalization of contra-continuity via Levine's g-closed sets, Chaos Solitons and Fractals, 32(2007), 1597-1603.

[10] M. Caldas, S. Jafari, K. Viswanathan and S. Krishnaprakash, On contra πgp-continuous functions, Kochi J. Math., 5(2010), 67-78.

[11] J. Dontchev, Contra-continuous functions and strongly S-closed spaces, Internat. J. Math. and Math. Sci., 19(1996), 303-310.

[12] J. Dontchev and M. Przemski, On the various decompositions of continuous and some weakly continuous functions, Acta Math. Hungar., 71(1-2)(1996), 109-120.

[13] E. Ekici, On γ-normal spaces, Bull. Math. Soc. Sci. Math. Roumanie (N.S), 50(98)(2007), 259-272.

[14] A. A. El-Atik, A study on some types of mappings on topological spaces, Master's Thesis, Tanta University, Egypt, 1997.

[15] M. Ganster and D. Andrijevic, On some questions concerning semi-preopen sets, J. Inst. Math. Compu. Sci. Math., 1(1988), 65-75.

[16] M. Ganster and M. Steiner, On bτ-closed sets, App. Gen. Topol., 2(8)(2007), 243-247.

[17] S. Jafari and T. Noiri, On contra precontinuous functions, Bull. Malaysian Math. Sci. Soc., 25(2002), 115-128.

[18] S. Jafari and T. Noiri, Contra-super-continuous functions, Ann. Univ. Sci. Budapest. Eotvos Sect. Math., 42(1999), 27-34.

[19] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70(1963), 36-41.

[20] N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo, 19(2)(1970), 89-96.

[21] N. Levine, On the commutivity of the closure and interior operator in the topological spaces, Amer. Math. Monthly, 68(1961), 474-477.

[22] A. S. Mashour, M. E. Abd El-Monsef and S. N. El-Deeb, On precontinuous and weak precontinuous mappings, Proc. Math. Phys. Soc. Egypt., 53(1982), 47-53.

[23] M. Mrsevic, On pairwise R_0 and pairwise R_1 bitopological spaces, Bull. Math. Soc. Sci. Math RS Roumano. (N.S) 30(78)(1986), 141-148.

[24] A. A. Nasef, Some properties of contra-γ-continuous functions, Chaos Solitons and Fractals, 24(2)(2005), 471-477.

[25] T. Noiri, Characterizations of extremally disconnected spaces, Indian J. Pure Appl. Math., 19(4)(1988), 325-329.

[26] T. Noiri, Super-continuity and some strong forms of continuity, Indian J. Pure Appl. Math., 15(1984), 241-250.

[27] T. Noiri, H. Maki and J. Umehara, Generalized preclosed functions, Mem. Fac. Sci. Kochi Univ. Ser. A Math., 19(1998), 13-20.

[28] J. H. Park, On πgp-closed sets in topological spaces, Indian J. Pure Appl. Math., (In press).

[29] O. Ravi, I. Rajasekaran, A. Pandi and S. Murugesan, On contra πgγ-continuous functions, Journal of Informatics and Mathematical Sciences, 6(2)(2014), 109-121.

[30] O. Ravi, S. Margaret Parimalam, S. Murugesan and A. Pandi, Slightly πgγ-continuous functions, Journal of New Results in Science, 3(2013), 60-71.

[31] T. Soundararajan, Weakly Hausdroff spaces and the cardinality of topological spaces, 1971 General Topology and its Relation to Modern Analysis and Algebra. III, (Proc. Conf. Kanpur, 1968). Academia. Prague 1971, 301-306.

[32] D. Sreeja and C. Janaki, On πgb-closed sets in topological spaces, International Journal of Mathematical Archive, 2(8)(2011), 1314-1320.

[33] R. Staum, The algebra of bounded continuous functions into a nonarchimedean field, Pacific J. Math., 50(1974), 169-185.

[34] M. H. Stone, Applications of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc., 41(1937), 375-481.

[35] K. V. TamilSelvi, P. Thangaraj and O. Ravi, Slightly gγ-continuous functions, Applied Mathematical Sciences, 8(180)(2014), 8987-8999.

[36] V. Zaitsev, On certain classes of topological spaces and their bicompactifications, Dokl. Akad. Nauk. SSSR, 178(1968), 778-779.