# From Beam to Chassis: How to Increase NVH Performances with an Optimized Moment of Inertia Distribution

E. Torricelli, L. D'Agostino, A. Baldini and P. Moruzzi

***Abstract*— Car weight reduction is becoming more and more important for every kind of vehicle: minor mass implies, in fact, minor consumption, makes easier to fulfill homologation rules and assures a better handling behavior. Despite that, several vehicle missions have always been solved by adding more mass, e.g. NVH. In this paper, a methodology to optimize the stiffness distribution is proposed in order to obtain better vibrational performances without increasing the mass. At first, the problem has been solved for a simple beam using finite element and optimization algorithms. At a second stage, the optimal moment of inertia distribution found has been applied to a chassis thanks to a topometry optimization. Finally, the improvement in NVH performances has been verified comparing the inertances of the optimized model with those of the non-optimized one.**

***Index Terms* — optimization, NVH, torsional and bending stiffness, chassis, FEM**

## I. Introduction

Light-weight design is becoming a key factor for the whole automotive industry: not only sport car but also economy car manufacturers have to reduce mass. Whereas the equation "minor weight implies better handling performance" has always mattered for the racing world, mass problem now concerns every utility car. Less weight, in fact, implies less fuel and less pollution and helps to pass the more and more stringent homologation rules.

Despite helping in fuel consumption's reduction, a minor car-weight can be negative for performance in some fields, which have always taken advantage from the mass of the car, e.g. NVH. As the vehicle mass reduction process cannot be avoided, finding the best mass and stiffness distributions can keep vibration performance acceptable.

In this paper, a guideline to determinate the optimal distribution of inertia moments for an automotive chassis is presented. At first, the chassis has been reduced to a simple beam and the best moment of inertia distribution is found through optimization. The use of beam in NVH analysis is widely supported by literature: introducing simple model, in fact, reduces the computational effort for complex structure, [1] and [2]. Finally, the optimized moment of inertia has been applied to the chassis and the NVH behavior has been evaluated in terms of inertances. The methodology relies on finite element coupled with different optimization techniques.

Manuscript received March 22, 2011; revised April 6, 2011. This work was supported by Ferrari S.p.A.

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## II. Simple Beam

In order to find the best moment of inertia distribution the chassis has been reduced to a beam. The beam length dimension is equal to the chassis wheelbase. To improve the NVH behavior of the structure, normal modes and inertance had to be considered. The normal modes should be increased in order to avoid resonance peaks within the frequency range of interest. Inertances, which can be defined as the transfer function of a dynamic system with force as input and acceleration as output, both with the same application point, should be reduced. Concerning an automotive chassis, a high value of inertances on the suspension joints assures a better response against wearing course roughness.

### *A. Analytical approach*

In static loadcase, the moment of inertia can be expressed according to the Euler-Bernoulli beam theory [1]

$$\frac{M(x)}{EI(x)} = \frac{d^2z}{dx^2} \qquad (1)$$

where *M(x)* is the bending moment, *EI(x)* is the flexural rigidity, in which *E* is the modulus and *I(x)* is the moment of inertia, and *z(x)* is the vertical displacement.

Considering a simply supported beam with constant section and length L subjected to a central load, the bending moment is bilinear and the displacement is:

$$z(x) = \frac{M(x)(4x^2 - 3L^2)}{24EI} \quad for\ 0 \le x \le \frac{L}{2} \qquad (2)$$

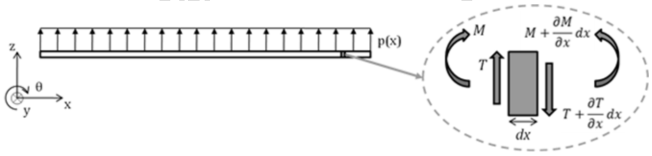


Fig. 1. Beam in bending vibration and the free body diagram for a beam element

ISBN: 978-988-19251-5-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

Fig.1 shows a simple beam subjected to bending vibration. The force equilibrium in *z* and the rotation equilibrium on *y* are:

$$\begin{cases} \rho S(x)\dfrac{\partial^2 z(x)}{\partial t^2} = -\dfrac{\partial T}{\partial x} + p(x) \\ EJ(x)\dfrac{\partial^2 \theta(x)}{\partial t^2} = \dfrac{\partial M}{\partial x} - T \end{cases} \tag{3}$$

where $\rho$ is the density, *S(x)* is the generic area of each section, *T* is the shearing force, *p(x)* is the transverse force per unit length and $\theta$ is the rotation angle.
Combining the two equations of equilibrium (3) with the static beam equation (1) and introducing some simplifications is possible to obtain, [3] and [4]:

$$\frac{\partial^2}{\partial x^2}\left(EJ(x)\frac{\partial^2 z(x)}{\partial t^2}\right) + \rho S(x)\frac{\partial^2 z(x)}{\partial t^2} - p(x) = 0 \tag{4}$$

which is a partial differential equation of order four.
To increase the inertances of the beam means to reduce the accelerations. As a consequence the equation (4) has to be solved with the condition that the moment of inertia minimizes the third derivative of displacement with respect of time:

$$J(x) \quad \rightarrow \quad \frac{\partial^3 z(x)}{\partial t^3} = 0 \tag{5}$$

The equation is not solvable analytically and also numeric methods as Runge-Kutta fail if the moment of inertia *J* depends on *x* coordinate. In order to solve this problem, trial functions are needed and are extensively used in NVH research, e.g. [5].

### B. Finite Element Method

To avoid a preconceived distribution of the moment of inertia, a finite element approach has been chosen.
The beam has been discretized in 50 elements with rectangular section, with equal dimensions, the thickness is 2mm and the width is 60mm, except from the height. Changing this value allow to control and vary the moment of inertia of each section. The beam has been tested with two different loadcases:

- normal modes analysis
- modal frequency response analysis with 51 subcases, everyone with one dynamic force on a different node of the beam.

Both the analyses have been carried out between 70 and 400Hz and the structural damping for the frequency response has been set to 1.5%.

## III. Optimization

The finite element method has been coupled with optimization algorithms in order to find the best moment of inertia for each element of the beam.

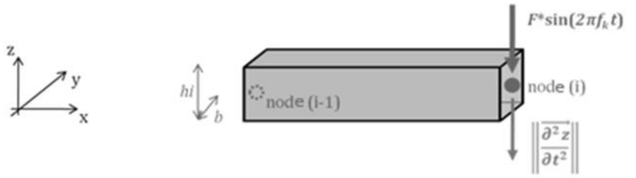


Fig. 2. I-th element of the beam: input force and acceleration output for the modal frequency response analysis.

An optimization problem can be defined by three different entities: the design variables, one or more objectives and the constraints. Basically, the design variables are the independent variables that can be changed during the optimization process to reach the objectives without breaching some conditions, i.e. the constraints. The domain of all possible design variables is called design space.
In this paper, the design variables are the heights of each beam element, whereas two objectives have been set:

- increase the first bending normal mode of the structure,
- minimize the average value of the acceleration on each node of the beam when loaded by a force inside the range of analysis (figure 2)

$$\begin{cases} max\{f^{I}_{nat,flex}\} \\ min\left\{\sum_{i=1}^{51}\left[\sum_{k=70}^{400}\left(\left\|\dfrac{\partial^2 \vec{z_{i,k}}}{\partial t^2}\right\| \Big/ (2\pi f_k)\right)/331\right]/51\right\} \end{cases} \tag{6}$$

The only constrain has been set on the mass of the beam by admitting a maximum 5% change of its value.
Even though the objectives are two, the first analyses have been carried out with a single objective and with a very large design space (0-240mm) to evaluate the best moment of inertia distribution for each loadcase.
The preliminary optimizations have shown that the two objectives give opposite results: the solver moves, in fact, all the inertia of the structure on the middle to counteract the first bending normal mode and increase its frequency. On the other hand, the heights are higher on the two side of the beam to minimize the inertances. Moreover, a reduction of the design space in the range 20-100mm has not worsened the analysis results.
The true optimization with two objectives has been carried out with a peculiar methodology to obtain good results. With a simple approach, in fact, only the increase of the first bending normal mode frequency was significant and on the contrary the inertances did not increase a lot.
In order to improve the inertances too, the methodology consists on three different steps of optimization:

- *MOGT*
- *MOGA-II*
- *Simplex*

The first step is done with the *MOGT*, *multi-objective game theory* [6]. This algorithm sweeps the design space simulating human behavior in strategic situation. The input variables and the objective function to be minimized are

ISBN: 978-988-19251-5-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

subdivided among the players. At every turn of the game, each player has at his disposal a few iterations of the simplex method, which is explained further in the paper, to be carried out on the design subspace of the input variables which have been assigned to him. With these simplex iterations he tries to minimize its only objective function. At the end, equilibrium is met as a compromise between the objectives since the players' strategy is influenced mutually. If the convergence criterion is not satisfied, another turn starts, with a different subdivision of the input variables. In this step the starting configuration was a constant section beam with 60mm height and the design space included a range between 40 and 80mm.

The second step uses the *MOGA-II*, *multi-objective genetic algorithm II* [7]. In this genetic algorithm, a sample (individual) is encoded putting side by side the binary representation of all input variables (chromosome). The research of the optimal sample is performed letting groups of individuals (population of a generation) evolve in the design space towards better solutions. At every new generation, the objective function on every individual is evaluated. According to these performances, multiple individuals are selected from the population and modified through cross-over method and random mutations in order to build a new generation with some others, stochastically generated. The new population is then used in the next iteration of the algorithm. Commonly, the algorithm terminates when a maximum number of iteration has been carried out, but an acceptable level can also be imposed. In the beam optimization, this algorithm has been used with a 20-100mm range for the design space. The starting population has been built with 150 samples consisting in 50 individuals from the Pareto distribution of the *MOGT* and 100 randomly created. The number of maximum generations has been set to 600. The objective was only to maximize the inertances and the normal mode frequency has been constrained to remain above the 50 best average value of the previous generation.

The last step is realized with the *Simplex* algorithm, [8].

In order to find the optimum of n-dimensional problem, the *Simplex* method uses a regular simplex, which is a geometrical enclosed figure within n+1 equidistant vertices.

The first introduced was the *Spendley Simplex Method*: it starts evaluating the objective function on a set of samples locating a regular simplex in the design space. Then, it generates a new figure reflecting the vertex at which the response is the worst. The process is then iterated checking if each vertex has been in the simplex for more than a fixed number of iterations; when it happens, the simplex is contracted by replacing all the other samples. The procedure is generally stopped after a fixed number of cycles but the convergence can be imposed also on the length of the edge connecting two vertices. In the current methodology, the simplex algorithm has been used to refine the convergence of the genetic optimization result and to test its stability.

## IV. RESULTS

The optimized heights have been interpolated with a 6th degree polynomial in order to smooth the results and avoid the checkerboarding phenomena. Afterwards, the moment of inertia has been calculated for each beam element according to the expression (7):

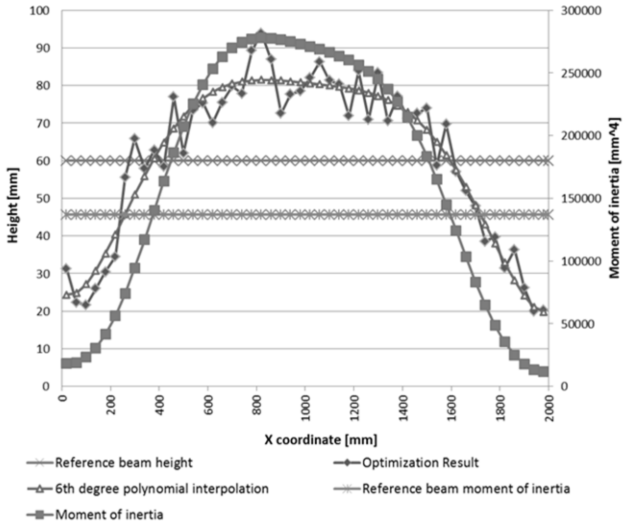


Fig. 3 Optimization results in terms of heights and moments of inertia distribution

$$J_i = \frac{1}{12} w h^3 - \frac{1}{12}(w-t)(h-t)^3 \qquad (7)$$

where $w$ is the constant dimension of the rectangular section, $h$ is the optimized height and $t$ is the thickness.

Optimization results are shown in figure 3: the beam heights trend and the moment of inertia distribution are compared with the respective ones of the constant section beam.

The optimized distribution is completely different from the constant value: on one side, the height is below the constant value on both ends of the beam (-60%); on the other side, the height reaches its maximum in the middle (+33%).

As stated previously, the two objectives give different optimal distribution, if considered alone. However, the proposed methodology allows finding a distribution able to improve both. The first normal bending mode, in fact, is increased by 40% and the average inertances are 8% better.

## V. OPTIMIZATION II

The distribution of moment of inertia can be seen as the stiffness distribution on the beam. Nevertheless, changing the height of each beam element means also to change its mass. Mass is really important in dynamic analysis, but can affect the optimal distribution found: in the simple problem of the beam, mass and moment of inertia are linked, but in a chassis a stiffness increase can be obtained without adding more mass (for example by changing material).

As a consequence, the NSM, *non-structural mass*, has been used for a new optimization. As the name suggests, NSM is a mass per unit length, if applied to beams, that has no structural stiffness. The density of the beam has been changed to maintain the value of the whole mass constant.

Two different optimizations have been carried out with different percentages of *non-structural mass*:

- 50% of NSM and 50% of common mass
- 99% of NSM and 1% of common mass

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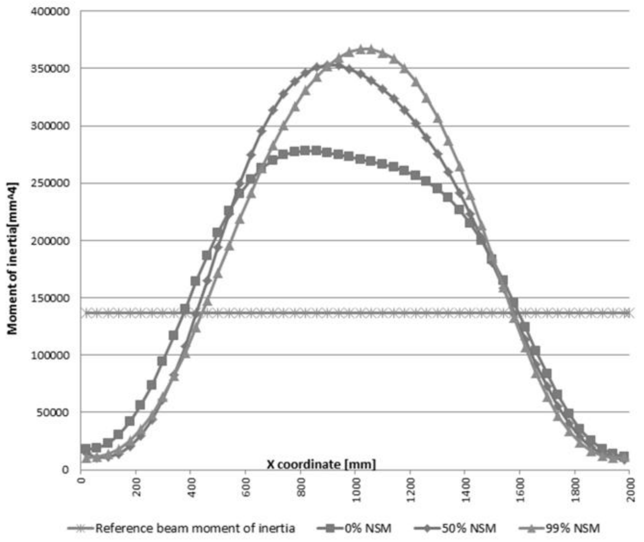


Fig. 4 Moment of inertia distribution after NSM introduction

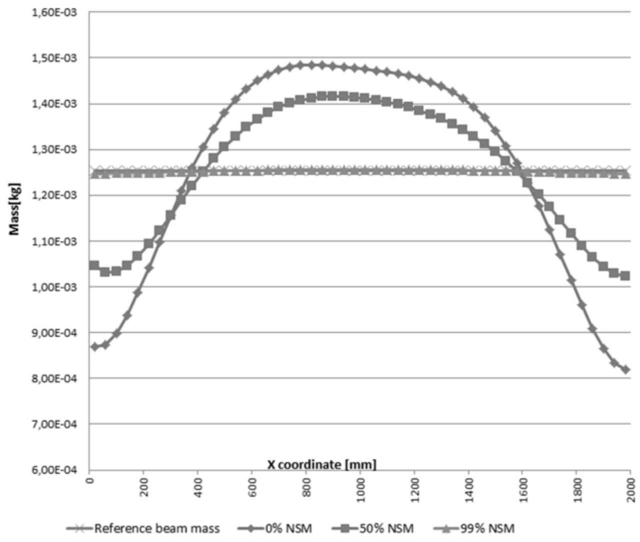


Fig. 5 Mass distribution

## VI. RESULTS II

Optimized moment of inertia distributions are shown in figure 4. The 0% NSM case is the previous one, where all the mass is assigned through density. The three optimized beams present the same moment of inertia distribution on the ends, but in the middle the trend is different. Both beams with non-structural mass present higher value of height on the central part. The difference in distribution can be easily explained by the mass distribution in the three cases.

Figure 5 shows the mass distribution for every beam: while the mass trend is similar to moment of inertia distribution for the reference model with constant section and for the 0% NSM case, it is really different for the other two beams. In 50% NSM case, in fact, half of the total mass remains equally spread on the beam while the remaining part can be moved according to the height assigned to each element. With 99% of non-structural mass, instead, almost all the mass is equally distributed on the beam. To conclude, without non-structural mass, the solver optimizes heights throughout the beam and not only changes the stiffness trend, but also moves the mass. If the normal mass is replaced with 50% NSM, the optimization modifies the moment of inertia of each element but a minor part of its mass, while with 99% NSM the solver can change only the stiffness distribution.

NVH results are listed on table I.

Both objectives increase their value in all optimized cases, but the importance of *non-structural mass* introduction can be seen in inertances results: the 99% NSM case doubles the improvement of the 0% NSM beam in inertances, losing only 5 point percentages in the first normal mode.

The results corroborate what stated before: adding *non-structural mass* allows the solver to focus only on stiffness and the solution found is the true optimal distribution of inertia moments.

TABLE I
RESULTS

| Case | First bending normal mode[a] | Average value of inertances in the range 70-400Hz[a] |
|---|---|---|
| Reference model | 100% | 100% |
| 0% NSM | 140.54% | 108.87% |
| 50% NSM | 140.50% | 112.47% |
| 99% NSM | 135.79% | 116.88% |

[a]Results are in percentage in respect to the reference model with constant section.

## VII. CHASSIS OPTIMIZATION

The optimal moment of inertia distribution found assures an outstanding improvement of NVH performances for a beam. However, the beam is only a simplification of the actual model, which is an automotive chassis. Thus, the best stiffness distribution of the 99% NSM case should be then applied to the model, which is a FEM shell structure shown in figure 6.

Usually, the first stage of chassis design ends achieving given values of torsional and bending stiffness. However, the previous work on the beam has shown that not only the value of global stiffness is important, but also its distribution throughout the model. In order to apply the best moment of inertia distribution, a peculiar optimization technique, called topometry optimization, and the software Altair® Optistruct have been carried out in the chassis. Topometry optimization aims to find the best thickness of every shell element of the model according to given loading conditions, i.e. torsional and bending loadcases.

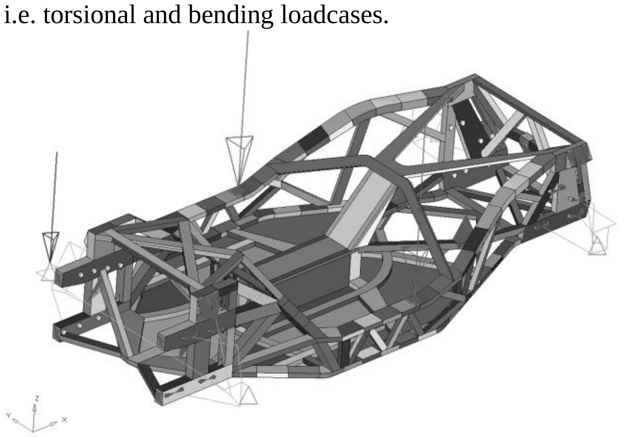

Fig. 6 Reference chassis

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ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

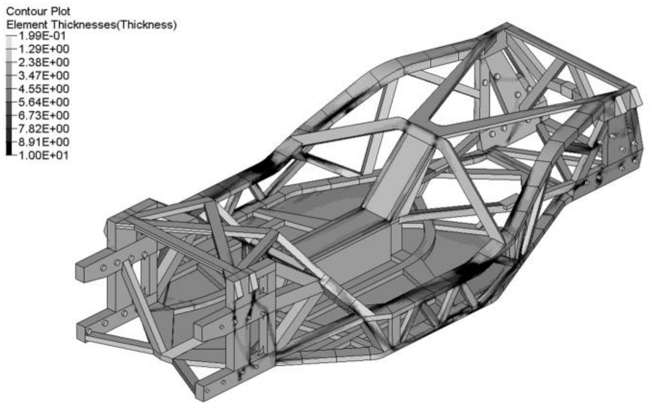


Fig. 7 Topometry optimization results

Obviously, these numerical tests replicate the experimental analysis, where torsional and bending stiffnesses are calculated from the displacements measured by infra-red sensors placed under the car. In the FEM analysis too, the data collected refer to points underneath the chassis, to meet Ferrari internal regulations.

The displacements detected at the front and rear track are used to compute the stiffness of the chassis, not including the suspensions contribution.

The optimization objective has been set to minimize the mass assuring the given value of bending and torsional stiffnesses and the best moment of inertia distribution.

## VIII. Chassis Results

The results of topometry optimization can be seen in figure 7, where the thickness contours are shown. At a later stage, the chassis with optimal thickness distribution, model C, has been tested for inertances in the range of interest, 70-400Hz, with a 1.5% structural damping. In order to evaluate the difference in performance with the 99% NSM case moment of inertia distribution, the inertance analysis has been carried out also on two other different optimized chassis, model A and model B. Concerning the model A, the optimization constraints have been set only on stiffness values, whereas model B has been optimized with the moment of inertia distribution of a constant section beam, which has been the benchmark during previous beam analysis.

The inertance results are listed in table II and table III.

Concerning the front suspensions joints, model B and C show better performances than the reference model in the whole frequency range: the average improvement is 2.6 points percentages for model B and 3 points for model C.

The behavior for rear suspensions joints, on the other hand, is different for the two optimized models. The average values of inertances, in fact, decrease for model B, whereas the improvements are up to 12 points percentages in model C thanks to the new optimized moment of inertia distribution.

TABLE II
Results for the front suspensions joints

| Case | Mean inertance in range 70-120Hz[a] | Mean inertance in range 120-250Hz[a] | Mean inertance in range 250-400Hz[a] |
|---|---|---|---|
| Model A | 100% | 100% | 100% |
| Model B | 105% | 102% | 101% |
| Model C | 104% | 104% | 101% |

[a]Results are in percentage in respect to Model A

TABLE III
Results for the rear suspensions joints

| Case | Mean inertance in range 70-120Hz[a] | Mean inertance in range 120-250Hz[a] | Mean inertance in range 250-400Hz[a] |
|---|---|---|---|
| Model A | 100% | 100% | 100% |
| Model B | 96% | 81% | 32% |
| Model C | 103% | 112% | 112% |

[a]Results are in percentage in respect to Model A

## IX. Conclusion

A methodology to determine the optimal distribution of moment of inertia for an automotive chassis has been presented. Optimizing the distribution of stiffness, in fact, can lead to an improvement in NVH performance. Even though the automotive chassis has been reduced to a beam in order to find, with a simpler approach, the correct moment of inertia for each section, the inertances have been increased without adding a large quantity of mass. Despite that, the difference in performance improvement between the beam and the chassis with the same moment of inertia distribution testifies the great simplification that has been made.

Since the importance of stiffness distribution has been proven, the methodology can be improved using a more sophisticated approach. The use of an accurate reduced model, in fact, can lead to more realistic results and avoid the reduction in NVH performance when, at the final stage, the optimal moment of inertia is applied to the chassis.

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ISBN: 978-988-19251-5-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)