Integer Optimization Model for a Location-Allocation-Routing in a Distribution Network with Restrictions on Route

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Abstract. In a distribution network, locating a facility such as supplier is important to decide that could impact not only the profitability of an organization but the ability to serve customers in short time. This paper considers the integrated Location-Allocation-Routing problem such that to minimize the overall cost by simultaneously selecting a subset of candidate facilities and constructing a set of delivery routes that satisfy some restrictions. In this paper we impose restrictions on route, i.e., distance and forbidden route. We use integer programming model to describe the problem. A feasible neighbourhood search is proposed to solve the result model.

Keywords*: Distribution system, integer programming, modeling, direct search*

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I. INTRODUCTION

The design of a distribution system involves a decision to select and find the best locations for facilities and to allocate customers to the selected facilities. These decision problem can be solved using location-allocation models. The objective of such models is to select the optimal locations of facilities from a list of candidate such that the total transportation cost from facilities to customers is minimized and an optimal number of customers have to be allocated in an area of interest in order to satisfy the customer demands. Therefore determining the locations of facilities within a distribution network is an important decision that impacts not only the profitability of an organization but the ability to serve customers. The term allocation implies rules which specify how demands are allocated to the candidate locations. There are three primary components in the location-allocation models, viz., the customer (or demand) locations, the list for candidate location, and a distance or travelling time between facilities and customer locations.

Facility location problems have attracted many researchers and have been applied to many real world problems. At the beginning , facility location problem is proposed by [1]. He introduced, what is called, a Weber facility location problem, to decide location of a warehouse in such a way to minimize the distance traveled between the warehouse and its customers. [2] used facility location model to improve geographical accessibility to public schools in rural area in India. Location-allocation models play important roles for designing health facilities such as locating the best sites for service facilities in a new area, evaluating the efficiency of the past location decisions and improving existing location patterns [3]. The authors provide an excellent review of location-allocation literature that employed location-allocation models in planning health care systems in developing nations. [4] also used this model to locate the primary health care centers in the city of Lujan, Argentina. [5] integrate location-allocation model with accessibility in order to improve the spatial planning of public health services.

A basic method which is usually used for location-allocation models is the P-median problem. This problem identifies the median points among the potential facility points such that total cost is minimized [6]. Another method used for location-allocation problem mentioned much in literature is covering problem. This objective of this problem is to find location of facilities which provide customers the access to facility services within a specified distance.

In location modeling deliveries are made on out-and-back routes visiting a single customer or most frequently the customer who travel individually to the facility site. The consequence is, the cost of delivery is independent of other deliveries made. In many contexts, however, deliveries are made along multiple stop routes visiting two or more customers; in this case, the cost of delivery depends on the other customers on the route and the sequence in which they are visited. In order to capture accurately the cost of multiple stop routes within a location model, the routing problem must be solved at the same time as the location problem. This type of problem is called location-routing problem.

Generally, the objective of the location-routing problem (LRP) is to select location from a subset of candidate facilities and to construct a set of delivery routes that satisfy:

i. Customer demands without exceeding vehicle or facility capacities.
ii. The number of vehicles, the route lengths and the route durations and
iii. Each route begins and ends at the same facility

Location-routing problems are clearly related to both the classical location problem and the vehicle routing problem. In fact, both of the latter problems can be viewed as special cases of the LRP. If we require all customers to be directly linked to a depot, the LRP becomes a standard location problem. If, on the other hand, we fix the depot locations, the LRP reduces to a VRP. From a practical viewpoint, location-routing forms part of *distribution management*, while from a mathematical point of view, it can usually be modeled as a *combinatorial optimization* problem. We note that this is an NP-hard problem, as it encompasses two NP-hard problems (facility location and vehicle routing). Since a number of problem versions exist, we cannot reproduce all the formulations here. In the first instance, the reader is referred to [7] for an excellent review of various formulations.

Most of the research to date has focused on heuristic methods since LRPs merge two NP-hard problems. The heuristics generally decompose the problem into its three components, facility location, customer allocation to facilities and vehicle routing, and solve a series of well-known problems such as *p*-median, location-allocation and vehicle routing. Exact methods have been developed for a small number of LRP models that are derived from two-index flow formulations for the vehicle routing problem (VRP). [8] solve a single depot model by a constraint relaxation method.

[9] develops an equivalent model and also extends the model to the case where the number of vehicles used is a variable in the model. [10] solve a multi-depot problem in which at most *p* facilities are located. The largest problems solved have seven candidate facilities and 40 customers. [11] solve a multi-depot capacitated LRP using a constraint relaxation method. In their work, the largest problem solved to optimality has eight candidate facilities and 20 customers. [11] use a branch and-bound procedure to solve asymmetric LRPs that include as many as three candidate facilities and 80 customers. [14] use two Meta-Heuristic algorithms of Genetic and Tabu Search algorithm. Since the performance of these heuristic algorithms is significantly influenced by their parameters, Taguchi Method is used to set the parameters of developed algorithms.

Forbidden route involving pairs of edges occur frequently and can occur dynamically due to rush hour constraints, lane closures, construction, etc. Longer forbidden subpaths are less common, but can arise, for example if heavy traffic makes it impossible to turn left soon after entering a multi-lane roadway from the right. If we are routing a single vehicle it is more natural to find a detour from the point of failure when a forbidden path is discovered.

In this paper we address the integrated model for Location-Allocation-Routing problem. We impose another restrictions, i.e., distance and forbidden route. The integrated problem can be formulated as a large-scale integer programming model. We solve the model using an exact method called Feasible Neighborhood Search Approach.

The rest of this paper is organized as follows. In the next Section we present the location-allocation model. In Section 3 we address the location-routing problem. Section 4 describes the definition of Forbidden Route. In Section 5, we introduce the mathematical model for the location-allocation-routing problem which consider distance and forbidden route. The meaning of Neighborhood Search is mentioned in Section 6. In Section 7, we propose the algorithm. We conclude the paper in Section 8.

II. LOCATION-ALLOCATION MODEL

The basic forms of location-allocation models for private sector is the *P*-median problem. The model is to minimize the total of travel distances between the customer points and the nearest servicing facilities.

We define set of Notations as follows.

Set

I Set of customer nodes

J Set of potential facility sites

M Number of customer points in the considered area

N Number of potential facility locations

Parameters

a_i Demand at node

d_{ij} Distance between node and

Q Number of facilities to be located

Variables

X_{ij} Binary variable whether customer is assigned to a facility

The model can be formulated as follows.

The objective is to minimize the total distance or travel time between customer node i and facility site node j.

$$Minimize \sum_{i\in I}\sum_{j\in J} a_{ij} d_{ij} x_{ij} \tag{1}$$

There are constraints need to be satisfied.

In order to make sure that every customer (or demand) is assigned to one and only one facility, we need the following expression.

$$\sum_{j\in J} x_{ij} = 1, \qquad \forall i \in I \tag{2}$$

$$x_{ij} \le x_{jj}, \qquad \forall i \in I, \forall j \in J \tag{3}$$

The next equation is to limit the number of facilities to be located

$$\sum_{j\in J} x_{jj} = Q \tag{4}$$

III. LOCATION-ROUTING MODEL

Next, we present a formulation of the LRP based on set-partitioning with distance constraints. The objective is to select a set of locations and to construct a set of associated delivery routes in such a way as to minimize facility costs plus routing costs. The set of routes must be such that each customer is visited exactly once by one route and that the length of each route does not exceed the maximum distance.

The model developed in this paper is based on [12]. Let I be the set of customer location nodes and J be the set of candidate facility location nodes. We define the graph $G=(N,A)$, where is the set of nodes and $A=N\times N$ is the set of arcs. We let d_{ij} for all $(i, j)\in A$ be the distance between nodes i and j. The distances satisfy the triangle inequality. The distances satisfy the triangle inequality. For applications in which the distance constraint applies to the length of the route to the last customer instead of the length of the return trip to the depot, we set to 0 for all (i, j) with $i\in I$ and $j \in J$. We define a feasible route k associated with facility j as a simple circuit that begins at facility j, visits one or more customer nodes and returns to facility j and that has a total distance of at most the maximum distance, denoted M. Then, we let P_j denote the set of all feasible routes associated with the facility j for all $j \in J$. The cost of a route $k\in P_j$ is the sum of the costs of the arcs in the route. The cost of an arc $(i, j) \in A$ is proportional to the distance d_{ij} to reflect distance related with operating costs.

Parameters

$$a_{ijk} = \begin{cases} 1, & \text{if route k associated with facility } j \text{ visits customer } i, \forall i\in I, \forall j \in J, \forall k\in P_j \\ 0, & \text{otherwise} \end{cases}$$

c_{jk} cost of route k associated with facility $j, \forall j\in J, \forall k\in P_j$

f_j fixed cost associated with selecting facility $j, \forall j\in J$

α object weighted factor

Decision Variables

$$X_j = \begin{cases} 1, & \text{if facility } j \text{ is selected}, \forall j\in J \\ 0, & \text{otherwise} \end{cases}$$

$$Y_{jk} = \begin{cases} 1, & \text{if route } k \text{ associated with facility } j \text{ is selected}, \forall j\in J, \forall k\in P_j \\ 0, & \text{otherwise} \end{cases}$$

The objective is to minimize cost

Minimize $$\alpha.\sum_{j\in J} f_j X_j + \sum_{j\in J}\sum_{k\in P_j} c_{jk} Y_{jk} \quad (5)$$

s.t. $$\sum_{j\in J}\sum_{k\in P_j} a_{ijk} Y_{jk} = 1 \quad \forall i\in I \quad (6)$$

$$X_j - Y_{jk} \geq 0 \quad \forall j\in J, \forall k\in P_j \quad (7)$$

$$X_j \in \{0,1\} \quad \forall j\in J \quad (8)$$

$$Y_{jk} \in \{0,1\} \quad \forall j\in J, \forall k\in P_j \quad (9)$$

The objective function (1) seeks to minimize the weighted sum of the facility costs and the routing costs. Constraints (2) are the set partitioning constraints that require each customer i be served by exactly one of the selected routes. Constraints (3) require that facility j be selected if a route k associated with facility j is selected. Constraints (4) and (5) are standard binary restrictions. The LRP with distance constraints is NP-hard. By placing very large costs on the arcs connecting two customer nodes, we obtain a special case of the model in which the selected routes contain exactly one customer.

As presented, the formulation LRP potentially contains an exponential number of variables and an exponential number of constraints (6). Thus, for instances of practical size, enumerating all of the feasible routes and solving the resulting integer program is unlikely to be effective. Instead, we will use feasible neighbourhood search for solving the model.

IV. FORBIDDEN ROUTE

We are given an directed graph $G(V, A)$ with $n = |V|$ vertices and $m = |A|$ edges where each edge $e \in A$ has a positive weight denoting its length. We are also given a source vertex $s \in V$, a destination vertex $t \in V$, and a set X of forbidden route in G. The graph G together with X models a vehicle routing network in which a vehicle cannot follow any route in X because of the physical constraints . We want to find a shortest route from s to t that does not contain any route in X as a subpath—we make the goal more precise as follows. A route is a sequence of vertices each joined by an edge to the next vertex in the sequence. Note that we allow a route to visit vertices and edges more than once. If a route does not visit any vertex more than once, we explicitly call it a simple route. *A* simple directed route from vertex v to vertex w in G is called a forbidden route or an exception if a vehicle cannot follow the route from v to w because of the physical constraints. Given a set X of forbidden route, a route $(v_1, v_2, v_3, \ldots, v_l)$ is said to avoid A if $(v_i, v_i + 1, \ldots, v_j) \notin A$ for all i, j such that $1 \leq i < j \leq$ l.

V. LOCATION-ALLOCATION-ROUTING PROBLEM WITH DISTANCE AND FORBIDDEN ROUTE

A. Problem formulation

Given a set of products L need to be distributed to a set of suppliers. The company has determined a list of candidate as potential suppliers (J). There is a set of customer nodes I with given demands spread across the city. A set of vehicle (M) is available to deliver the product. Each vehicle has a maximum capacity, Q. As mentioned in the problem description of location-routing model, we define a feasible route r associated with facility j as a simple directed graph that begins at facility j, visits one or more customer nodes and returns to facility j. with maximum distance of travelling N. Then, we let P_j denote the set of all feasible routes associated with the facility j for all $j \in J$. Unfortunately, due to physical constraint, there are forbidden route in which a vehicle cannot pass by.

The Model

The Location-Allocation-Routing Problem can be formulated mathematically as follows.

Notations used.

Sets

L Set of product

J Set of potential suppliers

I Set of customers' nbode

M Set of vehicles

R Set of feasible route

X Set of forbidden route

Parameters

a_i Demand at node $i \in I$

d_{ij} Distance from node $i \in I$ to node $j \in J$

Q Maximum weight capacity of a vehicle

q_{ijrm} Weight demand of customer i delivered from location j of vehicle m using route r

λ, ρ Costs

γ^l_{ijrm} Cost of transportation of vehicle m to deliver product l from supplier j to customer i using route r

Variables

x_{ij} Binary variable whether supplier j will serve customer i

y^l_j Binary variable if product l is located to supplier j

z^l_{ijrm} Binary variable if product l will be delivered to customer i from supplier j through route r using vehicle m

The objective function of this model is to minimize total cost.

$$\lambda \sum_{j\in J}\sum_{l\in L} y^l_j + \rho \sum_{i\in I}\sum_{j\in J} d_{ij} a_i x_{ij} + \sum_{i\in I}\sum_{j\in J}\sum_{r\in R, r\notin X}\sum_{l\in L} \gamma^l_{ijrm} z^l_{ijrm} \qquad (10)$$

Subject to constraints

The following expression is to make sure that every customer is assigned to one and only one supplier.

$$\sum_{j\in J} x_{ij} = 1, \qquad \forall i \in I \qquad (11)$$

$$x_{ij} \le x_{jj}, \qquad \forall i \in I, \forall j \in J \qquad (12)$$

The next constraint is to guarantee that product $l \in L$ is only located at supplier $j \in J$

$$\sum_{j\in J} y^l_j = 1, \qquad \forall l \in L \qquad (13)$$

Eq. (14) presents the requirement that each customer i is served exactly by one of the selected routes but not the forbidden routes.

$$\sum_{i\in I}\sum_{j\in J}\sum_{r\in R, r\notin X} b_{ijrm} z^l_{ijrm} = 1, \qquad \forall l \in L, \forall m \in M \qquad (14)$$

Constraints (15) state that supplier j be selected if a route r, as long as $r \notin X$, associated with supplier j is selected.

$$Y^1_j - Z^1_{ijrm} \ge 0, \forall i \in I, \forall j \in J, \forall r \in R, \forall r \notin X, \qquad (15)$$

Constraints (16) guarantee that vehicle capacities are respected in weight.

$$\sum_{i\in I}\sum_{j\in J}\sum_{r\in R, r\notin X} q_{ijrm} z^l_{ijrm} \le Q_m \qquad \forall l \in L, \forall m \in M \qquad (16)$$

$$x_{ij}, y^l_j, z^l_{ijrm} \in \{0,1\}$$
$$\forall i \in I, \forall j \in J, \forall m \in M, \forall r \in R, \forall l \in L \qquad (17)$$

The model is a large scale Integer programming problem. We develop the following method for solving the model.

VI. NEIGHBOURHOOD SEARCH

It should be noted that, generally, in integer programming the reduced gradient vector, which is normally used to detect an optimality condition, is not available, even though the problems are convex. Thus we need to impose a certain condition for the local testing search procedure in order to assure that we have obtained the "best" suboptimal integer feasible solution.

Scarf (1986) has proposed a quantity test to replace the pricing test for optimality in the integer programming problem. The test is conducted by a search through the neighbours of a proposed feasible point to see whether a nearby point is also feasible and yields an improvement to the objective function.

Let $[\beta]_k$ be an integer point belongs to a finite set of neighbourhood $N([\beta]_k)$. We define a neighbourhood system associated with $[\beta]_k$, that is, if such an integer point satisfies the following two requirements

1. If $[\beta]_j \in N([\beta]_k)$ then $[\beta]_k \in [\beta]_j), j \neq k$.
2. $N([\beta]_k) = [\beta]_k + N(0)$

With respect to the neighbourhood system mentioned above, the proposed integerizing strategy can be described as follows.

Given a non-integer component, x_k, of an optimal vector, x_B. The adjacent points of x_k, being considered are $[x_k]$ dan $[x_k]$

+ 1. If one of these points satisfies the constraints and yields a minimum deterioration of the optimal objective value we move to another component, if not we have integer-feasible solution.

Let $[x_k]$ be the integer feasible point which satisfies the above conditions. We could then say if $[x_k] + 1 \in N([x_k])$ implies that the point $[x_k] + 1$ is either infeasible or yields an inferior value to the objective function obtained with respect to $[x_k]$. In this case $[x_k]$ is said to be an "optimal" integer feasible solution to the integer programming problem. Obviously, in our case, a neigbourhood search is conducted through proposed feasible points such that the integer feasible solution would be at the least distance from the optimal continuous solution.

VII. THE ALGORITHM

First we solve the relaxed problem, the procedure for searching a suboptimal but integer-feasible solution from an optimal continuous solution can be described as follows.

Stage 1.

Step 1. Get row $i*$ the smallest integer infeasibility, such that

$$\delta_{i*} = \min\{f_i, 1 - f_i\}$$

(This choice is motivated by the desire for minimal deterioration in the objective function, and clearly corresponds to the integer basic with smallest integer infeasibility).

Step 2. Do a pricing operation

$$v_{i*}^T = e_{i*}^T B^{-1}$$

Step 3. Calculate $\sigma_{ij} = v_{i*}^T \alpha_j$

With corresponds to

$$\min_j \left\{ \left| \frac{d_j}{\alpha_{ij}} \right| \right\}$$

Calculate the maximum movement of nonbasic j at lower bound and upper bound.

Otherwise go to next non-integer nonbasic or superbasic j (if available). Eventually the column $j*$ is to be increased form LB or decreased from UB. If none go to next $i*$.

Step 4.

Solve $B\alpha_{j*} = \alpha_{j*}$ for α_{j*}

Step 5. Do ratio test for the basic variables in order to stay feasible due to the releasing of nonbasic $j*$ from its bounds.

Step 6. Exchange basis

Step 7. If row $i* = \{\varnothing\}$ go to Stage 2, otherwise

Repeat from step 1.

Stage 2. Pass1 : adjust integer infeasible superbasics by fractional steps to reach complete integer feasibility.

Pass2 : adjust integer feasible superbasics. The objective of this phase is to conduct a highly lovalized neighbourhood search to verify local optimality.

VIII. CONCLUSIONS

This paper presents a Location-Allocation-Routing problem model in which we consider the distance and there are some forbidden route. The framework of the model stems from Location-Allocation problem, Location-Routing problem and VRP with time windows with forbidden route. Then we exclude the forbidden route from the previous assigned route. We solve the model using a feasible neighbourhood search.

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